CONF-870947--8

MASTER

RAY TRACING IN ATF USING 3-D SPLINES[*]

CONF-870947--8

DE88 000633

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The ray tracing code, RAYS,[1] requires a subroutine module which supplies magnetic field and density as well as their gradients ($\bar{B}$,$\bar{\nabla} B$,n_e, and $\bar{\nabla} n_e$), to propagate rays using the cold plasma approximation. In order for the ray trajectory to remain a solution of the dispersion relation, $D(\omega,\bar{k},\bar{r}) = 0$, the spatial derivatives of $\bar{B}$ and n_e numerically defined by the module must be a good approximation to the analytic derivatives of the field quantities themselves. This consistency, coupled with initializing the ray on a root of the dispersion relation, $D(\omega,\bar{k}_0,\bar{r}_0) = 0$, is sufficient for the ray to satisfy the dispersion relation along the ray. An additional requirement is that n_e be constant on a magnetic field line. Although this is not a necessary condition for the ray to satisfy the dispersion relation, the physics of the problem may dictate that this constraint be satisfied. The example of lower hybrid (LH) heating is described below where this requirement should be satisfied in order to get meaningful results. A computationally efficient and robust way to solve these problems is to use 3-D cubic spline interpolation of a data set with the desired model for $\bar{B}$ and the accurate calculation of toroidal flux, Ψ. The advantages and use of splines is the subject of this paper.

In the electrostatic limit for LH waves, the parallel and perpendicular components of the ray group velocity are given by $V_{g_\parallel}/V_{g_\perp} = k_\perp/k_\parallel \gg 1$. Thus the LH ray path basically follows a field line of $\bar{B}$ as it migrates slowly towards the plasma center. Since the wavevector, $\vec{k}$, is strongly influenced by the local value of n_e, the model chosen for n_e has a significant effect on the ray trajectory. In order to do LH ray tracing, we therefore require that Ψ and its spatial derivatives be modeled consistently with $\bar{B}$, such that Ψ is indeed constant along a field line of $\bar{B}$. This stringent requirement, coupled with the high computational cost (in general) of computing Ψ, dictates the use of an interpolatory method with a one-time initialization of an accurate computation of Ψ.

In our use of 3-D splines we set the spline tension factor to zero, which results in an interpolating function that is the tensor product of three cubic spline polynomials. In addition to the fact that the splines take on the data values at the grid points, first and second derivatives are continuous everywhere. The spline coefficients for the three components of $\bar{B}$ and for Ψ are stored in three dimensional cylindrical coordinates (to conveniently model the vessel of most plasma fusion experiments of interest). Storing all the spline coefficients on our VAX-8600 requires only 1000 blocks of VAX disc space for a reasonable density of grid points to define the stellarator field in the ATF device under construction at Oak Ridge National Laboratory[2] ($R \times \phi \times Z = 30 \times 35 \times 30 = 31K$ points). The calculation of these coefficients is done only once, and they are read in during subsequent runs. Approximately 10^4 spline evaluations of $\bar{B}$ and Ψ require one minute of VAX-8600 cpu-time, while the read-in time from disk at the start of each run is about half a second.

The initial setup calculation which defines $\bar{B}$ and Ψ is done using the Biot-Savart Law to evaluate the vacuum field due to the helical windings and the vertical field coils in ATF. The calculation of Ψ is done by following a field line originating from the field point to produce a puncture plot over one poloidal rotation in the toroidal plane containing the point. The flux at the desired point is then calculated by doing the two dimensional integral of B_ϕ flux through this surface. This is a very time consuming process to do for 3.1×10^4 points, requiring about 11 hours of CRAY-1 time. Early on in the work it was recognized that the amount of computation could be significantly reduced by using the following technique. First the Ψ_{ij0} $(R_i, Z_j, \phi = 0)$ are computed over the grid points in the $\phi = 0$ plane. The Ψ_{ijk} at all points with $\phi \neq 0$ are determined by following a field line from the point back to $\phi = 0$. The value of Ψ at this point (determined by a 2-D spline of the Ψ_{ij0}) is then equal to Ψ_{ijk} due to the constancy of Ψ along a field line. Since following a field line over at most one period of ATF is very fast, a 3-D calculation has been reduced to nearly 2-D. Using this trick, we can set up B_R,B_ϕ,B_Z and Ψ at 3.1×10^4 points in under two hours on the VAX.

