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# EFFECTS OF LOSSES ON ION VELOCITY DISTRIBUTIONS IN THE PRESENCE OF ICRH

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## EFFECTS OF LOSSES ON ION VELOCITY DISTRIBUTIONS

IN THE PRESENCE OF ICRH

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## Abstract

Effects of particle and energy losses on the velocity distribution function of minority ions heated by ICRH are considered. An approximate model, including such losses, is derived for the stationary pitch angle averaged distribution function. Energy losses are shown to be important when the energy confinement time is of the same order or smaller than the slowing down time for ion-electron collisions. The criterion for when particle losses are important is shown to be more complicated. It does not only depend on the slowing down time but also, among other things, on the applied RF-power. Effects of a velocity dependent particle confinement time are also studied.

## 1. Introduction

The evolution of the ion velocity distribution function, f, in the presence of ICRH driven, quasi-linear, velocity space diffusion is determined by the Fokker-Planck equation [1-10]

$$\frac{\partial f}{\partial f} = C(f) + Q(f) \tag{1}$$

where C(f) is the collision operator and Q(f) is the quasi-linear ICRHdiffusion operator.

Extensive computer studies have been made of the Fokker-Planck equation describing ICRH-driven, quasi-linear velocity space diffusion [1-5]. However, the numerical codes needed to obtain a full 2D solution to the Fokker-Planck equation are cumbersome and demands large computer resources. Approximate models that provide a short computational procedure and contributes to the physical understanding can therefore be of particular interest. Analytical and/or semi-analytical [6-10] investigations have resulted in approximate solutions for the steady-state velocity distribution function of the RF-heated ions.

On the other hand, most analytical investigations of eq. (1), e.g. Refs [6-10], have neglected particle and energy losses, i.e. the particle  $(\tau_p)$  and energy  $(\tau_E)$  confinement times of the heated ion species are assumed to be infinite. Nevertheless, steady-state solutions of eq. (1) are still consistent since the necessary energy sink is provided by collisions with

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the background plasma particles. The RF-power collisionally transferred to the background plasma particles is then assumed to be balanced by the losses of these background particles.

When investigating the effect of finite particle and energy confinement times of the heated ion species eq. (1) should be augmentet with loss terms representing particle and energy transport. In a previous numerical investigation,  $[\hat{1}]$ , it has been shown that the inclusion of such loss terms can have a significant effect on the velocity distribution.

Qualitatively one expects the form of the velocity distribution function to be significantly affected by the loss terms when  $\tau_p$  and  $\tau_E$  are of the same order or smaller than the slowing down time,  $t_s$ . The main effect of the inclusion of particle and energy loss terms should be to suppress the high energy tails created by ICRH.

The purpose of the present paper is to study the effect of finite particle and energy confinement times on the pitch angle averaged distribution F(v), where

$$F(v) = \frac{1}{2} \int_{-1}^{1} f(v,\mu)d\mu$$

The pitch angle averaged distribution, F(v), is an important quantity to evaluate, since many physically significant velocity space averages, like collisional power transfer to background plasma particles, fusion reactivity etc, only depends on F(v).

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#### 2. Fokker-Planck equation

The Fokker-Planck equation, including loss terms, for the heated ions can be written as

$$\frac{\partial f}{\partial f} = C(f) + Q(f) - L + S$$
(3)

where L represents losses and S is a source term.

The analysis will be restricted to minority heating, i.e., heating of a minority ion species at the fundamental ion cyclotron frequency. Self collisions between ions of the heated species can then be neglected compared to collisions with the background species. Furthermore, the back-ground plasma is assumed to be almost Maxwellian with a given temperature, which implies the presence of an energy sink, as discussed in the introduction. The collision operator for a test particle distribution thermalizing against a Maxwellian background can thus be used.

With the notation used by Stix, [6], the collision operator can be written as

$$C(f) = \frac{1}{v^2} \frac{\partial}{\partial v} \left\{ -\alpha(v)v^2 f + \frac{1}{2} \frac{\partial}{\partial v} \left[ \beta(v)v^2 f \right] \right\} + \frac{\gamma(v)}{4v^2} \frac{\partial}{\partial \mu} \left[ (1-\mu^2) \frac{\partial f}{\partial \mu} \right]$$
(4)

where v is the velocity and  $\mu = v_{\parallel}/v$  is the cosine of the pitch angle. The collision coefficients  $\alpha$ ,  $\beta$  and  $\gamma$  describe dynamical friction on the background species, energy diffusion and pitch angle scattering respectively, [6].

