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SIMPLE THEORY OF NONLEPTONIC KAON DECAYS *

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ABSTRACT

We first summarize (a) why the quark $\hat{s}$-$\bar{d}$ loop transition dominated by the physical W^+ exchange controls the large $\Delta I = 1/2$ K_π and $K^0_{2\pi}$ nonleptonic decay amplitudes, and (b) why the vacuum-saturated hadronic (implied W^+) current-current hamiltonian correctly explains the small $\Delta I = 3/2$ $K^+_{2\pi}$ decay. Then we study in greater detail a more complete hadronic D,K,π meson-$W^\pm$ loop calculation of the $\Delta I = 1/2$ and $\Delta I = 3/2$ $K_{2\pi}$ amplitudes and show that this picture further reinforces our original quark $\Delta I = 1/2$ and hadron vacuum-saturated $\Delta I = 3/2$ (long distance) scheme.

I. INTRODUCTION

Over the past two decades, elementary particle physicists have only partially understood the dynamical mechanisms which drive the six large $\Delta I = 1/2$ $K^0_{2\pi}$ and $K_{3\pi}$ decay amplitudes and also account for the much smaller $\Delta I = 3/2$ $K^+_{2\pi}$ decay. The current algebra-partially conserved axial current (PCAC) techniques[1,2] of the 1960s and also quark model notions[3,4] of the 1970s have focused on different aspects of the problem. However, an overview which completely explains all kaon decay amplitudes (in terms of no additional parameters) and links together the underlying quark model with the hadronic current picture is lacking at present. Such a complete yet "simple" description is the intended goal of this paper.

In Sec. II we summarize the quark model theory of $\Delta I = 1/2$ $K^0_{2\pi}$ and $K_{3\pi}$ decays. First we consider the underlying $\bar{s}$-$\bar{d}$ quark loop transition dominated by the physical W^+ exchange but also include the unphysical Higgs X^+ exchange for gauge invariance reasons. Then using light plane wave functions, the nonperturbative strong interaction binding of the above s-d quark transition in $\langle\pi|H^{eff}_{w,1/2}|K\rangle$ provides a successful explanation of $K_{L\gamma\gamma}$ decay. Finally, pion PCAC extends this quark prediction to the observed $\Delta I = 1/2$ $K^0_{2\pi}$ and $K_{3\pi}$ weak decay amplitudes. The heavy charmed quark mass is seen to drive the $\Delta I = 1/2$ rule.

Next in Sec. III we temporarily suppress the quark picture and consider instead the surprisingly accurate prediction of "vacuum saturation" (VS) of the original Cabibbo[5] hadronic current-current hamiltonian applied to the $\Delta I = 3/2$ $K^+_{2\pi}$ weak decay amplitude. Non-elementary partially conserved hadron currents are employed here, still manifesting gauge invariance of the implied W exchange while scaling the $K^+_{2\pi}$ amplitude to the pion decay constant. Pion PCAC also checks the consistency of this hadronic VS procedure.

Finally, in Sec. IV we attempt to explain why the large $\Delta I = 1/2$ amplitudes are most easily understood at the quark level, but the small $\Delta I = 3/2$ amplitude is more transparent at the hadron level. To this end, we employ a low-energy field-theoretic approach in analogy to the dispersion-theoretic Cottingham-type treatment[6,7] to these current-current weak $K_{2\pi}$ decay amplitudes. The latter low-energy version[7] suggests that we follow a hadronic D,K,π saturation

procedure for $\Delta I = 1/2$ and $\Delta I = 3/2$ $K_{2\pi}$ decays. The resulting amplitudes are in close agreement with our original $\Delta I = 1/2$ quark self-energy and $\Delta I = 3/2$ hadron vacuum saturation approach and also with experiment, both in magnitude and in relative sign.

We summarize our results in Sec. V and suggest that the quark-based s-d $\Delta I = 1/2$ transition combined with the hadronic current-current VS amplitude for $K^+_{2\pi}$ gives the most natural and simple explanation of all $\Delta I = 1/2$ and $\Delta I = 3/2$ nonleptonic weak kaon decays.

II. QUARK MODEL THEORY FOR $\Delta I = 1/2$ KAON DECAYS

A. Finite $\Delta I = 1/2$ s-d Quark Loop

It is a straightforward matter[8-10] to show that the physical (left-handed) W^+-$\bar{s}\bar{d}$ quark loop graph of Fig. 1 must produce an effective $\Delta S = 1$ weak hamiltonian density of the $\Delta I = 1/2$ left-handed (LH) form

$$H^{eff}_{w,1/2} = b\left(\bar{d}\, \not{p}_L s + \bar{s}\, \not{p}_L d\right) \quad . \tag{1a}$$

where $\not{p}_L = \not{p}(1-i\gamma_5)$. In the 'tHooft-Feynman ($\xi_w = 1$) gauge, the dimensionless scale of b for low $p^2 \leq 1$ GeV^2 is simply (using $\theta_c \approx 13.1°$, $s_c c_c \approx 0.221$, $m_c \approx 1.6$ GeV >> m_u)

$$b \approx b(p^2 = 0) = -\frac{G_F s_c c_c}{\sqrt{2}\, 8\pi^2}(m_c^2 - m_u^2) \approx -5.9\times10^{-8} \quad . \tag{1b}$$

The finiteness of (1b) is signaled by the characteristic GIM structure $m_c^2 - m_u^2$ and is obtained from the weak quark current[11]

$$j^w_\mu = \bar{u}\,\gamma^L_\mu(d\cos\theta_c + s\sin\theta_c) + \bar{c}\,\gamma^L_\mu(-d\sin\theta_c + s\cos\theta_c) \quad . \tag{2}$$

Although the unphysical X^+ Higgs scalar quark loop of Fig. 2 is needed to complete the gauge invariance of Fig. 1 at the current mass pole position[12] for $m_s = m_d$, this unphysical X^+ Higgs does not couple to quarks in a left-handed manner and must not be allowed to shift the pole position by contributing to (1b) when $m_s \neq m_d$. In fact, the operator regularization procedure[13] has been applied to this s-d quark "self-energy,"[14] and as anticipated above, Fig. 1 plus Fig. 2 remains gauge invariant while the effective weak hamiltonian density (1) is unaltered to leading order in m_c^2/m_W^2. Thus, we shall continue to take the effective weak hamiltonian Eq. (1) as valid in any gauge. As such, (1) should be of direct physical significance.

