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FADING OF CLASSICAL OSCILLATIONS IN QUANTUM DECAY OF FALSE VACUUM AND THE INVISIBLE AXION CONSTANT

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It is shown that the ultrarelativistic growth of bubbles of new phase in the course of false vacuum decay gives rise to effective fading of residual oscillations of fields around the stable values in the new phase. The damping rate corresponds to constant total energy of the oscillations inside the bubble. It is argued that as a possible consequence of this behavior under certain assumptions about phase transition in QCD the amplitude of the coherent invisible axion wave can be at present much less than it is usually estimated. As a result the upper bound for the axion constant can be relaxed from $f_a \lesssim 10^{12}$ GeV up to $f_a \lesssim 3\cdot10^{16}$ GeV.

Fig. -, ref. - 9

In the course of its expansion and cooling the universe underwent a succession of phase transitions in which non-trivial vacuum structures on different scales were formed, i.e. the confinement phase transition on the scale of the strong interaction Λ_{QCD}, formation of the electroweak symmetry breaking condensate on the Fermi scale and, possibly, analogous transitions at higher temperature induced by new hypothetical interactions. In the process of such a transition fields are approaching their new equilibrium position by oscillations which then fade out due to various mechanisms. Consequences of these oscillations at least in some of the past phase transitions can in principle be observable at present. For instance, in the so-called "new inflationary scenario" /1/ damping of the scalar field oscillations by particle creation was considered as a mechanism for reheating /1/ and as a possible sourse of baryon asymmetry of the universe /2/. Another interesting theoretical subject of this kind is the bound /3/ on the constant f_a of invisible axion:

$$f_a \lesssim 10^{12}\ \mathrm{GeV} \qquad (1)$$

which is obtained from considering energy density associated with coherent oscillations of the axion field induced in the QCD confinement transition.

The main goal of this paper is to point out one more mechanism of fading of the residual oscillations, which somehow was ignored so far. This mechanism operative at least in the first order phase transitions driven by quantum rather than thermal fluctuations is due to ultrarelativistic growth of bubbles of the new phase. Under the assumption that the confinement transition in QCD is of this type (and thus goes in the early universe with sufficient supercooling) the bound (1) can be substantially relaxed. As a result the present energy density of the coherent axion wave would not exceed the critical one under a much looser than (1) condition

$$f_a \lesssim 3\cdot 10^{16}\ \mathrm{GeV} \tag{2}$$

We start with reminding the reader few points of the theory of false vacuum decay. The initial state before the decay is the metastable vacuum in which mean values of the fields ϕ_i are ϕ_+ and correspond to a local rather than global minimum of the potential (or of the effective potential if some of the fields ϕ_i are composite, as in case of the phase transition in QCD). Such a state decays /4/ through nucleation due to quantum fluctuations of bubbles inside which fields are near the lower minimum of the potential and subsequent growth of thise bubbles. To start expanding the bubble has to have a finite critical size at which the energy spent on creation of the bubble wall is compensated by the gain in the volume energy.

One can find the profile of the fields in the critical bubble simultaneously with the quasiclassical action S_0

which enters in the exponential factor exp($-S_0$) in the probability of the critical bubble nucleation. To this end one should consider /5/ the Euclidean version of the theory, i.e. with the action

$$S=\int d^4x\left(\frac{1}{2}\sum_i(\partial_\mu\phi_i)^2+V(\phi_i)\right), \qquad (3)$$

and find a solution $\bar\phi_i(x_\mu)$ (called bounce /5/) to the equations of motion which satisfy the following two conditions: i) when $|x_\mu|\to\infty$ the fields $\bar\phi_i(x_\mu)$ tend to their values in the false vacuum ϕ_+ , ii) the second variation of the action (3) around the configuration $\bar\phi(x_\mu)$ should have exactly one negative mode. Then the WKB exponent is given by $S_0=S[\bar\phi]$. (Effects of gravity are neglected for a while).