Figs. 1 and 2 show contour plots of $|\bar{B}|$ and Ψ in ATF which were generated using the 3-D spline program. Several contours of Ψ have been plotted with the outermost one representing the largest value of Ψ which the splines can represent before the surfaces begin to break down. This corresponds to a contour approximately 2 cm smaller in radius than the last closed flux surface. The mod-B contours superimpose accurately with those generated directly from an evaluation of the Biot-Savart Law. Figs. 3 and 4 show ray trajectories for two orthogonal projections of an ensemble of rays propagating in the LH frequency regime. As pointed out above, the density profile strongly affects the LH rays and it is necessary to have Ψ, and therefore $n_e(\Psi)$, constant on a field line of $\bar{B}$. Fig. 5 shows that the value of n_e increases monotonically along the ray trajectories, as desired. Earlier work, with an ad hoc density model which was not constant on field lines, produced significantly different trajectories and caused the rays to have excursions through regions of decreasing density. Thus, the 3-D spline package allows efficient and accurate calculation of $\bar{B}$ and Ψ for a problem which would otherwise be computationally overwhelming.

* Research sponsored by the Office of Fusion Energy, U.S. Department of Energy, under Contract No. DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

REFERENCES

1. D. B. Batchelor, R. C. Goldfinger, "RAYS: A Geometrical Optics Code for EBT," Oak Ridge National Lab. Report ORNL/TM-6844 (1982).
2. J. F. Lyon, et al., "The Advanced Toroidal Facility," Fusion Technology, **10** 179 (1986).

FIGURE CAPTIONS

Fig. 1 Mod-B and toroidal flux contours in the $\phi = 0°$ plane of ATF. The dotted lines represent contours of constant toroidal flux and density. The value of $|\bar{B}|$ on each contour is indicated, normalized to unity at the magnetic axis.

Fig. 2 Mod-B and flux contours in the equatorial plane, $Z = 0$.

Fig. 3 Ray trajectories for an ensemble of lower hybrid rays ($n_e = 2 \times^{13}\ cm^{-3}$, $B_0 = 1\ T$, $f = 800\ MHz$) projected into the $\phi = \mathrm{TAN}^{-1}(y/x) = 0$ plane.

Fig. 4 Orthogonal projection of the rays into the $R = \sqrt{x^2 + y^2} = 210$ cm surface.

Fig. 5 Normalized local density value along the rays as a function of ϕ. The rays migrate up the density profile indicating that the model for n_e has Ψ constant on a field line of $\bar{B}$.

