FR 8801571

LAPP-TH-203/87
October 1987

THE ROLE OF LOCALITY IN STRING QUANTIZATION

Raymond Stora

LAPP, BP.909, F-74019 Annecy-le-Vieux Cedex
and
CERN, TH-Division, CH-1211 Geneva 23

Lecture given at the Cargese Summer Study Institute
on Non Perturbative Quantum Field Theory,
Cargese, July 16-30, 1987

ERRATUM LAPP-TH-203/87

October 1987

THE ROLE OF LOCALITY IN STRING QUANTIZATION

by R. STORA

Two misprints :

Eq.22 should read $\mathfrak{A} \sim \int \frac{dz \wedge d\bar{z}}{2i} \mu \, \partial_z^3 C^z + \text{"c.c."}$

Eq. 35 should read $(\partial_{\bar{z}} - \mu\partial_z - 2\partial_z\mu) \, b_{zz} = 0$

THE ROLE OF LOCALITY IN STRING QUANTIZATION

Raymond Stora

LAPP, BP. 909, F-74019 Annecy-le-Vieux Cedex
and
CERN, TH-Division, CH-1211 Geneva 23

INTRODUCTION

Whereas free string quantization is by now part of the common knowledge[1], it still calls for some comments : it is a gauge theory, for the gauge group "Diff × Weyl" ; gauge fixing has then to be performed according to the Faddeev Popov procedure and the ensueing Slavnov symmetry can be found, given local gauge functions. It is customary to work in a Landau type conformal gauge and furthermore to eliminate the Weyl ghost and antighost as well as the multiplier field so that only a pair of diffeomorphism ghost and antighost are left over, besides the string field. These eliminations lead to a loss of nilpotency of the Slavnov symmetry which, in the existing versions only holds modulo the ghost equations of motion. The alternative use of the Hamiltonian formalism[2] not only spoils the world sheet geometry but prevents a systematic use of world sheet locality which is necessary for an unambiguous classification of the anomalies. These questions have been straightened out recently[3),4),5)], including the study of the localized Slavnov symmetry through the corresponding current algebra study[5),6)] and the supersymmetric extension[7]. It is the aim of these notes to review some of these recent constructions.

We shall mostly be concerned with the one orbit theory in the conformal gauge, i.e. assume no global zero mode. In this framework, we describe the Slavnov symmetry and the corresponding Slavnov identity (section II), the covariance under diffeomorphisms, and the corresponding Ward identity (section III), the localized Slavnov symmetry and the corresponding current algebra (section IV). Locality is used throughout and provides unambiguous answers, e.g. on the existence and form of the anomalies.

In general, however, the conformal gauge is not a good gauge. In the present framework, this is signalled by the presence of global zero modes which depend on the boundary conditions and have to be gauge fixed. As an example, the case where the world sheet is a compact Riemann surface of genus $g > 1$ is treated in some details (section V).

Some concluding remarks are gathered in section VI.

II. THE FREE BOSONIC STRING IN THE CONFORMAL GAUGE : THE SLAVNOV SYMMETRY

We start with a string with world sheet Σ mapped into R^D equipped with a metric $(\, , \,)$: the usual (σ,τ) variables will be collectively denoted $x = (x^1, x^2)$ and the string field by $\vec{X}(x) \in R^D$. $g^{\alpha\beta}(x)$ $\alpha,\beta = (1,2)$ is a metric on Σ. Unless otherwise stated we shall work in the euclidean framework, i.e. both $g^{\cdot\cdot}$ and $(\, , \,)$ are euclidean. The string action is

$$S_{inv}(X.g) = \frac{1}{2} \int_\Sigma d^2x \, \sqrt{g} \, g^{\alpha\beta}(x) \, (\partial_\alpha \vec{X}, \, \partial_\beta \vec{X})(x) \tag{1}$$