The profile of the fields in the Minkowski space in the moment of bubble nucleation and during its subsequent classical growth is determined from the Euclidean solution $\bar\phi(x)$ by analitycal continuation.

For what follows the most essential property of the solution $\bar\phi(x)$ is its O(4) symmetry /5/. In the absense of gravity this symmetry property is proven in the paper /6/, while with account of gravity no general mathematical proof of this behavior is given so far. However, physical arguments for the O(4) symmetry of bounce are quite transparent. Namely, when continued to the Minkowski space the O(4) symmetry transforms into O(3,1), which implies that the expanding critical bubble looks the same for any moving observer (moreover its center is at rest in any Minkowski frame). If this were not true one should have to sum nucleation probability over velocities of the bubble when calculating the false vacuum decay rate[*], i.e. to integrate over the non-compact group O(3,1) which would give physically senseless infinite probability. Clearly this argument is applicable for the case of O(3,1) symmetric gravitational background of the false vacuum, in particular for tunneling from the De Sitter space.

The symmetry O(4) implies that the fields $\bar{\phi}_i$ are functions of the variable $x_\mu^2 = r^2 + \tau^2$ (where $r^2 = |\vec{x}|^2$) -. Therefore in Minkowski space $\bar{\phi}_i(x_\mu^2)$ become $\bar{\phi}_i(r^2 - t^2)$, and the Euclidean field configuration identically maps on the exterior of the light cone, i.e. on r and t such that $r > t$. (The coordinate system origin is obviously placed in the center of the bounce). Notice also that the $\tau = 0$ spatial cross section of the bounce gives the initial configuration of the critical bubble in Minkowski space (at $t = 0$), and that the coordinate s system origin in the Euclidean space ($r^2 + \tau^2 = 0$) maps on the whole light cone ($r^2 - t^2 = 0$). Thus the value of the fields are constant on the light cone and provide the boundary conditions for evolution of the fields inside the light cone which contains the expanding region of the stable phase with the residual oscillations around new vacuum mean values. It can also be mentioned that we do not use here the so-called thin wall approximation /4,5/, i.e. the difference between $\bar{\phi}_i(0)$ and the new equilibrium values ϕ_- is not assumed to be small so that apriori the oscillations could be large.

Since the boundary conditions are defined on the Lorentz

*) It is this integration which was erroneously suggested in ref./4/. To avoid infinite result an artificial cutoff was introduced.

invariant surface the solution inside the light cone depends only on the invariant $\xi=(t^2-r^2)^{1/2}$. The equations of motion in ξ look as follows

$$\ddot{\phi}_i + \frac{3}{\xi}\dot{\phi}_i + \frac{\partial V}{\partial \phi_i} = 0 , \qquad (4)$$

where the dot denotes derivative over ξ. The term $3\dot{\phi}/\xi$ clearly implies energy dissipation. In the large ξ asymptotics when one can use linear approximation for $\partial V/\partial \phi_i$ near the equilibrium position ϕ_-, the amplitudes a_i of deviations of the fields from ϕ_- are given by the Bessel function

$$a_i \propto J_1(m\xi)/\xi , \qquad (5)$$

where m is the mass matrix:

$$m_{ij} = \partial^2 V/\partial\phi_i \partial\phi_j .$$

This means, that average over the period amplitudes of oscillations fade as $\xi^{-3/2}$. In other words all the energy excess $\propto (V(\phi_+) - V(\phi_-))\cdot t^3$ is spent on acceleration of the bubble walls, i.e. this energy flows near the light cone $r=t$, while the total energy of the residual oscillations inside the cone $r<(1-\varepsilon)t$ with arbitrary small positive ε tends to a constant value $\propto a^2(t)t^3$ when t goes up to infinity. Notice that the volume occupied by the stable phase grows as t^3 and apriori one would expect that energy of oscillations constitutes some finite fraction of the latent heat, i.e. that it also grows as t^3. We see however that his fraction in fact goes to zero.

