IC/88/168

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

# NEUTRAL PION PHOTOPRODUCTION ON THE PROTON NEAR THRESHOLD AND THE CHIRAL SYMMETRY

L.M. Nath

and

S.K. Singh

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

IC/88/168

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

# NEUTRAL PION PHOTOPRODUCTION ON THE PROTON NEAR THRESHOLD AND THE CHIRAL SYMMETRY *

L.M. Nath ** and S.K. Singh ***
International Centre for Theoretical Physics, Trieste, Italy.

## ABSTRACT

The recent precise measurement of the neutral pion photoproduction on the proton near threshold seems to be inconsistent with the predictions of the low-energy theorems on photoproduction of pions. In this note, we have sought to resolve this problem, in the framework of an approximate SU(2) x SU(2) symmetry of the pion-nucleon dynamics, by incorporating the contribution from the nucleon matrix elements of the equal-time commutator between the divergence of the axial-vector current and the electromagnetic current, which was introduced earlier by Furlan et. al. We find that a mean value of the non-strange quark mass $\hat{m} = \frac{1}{2}(m_u+m_d) = (7\pm 2)$ MeV, leads to an estimate of the symmetry breaking terms compatible with experiment.

MIRAMARE - TRIESTE
July 1988

* To be submitted for publication.

** Permanent address: Physics Department, Dhaka University, Dhaka-2, Bangladesh.

*** Permanent address: Physics Department, Aligarh Muslim University, Aligarh, India.

The long-awaited experimental results on the photoproduction of neutral pions off proton near threshold are now available [1]. Precise experimental information on this process near threshold is very important for our understanding of the dynamics of photoproduction at low-energy. However, our knowledge of the neutral pion photoproduction on the proton relied, until recently, on the inferences and extrapolations from experiments performed at photon energies exceeding the threshold for this reaction by more than 15 MeV. The recent experiment at Saclay [1] has been carried out in the energy region from 144.7 to 173 MeV, including the threshold for the neutral pion photoproduction on the proton. Performing the multipole analysis of the measured cross sections, differential in the recoiling-proton energy, the electric dipole amplitude at threshold is found to be $E_{0+}(p\pi^0) = (-0.5 \pm 0.3) \times 10^{-3}\ \mu^{-1}$, which is neither compatible with the predictions from the low-energy theorems (LET) [2] which imply that $E_{0+}(p\pi^0) = -2.60 \times 10^{-3}\ \mu^{-1}$, nor does it agree with the previously inferred experimental value [1),3)], that is, $E_{0+}(p\pi^0) = (-1.8 \pm 0.6) \times 10^{-3}\ \mu^{-1}$. The data from this experiment have been analyzed elsewhere also [4] and the major conclusions have been confirmed, emphasizing again the strong disagreement between the LET predictions and the measured value of the electric dipole amplitude $E_{0+}(p\pi^0)$.

We have investigated this problem, using the chiral SU(2) x SU(2) symmetry of the pion-nucleon system [5] and the gauge invariance of the electromagnetic interactions. We have also considered in detail the contributions from the $N^*$ resonances and the vector mesons, which turned out to be hardly appreciable at threshold. On the other hand, we find that the discrepancy between the recent experimental results and the predictions of the LET can be adequately accounted for by incorporating a symmetry breaking term analogous to the $\sigma$-term in the low-energy pion-nucleon scattering. The possibility that an additional term proportional to the nucleon matrix elements of the commutator between the divergence of the axial-vector current and the electromagnetic current may play a significant role in the neutral pion photoproduction on the proton was pointed out first by Furlan, Paver and Verzegnassi (FPV) [6] and discussed further by MacMullen and Scadron [7] in a somewhat different form. In Ref.6, the symmetry breaking term was used to place an upper limit of 40 MeV on the mass of the non-strange, light quarks. Using the FPV approach, we find that a mean value of the non-strange quark mass $\hat{m} = \frac{1}{2}(m_u + m_d) = (7 \pm 2)$MeV [8] leads to estimates of the $\sigma$-like terms, which are compatible with those deduced from the latest experiment [1].

In order to describe the dynamics of the reaction $\gamma N \to \pi N$ at low-energy, we start with the Weinberg Lagrangian [5] for the pion-nucleon system which is invariant under the transformations of the chiral SU(2) x SU(2) and make the gauge-invariant replacement, $\partial_\mu \rightarrow \partial_\mu \mp ieA_\mu$ for the charged fields. Then, we introduce the Pauli-type interaction between the nucleon magnetic moment and the electromagnetic field to take into account the anomalous magnetic moment of the nucleon. The resulting interaction Lagrangian and the contribution it makes to the photoproduction of pions are given in the literature [9),10),11)].