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The RF-diffusion operator will be taken in the following form, cf [6,8]

$$Q(f) = K \frac{1}{v_{\perp}} \frac{\partial}{\partial v_{\perp}} \left[ v_{\perp} H(\frac{K_{\perp} v_{\perp}}{\omega_{ci}}) \frac{\partial f}{\partial v_{\perp}} \right] =$$

$$= K \frac{1}{v^{2}} \left\{ (1 - \mu^{2}) \frac{\partial}{\partial v} \left[ v H(\bar{v}_{\perp}) \frac{\partial}{\partial v} (vf) \right] + \frac{\partial}{\partial \mu} \left[ \mu (1 - \mu^{2}) H(\bar{v}_{\perp}) \frac{\partial}{\partial \mu} (\mu f) \right] \right]$$

$$= \frac{\partial}{\partial \mu} \left[ \mu (1 - \mu^{2}) H(\bar{v}_{\perp}) \frac{\partial}{\partial v} (vf) \right]$$

$$= \frac{\partial}{\partial v} \left[ (1 - \mu^{2}) v H(\bar{v}_{\perp}) \frac{\partial}{\partial \mu} (\mu f) \right]$$
(5)

where

$$H(\vec{v}_{\perp}) = J_{o}^{2}(\vec{v}_{\perp}) + \Lambda J_{2}^{2}(\vec{v}_{\perp})$$
(6)

Fugther notation is as follows:  $\bar{\mathbf{v}}_{1} = K_{1}\mathbf{v}_{1}/\omega_{ci} = K_{1}\mathbf{v}_{1}/\omega_{ci}$ ,  $\Lambda = |\mathbf{E}_{-}|^{2}/|\mathbf{E}_{+}|^{2}$  where  $\mathbf{E}_{+}$  and  $\mathbf{E}_{-}$  are the left and right hand polarized components of the RF-wave and K is a constant proportional to  $|\mathbf{E}_{+}|^{2}$ . For further notations see Refs [6,7]. The effects of a finite aspect ratio, in the form of trapped particles, are not included in the analysis. This is a reasonable approximation if the RF-power is deposited near the centre of the plasma, where the inverse aspect ratio  $\varepsilon$  is small. Furthermore, in ref. [5] the influence of a finite aspect ratio has been shown to be small on weighted velocity space averages of the distribution function. The RF-operator eq.

(5) has been further simplified by assuming that the effects due to finite  $E_{\parallel}$  and  $K_{\parallel}$  can be neglected. It is justified to neglect  $E_{\parallel}$  since typically  $|E_{\parallel}| << |E_{\perp}|$ ,  $|E_{\perp}|$ . Effects due to finite  $K_{\parallel}$  can be neglected if  $\omega/K_{\parallel} << v_{\parallel}$ , cf [6]. This condition is easily satisfied, in the velocity range of interest, for most ICRH scenarios, [7].

In order to account for radial particle and energy transport, the following loss terms have been used in several numerical studies of the Fokker-Planck equation

$$L = -\frac{f}{\tau_{p}} + \frac{1}{2v^{2}} \frac{\partial}{\partial v} \left[ \frac{1}{\tau_{E}} v^{3} f \right]$$
(7)

where  $\tau_p$  and  $\tau_E$  are the particle and energy confinement times for the heated ion species respectively. The first term in eq. (7) represents the loss of ions by diffusion and charge exchange. The second term, which is particle conserving, represents the loss of energy by thermal conduction. In general,  $\tau_p$  and  $\tau_E$  will be velocity dependent, but they are often approximated as constants, cf [1,2].

Since the first term in eq. (7) represents particle losses a source term is needed in order to achieve a steady-state distribution. The source term is taken in the following form

$$S = \frac{S_{o}}{2\pi^{3/2} v_{s}^{3}} e^{-\left(\frac{v}{v_{s}}\right)^{2}} e^{-\left(\frac{v}{v_{s}}\right)^{2}}$$
(8)

where S is the number of ions created per unit time, and  $v_{a}$  is the

"thermal velocity" of the created ions. This source term should be a reasonable approximation for ions created by ionization of a background neutral gas.