We proceed now to a discussion of gravity effects on the residual oscillations, in which we restrict ourselves with the realistic case when the transition occurs from the De Sitter space to the same space with a smaller cosmological constant or to the Minkowdki one, i.e. we assume that $V(\phi_+) > 0$ and $V(\phi_-) \geq 0$. The equations governing critical bubble nucleation were obtained in ref./7/. They read as follows.

In the Euclidean space:

O(4) invariant metrics -

$$ds^2 = d\xi^2 + \rho^2(\xi)\, d\Omega_S^2 \quad , \tag{6}$$

where $d\Omega_S^2$ is the squared length element on unit sphere S_3 ; equation for the scale factor -

$$\dot{\rho}^2 = 1 + \frac{8\pi G}{3}\rho^2\left(\frac{1}{2}\sum_i \dot{\phi}_i^2 - V(\phi_i)\right), \tag{7}$$

where G is the Newton's gravity constant; equations of motion of the fields ϕ_i -

$$\ddot{\phi}_i + \frac{3\dot{\rho}}{\rho}\dot{\phi} - \frac{\partial V}{\partial \phi_i} = 0 . \tag{8}$$

In the Minkowdki space:

O(3,1) invariant metrics -

$$ds^2 = d\xi^2 - \rho^2(\xi)\, d\Omega_H^2 \quad , \tag{9}$$

where $d\Omega_H^2$ is the metrics of unit hyperboloid;

$$\dot{\rho}^2 = 1 + \frac{8\pi G}{3}\rho^2\left(\frac{1}{2}\sum_i \dot{\phi}_i^2 + V(\phi_i)\right), \tag{10}$$

$$\ddot{\phi}_i + \frac{3\dot{\rho}}{\rho}\dot{\phi}_i + \frac{\partial V}{\partial \phi_i} = 0 . \tag{11}$$

The light cone on which the Euclidean solution matches the Minkowski one is defined by $\rho(\xi)=0$. Obviously, by shifting ξ one can make this cone to correspond to $\xi=0$. Notice that the scale factor ρ plays the role which ξ plays in the gravity - less case. Indeed if $C=0$ in eq.(7) or (10) one finds $\rho=\xi$.

Equation (11) formally coinsides with that for a spatially uniform scalar field in expanding universe, where the friction term is $3H\dot{\phi}$ and H is the Hubble constant. However, this correspondense is not literal since ξ has the meaning of time only locally at $r=0$, and the fields ϕ_i are not uniform. The distinction from the uniform case is that the friction is induced not only by the Hubble expansion but also by the growth of the bubble.

One can readily see from eq.(11) that in the large ξ asymptotics when linear approximation for $\partial V/\partial\phi_i$ is justified, the average energy of the residual oscillations in the comoving volume tends to a constant, i.e.

$$\rho^3\left(\frac{1}{2}\sum_i \dot{\phi}_i^2 + V(\phi_i) - V(\phi_-)\right) \underset{\rho\to\infty}{\longrightarrow} const. \qquad (12)$$

It should be underlined that this equation refers to the energy of only the oscillations, i.e. the vacuum energy density $V(\phi_-)$ is subtracted, since the total energy associated with a possible non-zero cosmological constant in the final state, naturally, grows as ρ^3.

Thus, one arrives at the conclusion that the quantum decay of metastable vacuum is accompanied by classical creation of only finite number of particles. This conclusion complements the result /8/ about strong suppression of quantum creation of particles in the course of false vacuum decay. This behavior is natural for physical reasons. Indeed, the exponential factor exp($-S_0$) in the rate of the decay is determined by ballance of the gain in the volume energy and the loss in the surface energy of the bubble. Copious particle creation would have reduced energy gain and thus suppressed the decay rate.