An immediate consequence of this continued left-handed structure of (1) is to insure that $H^{eff}_{w,1/2}$ (and also the smaller $\Delta I = 3/2$ component of H_w, to be discussed in Sec. III) obeys the original equal-time chiral charge commutation relation[1,2]

$$[Q + Q_5, H_w(V\text{-}A)] = 0 \quad , \qquad [Q_5, H_w] = -[Q, H_w] \quad . \tag{3}$$

The general property (3) combined with pion PCAC leads directly to the chiral-limiting $K^0_{3\pi}$ to $K^0_{2\pi}$ amplitude ratio[1,2]

$$|M_{K^0 3\pi^0}/M_{K^0 2\pi^0}| = 1/2f_\pi \approx 5.6\ \mathrm{GeV}^{-1} \tag{4}$$

for $f_\pi \approx 90$ MeV in the chiral limit. This predicted value (4), being close to the observed ratio[9,15] 6.7 GeV^{-1}, in turn supports the left-handed structure of (1), as well as the current algebra-PCAC procedure based on (3) for nonleptonic weak interactions.

B. Nonperturbative Strong $\bar{q}q$ Hadronization via Light Plane Wave Functions

Apart from (4), the physical consequences of the scale of the effective $\Delta I = 1/2$ hamiltonian (1) can be most easily seen by "cementing" Fig. 1 and (1) into the tightly bound $\bar{q}q$ pseudoscalar meson K-π transition, as depicted by the quark "submarine" graph of Fig. 3. Using nonperturbative light plane wave functions for the π and K, normalized[16,17] to the strong interacton decay constants $f_\pi \approx 93$ MeV and $f_K/f_\pi \approx 1.25$, the Nambu-Goldstone transition of Fig. 3 leads to the approximate relation[17]

$$\langle\pi^0|H^{eff}_{w,1/2}|K^0\rangle \approx \sqrt{2}\, b\, m_K^2\, f_K/f_\pi \approx -\frac{G_F s_c c_c}{8\pi^2}(m_c^2 - m_u^2)\, m_K^2\, f_K/f_\pi \tag{5a}$$

$$\approx -2.6 \times 10^{-8}\ \mathrm{GeV}^4 \quad . \tag{5b}$$

The result (5) is in "long-distance approximation" (long-distance strong interactions to all orders, but very short-distance weak interactions to first order) — no implied gluon interacts with the W in Fig. 3 as in a (non left-handed) short-distance "penguin" graph.

This prediction (5) can be directly tested by the simple π^0 (long distance) pole model for $K_{L\gamma\gamma}$ of Fig. 4, which gives the $K_{L\gamma\gamma}/\pi^0_{\gamma\gamma}$ amplitude ratio[9]

$$\left|\frac{F_{K_{L\gamma\gamma}}}{F_{\pi^0\gamma\gamma}}\right| = \left|\frac{\sqrt{2}\,\langle\pi^0|H^{eff}_{w,1/2}|K^0\rangle}{m_K^2 - m_\pi^2}\right| \approx 16 \times 10^{-8} \quad . \tag{6a}$$

The theoretical ratio (6a) compares well with the observed amplitude ratio[15]

$$\left|\frac{F_{K_{L\gamma\gamma}}}{F_{\pi^0\gamma\gamma}}\right|_{exp} = \sqrt{\frac{\Gamma_{K_{L\gamma\gamma}}}{\Gamma_{\pi^0\gamma\gamma}}\left(\frac{m_\pi}{m_K}\right)^3} \approx 13 \times 10^{-8} \quad . \tag{6b}$$

An even closer match to (6b) can be obtained by inclusion in (6a) of the smaller η, η' pole contributions[18] to Fig. 4. Furthermore, the $\Delta I = 1/2$ prediction (5) can also roughly explain the observed $K_{L\mu\bar{\mu}}$, $K^+_{\pi^+ e\bar{e}}$ and $K^0_{2\pi\gamma}$ decay amplitudes.[18]

C. PCAC Extension to $K^0_{2\pi}$ Decays

The theoretical $\Delta I = 1/2$ K-π transition (5) (based on (1)) can also be extended to the $\Delta I = 1/2$ dominated $K^0_{2\pi}$ decays by use of the pion PCAC relation (for $f_\pi \approx 93$ MeV)

$$\langle\pi\pi|H^{eff}_{w,1/2}|K^0\rangle_{PCAC} = (i/f_\pi)\,\langle\pi^0|H^{eff}_{w,1/2}|K^0\rangle\,(1-m_\pi^2/m_K^2) \quad . \tag{7}$$

Note that the $(1-m_\pi^2/m_K^2)$ factor in (7) must occur because of the kaon decay SU(3) null theorem[19] due to CP conservation of H_w combined with SU(3) symmetry. The PCAC factor (i/f_π) in (7) is twice that in the original PCAC analyses[1,2] and is due to rapid momentum variation of $\langle\pi\pi|H^{eff}_{w,1/2}|K^0\rangle$. The result (7) can be deduced in three independent ways:

(i) from the effective chiral lagrangian of Cronin,[20], once the π^0 on the RHS Cronin version of (7) is extrapolated to the K^0 mass shell[18];

(ii) from the K^0-vacuum tadpole graphs,[6-10,21] which are the natural extension of the s-d self-energy diagram of Fig. 1; and

(iii) from a Weinberg-type of low energy expansion.