The particle source term and the loss term must balance each other in steady-state, i.e.,

$$\int_{0}^{\infty} \frac{F(v)}{\tau_{p}} v^{2} dv = \frac{S_{o}}{4\pi}$$
(9)

Thus, if  $\tau$  is approximated as constant, S and  $\tau$  are related, in steadystate, by

$$\mathbf{n} = \mathbf{S}_{\mathbf{T}} \tag{10}$$

where n is the density of the heated species.

## 3. Approximate models for F(v)

We write the distribution function, f, as

$$f(v,\mu,t) = F(v,t) [1+g(v,\mu,t)]$$
(11)

where F(v,t) is the pitch angle averaged distribution and  $g(v,\mu,t)$  accounts for the anisotropy. The function  $g(v,\mu,t)$  satisfies the condition

$$\int_{-1}^{1} g(v,\mu,t) d\mu = 0$$
(12)

Inserting eq. (11) into the Fokker-Planck equation (3), and averaging the resulting equation over the cosine of the pitch angle,  $\mu$ , one finds

$$\frac{\partial F}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left[ AF + B \frac{\partial F}{\partial v} \right] - \frac{F}{\tau_p} + \frac{S_o}{2\pi^{3/2} \frac{v^3}{v_s^3}} \exp\left[-\left(\frac{v}{v_s}\right)^2\right]$$
(13)

where

$$A = -\alpha v^{2} + \frac{1}{2} \frac{d}{dv} (\beta v^{2}) + \frac{1}{\tau_{E}} v^{3} + K v R_{2}(v,t)$$
(14)

$$B = \frac{1}{2} \beta v^{2} + K v^{2} [G(v) + R_{1}(v,t)]$$
(15)

$$G(v) = \frac{1}{2} \int_{-1}^{1} (1-\mu^2) H(\bar{v}_{\perp}) d\mu$$
 (16)

$$R_{1}(v) = \frac{1}{2} \int_{-1}^{1} (1-\mu^{2})H(\bar{v}_{1})g(v,\mu,t)d\mu$$
(17)

$$R_{2}(v) = \frac{1}{2} \int_{-1}^{1} (1-\mu^{2})H(\overline{v}_{\perp}) \left[v \frac{\partial g(v,\mu,t)}{\partial v} - \mu \frac{\partial g(v,\mu,t)}{\partial \mu}\right] d\mu$$
(18)

Furthermore, in steady state we obtain

$$\frac{\mathrm{dF}}{\mathrm{dv}} + \frac{\mathrm{A}}{\mathrm{B}} \mathbf{F} = \frac{\mathrm{S}}{4\pi\mathrm{B}} \left\{ \mathrm{erfc}\left(\frac{\mathrm{v}}{\mathrm{v}_{\mathrm{s}}}\right) + \frac{2}{\sqrt{\pi}} \frac{\mathrm{v}}{\mathrm{v}_{\mathrm{s}}} \exp\left[-\left(\frac{\mathrm{v}}{\mathrm{v}_{\mathrm{s}}}\right)^{2}\right] \right\} - \frac{1}{\mathrm{B}} \int_{\mathrm{v}}^{\infty} \frac{\mathrm{F}}{\mathrm{\tau}_{\mathrm{p}}} \mathrm{v}^{2} \mathrm{dv}$$
(19)

This equation can be split up into two coupled ordinary differential equations, viz.

$$\frac{dF}{dv} + \frac{A}{B}F = \frac{1}{4\pi B} \left\{ \ell(v) - S_0 \left[ erf\left(\frac{v}{v_s}\right) - \frac{2}{\sqrt{\pi}} \frac{v}{v_s} exp\left[-\left(\frac{v}{v_s}\right)^2\right] \right\}$$
(20)  
$$\frac{d\ell(v)}{dv} = \frac{1}{\tau_p}F 4\pi v^2 , \quad \ell(0) = 0 \quad \text{and} \quad \ell(v + \infty) + S_0$$

where l(v) is the number of particles lost per unit time with velocities  $\leq v$ .

If the particle confinement time can be approximated as constant we obtain

$$\frac{\mathrm{d}F}{\mathrm{d}v} + \frac{\mathrm{A}}{\mathrm{B}} \mathbf{F} = \frac{1}{4\pi\tau} \frac{1}{\mathrm{p}} \left\{ n(\mathbf{v}) - n_{\mathrm{tot}} \left[ \mathrm{erf}\left(\frac{\mathbf{v}}{\mathbf{v}}\right) - \frac{2}{\sqrt{\pi}} \frac{\mathbf{v}}{\mathbf{v}} \exp\left[-\left(\frac{\mathbf{v}}{\mathbf{v}}\right)^{2}\right] \right\} (21)$$

$$\frac{\mathrm{d}n(\mathbf{v})}{\mathrm{d}v} = \mathbf{F} 4\pi v^{2} , \quad n(0) = 0 \quad \text{and} \quad n(\mathbf{v} \neq \infty) \neq n_{\mathrm{tot}}$$

where n(v) is the density of particles with velocities  $\leq v$ .