Naturally the relevance of the conclusion about classical fading of the residual oscillations depends on details of specific transition, since it may be that the fading is slow on the relevant time scale. For instance, creation by the oscillating fields of secondary particles can be more efficient, or if the decay rate is sufficiently large the bubbles are copiously nucleating and start coalesce before the oscillations fade out. In what follows two examples are considered in one of which the mechanism discussed here seems to be irrelevant while in the other one, i.e. in the case of the axion coherent wave, this mechanism can substantially affect estimates of the axion constant.

The term with friction in eqs.(4) and (11) is efficient if $\dot{\phi}$ is large at small ξ, i.e. if the value of $\bar{\phi}_i(0)$ emerging from the Euclidean solution is such that the derivative $\partial V/\partial\phi_i \,\big|_{\bar{\phi}_i(0)}$ is sufficiently large, so that already at small ξ the fields approach the equilibrium position $\phi \approx \phi_-$ in which asymptotic relation (5) or (12) is justified. An example when this is not true is the tunneling in approximately Coleman - Weinberg potential:

$$V(\phi)=\frac{1}{2}\mu^2\phi^2+\frac{\lambda}{4}\phi^4\left(\ln\frac{\phi^2}{\eta^2}-\frac{1}{2}\right) \tag{13}$$

with $\mu^2 \ll \lambda\eta^2$. (Tunneling in such potential was considered /1,2/ in the "new inflationary scenario".) In this case $\bar{\phi}(0)\sim \mu^2/\lambda \ln(\lambda\eta^2/\mu^2)$ and the field $\phi(\xi)$ slowly rolls down the flat part of the potential before it reaches the oscillation region $\phi \simeq \eta$. As a result the oscillations start at large ξ at which friction is already small. Of course in this case the asymptotic behavior (12) also sets in, which corresponds to finite, but in this case very large number of created particles. However with the parameters considered in refs./1/ and /2/ (effectively these correspond to $\mu \sim \sim 10^{-8}\lambda^{1/2}\eta$) oscillation damping due to production of secondary particles /2/ is far more effective than due to the bubble growth.

Let us proceed to discussion of the coherent axion wave. The standard scenario /3/ is the following (see also in ref. /9/). Before the confinement phase transition in QCD the axion field $a(x)$ is massless and its spatial average value a_0 is arbitrary, and is naturally assumed to be of the order of the axion constant f_a (recall that the Hamiltonian is periodic in a with the period $2\pi f_a$). After the phase transition non-perturbative QCD generate temperature-dependent axion mass which grows in the process of cooling from zero at $T_c \sim \Lambda_{QCD}$ to $\mu \sim m_\pi f_\pi/f_a$ at $T\to 0$. It is assumed that the phase transition occurs uniformly in the space at $T=T_c$, and the relaxation of the average value $a(t)$ to the position corresponding to minimum of the mass term (chosen as $a = 0$) is governed by the standard equation

$$\frac{d^2a}{dt^2} + 3H\frac{da}{dt} + \mu^2(t)\,a = 0\,, \tag{14}$$

where $H \sim T^2(t)/m_{Pl}$ is the Hubble constant ($m_{Pl} \sim G^{-1/2}$ $\sim 10^{19}$ GeV is the Planck mass). Once $\mu(t)$ becomes larger than $H(t)$ adiabatic regime of oscillations starts off (before that the field does not go far away from a_0 /3/). In the adiabatic regime one has

$$\mu(t)\,a^2(t)\,R^3(t) = const\,, \tag{15}$$

where $R(t)$ is the scale factor, $R(t) \propto T(t)^{-1}$. As found in refs./3/, this regime in fact sets in at a temperature close to T_c , so that at that moment $\mu(t)$ is given by

$$\mu_0 \sim H \sim T_c^2/m_{Pl} \sim \Lambda_{QCD}^2/m_{Pl}\,. \tag{16}$$

Thus, from eq.(15) one can estimate the present energy density associated with oscillations of the axion field:

$$\rho_a = \mu^2 a^2 \approx \mu\,\mu_0\, a_0 \left(\frac{T}{T_c}\right)^3 \sim \frac{m_T f_T\, T^3}{f_a\, m_{Pl}\, \Lambda_{QCD}}\, a_0^2\,, \tag{17}$$

where T is the present temperature: $T \approx 3$K. Therefore, assuming that $a_0 \sim f_a$ and requiring that the density ρ_a does not exceed the critical one: $\rho_{crit} \approx 2\cdot10^{-29}\,g\cdot cm^{-3}$ 10^{-46} GeV^4, one finds the bound (1) for f_a .