Since $S_{inv}(X,g)$ only depends on the conformal class of the metric g (i.e. is invariant under the Weyl scaling $g \to e^{\varphi} g$) it is convenient to use the following parametrization : z, $\bar{z}$ denote some complex analytic coordinates associated with a background (conformal class of) metric $\mathring{g}$ and the conformal class of g will be parametrized by the Beltrami differential μ ($|\mu| < 1$) such that

$$ds^2 = \frac{1}{2} g_{\alpha\beta} \, dx^\alpha \, dx^\beta \propto |dz + \mu d\bar{z}|^2 \, , \quad |\mu| < 1 \tag{2}$$

S_{inv} can then be rewritten as

$$S_{inv}(\vec{X}, \mu, \bar{\mu}) = \int_\Sigma \frac{dz \wedge d\bar{z}}{2i} \, \frac{1}{1-\mu\bar{\mu}} \, (\partial_z - \bar{\mu}\partial_{\bar{z}} \vec{X}, \, \partial_{\bar{z}} - \mu\partial_z \vec{X}) \tag{3}$$

The geometrical object associated with μ is

$$\hat{\mu} = \mu \, d\bar{z} \otimes \frac{\partial}{\partial z} \tag{4}$$

It is called a Beltrami differential. It is a type $(0,1)$ one form with value a type $(1,0)$ vector field. It is convenient, in order to check the homogeneity of formulae under holomorphic changes of variables (e.g. change of chart on Σ) to think of μ as bearing a lower index $\bar{z}$ and an upper index z $(\mu \sim \mu_{\bar{z}}^{z})$. μ is assumed to be C^∞.

It is easy to work out the transformation of both $\vec{X}$ and μ under an infinitesimal diffeomorphism represented by a vector field

$$\gamma = \gamma^z \partial_z + \gamma^{\bar z} \partial_{\bar z} = (\gamma \cdot \partial) \quad (5)$$

$$\delta_\gamma X = (\gamma \cdot \partial)\, X$$
$$\delta_\gamma \mu = \partial_{\bar z} C^z + C^z \partial_z \mu - \mu \partial_z C^z \quad (6)$$

where

$$C^z = \gamma^z + \mu \gamma^{\bar z} \,. \quad (7)$$

All formulae have to be completed through the substitution $\mu \leftrightarrow \bar\mu$ $z \leftrightarrow \bar z$ $i \leftrightarrow -i$, denoted by "c.c.". It is easy to check that

$$[\delta_\gamma, \delta_{\gamma'}] = \delta_{[\gamma,\gamma']} \quad (8)$$

where $[\gamma,\gamma']$ is the Lie bracket.

The variable C^z (eq. 7) has been discovered by C. Becchi[4] in connection with the question of holomorphic factorization of the Green functional in the conformal gauge. We shall return to this point later.

Turning γ into a Faddeev Popov ghost (6) is transformed into

$$s\,\vec X = (\gamma \cdot \partial)\, \vec X$$
$$s\,\mu = \partial_{\bar z} C^z + C^z \partial_z \mu - \mu \partial_z C^z \quad (9)$$

and eq. (8) becomes

$$s\, C^z = C^z\, \partial_z\, C^z \quad (10)$$

Note that in eq. (10), there is no summation over $z, \bar z$.

These formulae are easily obtained in terms of complex analytic coordinates Z pertaining to the complex structure parametrized by μ : one has

$$d\,Z = \lambda(dz + \mu d\bar z) \quad (11)$$

where the non local integrating factor λ fulfills (as a consequence of $d^2 = 0$):

$$-\,\partial_{\bar z}\lambda + \mu \partial_z \lambda + \lambda \partial_z \mu = 0 \quad (12)$$

The action of diffeomorphisms is easily worked out in terms of the Z variables, after an easy elimination of λ, using eq. (12). Note that in terms of C^z, all formulae are local, as a consequence of the elimination of the only non local item which occurs in the geometry, namely the integrating factor λ.

Of course we have

$$s^2 = 0 \qquad (13)$$

since all we have done is to represent the Lie algebra of diffeomorphisms.

Gauge fixing is carried out as usual. The conformal gauge is characterized by the gauge function

$$\mu - \overset{\circ}{\mu} \qquad (14)$$

where $\overset{\circ}{\mu}$ is a prescribed Beltrami differential. If one stays in a Landau type gauge, the multiplier field can be eliminated and μ must be replaced by $\overset{\circ}{\mu}$ everywhere. From now on we shall suppress the upper script $\circ$ on $\overset{\circ}{\mu}$, remembering that μ is now a classical field.