In applications where the effect of a finite particle confinement time can be neglected one obtains the following formal solution

$$F(v) = F(0) \exp\left[-\int_{0}^{v} \frac{A}{B} dv\right]$$
(22)

The problem with equations (2), (21) and (22) is that we do not know the function  $g(v,\mu)$ . In principle,  $g(v,\mu)$  can be determined from the Fokker-

Planck equation, but it has been found very difficult to obtain an analytic solution which can be used to evaluate  $R_1(v)$  and  $R_2(v)$ . We are therefore forced to consider approximate models.

3.1 "Isotropic" approach

A widely used approximation is to neglect the influence of anisotropy by assuming  $g(v,\mu) \approx 0$ . This approximation is valid in the limit of weak anisotropy, i.e., it should be reasonable for velocities below  $v_{\gamma}$ , where  $v_{\gamma}$ is the characteristic velocity associated with the pitch angle scattering [6]. In Ref [8] this approximation has been shown to provide useful and accurate results for calculating most of the physically meaningful velocity space averages, including such high energy characteristics as power transfer to electrons and fusion reactivity. Furthermore, the losses considered in this work will tend to suppress the anisotropic high energy tails. Thus, the neglect of  $g(v,\mu)$  can be expected to be a reasonable approximation in the present context, in particular for calculating velocity space averages.

If we neglect  $g(v,\mu)$  then  $R_1(v) = 0$  and  $R_2(v) = 0$ . The system of first order differential equations (20) or (21) is then fairly easy to solve numerically, and provides a short computational procedure. The computations become particular simple if the solution eq. (22) is applicable (i.e.  $\tau_p \neq \infty$ ). Furthermore, the function G(v) can be calculated analytically to the required accuracy by using the following expansion.

 $\frac{1}{2} \int_{-1}^{1} (1-\mu^2) J_n^2 (\bar{v} \sqrt{1-\mu^2}) d\mu =$ 

$$= \sum_{k=0}^{\infty} \frac{(-1)^{k} (2n+2k)! (2k+2)!!}{2^{2(k+n)} [(n+k)!]^{2} (2n+k)!k! (2k+3)!!} - \frac{2^{2(k+n)}}{v}$$
(23)

3.2 Approximate solution in the high energy range

It is possible to derive an approximate solution of eq. (19) in the high energy range. This solution gives a hint about how the losses affect the tail of the distribution. In order to derive this solution we write the pitch angle averaged distribution as

$$F(v) = F_{\star} \exp\left[-\int_{0}^{v^{2}} h(v') dv'^{2}\right]$$
(24)

The function h(v) gives a measure of the local temperature, i.e. h(v) is equal to the slope of lnF(v) against  $v^2$ , and  $F_*$  is a constant.

In the high energy range one can now approximate the particle loss and source terms as follows (v >>  $v_{e}$ )

$$\int_{v}^{\infty} \frac{F}{\tau_{p}} v^{2} dv - \frac{S_{o}}{4\pi} \left\{ erfc\left(\frac{v}{v_{s}}\right) - \frac{2}{\sqrt{\pi}} \frac{v}{v_{s}} exp\left[-\left(\frac{v}{v_{s}}\right)^{2}\right] \right\} \approx$$

$$\approx \frac{v}{2h\tau_{p}} F + \int_{v}^{\infty} \frac{d}{dv'} \left(\frac{v'}{2h\tau_{p}}\right) F dv' + \frac{v}{2h\tau_{p}} F as v + \infty$$
(25)

The asymptotic expression is valid when

$$\left|\frac{\tau_{p}}{v^{2}}\frac{d}{dv}\left(\frac{v}{2h\tau_{p}}\right)\right| \ll 1$$
(26)

Substituting eq. (24) into eq. (19) and using the asymptotic expression eq. (25) one finds

$$h(v) = \frac{A}{4vB} \left[ 1 + \sqrt{1 + \frac{4v^2 B}{\tau_p A^2}} \right]$$
(27)

This expression will be analyzed in section 4. It is convenient for computational reasons to use eq. (27) for the evaluation of the solutions of eq. (20) or eq. (21) in the velocity range where eq. (26) is satisfied.