We define the invariant amplitudes A, B, C, D following CGLN [12)] and in the isospin space they are further decomposed as

$$A_\beta(s,t) = \delta_{\beta 3}A^{(+)}(s,t) + \frac{1}{2}\left[\tau_\beta, \tau_3\right] A^{(-)}(s,t) + \tau_\beta A^{(0)}(s,t) \quad , \qquad (1)$$

with similar relations for other amplitudes, where s, t, u are the Mandelstam variables. The s-wave amplitude at theshold is given by

$$E_{0+} = \left\{ \frac{1}{x} \left[ A + (W - m)D - \frac{2m\nu_B}{(W-m)} (C - D) \right] \right\}_{\underset{\sim}{q}=0} \quad , \qquad (2)$$

where

$$X = 4\pi \frac{2W}{(W-m)} \cdot \frac{1}{[(m+E_1)(m+E_2)]^{1/2}} \quad ,$$

$$W = \sqrt{s} \, , \quad \nu_B = \frac{(t-\mu^2)}{4m} \quad .$$

In the s-channel c.m. frame, $E_1$ and $E_2$ are, respectively, the energies of the incoming and the outgoing nucleon with K and q as the momenta of the photon and the pion, respectively. We wish to observe here that the results of the low-energy theorems [2)] can be exactly reproduced if the electric dipole amplitudes $E_{0+}^{(+,0,-)}$ obtained from the chiral-invariant Lagrangian as discussed here are expanded in powers of $\mu/m$, where $\mu$ is the pion mass and m the nucleon mass. However, as it has been already mentioned, the LET predictions are in strong disagreement with the experimental value of the $E_{0+}(p\pi^0)$ at threshold [1)], although for the charged pion photoproduction off the nucleon near threshold the LET offer a satisfactory explanation of the current experimental results.

The $\Delta(1232)$, which plays an important role in the dynamics of the pion photoproduction, can be incorporated in our formalism without spoiling the chiral symmetry and the gauge invariance of the system. This problem has been discussed in detail in the literature [9),10),11),13)]. In particular, the $\Delta$-contributions to the invariant amplitudes have been evaluated explicitly, using the full spin-3/2 propagator and the most general forms of the $\pi N\Delta$ and $\gamma N\Delta$ interaction Lagrangians. Expressions for the $\Delta$-exchange contain two



arbitrary parameters, Z and Y, which measure the off-mass-shell effects of the $\Delta(1232)$. From purely theoretical considerations, the values of Z and Y are found to be $\frac{1}{2}$ and 0, respectively [10),14)]. Considerable work has been done to obtain the values of these parameters from the phenomenological analysis of the pion-nucleon scattering [15)] and photoproduction of pions [11)]. Although it is rather difficult to fix Z and Y uniquely from the phenomenology, it has been possible to place bounds on the values of Z and Y, which are

$$|Z| \leq \frac{1}{2} \, , \quad |Y| \leq \frac{1}{2} \quad . \qquad (3)$$

The numerical values of the $\Delta$-contribution to the s-wave amplitudes at threshold have been computed for different values of Z and Y compatible with the constraints (3) and they are given in Table 1. We find that our predictions depend rather weakly on the off-mass-shell parameters and, more importantly, the $\Delta$-contribution to the electric dipole amplitude at threshold is not appreciable compared with the nucleon contribution. The higher resonances such as $N^*(1440)$, $N^*(1520)$ do not make any appreciable contribution to $E_{0+}$ at threshold, because their masses are larger and the coupling parameters are much smaller [10)].

We have also considered the vector meson exhanges in the t-channel. The contributions form the $\omega$ and $\rho$-mesons have been evaluated explicitly, using the currently available information on the $VN\gamma$ and VNN coupling parameters [11),16,17)]. There are some uncertainties (of nearly 20%) regarding the values of the VNN couplings. However, the vector meson contributions to $E_{0+}(p\pi^0)$ at threshold are so small that our conclusions are not affected by these uncertainties.