The gauge fixed action is then

$$\begin{aligned} &S_{gf}(X, \mu, \bar{\mu}, C^z, C^{\bar{z}}, b_{zz}, b_{\bar{z}\bar{z}}) \\ &= S_{inv}(X, \mu, \bar{\mu}) + S_{gh}(\mu, \bar{\mu}, C^z, C^{\bar{z}}, b_{zz}, b_{\bar{z}\bar{z}}) \end{aligned} \qquad (15)$$

where b_{zz} is the antighost, bearing the indices of a quadratic differential to insure the homogeneity of the formulae.
S_{inv} is given by eq. (3), and

$$S_{gh} = \int \frac{dz \wedge d\bar{z}}{2i} \; \bar{b}_{zz} \; s\mu + \text{"c.c"} \qquad (16)$$

with $s\mu$ as in eq.(9).

Introducing sources coupled to the s-variation:

$$S_{source} = \int \frac{dz \wedge d\bar{z}}{2i} \, [(\mathfrak{X}, sX) + \Gamma \, s \, C^z + \text{c.c.}] \qquad (17)$$

the Slavnov symmetry eqs (9),(10) extended by

$$\begin{aligned} &s \, b_{zz} = 0 \quad \& \; (\text{"c.c."}) \\ &s \, \mathfrak{X} = s \, \Gamma = 0 \end{aligned} \qquad (18)$$

is expressed through the Slavnov identity

$$\int \frac{dz \; d\bar{z}}{2i} \left[\left(\frac{\delta S}{\delta X} , \frac{\delta S}{\delta \mathfrak{X}} \right) + \frac{\delta S}{\delta \Gamma} \frac{\delta S}{\delta C^z} + \frac{\delta S}{\delta b_{zz}} \frac{\delta S}{\delta \mu} + \text{"c.c."} \, .. \right] (z,\bar{z}) = 0 \qquad (19)$$

For

$$S = S_{tot} = S_{inv} + S_{gf} + S_{source} \qquad (20)$$

The Slavnov identity is a current algebra Ward identity for the energy momentum tensor components Θ_{zz}, $\Theta_{\bar{z}\bar{z}}$ whose correlation functions are generated by functional differenciation with respect to μ, $\bar{\mu}$ respectively.

Note, in order to compare with the usual formulae, that eq.(19), derived as a consequence of the nilpotency property eq.(11) can also be interpreted as the invariance of S under the transformation $\tilde{s}$:

$$\begin{aligned} &\tilde{s} = s \quad \text{on} \quad X, \mathfrak{X}, \Gamma \\ &\tilde{s}\,\mu = 0 \qquad \tilde{s}\, b_{zz} = \frac{\delta S}{\delta \mu} = \Theta_{zz} \end{aligned} \tag{21}$$

However $\tilde{s}$ is only nilpotent modulo the ghost equation of motion.

Passing to the quantum level amounts to replacing the classical action S by the vertex functional Γ which also takes into account one loop diagrams (and, here, only those) and requiring eq.(19) with S replaced by Γ.

The "holomorphy" anomaly is then found on the right hand side with a coefficient proportional to D-26 and the local form

$$\mathfrak{A} \sim \int \frac{dz \wedge d\bar{z}}{2i}\, \mu\, \partial_z^z\, C^z + \text{"c.c."} \tag{22}$$

If several charts are needed to cover Σ eq.(22) is no good. One should then use e.g. eq.(31) of ref. 3) and convert it into the present notations.

III. THE DIFFEOMORPHISM WARD IDENTITY[4),5),12),13)]

Besides the Slavnov symmetry associated with Diffeomorphisms and their gauge fixing, the classical action admits another invariance under the action of diffeomorphisms, which commutes with the Slavnov symmetry. Actually the action of diffeomorphisms is uniquely defined on $\vec{X}$ and μ, but not on C^z, b_{zz} and the other fields. One set of transformations was proposed by C. Becchi[4)], another one by L. Baulieu and M. Bellon[5)]. They are equivalent modulo the equations of motion. We shall limit ourselves to the latter.