Since the third method can be directly applied to $\Delta I = 1/2$ and also easily extended to $\Delta I = 3/2$ transitions, we shall briefly review the latter PCAC approach to $K_{2\pi}$ in Appendix A.

Given (7), we substitute in $\langle\pi^0|H^{eff}_{w,1/2}|K^0\rangle$ as derived from (5) to find[6-10,17]

$$i\langle\pi\pi|H^{eff}_{w,1/2}|K^0\rangle_{PCAC} \approx \frac{G_F s_c c_c f_K}{8\pi^2 f_\pi^2}\,(m_c^2 - m_u^2)\,(m_K^2 - m_\pi^2) \tag{8a}$$

$$\approx 26 \times 10^{-8}\ \text{GeV} \quad . \tag{8b}$$

valid for both the $K^0 \to \pi^+\pi^-$ and $\pi^0\pi^0$ $\Delta I = 1/2$ amplitudes. Note the mixture of quark and hadron parameters in (5) and (8): $m_c^2 - m_u^2$ refers to the GIM quark structure of Fig. 1, while $(m_K^2 - m_\pi^2)$ represents the required hadronic SU(3) suppression of $K_{2\pi}$ decays. Equation (8) is in fact close to the average experimental[15] amplitude

$$|M_{K^0_{2\pi}}|_{exp} = m_K\,\sqrt{8\pi\,\Gamma_{K^0_{2\pi}}/p} \approx 27 \times 10^{-8}\ \text{GeV} \quad . \tag{9}$$

Since the PCAC relation (4) extends the scale of (8) to all four $\Delta I = 1/2$ dominated $K_{3\pi}$ decays, the <u>one</u> pre-PCAC theoretical scale (5) (based on Eq. (1)) explains <u>ten</u> observed kaon weak decays[18]: $K^0_{2\pi}$ (two), $K_{3\pi}$ (four), $K_{L\gamma\gamma}$, $K_{L\mu\bar{\mu}}$, $K^+_{\pi^+ e\bar{e}}$, $K^0_{2\pi\gamma}$.

III. HADRON VACUUM SATURATION FOR $\Delta I = 3/2$ $K^+_{2\pi}$ DECAY

A. Hadronic Current-Current Hamiltonian

Here we treat $\Delta I = 3/2$ $K^+ \to \pi^+\pi^0$ decay by returning to the Cabibbo form[5] of the current-current nonleptonic weak hamiltonian density used extensively in the 1960s[1,2,9]:

$$H_w = \frac{G_F}{2\sqrt{2}}\left(J^\dagger_\mu J^\mu + J^\mu J^\dagger_\mu\right) \tag{10a}$$

$$J_\mu = (V-A)^{1-i2}_\mu \cos\theta_c + (V-A)^{4-i5}_\mu \sin\theta_c \quad . \tag{10b}$$

We note four important features of the long-distance H_w in (10):

(i) the well-known[2] scale factor of $G_F/2\sqrt{2}$ in (10a) guarantees that the semileptonic and leptonic versions accordingly "double up" to $G_F/\sqrt{2}$ and correctly fit neutron and muon decay rates;

(ii) the nonelementary hadron currents in (10), while nonperturbatively generated by strong interactions at energy 1 GeV, are of the left-handed (V-A) form due to the $W^{\pm}$; as such, (10) also satisfies the charge algebra relation (3) together with (pion) PCAC;

(iii) the $\Delta I = 1$ hadron current of (10b) is partially conserved; as such, the $\Delta S = 1$ H_w of (10a) is gauge invariant (the gauge-dependent propagator component of W proportional to $q_\mu q_\nu$ then vanishes when contracted with $J^\mu J^\nu$);

(iv) the implied very heavy (point-like) $W^{\pm}$ exchanged in (10a) with $M_W^2 >> m_c^2 >> m_{s,d,u}^2$ requires a decoupling of the weak (W) cloud of wavelength $\lambda_w \sim M_W^{-1} \sim (1/400)$fm and the strong (gluon) cloud with $\lambda_{st} \sim \hat{m}_{con}^{-1} \sim (1/2)$fm surrounding low mass hadrons (and quarks) undergoing nonleptonic decays, as follows from gauge invariance considerations.[14] The latter is a restatement of the long-distance approximation.

B. Hadronic Vacuum Saturation

These four features of the hadronic current-current hamiltonian (10) suggest that $\langle\pi^+\pi^0|H_w|K^+\rangle$ factors (vacuum saturates) into hadron current transitions (nonperturbatively measured as all orders strong) and is then multiplied by $G_F/2\sqrt{2}$ (indicating lowest order weak). This VS long-distance approximation is depicted in Fig. 5. Once again all implied strong interaction gluons either build up the nonperturbative pion decay constant f_π or the nonperturbative Ademollo-Gatto[22] vector current vertex $\langle\pi^0|V_\mu|K^+\rangle$ in this long-distance approximation. The explicit gauge invariance of the implied decoupled strong and weak interaction clouds further suggests that this VS amplitude is of direct physical significance and will control the nonleptonic decay in question provided the dominant $\Delta I = 1/2$ mode is suppressed. Since this is the case for the $\Delta I = 3/2$ $K_{2\pi}^+$ decay, it should not be surprising that the VS amplitude (obtaining contributions from $J^\dagger J$ but not $JJ^\dagger$ in (10a)),

$$i\langle\pi^+\pi^0|H_w|K^+\rangle_{VS} = i\left(G_F s_c c_c/2\sqrt{2}\right)\langle\pi^+|-A_\mu^\dagger|0\rangle\langle\pi^0|V^\mu|K^+\rangle$$
$$= \left(G_F s_c c_c/2\sqrt{2}\right) f_\pi f_+(0)\,(m_K^2 - m_\pi^2) \approx 1.86 \times 10^{-8}\ \text{GeV} \quad , \qquad (11a)$$

dominates the experimental amplitude[15]

$$|M_{K^+2\pi}|_{exp} = m_K\sqrt{8\pi\,\Gamma_{K^+2\pi}/p} = (1.83 \pm 0.01) \times 10^{-8}\ \text{GeV} \quad . \qquad (11b)$$