Our results displayed in Table 2, imply that the contributions from the $N^*$resonances and the vector mesons cannot certainly account for the disagreement between the LET predictions and the recent experimental results on $E_{0+}(p\pi^0)$. In view of these considerations, we propose, following FPV [6)], that a chiral symmetry breaking term analogous to the $\sigma$-term in pion-nucleon scattering be incorporated in our description of the photoproduction dynamics. Using the experimental information on the charged and neutral pion photoproduction at threshold [1),3)], we obtain [*)]

---

* The cusp effect [18)] in $E_{0+}(p\pi^0)$ caused by the final state interaction, which equals approximately $-0.82 \times 10^{-3}\ \mu^{-1}$ at threshold, increases further the disagreement between the LET predictions and the latest experimental results [1)]. Inclusion of the cusp effect in our calculations gives: $E_{0+}^{(+)}(\Sigma^V) = (2.22 \pm 0.70) \times 10^{-3}\ \mu^{-1}$.

$$E_{0+}^{(+)}(\Sigma^{v}) = (1.40 \pm 0.70) \times 10^{-3}\ \mu^{-1} \quad ,$$

$$E_{0+}^{(0)}(\Sigma^{s}) = (0.40 \pm 0.40) \times 10^{-3}\ \mu^{-1} \quad , \tag{4}$$

where $E_{0+}(\Sigma^{v,s})$ are the contributions from the symmetry breaking term to the dipole amplitudes $E_{0+}^{(+,0)}$.

In the nucleon Breit frame in which the produced pion is at rest, the additional input to the photoproduction amplitude can be represented as [6]

$$T_{\lambda}^{(\alpha)}(\text{B.th.}) = \frac{i}{\mu f_{\pi}} < N(\underset{\sim}{p}) | \left[ \dot{Q}_{\alpha},\ V_{\lambda}^{e.m.}(0) \right] | N(-\underset{\sim}{p}) > \quad , \tag{5}$$

where

$$\dot{Q}_{\alpha} = \int \partial_{\nu}\ A_{\alpha}^{\nu}(x) d\underset{\sim}{x} \quad ,$$

and $f_{\pi}$ is the pion decay constant.

Writing the currents as the bilinear quark densities and taking the symmetry breaking term proportional to the quark mass, the commutator in Eq.(5) is evaluated in terms of the tensor currents defined by *)

$$J_{\mu\nu}^{(\beta)} = \bar{\psi}\sigma_{\mu\nu} \frac{\lambda_{\beta}}{2} \psi \ , \quad \sigma_{\mu\nu} = \frac{i}{2}\left[ \gamma_{\mu},\ \gamma_{\nu} \right] \ ; \ \beta = 0, \ldots, 8 \quad . \tag{6}$$

Then, introducing the tensor current form factors $G_i(t)$, the nucleon matrix elements of the commutator are obtained. More explicitly, the contribution from the symmetry breaking term to the transverse amplitude $T_1$ at the Breit threshold, if we ignore the t-dependence of the σ-like terms, can be written as

$$T_1^{(+,0)} \propto - \frac{2\hat{m}}{\mu f_{\pi}}\ G_1^{(v,s)} \quad . \tag{7}$$

The $SU(6)_W$ in the static limit leads to the following estimates of the tensor current form factors:

$$G_1^{(v)} \approx -\frac{1}{2} \ , \qquad G_1^{(s)} \approx -\frac{5}{18} \quad . \tag{8}$$

---

* The commutator in Eq.(5) is evaluated by using the free-quark algebra. However, according to QCD, such calculations are not always free from anomalous dimensions at the loop level [19].



Taking into account the appropriate normalization factor and using $\hat{m} = (7 \pm 2)$ MeV [8] as input, we obtain *)

$$E_{0+}^{(+)}(\Sigma^{v}) = (1.60 \pm 0.46) \times 10^{-3}\ \mu^{-1} \quad ,$$

$$E_{0+}^{(0)}(\Sigma^{s}) = (0.88 \pm 0.25) \times 10^{-3}\ \mu^{-1} \quad . \tag{4a}$$

Eqs.(4) and (4a) show that the agreement between the theoretical expectations and the empirical determinations of the symmetry breaking terms is quite reasonable, taking into account that the form factors $G_1^{(v,s)}$ have been estimated, using ultimately the SU(6) which is known to be only an approximate symmetry of hadrons.

We conclude that a symmetry breaking interaction term like the σ-term in pion-nucleon scattering is necessary as well as adequate to resolve the discrepancy between the LET predictions and the recent experimental results on the neutral pion photoproduction off the proton **).

## ACKNOWLEDGMENTS

The authors would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where this work has been done. They are grateful to Professors G. Furlan, N. Paver, C.A. Dominguez and Riazuddin for some useful discussions and valuable comments.

---

* The value of the amplitude $T_1$ at the Breit threshold is not quite the same as that of the conventional amplitude at the threshold in the c.m. frame. However, the difference is ignorable, at least, for the calculation of the chiral symmetry breaking term. This point has been discussed in detail in Ref.6.

** The experiment has also been carried out in Mainz by Breitbach et al.[20]. The estimated value of $E_{0+}(p\pi^0)$ at threshold, based on the preliminary analysis, is compatible with the Saclay results [1].

## REFERENCES


1) E. Mazzucato et al., Phys. Rev. Lett. 57, 3144 (1986).