The symmetry in question reads:

$$\begin{aligned} \partial_\Lambda X &= (\lambda.\partial)\, X \\ \delta_\Lambda \mu &= \partial_{\bar{z}}\Lambda^z + \Lambda^z \partial_z \mu \quad \mu \partial_z \Lambda^z \\ \delta_\Lambda C^z &= \Lambda^z \partial_z C^z - C^z \partial_z \Lambda^z \equiv [\Lambda, C] \\ \delta_\Lambda b_{zz} &= (\Lambda^z \partial_z + 2\partial_z \Lambda^z)\, b_{zz} \end{aligned} \tag{23}$$

and "c.c.".

The commutation properties are

$$[\delta_\Lambda, \delta_{\Lambda'}] = \delta_{[\Lambda,\Lambda']} \tag{24}$$

with

$$[\Lambda,\Lambda'] = \Lambda^z \partial_z \Lambda'^z - \Lambda'^z \partial_z \Lambda^z \tag{25}$$

We have used the obvious notation

$$\Lambda^z = \lambda^z + \mu\lambda^{\bar z} \Leftrightarrow \lambda^z = \frac{\Lambda^z - \mu\Lambda^{\bar z}}{1 - \mu\bar\mu} \tag{26}$$

The last of eq.(23) is obtained by duality, demanding that S_{gh} be invariant. One may similarly infer the formula for $\delta_\Lambda \Gamma$. $\delta_\Lambda X$ is not completely computable by duality. It is only so modulo the ghost equation of motion, which nevertheless allows to write a Ward identity e.g. for the connected Green's functional (the Legendre transform of the vertex functional). Details will appear elsewhere. Either the Ward identity or the Slavnov identity can be used to prove the holomorphic factorization of the connected Green functional[4),5)]:

$$Z^c(J_x, J_c, J_b, \mu, \text{"c.c."}) = Z^c(J_x, J_c, J_b, \mu) + Z^c(J_{\bar x}\ J_{\bar c}\ J_{\bar b}, \bar\mu) \tag{27}$$

where J_{field} is the source argument corresponding to the field in question. It is in the course of requiring the holomorphic factorization property that C. Becchi discovered that the proper definition of the correlation functions of the Θ_{zz}, $\Theta_{\bar z\bar z}$ components of the energy momentum tensor goes through differenciation with respect to μ, $\bar\mu$ and that the proper ghost fields are C^z, $C^{\bar z}$, rather than γ^z, $\gamma^{\bar z}$.
$Z^c(J_x, J_c, J_b, \mu)$ may be derived from the action eq.(20) by putting $C^{\bar z} = b_{\bar z\bar z} = \bar\Gamma = \bar\mu = 0$ (i.e. forgetting the "(c.c.)" world) which reduces to the FMS action[1)], including terms coupled to the stress energy tensor and to the Slavnov variations, and the S operation reduces to s_{FMS}, except for the interchange $s\mu$, $sb = 0$ into $s_{FMS}\mu = 0$ $s_{FMS}b = \delta S/\delta\mu$. s_{FMS} is nilpotent. One may similarly define a nilpotent $\bar s_{FMS}$. $s_{FMS} + \bar s_{FMS}$ leaves invariant $(S_{inv} + S_{gh})|_{\mu=\bar\mu=0}$ but $(s_{FMS} + \bar s_{FMS})^2$ only vanishes modulo the ghost equations of motion.

IV. LOCALIZED SLAVNOV SYMMETRY AND THE SLAVNOV CURRENT ALGEBRA

The Slavnov symmetry eqs.(9),(10) can be thought of as involving an "infinitesimal space time independent anticommuting parameter". This point of view which is conveniently replaced by the present one where s is a graded derivation, calls for the corresponding generalization: can one construct an extension of s into s_{loc}, involving, besides the former variables, a space time dependent even field $\lambda(x)$ (the localization of the ghost of the "small anticommuting parameter"), together with an odd gauge field

$$\alpha = \alpha_\mu(x)\; dx^\mu \;, \qquad \alpha_\mu(x) \text{ odd} \tag{28}$$

serving as a source of the Noether current corresponding to the Slavnov symmetry, in such a way that $s^2_{loc} = 0$, and that the action S can be extended into an action S_{loc}, depending on α, but not on λ, left invariant by s_{loc}. Both s_{loc} and S_{loc} are assumed to reduce to the "global" Slavnov symmetry and the former S for $\lambda = 1$, $\alpha = 0$.