This standard estimate is considerably modified under the assumption that the confinement phase transition proceeds with a certain supercooling. If one assumes that the probability W of critical bubble nucleation per unit volume per unit time is such that

$$H \lesssim W^{1/4}, \tag{18}$$

where $H \sim \Lambda^2_{QCD}/m_{Pl}$ is the Hubble constant corresponding to a cold metastable deconfinement phase, then the Hubble expansion will not have enough time to drag growing bubbles apart and the transitinn will be completed after the time $t_o \sim W^{-1/4}$ by collision of bubbles. In this case the overcooling factor exp(Ht_o) can on one hand be not too large so that additional entropy will not dilute the baryon asymmetry, and on the other hand if the inequality (18) is close to equality, the overcooling will be sufficient to prevent temperature from approaching T_c after reheating. Therefore the axion mass can be taken as $\mu \sim m_\pi f_\pi/f_a \sim$ const both inside the bubbles and after their coalescense.

The condition $f_a \ll m_{Pl}$ implies then that $\mu \gg H$, and also under the assumed strength of inequality (18) $\mu \gg W^{1/4}$. According to eq.(5), the average axion amplitude inside a growing bubble behaves at $t \gg \mu^{-1}$ as

$$a^2(t) \sim a_o^2/(\mu t)^3. \tag{19}$$

When the bubbles coalesce i.e. at $t_o \sim W^{-1/4}$ the resulting average amplitude arises from superposition of axion waves from different bubbles, therefore this resulting amplitude can be estimated from eq.(19) with $t \approx t_o$:

$$a_1^2 \sim a_o^2 W^{3/4}/\mu^3. \tag{20}$$

Since in the discussed scenario $\mu(t) \approx$ const, the present energy density of the axion field can be estimated as follows

$$\rho_a = \mu^2 a^2 \sim \frac{a_0^2 W^{3/4}}{\mu}\left(\frac{T}{T_1}\right)^3 \sim \frac{a_0^2 f_a W^{3/4}}{m_\pi f_\pi}\left(\frac{T}{T_1}\right)^3 \qquad (21)$$

With $a_0 \sim f_a$, $W^{1/4} \sim H \sim \Lambda_{QCD}^2/m_{Pl}$ and $T_1 < T_c \sim \Lambda_{QCD}$ (notice, that $T_1 \sim T_c \exp(-H/W^{1/4}) < T_c$) one finds

$$\rho_a \gtrsim \left(\frac{f_a}{m_{Pl}}\right)^3 \frac{\Lambda_{QCD}^3}{m_\pi f_\pi} T^3 .$$

Therefore the condition $\rho_a < \rho_{crit}$ is satisfied if the bound (2) on f_a is fulfilled.

Of course the scenario discussed here involves special assumptions about the confinement phase transition in QCD. In particular, it assumes that a metastable cold deconfinement phase is possible, which to my knowledge, is by no means ruled out. On the other hand, this scenario demonstrates that in some cases the fading of residual oscillations can be quite essential.