In (11a), $f_+(0) = 1 - O(\epsilon^2) \approx 0.97$ is near unity due to the nonrenormalization theorem.[22] What is surprising is the almost exact agreement[10,17] between (11a) and (11b). Even the sign of the VS amplitude (11a) relative to the $\Delta I = 1/2$ prediction (8a) is the same as the observed $\Delta I = 1/2$ and $\Delta I = 3/2$ interference sign in the $K^0 \to \pi^+\pi^-$ and $K^0 \to 2\pi^0$ amplitudes[10,17] $(M_{1/2}/M_{3/2} > 0)$.

C. PCAC Check of Vacuum Saturation

Just as the $\Delta I = 1/2$ $K_{2\pi}^0$ amplitudes obey the pion PCAC relation (7), the $\Delta I = 3/2$ $K_{2\pi}^+$ amplitude satisfies the pion PCAC equation (derived in Appendix A),

$$i\langle\pi^+\pi^0|H_{w,3/2}|K^+\rangle_{PCAC} = (3/2f_\pi)\,\langle\pi^+|H_{w,3/2}|K^+\rangle\,(1 - m_\pi^2/m_K^2) \quad . \qquad (12a)$$

Then scaled to the observed $K_{2\pi}^+$ amplitude (11b), we deduce from (12a) that[21]

$$\left|\langle\pi^+|H_{w,3/2}|K^+\rangle\right|_{PCAC}^{exp} \approx 0.123 \times 10^{-8}\ \text{GeV}^2 \quad . \qquad (12b)$$

On the other hand, direct vacuum saturation of the K^+-π^+ matrix element of the hadronic weak current-current hamiltonian (10) on the kaon mass shell leads to

$$\langle\pi^+|H_w|K^+\rangle_{VS} = \left(G_F/2\sqrt{2}\right)\langle\pi^+|J^\dagger|0\rangle\cdot\langle 0|J|K^+\rangle = \left(G_F/\sqrt{2}\right)s_c c_c\, f_\pi\, f_K\, m_K^2 \quad . \tag{13a}$$

The corresponding $\Delta I = 3/2$ component of this entire VS amplitude is 1/3 of (13a) (the $\Delta I = 1/2$ VS component being 2/3 - see Appendix B), giving for $f_K = f_\pi$,

$$\langle\pi^+|H_{w,3/2}|K^+\rangle_{VS} = \left(G_F/3\sqrt{2}\right)s_c c_c f_\pi^2 m_K^2 \approx 0.129 \times 10^{-8}\ \mathrm{GeV}^2 \quad , \tag{13b}$$

in near agreement with the experimental-PCAC amplitude (12b).

It is no accident that (13b) is so close to (12b). In fact, if (13b) is substituted into the $\Delta I = 3/2$ PCAC relation (12a), then the entire VS $K^+_{2\pi}$ amplitude (11) for $f_+(0) = 1$ and $m_\pi = 0$ is precisely recovered. This further confirms the natural theoretical and experimental consistency of the hadronic PCAC and VS procedures.

IV. HADRONIC VIEW OF COMBINED $\Delta I = 1/2$ AND $\Delta I = 3/2$ $K_{2\pi}$ DECAYS

The remaining issues are: (a) Why are the $K^0_{2\pi}$ $\Delta I = 1/2$ decays most easily understood at the quark (self energy) level, while the $K^+_{2\pi}$ $\Delta I = 3/2$ decay is most simply explained at the hadron (vacuum saturation) level? (b) Why does only the $J^\dagger J$ weak current product dominate the vacuum-saturated $K^+_{2\pi}$ decay amplitude? To answer these questions, we reformulate the entire $K_{2\pi}$ problem completely in terms of hadron states but still continue to work within the long-distance approximation.

A. Hadronic Version of the $\Delta I = 1/2$ to $\Delta I = 3/2$ Amplitude Ratio

First we generalize the current algebra-pion PCAC theorems derived in Appendix A to obtain all three $K_{2\pi}$ amplitudes systematically from the two K_π weak transitions:

$$\langle\pi^0\pi^0|H_w|K^0\rangle = (i/f_\pi)\,\langle\pi^0|H_w|K^0\rangle\,(1-m_\pi^2/m_K^2) \quad . \tag{14a}$$

$$\langle\pi^+\pi^-|H_w|K^0\rangle = \left(-i/\sqrt{2}\, f_\pi\right)\langle\pi^+|H_w|K^+\rangle\,(1-m_\pi^2/m_K^2) \quad . \tag{14b}$$

$$\langle\pi^+\pi^0|H_w|K^+\rangle = (-i/2f_\pi)\left[\langle\pi^+|H_w|K^+\rangle + \sqrt{2}\,\langle\pi^0|H_w|K^0\rangle\right](1-m_\pi^2/m_K^2) \quad . \tag{14c}$$

Owing to the rapid variation of the $K_{2\pi}$ amplitudes with pion momenta, these PCAC relations (14) follow directly from the <u>sum</u> of the single soft pion reductions combined with the chiral charge algebra structure (3) satisfied by H_w. Alternatively, Eqs. (14) respectively reduce to the $\Delta I = 1/2$ and $\Delta I = 3/2$ PCAC relations (7) and (12a) due to the isospin identities

$$\langle\pi^+|H_{w,1/2}|K^+\rangle = -\sqrt{2}\,\langle\pi^0|H_{w,1/2}|K^0\rangle \tag{15a}$$

$$\langle\pi^0|H_{w,3/2}|K^0\rangle = \sqrt{2}\,\langle\pi^+|H_{w,3/2}|K^+\rangle \quad . \tag{15b}$$