Besides being a natural question to ask, this sheds a new light on the following little puzzle alluded to in the Introduction : "the anomaly" ($\propto$ D-26) is found in the lack of nilpotency, at the quantum level, of the Q operator obtained by integrating the time component of the Slavnov Noether current[2]. This result relies on a renormalization procedure needed to pass from the classical to the quantum situation, which is performed by using Wick ordering in a way which is not known to be unique and whose ambiguity is not known by virtue of admitted principles.

Formally Q^2 appears as an equal time commutator term for the Slavnov current algebra Ward identity. A study of that Ward identity, using the locality principle which leads to a parametrization of the ambiguities, should therefore show the result that its anomalies are rigidly linked to the "global" holomorphy anomaly, thus explaining unambiguously why the nilpotency anomaly is also proportional to D-26.

One solution[5),6)], which exhibits this phenomenon, has been constructed. This does not completely settle the question, but shows that the Slavnov current algebra cannot have more anomalies. The construction follows the lines of Ref. 8), which deals with the Yang-Mills case. The extension thus obtained extends the holomorphic factorization property to correlation functions involving the Slavnov Noether current.

"Minimal coupling" is used throughout, i.e. the following replacement both in S eqs.(3),(16),(17) and in s eqs.(9),(10):[6)]

$$u \to \hat{u} = u\,(1 + \beta_z \mathfrak{C}^z) - \beta_z \mathfrak{C}^z \qquad (29)$$

$$s \to s_{loc} \qquad S \to S_{loc}$$

together with the adjunction of

$$s_{loc}\, \beta_z = -\partial_z \lambda/\lambda \qquad s_{loc}\, \lambda = 0 \qquad (30)$$

The correlation functions of the components $J_z\ J_{\bar z}$ of the Noether current are obtained by functional differenciation with respect to β_z, $\beta_{\bar z}$. (Here, $\mathfrak{C}^z = \lambda C^z$ $\alpha_z = \lambda\beta_z$). One can easily check that $\left.\frac{\delta S}{\delta\beta_z}\right|_{\beta=0}$ is the known Noether current.

The algebra thus constructed is isomorphic with a trivial extension of the Slavnov algebra. The anomaly of the corresponding Noether current Ward identity is thus

$$\mathfrak{A}_{loc} \sim \int \frac{dz\, d\bar z}{2i}\, \partial^z\, \mathfrak{C}^z\, \Omega + \text{"c.c."} \qquad (31)$$

and its coefficient is the same as that of $\mathfrak{A}$ (eq.(22)) to which it reduces for $\lambda = 1$, $\beta = 0$.

V. GLOBAL ZERO MODES

So far we have constructed an action which is well defined in the absence of global zero modes, which is the case for f...d theory ($\Sigma = R^2$). For other types of surfaces, occuring in string theory, the global zero modes have to be discussed in each particular case, and gauge fixed. We propose to do this, as an exercise, for Σ compact. The constant zero modes for $\vec X$ lead to the extension of the s operation according to

$$s\,\vec X = (\gamma.\partial)\,\vec X + \vec c \qquad s\,\vec c = 0 \qquad (32)$$

$\vec c$ independent of x

A convenient gauge function is $\int \vec X\, \rho_{z\bar z}\, dz\, d\bar z$ where ρ is the density which describes $\overset{\circ}{g}$ in terms of the z, $\bar z$ variables. The corresponding Faddeev Popov term is

$$\bar{\vec c} \int [(\gamma.\partial)\vec X + \vec c]\rho_{z\bar z}\, dz\, d\bar z \qquad (33)$$

and the integration over $\bar{\vec c}$, $\vec c$ yields a determinant factor $(\int \rho\, dz\, d\bar z)^{D/2}$, whereas the Laplacian has to be restricted to $\vec X$ orthogonal to constants with respect to the metric ρ.