2) S. Fubini, G. Furlan and C. Rossetti, Nuovo Cimento 40, 1171 (1965); P. DeBaenst, Nucl. Phys. B24, 633 (1970); N. Paver, Electromagnetic Interactions of Hadrons, Eds. A. Donnachie and G. Shaw (Plenum Press, New York, 1978) Vol.II, p.353; A. Donnachie and G. Shaw, Electromagnetic Interactions of Hadrons, Eds. A. Donnachie and G. Shaw (Plenum Press, New York, 1978) Vol.I, p.143.

3) M.I. Adamovitch, Proc. P.N. Lebedev Physics Institute 71, 119 (1976).

4) R. Davidson and N.C. Mukhopadhyay, Phys. Rev. Lett. 60, 748 (1988). Please see also, E. Mazzucato et. al., Phys. Rev. Lett. 60, 749 (1988).

5) S. Weinberg, Phys. Rev. 166, 1568 (1968).

6) G. Furlan, N. Paver and C. Verzegnassi, Nuovo Cimento 20A, 295 (1974); 62A, 519 (1969); 70A, 247 (1970).

7) J.T. MacMullen and M.D. Scadron, Phys. Rev. D20, 1069, 1081 (1979).

8) C.A. Dominguez and E.D. Rafael, Ann. Phys. (N.Y.) 174, 372 (1987); J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982).

9) R.D. Peccei, Phys. Rev. 181, 1902 (1969).

10) B. Etemadi, A.J. Phares and L.M. Nath, ICTP, Trieste, Report No. IC/74/135; L.M. Nath and B.K. Bhattacharjee, Z. Phys. C5. 9 (1980).

11) M.G. Olsson and E.T. Osypowski, Nucl. Phys. B87, 399 (1975); Phys. Rev. D17, 174 (1978); M.G. Olsson, E.T. Osypowski and E.H. Monsay, Phys. Rev. D17, 2938 (1978).

12) G.F. Chew, M.L. Goldberger, F.E. Low and Y. Nambu, Phys. Rev. 106, 1345 (1957).

13) B.J. Read, Nucl. Phys. B52, 565 (1973).

14) L.M. Nath, B. Etemadi and J.D. Kimel, Phys. Rev. D3, 2153 (1971).

15) M.G. Olsson, L. Turner and E.T. Osypowski, Phys. Rev. D7, 3444 (1973); M.G. Olsson and E.T. Osypowski, Nucl. Phys. B101, 136 (1975); L.M. Nath and A.Q. Sarkar, Phys. Lett. 52B, 213 (1974); Q. Haider, L.M. Nath and A.Q. Sarkar, Phys. Rev. D15, 766 (1977).





16) O. Dumbrajs et al., Nucl. Phys. B216, 277 (1983).

17) L. Capraro et al., Nucl. Phys. B288, 659 (1987).

18) J.M. Laget, Phys. Rep. 69, 1 (1981); G. Fäldt, Nucl. Phys. A333. 357 (1980).

19) S.S. Shei and H.S. Tsao, Phys. Rev. D15, 3049 (1977).

20) G. Breitbach et al., Report of the Institute for Kernphysik, Mainz, May 1988, p.104.

Table 1

| | Contributions from $\Delta(1232)$ | | | |
|---|---|---|---|---|
| | $Z = Y = \frac{1}{2}$ | $Z = \frac{1}{2}$, $Y = 0$ | $Z = Y = -\frac{1}{4}$ | $Z = Y = -\frac{1}{2}$ |
| $E_{0+}^{(+)}$ | 0.34 | 0.10 | 0.04 | -0.10 |
| $E_{0+}^{(-)}$ | -1.09 | -0.02 | 0.72 | 0.90 |

Units are $10^{-3}\ \mu^{-1}$ .

Table 2

| | Contributions from | | | Total | Exp [1,4] |
|---|---|---|---|---|---|
| | Nucleon | $\Delta(1232)$ | Vector Mesons | | |
| $E_{0+}(\pi^{+})$ | 27.60 | -0.03 | 0.08 | 27.65 | 28.6 ± 0.14 |
| $E_{0+}(\pi^{-})$ | -31.80 | 0.03 | 0.08 | -31.69 | -31.5 ± 1.0 |
| $E_{0+}(p\pi^{0})$ | - 2.60 | 0.10 | 0.20 | - 2.30 | - 0.5 ± 0.3 |

The $\Delta(1232)$ contributions shown in Table 2 correspond to the theoretically preferred values of Z and Y, i.e. $Z = \frac{1}{2}$ , $Y = 0$.
Units are $10^{-3}\ \mu^{-1}$ .