Next, in Figs. 6 and 7 we display the leading low-energy graphs which respectively contribute to $\langle\pi^0|H_w|K^0\rangle$ and to $\langle\pi^+|H_w|K^+\rangle$. The relative signs of these various K^+, D^-, π^0, $\bar{D}^0$ "self energy" graphs are determined by the vector vertices, with $\langle P^f|V^j|P^i\rangle \sim if^{fji}$ leading to

$$if_{3,4-i5,\frac{4+i5}{\sqrt{2}}}\ if_{\frac{4-i5}{\sqrt{2}},1+i2,\frac{6+i7}{\sqrt{2}}} = -\frac{1}{\sqrt{2}} \tag{16a}$$

$$-\,if_{3,11-i12,\frac{11+i12}{\sqrt{2}}}\ if_{\frac{11-i12}{\sqrt{2}},13+i14,\frac{6+i7}{\sqrt{2}}} = +\frac{1}{\sqrt{2}} \tag{16b}$$

for the K^+ and D^- intermediate-state diagrams of Figs. 6 and

$$if_{\frac{1-i2}{\sqrt{2}},\,1+i2,\,3}\;\; if_{3,\,4-i5,\,\frac{4+i5}{\sqrt{2}}} = +1 \tag{17a}$$

$$-\,if_{\frac{1-i2}{\sqrt{2}},\,11-i12,\,\frac{9+i10}{\sqrt{2}}}\;\; if_{\frac{9-i10}{\sqrt{2}},\,13+i14,\,\frac{4+i5}{\sqrt{2}}} = -1 \tag{17b}$$

for the π^0 and $\bar{D}^0$ intermediate-state diagrams of Fig. 7. Here the minus signs on the LHS of (16b) and (17b) are due to the $\bar{c}d$ minus sign in the GIM weak quark current (2).

Then breaking SU(4) symmetry in Figs. 6(c) and 7(c) only in the intermediate state propagator masses, the associated closed loop Feynman integrals are respectively proportional to

$$\langle\pi^0|H_w|K^0\rangle \propto \int d^4p \left[\left(-1/\sqrt{2}\right)(p^2-m_K^2)^{-1} + \left(1/\sqrt{2}\right)(p^2-m_D^2)^{-1}\right] \propto \left(1/\sqrt{2}\right)(m_D^2-m_K^2)\ , \tag{18a}$$

for Fig. 6(c) due to (16) and

$$\langle\pi^+|H_w|K^+\rangle \propto \int d^4p\, [(p^2-m_\pi^2)^{-1} - (p^2-m_D^2)^{-1}] \propto -(m_D^2-m_\pi^2) \tag{18b}$$

for Figs. 7(c) due to (17). Again, (18) holds only in the long-distance approximation. Finally, after substituting (18a,b) into the PCAC relations (14a,b,c), we are led to the approximate $K_{2\pi}$ amplitude ratios

$$M_{K^0\pi^+\pi^-}/M_{K^0\pi^0\pi^0} \approx (m_D^2-m_\pi^2)/(m_D^2-m_K^2) \approx 1.07 \tag{19a}$$

$$M_{K^0\pi^+\pi^-}/M_{K^+\pi^+\pi^0} \approx \sqrt{2}\,(m_D^2-m_\pi^2)/(m_K^2-m_\pi^2) \approx 21.7\ . \tag{19b}$$

These predictions are in rough agreement with the observed amplitude ratios[15]

$$M_{K^0\pi^+\pi^-}/M_{K^0\pi^0\pi^0} = \sqrt{B(K^0\to\pi^+\pi^-/\pi^0\pi^0)\, q_{00}/2q_{+-}} \approx 1.05 \tag{20a}$$

$$M_{K^0\pi^+\pi^-}/M_{K^+\pi^+\pi^0} = (m_{K^0}/m_{K^+}) \sqrt{\Gamma_{K^0\pi^+\pi^-}\, q_{+0}/\Gamma_{K^+\pi^+\pi^0}\, q_{+-}} \approx 15.1\ . \tag{20b}$$

The choice of positive signs in (20) is consistent [16] with the phase conventions of (15) as well as with the signs of (8) and (11).

B. Scale of K_π Hadronic Transitions

Continuing to treat the numerator currents in Figs. 6(c) and Figs. 7(c) in the chiral limit, the hadronic weak hamiltonian density (10) leads to the Feynman amplitudes based on (16)-(18).

$$\langle\pi^0|H_w|K^0\rangle \approx \frac{iG_F s_1 c_1}{\sqrt{2}} \frac{(m_D^2-m_K^2)}{\sqrt{2}\,(2\pi)^4} \int d^4p\, \frac{p^2}{(p^2-m_D^2)(p^2-m_K^2)} \tag{21a}$$

$$\sim -2.5 \times 10^{-8}\ \mathrm{GeV}^2\ , \tag{21b}$$

$$\langle\pi^+|H_w|K^+\rangle \approx \frac{-iG_F s_1 c_1}{\sqrt{2}} \frac{(m_D^2-m_\pi^2)}{(2\pi)^4} \int d^4p\, \frac{p^2}{(p^2-m_D^2)(p^2-m_\pi^2)} \tag{22a}$$

$$\sim +\,4.2 \times 10^{-8}\ \mathrm{GeV}^2\ . \tag{22b}$$

in long-distance approximation. To obtain the numerical estimates of the formally logarithmically ultraviolet divergent integrals (21a) and (22a), we have cut off these integrals at the heavy charmed D mass. Averaging between (21b) and (22b), we estimate the $\Delta I = 1/2$ $K^0_{2\pi}$ scale from (14a) and (14b) as

$$\left| M^{1/2}_{K^0 2\pi} \right| \sim 27 \times 10^{-8}\ \text{GeV} \quad , \tag{23}$$

reasonably close to the experimental value (9).