More serious are the zero modes occuring in the ghost sector:

$$s\,\mu \equiv \partial_{\bar z} C^z + C^z \partial_z \mu - \mu \partial_z C^z = 0 \tag{34}$$

describes the conformal Killing vectors for the complex structure defined by μ, as one can easily see by passing to the Z variables (cf. eq.(11)). As one knows the vector space of solutions has complex dimension 3 for genus $g = 0$ ($\Sigma \sim S^2$), 1 for $g = 1$ ($\Sigma \sim T^2$), 0 for $g > 1$. We shall henceforth assume $g > 1$.

The antighost zero modes fulfill

$$(\partial_{\bar z} - 2\mu\partial_z - \partial_z\mu)\, b_{zz} = 0 \tag{35}$$

as one can easily see by integration by parts in eq.(16),(9). In terms of the Z variables (cf. eq.(11)), one sees that b_{zz} is a holomorphic quadratic differential for the complex structure described by μ. One knows there is none for $g = 0$, 1 for $g = 1$ 3g-3 otherwise. Let ϕ^i, $i = 1,\dots,3g-3$ be a basis of solutions, functionals of μ.

Applying s (eqs.(9),(10)) to eq.(35), with b_{zz} replaced by ϕ^i, one easily finds that $s\phi^i$ is of the form

$$s\phi^i = (2\partial_z C^z + C^z\partial_z)\phi^i + A^i{}_j\, \phi^j \tag{36}$$

where $A^i{}_j$ is a matrix independent of z, $\bar z$, linear in C^z, and otherwise a functional of μ fulfilling

$$s\,A = A^2 \tag{37}$$

because $s^2 = 0$.

One then finds a new nilpotent s operation, including the invariance of the action under translation of b along its zero modes : it is sufficient to keep $s\vec{X}$, $s\mu$, sC^z and take

$$s\,\tilde b_{zz} = c_i\phi^i + b_i s\phi^i \tag{38}$$

with

$$s\,b_i = -c_i \qquad s\,c_i = 0 \tag{39}$$

c_i even, b_i odd

instead of $s\,b_{zz} = 0$.

One still has $s^2 = 0$. That s, extended by eq.(38) leaves S_{gh} invariant, is due to the identity

$$\int s\phi^i\, s\mu = 0 \tag{40}$$

with $s\phi^i$ given by eq.(36).

In order to perform gauge fixing, let us choose a set $\{h_i\}$, smooth in μ, dual to the system $\{\phi^j\}$:

$$\int \phi^j h_i \, dz \, d\bar{z} \equiv (\phi^j, h_i) = \delta^j{}_i \qquad (41)$$

and choose as gauge functions

$$(b, h_i) , \quad (b, sh_i) \qquad (42)$$

of the fermionic and bosonic type, respectively. In the corresponding Landau gauge, the corresponding Faddeev Popov Lagrangian involving the bosonic antighost $\bar{c}_i$ and the bosonic antighost $\bar{b}_i$, reads:

$$\bar{c}_i(c_i + b_j(s\phi^j,h_i)) + \bar{b}_i(c_j(\phi^j sh_i) + b_j(s\phi^j,sh_i)) \qquad (43)$$

with the constraints

$$(b, h_i) = (b, sh_i) = 0 \qquad (44)$$

The corresponding Faddeev Popov superdeterminant is

$$^{s}\mathrm{det} \begin{Vmatrix} \delta^j_i & (s\phi^j, h_i) \\ -(s\phi^j, h_i) & (s\phi^j, sh_i) \end{Vmatrix} \qquad (45)$$

VI. CONCLUDING REMARKS

We have reviewed recent reinvestigations of string first quantization, taking as an example the free bosonic string. Emphasis was put on locality, together with the construction of nilpotent Slavnov symmetries. These are two principles which lead to a class of "DET'"s resulting from gaussian integrations. The test for the correctness and usefulness of these principles is of course the possibility to construct sufficiently many observables. It is however encouraging to see some puzzles resolved, such as for instance the difficulty to construct an off shell nilpotent Slavnov symmetry[1),2),10)], as well as the tight relationship between the "nilpotency" anomaly[2)] and the trace or holomorphy anomaly.