If the effects of finite  $\tau_{p}$  can be neglected we obtain (valid for all v)

$$h(v) = \frac{A}{2vB}$$
(28)

i.e., this is the same as eq. (22) as expected.

Although the isotropic model discussed in 3.2 provides useful and accurate results for most velocity space averages, it cannot be expected to give an accurate description of the anisotropic high energy tail. In the high energy tail, where pitch angle scattering is weak  $(v > v_{\gamma})$ , the distribution becomes strongly anisotropic even for moderate RF-powers, and  $g(v,\mu)$  can no longer be neglected. For  $v > v_{\gamma}$  the distribution function will be peaked around  $\mu = 0$ , since the ICRH primarily increases the perpendicular velocity of the heated ions. The function  $g(v,\mu)$  will then have the following asymptotic property, cf [9],

$$\frac{1}{2} \int_{-1}^{1} \mu^{2n} g(v,\mu) d\mu \neq -\frac{1}{2n+1}$$
(29)

Using this approximation  $R_1(v)$  and  $R_2(v)$  simplifies to

$$R_{1}(v) = H\left(\frac{k_{\perp}v}{\omega_{ci}}\right) - G(v)$$
(30)

$$R_{2}(v) = H\left(\frac{k_{\perp}v}{\omega_{ci}}\right)$$
(31)

We can summarize as follows: The isotropic approximation, as discussed in sec. 3.2, is particularly useful for calculating velocity space averages. However, the detailed form of the anisotropic high energy tail is more accurately described if the approximate forms (30) and (31) are used.

## 4. Comparison and results

It is possible to qualitatively understand how the losses affect the high energy tail by analysing the asymptotic expression eq. (27). If we use the high energy approximations for  $\alpha$  and  $\beta$  given by Stix [6], we obtain from eq. (27)

$$h(v) = \frac{1}{2} \frac{v^{3} + v_{\alpha}^{3} + \frac{t_{s}}{2\tau_{E}} v^{3} + t_{s} KvR_{2}(v)}{v^{3} + v_{\beta}^{3} + 2t_{s} K \frac{v^{3}}{v_{T}^{2}} [G(v) + R_{1}(v)]} v_{T}^{-2} \cdot$$

$$\{1 + [1 + 2 \frac{v^{3} + v^{3}_{\beta} + 2t_{s}K \frac{v^{3}}{v_{T}^{2}} [G(v) + R_{1}(v)]}{v^{3} + v^{3}_{\alpha} + \frac{t_{s}}{2\tau_{E}} v^{3} + t_{s}KvR_{2}(v)} \frac{v v_{T}^{2} \frac{t_{s}}{\tau_{p}}}{v^{3} + v^{3}_{\alpha} + \frac{t_{s}}{2\tau_{E}} v^{3} + t_{s}KvR_{2}(v)}] \}$$

$$= \frac{1}{2} h(v)_{\tau_{p}=\infty} \left\{ 1 + \left[ 1 + 4 \frac{t_{s}}{\tau_{p}} - \frac{1}{h(v)_{\tau_{p}=\infty}} \frac{1}{v^{2}(1 + \frac{t_{s}}{2\tau_{E}} + \frac{v_{\alpha}^{3}}{v^{3}} + \frac{t_{s}KR_{2}(v)}{v})} \right]^{1/2}$$
(32)

where  $v_T = 2KT_e/m_i$  (m is the mass of the heated species) and h(v) is h(v) when the particle confinement time is infinit.

## 4.1 Energy losses

The energy loss term acts like a friction term, and if  $\tau_{\rm E}$  is constant it affects the tail in the same way as collisions with electrons do. Eq. (32) shows that t  $/[2(1+v_{\alpha}^3/v^3)]$  is the characteristic time with which  $\tau_{\rm E}$  should be compared, i.e. if  $\tau_{\rm E} >> t_{\rm s}/2$  then the effects of energy losses can be neglected. This also shows that energy losses mainly affect the high energy tail, unless  $\tau_{\rm E}$  is increasing with v, i.e. unless the energy confinement of the high energy particles is good.