That the dominant D meson graphs of Fig. 6(b) and Fig. 7(b) in fact correspond to $\Delta I = 1/2$ transitions can be inferred from the GIM weak quark current (2). More specifically the $\bar{c}d$ current has $I = 1/2$, while the $\bar{c}s$ current has $I = 0$, so that $H_w \sim J{\cdot}J$ transforms as $\Delta I = 1/2$.

C. Reason for $J^\dagger J$ Alone in Vacuum Saturation of $K^+_{2\pi}$

In the direct vacuum saturation of H_w for $K^+_{2\pi}$ decay as given in (11a), the almost exact matching with the observed $K^+_{2\pi}$ amplitude (11b) is because only the $J^\dagger J = J^{1+i2}J^{4-i5}$ but not the $JJ^\dagger$ term in (10a) contributes in vacuum saturation (VS) approximation for this $\Delta S = -1$ transition. The underlying reason for this asymmetric VS pattern is, with $J^\dagger J$ (or $JJ^\dagger$) corresponding to W^+ (or W^-) propagating forward in momentum space, there are three W^+ self-energy type ($D^-,\bar{D}^0,\pi^0$) loop graphs but only one W^- (K^+) loop in Figs. 6 and 7.

For $\Delta I = 1/2$ transitions in $K^0_{2\pi}$ decays, the dominant D meson W^+ loop graphs in the K_π amplitudes (21) and (22) reinforce one another in $K^0_{\pi^+\pi^-}$ and $K^0_{\pi^0\pi^0}$, as is seen by folding (18a) into (14a) and (18b) into (14b). These D meson W^+ loop graphs of Fig. 6(b) and Fig. 7(b) reflect the $\bar{s}d$ quark W^+ loop graph of Fig. 1 as hadronized into the K_π amplitude (5) represented by Fig. 3. (It is satisfying to note the consistency between the values of (5b) and 21b)). Thus one may claim the large $K^0_{2\pi}$ amplitude ($\Delta I = 1/2$ rule) is due to the large hadron GIM mass difference $m^2_D - m^2_\pi$ or due to the large quark GIM mass difference $m^2_c - m^2_u$. Alternatively one might say that the kaon $\Delta I = 1/2$ rule is a consequence of the two D meson W^+ loops reinforcing one another and dominating the smaller W^+ pion and W^- kaon loops.

In the small $\Delta I = 3/2$ $K^+_{2\pi}$ amplitude, however, these two D meson W^+ loops cancel each other in (14c) (ie. in the denominator of (19b)). This leaves only the one pion W^+ graph of Fig. 7(a) to cancel partially against the one kaon W^- graph of Fig. 6(a). The latter PCAC-cancellation picture can be replaced by the one W^+ non-PCAC VS graph of Fig. 5 for $K^+_{2\pi}$ decay involving the one $J^\dagger J$ term in H_w.

Another way to see such $\Delta I = 1/2$ - $\Delta I = 3/2$ reinforcement-cancellation patterns in $K_{2\pi}$ decays is to consider a (long distance) dispersion relation "Cottingham-type current-current formula" involving only the one W^+ exchange but then having both "disconnected" (disc) and "connected" (conn) amplitudes as depicted in Fig. 8.

$$M_{K2\pi} = M_{disc} + M_{conn} \quad . \tag{24}$$

The M_{disc} amplitude due to $|0\rangle\langle 0|$ and to $|K^+\pi^+\rangle\langle K^+\pi^+|$ intermediate states is double the VS $\Delta I = 3/2$ amplitude and experiment (11), as originally noted by Feynman.[23] The more complete long-distance program of Preparata and collaborators[6] focuses on the crossed-channel (dual) version of M_{conn} and finds that it is controlled by the reinforcement of nonexotic Regge poles for $\Delta I = 1/2$ $K_{2\pi}$ and the partial cancellation of exotic poles for $\Delta I = 3/2$ $K_{2\pi}$. In fact the net $M_{conn}(K^+_{2\pi})$ amplitude is negative relative to $M_{disc}(K^+_{2\pi})$ and effectively cancels off the disconnected $|K^+\pi^+\rangle\langle K^+\pi^+|$ amplitude,[6] again leaving the VS amplitude (11a) to dominate $K^+_{2\pi}$ decay. The larger $\Delta I = 1/2$ amplitude M_{conn} of Ref. 6 also appears to approximate numerically $K^0_{2\pi}$ decays and this is further supported by the recent low-energy direct-channel dispersive analysis[7] of Pham and Sutherland emphasizing the importance of charmed D meson intermediate states. It was the latter work which motivated us to seek a similar reinforcement-cancellation pattern of W^+ and W^- field theory meson loops in Sec. IVa and Sec. IVb.

V. CONCLUSION

We summarize our results, but first note that there is more than one method to obtain the observable large $\Delta I = 1/2$ $K^0_{2\pi}$ amplitudes while also predicting the small $\Delta I = 3/2$ $K^+_{2\pi}$ weak amplitude. Within the long-distance framework, we have summarized the "hybrid" program in Secs. II and III, solving the kaon $\Delta I = 1/2$ rule in terms of the s-d-W self-energy quark graph while extracting the $\Delta I = 3/2$ $K^+_{2\pi}$ amplitude from vacuum saturation of the hadronic current-current matrix element. Then in Sec. IV we developed an alternative field theory meson-loop long-distance scheme, for both $K^0_{2\pi}$ and $K^+_{2\pi}$, which is essentially equivalent to our above (hybrid) picture. In all cases, we correctly predict (with no free parameters) the $\Delta I = 1/2$ $K^0_{2\pi}$ and the $\Delta I = 3/2$ $K^+_{2\pi}$ amplitudes, both in magnitude and relative sign. This latter approach of Sec. IV is also compatible with the results of the long-distance dispersion theory Cottingham formalism (high-energy cross channel[6] or low-energy direct channel[7]).