Of course, the route followed here is a bit strange : the gauge function $\mu-\overset{\circ}{\mu}$ is only good if there is only one orbit under diffeomorphisms[11)]. Otherwise, one should choose an appropriate gauge function, which, *for topological reasons*, cannot depend on μ alone, but also on $\bar{\mu}$ thus spoiling holomorphic factorization from the start. One thus completely looses contact with holomorphic geometry. One possibility is to choose harmonic type gauges[11]. The procedure proposed here, i.e. to gauge fix the antighost zero modes signalling the inadequacy of the gauge functior may not be equivalent. On the other hand, it respects the contact with holomorphic geometry.

ACKNOWLEDGEMENTS

I wish to thank C. Becchi, L. Baulieu, M. Bellon, J.C. Wallet for numerous discussions as well as communicating their results prior to publication. Thanks are also due to L. Alvarez-Gaumé, O. Alvarez, J.M. Bismut, L. Bonora, D. Friedan, C. Gomez, C. Itzykson, C. Reina, P. Windey for keeping me informed on their work.

REFERENCES

1) M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Vol. I, Cambridge University Press (1987) and references therein.

(FMS) D. Friedan, E. Martinec, S. Shenker, Nucl. Phys. B271, 93 (1986).

2) M. Kato, K. Ogawa, Nucl. Phys. B212, 443 (1983).
S. Hwang, Phys. Rev. D28, 14 (1983) and Goteborg Dissertation (1986).

3) L. Baulieu, C. Becchi, R. Stora, Phys. Lett. 180B, 55 (1986) and references therein.

4) C. Becchi, "On the Covariant Quantization of the Free String : The Conformal Structure", Genova preprint 1987.

5) L. Baulieu, M. Bellon, "Beltrami Parametrization and String Theory", LPTHE 87-39, Paris, June 1987.

6) J.P. Ader, J.C. Wallet, Phys. Lett. 192B, 103 (1987);
J.P. Ader, J.C. Wallet, in preparation;
R. Marnelius, Phys. Lett. 186B, 351 (1987).

7) L. Baulieu, M. Bellon, R. Grimm, "Beltrami Parametrization for Superstrings", LPTHE 87-43, Paris, July 1987, and L. Baulieu's lecture, these proceedings.

8) L. Baulieu, B. Grossmann, R. Stora, Phys. Lett. 180B, 95 (1986).

9) C. Becchi, in preparation.

10) E. Cohen, C. Gomez, P. Mansfield, Phys. Lett. 180B, 55 (1986);
P. Mansfield, Nucl. Phys. B283, 551 (1987);
P. Mansfield, University of Oxford preprint 46/87.

11) L. Bonora, C.M. Viallet, CERN Th.4796/87.

12) A.A. Belavin, A.M. Polyakov, A.B. Zamolodchikov, Nucl. Phys. B241, 333 (1984).

13) T. Eguchi, H. Ooguri, Nucl. Phys. B282, 308 (1987).

NOTE ADDED

In section III we have for simplicity described the Diffeomorphism Ward identity proposed by L. Baulieu, M. Bellon. This seems to be a particularity of the "b-c" system, whereas the identity proposed by C. Becchi holds for all conformal systems. It reads:

$$\delta_\lambda \mu = \delta_\Lambda \mu \quad \text{as in eq.(23)}$$

$$\delta_\lambda \varphi_q = (\lambda.\partial)\varphi_q + q(\partial_z \lambda^z + \mu \partial_z \lambda^{\bar z})\varphi_q, \quad \text{and c.c.}$$

for φ_q of type $(q,0)$ ($q = -1$ for c^z, $q = +2$ for b_{zz}).
It expresses the invariance of the theory under a diffeomorphic change of the background complex structure parametrized by the complex analytic coordinates z.