Thus, to summarize, the main conclusion of this paper is that the ultrarelativistic growth of bubbles of stable phase in a phase transition by itself gives rise to fading of residual oscillations of the fields inside the bubble. The damping rate corresponds to finite energy of oscillations inside an unlimitedly growing bubble. This fading can be essential for considering consequences of at least some of the phase transitions through which our universe has passed

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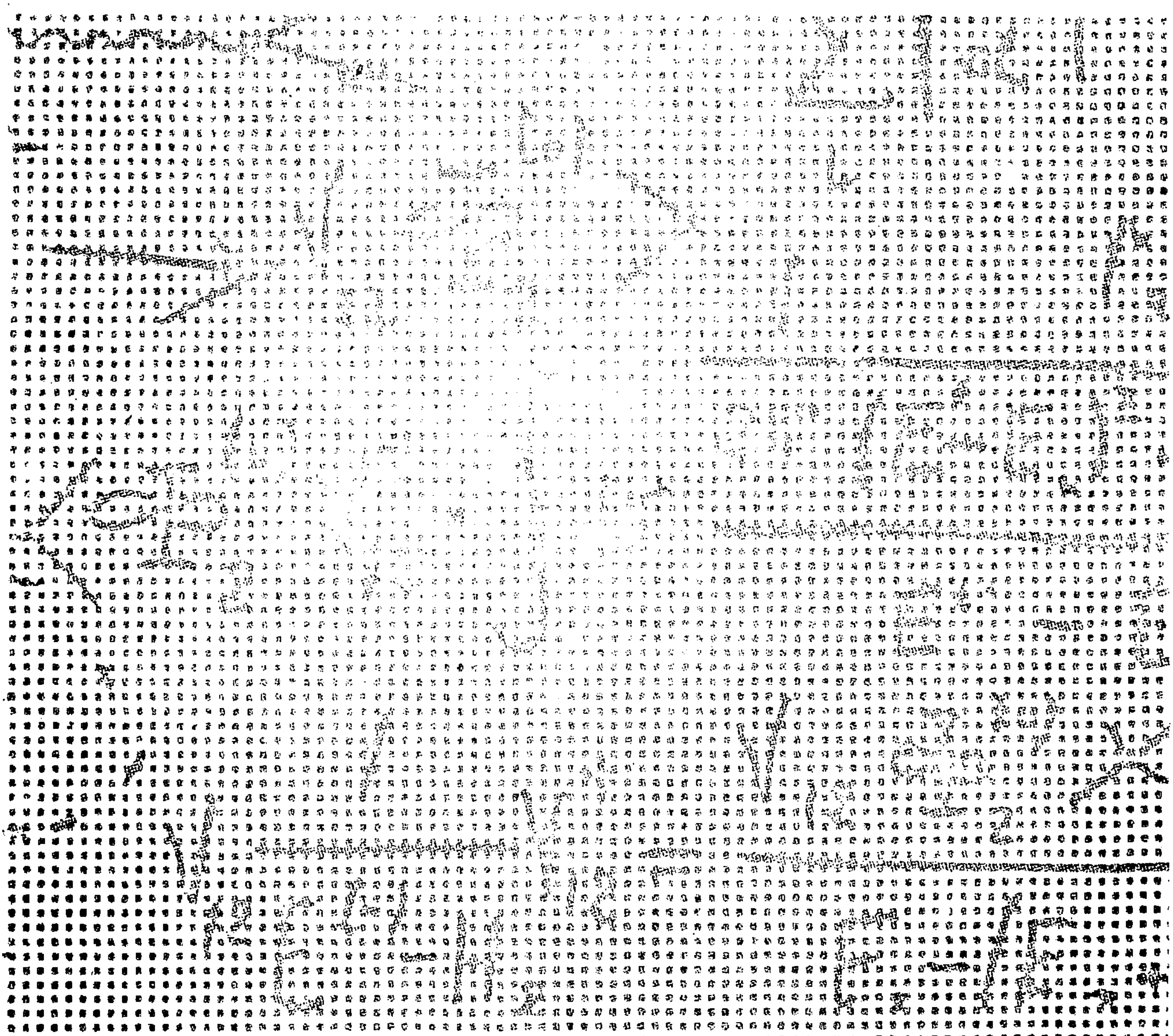

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