In passing, we remind the reader that of late there have been many new short-distance approaches to the $K_{2\pi}$ decays. There is a quite involved short-distance QCD program that claims[34] to compute $K^+_{2\pi}$, but admittedly[35] fails to recover $K^0_{2\pi}$. A more recent short-distance scheme[36] works in the large N limit to calculate $K_{2\pi}$ amplitudes dominated by "penguin" graphs, but even here there is honest debate[37] over its consistency with "chiral perturbation theory." To avoid in part such a problem, this short-distance program has been modified to include long-distance effects,[38] based on a truncated strong interaction chiral lagrangian. But regardless of the validity of these fundamentally short-distance descriptions of $K_{2\pi}$ amplitudes, we suggest that the alternative pure long-distance program presented in this paper offers the simplest solution of all kaon nonleptonic weak decays.

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APPENDIX A: PION PCAC FOR $\Delta I = 1/2, 3/2$ $K_{2\pi}$ TRANSITIONS

One way to exploit the consequences of the rapidly varying but conserved momentum ($k = q_i + q_j$) for $K_{2\pi}$ decays is to start with a low energy Weinberg-like expansion

$$\langle \pi^i \pi^j | H_w | K \rangle = A(k^2 + a_i q_i^2 + a_j q_j^2) \quad , \tag{A.1}$$

where $a_i + a_j = -1$ insures the known[19] CP invariant on-shell ($k^2 = m_K^2$, $q_i^2 = q_j^2 = m_\pi^2$) but SU(3)-breaking structure $(1 - m_\pi^2/m_K^2)$ of (A.1).

Specializing first to $\Delta I = 1/2$ $K^0_{2\pi}$ decay where $a_i = a_j = a_0 = -1/2$, pion PCAC and (A.1) imply with $q_i \to 0$, $q_j^2 \to m_K^2$,

$$\langle \pi^0 \pi^0 | H^{eff}_{w,1/2} | K^0 \rangle \to (-i/f_\pi) \langle \pi^0 | \left[Q_5^3, H^{eff}_{w,1/2} \right] | K^0 \rangle \tag{A.2a}$$

$$= (i/2f_\pi) \langle \pi^0 | H^{eff}_{w,1/2} | K^0 \rangle \tag{A.2b}$$

$$= A(m_K^2 - \tfrac{1}{2} m_K^2) \quad . \tag{A.2c}$$

Equation (A.2b) follows from (A.2a) and the usual chiral symmetry commutation relation (3). Then solving for A from (A.2c) and (A.2b) and substituting back into (A.1) for $k^2 = m_K^2$, $q_i^2 = q_j^2 = m_\pi^2$ leads to the desired pion PCAC relation (7):

$$\langle \pi\pi | H^{eff}_{w,1/2} | K^0 \rangle = (i/f_\pi) \langle \pi^0 | H^{eff}_{w,1/2} | K^0 \rangle (1 - m_\pi^2/m_K^2) \quad . \tag{A.3}$$

Here the two π^0's on the LHS of (A.3) are both on the pion mass shell, while the single π^0 on the RHS of (A.3) is on the kaon mass shell by use of pion PCAC. The kaon is always taken on mass shell.

For $\Delta I = 1/2$ $K^0 \to \pi^+ \pi^-$ decay one can show that[29] $a_+ = -1$, $a_- = 0$, so that again the pion PCAC relation (7) or (A.3) holds. However, for $\Delta I = 3/2$ $K^+ \to \pi^+\pi^0$ decay, one has $a_+ = -4/3$, $a_0 = 1/3$ and then the $\Delta I = 3/2$ pion PCAC relation following from (A.1) is[21,29]

$$\langle \pi^+\pi^0 | H^{eff}_{w,3/2} | K^+ \rangle = (-i3/2f_\pi)\, \langle \pi^+ | H^{eff}_{w,3/2} | K^+ \rangle\, (1 - m_\pi^2/m_K^2) \quad . \tag{A.4}$$

APPENDIX B: $\Delta I = 1/2,\ 3/2$ COMPONENTS OF HADRONIC VACUUM SATURATION

We first employ the group theory identity advocated in Refs. 3,4, but for hadron rather than quark flavor currents in (10) (we write J^{1-i2} as $J^{(ud)}$ etc. for simplicity):

$$\begin{aligned} J^\dagger J \to J^{(su)}\ J^{(ud)} &= \tfrac{1}{2}\left[J^{(su)}\ J^{(ud)} - J^{(ud)}\ J^{(su)}\right] \\ &+ \tfrac{1}{10}\left[J^{(su)}\ J^{(ud)} + J^{(ud)}\ J^{(su)} + 2J^{(dd)}\ J^{(sd)} + 2J^{(uu)}\ J^{(sd)}\right] \\ &+ \tfrac{1}{15}\left[J^{(su)}\ J^{(ud)} + J^{(ud)}\ J^{(su)} + 2J^{(dd)}\ J^{(sd)} - 3J^{(uu)}\ J^{(sd)}\right] \\ &+ \tfrac{1}{3}\left[J^{(su)}\ J^{(ud)} + J^{(ud)}\ J^{(su)} - J^{(dd)}\ J^{(sd)}\right] \quad . \end{aligned} \tag{B.1}$$