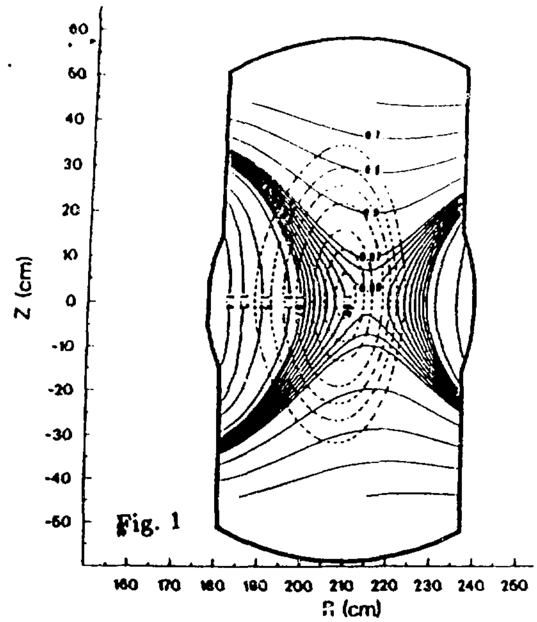

Fig. 1

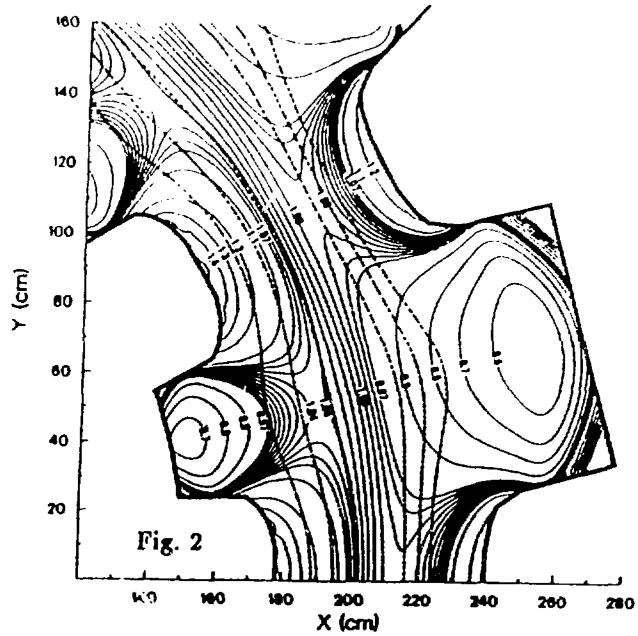

Fig. 2

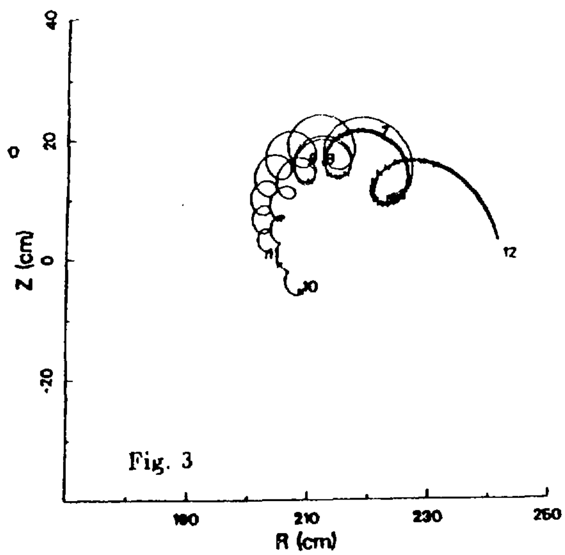

Fig. 3

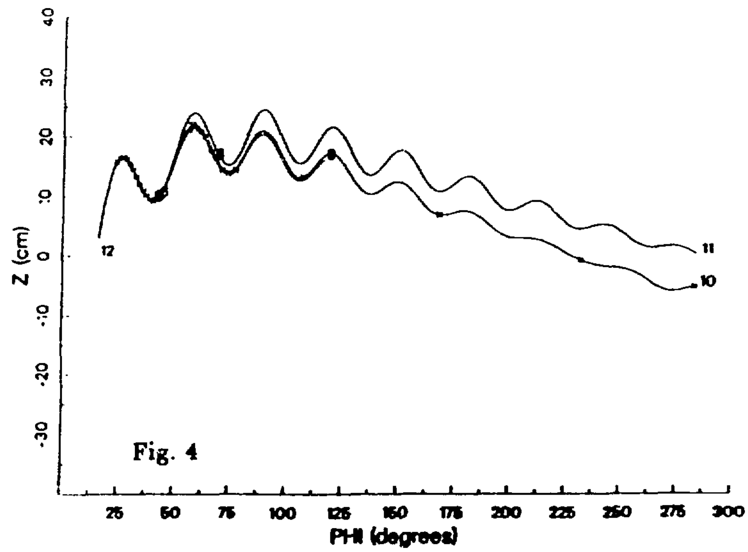

Fig. 4

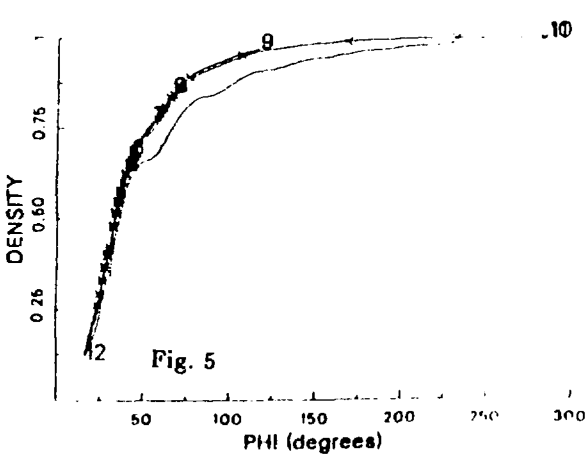

Fig. 5