On the RHS of (B.1), the hadronic currents respectively transform under SU(3)-SU(2) as $(8_A, 1/2)$, $(8_S, 1/2)$, $(27, 1/2)$ and $(27, 3/2)$. Then the π^+-K^+ (vacuum saturation) of (10) or (B.1) gives the $\Delta I = 1/2/\Delta I = 3/2$ amplitude ratio[29]

$$\frac{\langle \pi^+ | J^\dagger J_{1/2} | K^+ \rangle}{\langle \pi^+ | J^\dagger J_{3/2} | K^+ \rangle} = \frac{\frac{1}{2} + \frac{1}{10} + \frac{1}{15}}{\frac{1}{3}} = 2 \quad . \tag{B.2}$$

Then this VS ratio (B.2) can be translated to the entire VS amplitude:

$$\begin{aligned} \langle \pi^+ | H_w | K^+ \rangle_{VS} &= \langle \pi^+ | H_{w,1/2} | K^+ \rangle_{VS} + \langle \pi^+ | H_{w,3/2} | K^+ \rangle_{VS} \\ &= 2\langle \pi^+ | H_{w,3/2} | K^+ \rangle_{VS} + \langle \pi^+ | H_{w,3/2} | K^+ \rangle_{VS} = 3\langle \pi^+ | H_{w,3/2} | K^+ \rangle_{VS} \quad . \end{aligned} \tag{B.3}$$

This factor of 3 in (B.3) also appears in the $\Delta I = 3/2$ PCAC relation (12a) and follows more generally upon substitution of (15b) into (14c).

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FIGURE CAPTIONS

Fig. 1. Quark $\bar{s}$-$\bar{d}$ $\Delta I = 1/2$ transition due to physical W^+ exchange.

Fig. 2. Quark $\bar{s}$-$\bar{d}$ $\Delta I = 1/2$ transition due to unphysical χ^+ Higgs exchange.

Fig. 3. Hadronized quark s-d "submarine" graph (a) forming $\langle \pi^0 | H^{eff}_{w,1/2} | K^0 \rangle$, envisioned as long-distance quark graph (b).

Fig. 4. Dominant π^0 pole graph for $K^0 \to \gamma\gamma$ decay.

Fig. 5. Quark-spectator W^+ graph (a) nonperturbatively hadronized to the long-distance vacuum-saturation graph (b).

Fig. 6. Hadronic K^+ intermediate state (a) and D^- state (b) low-energy saturation of $\langle \pi^0 | H_w | K^0 \rangle$ envisioned as long-distance meson loop graphs (c).

Fig. 7. Hadronic π^0 intermediate state (a) and $\bar{D}^0$ state (b) low-energy saturation of $\langle \pi^+ | H_w | K^+ \rangle$ envisioned as long-distance meson loop graphs (c).

Fig. 8. Nonleptonic $K \to \pi^+\pi$ dispersive amplitude separated into disconnected part (a) and connected part (b).

Fig. 1

Fig. 2

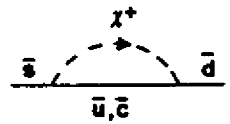

Fig. 3

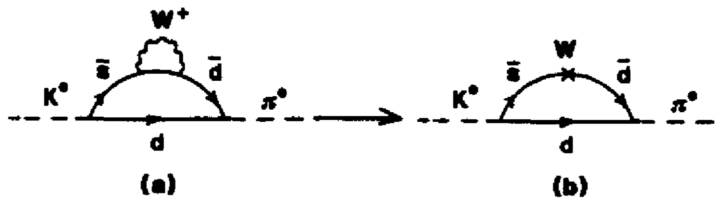

Fig. 4

Fig. 5

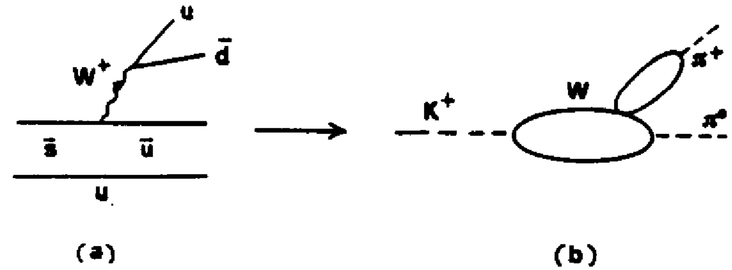

Fig. 6

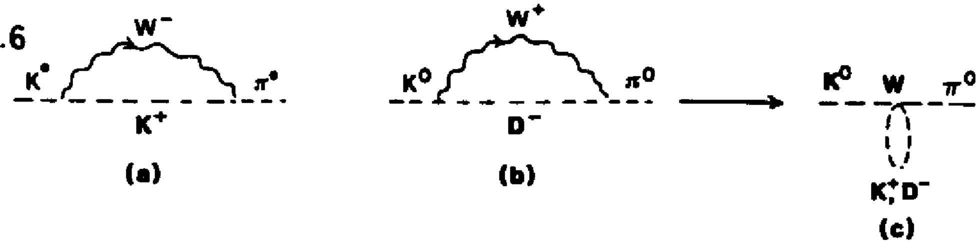

Fig. 7

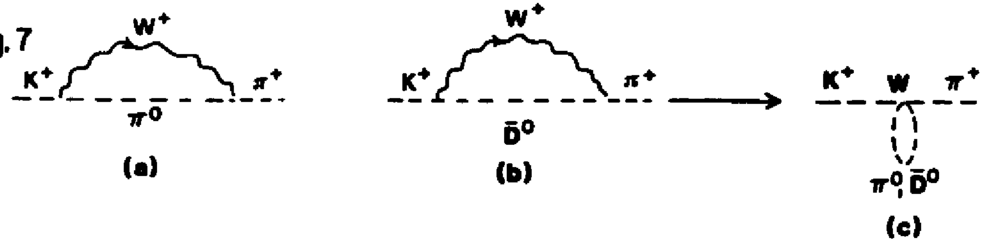

Fig. 8