These points are illustrated in Fig. 1, where the relative difference in power transfer to ions and electrons for finite (constant)  $\tau_{\rm E}$  as compared to the case  $\tau_{\rm E} = \infty$  is shown. In deriving the results in Fig. 1, as well as in the subsequent analysis, the following parameter values have been used: minority heating of <sup>3</sup>He in a deuterium background plasma,  $\eta_{\rm He} = 0.05 \ n_{\rm D}$ ,  $n_D = 3 \cdot 10^{13} \text{ cm}^{-3}$ ,  $T_D = T_e = 5 \text{ keV}$ ,  $k_\perp = 0.5 \text{ cm}^{-1}$ ,  $|E_\perp|/|E_+| = 7$  (estimated from eq. 15) in Ref. [6]). Note that  $P_M$  denotes the RF-power absorbed by an equivalent Maxwellian with density,  $n_3$  and temperature  $T_e$ . These He parameter values are representative for the plasmas in present-day large tokamak experiments, e.g. the JET plasma.

The differences in Fig. 1 become significant when  $\tau_{\rm E} \sim t_{\rm s}$ , as expected. Furthermore, we note that the difference in power transfer to background ions is much larger than the difference in power transfer to ions. Since the power transfer to background ions is mainly determined by the low energy part of the distribution whereas power transfer to electrons is more influenced by the high energy tail, the main effect of a finite  $\tau_{\rm E}$  is to suppress the high energy tail.

### 4.2 Particle losses

Eq. (32) shows that the effects of particle losses on the high energy tail become important when

$$\tau_{p} \sim \frac{4t_{s}}{h(v)_{\tau_{p}=\infty} v^{2} \left(1 + \frac{t_{s}}{2\tau_{E}} + \frac{v_{\alpha}^{3}}{v^{3}} + \frac{t_{s}KR_{2}(v)}{v}\right)}$$
(33)

Thus, the importance of particle losses depends sensitively on both  $t_s$  and  $h(v)_{\tau_p} = \cdots$ . Since  $h(v)_{\tau_p} = \cdots$  decreases for increasing RF-power, the influence of particle losses in the form of the velocity distribution function becomes stronger for increasing RF-power. This point is illustrated in Fig. 2, where the relative difference in fusion reactivity for  $\tau_p \simeq t_s$  as

compared to the case  $\tau = \infty$  is shown. Furthermore, Fig. 3 shows that the p main effect of particle losses is to suppress the high energy tail.

Since the confinement of high energy particles can be expected to be rather poor during ICRH, it is not realistic to approximate  $\tau_p$  as constant. Instead one would expect  $\tau_p$  to decrease with velocity in the high energy tail. Eq. (33) shows that a decreasing  $\tau_p$  can have a significant effect on the high energy tail, in particular if  $\tau_p$  decreases as  $1/v^2$  or faster. In order to investigate this point  $\tau_p$  has been assumed to have the following velocity dependence:

$$\tau_{p} = \begin{cases} \tau_{po} & v < v_{\star} \\ & & \\ \tau_{po} & \left(\frac{v_{\star}}{v}\right)^{n} & v > v_{\star} \end{cases}$$
(34)

Shown in Fig. 4 is the relative difference in fusion reactivity for  $\tau_p$  according to eq. (34) as compared to  $\tau_p = \tau_{po} = \text{const.}$  Since the velocity variation of the particle confinement,  $\tau_p$ , is not known, the parameters n and  $v_*$  have been varied to show the effect of different scalings. The result in Fig. 4 shows that losses of high energy particles can have a very significant effect on the fusion reactivity.

## 4.3 Effects caused by anisotropy

It is very difficult to treat the problem with anisotropy analytically. However, it is possible to get a qualitative understanding of how anisotropy, as compared to the isotropic approximation in 3.2, affects the pitch angle averaged distribution, F(v).

We know the asymptotic expressions for the functions  $R_1(v)$  and  $R_2(v)$ : In the low energy range where the distribution is almost isotropic, cf [6,7], we have  $R_1(v) \approx C$  and  $R_2(v) \approx 0$ , whereas in the anisotropic high energy tail  $R_1(v)$  and  $R_2(v)$  approach the expressions eq. (30) and eq. (31) respectively. There will be a gradual transition between these limiting forms for  $R_1(v)$  and  $R_2(v)$  in the intermediate velocity range. This transition should occur in a velocity range where the pitch angle scattering starts to become weak, a very rough estimate of this velocity range might be  $v \sim 0.5 v_v$ , cf [6,9].

The anisotropy will influence F(v) as compared to the isotropic model, in two different ways, (i) The diffusive RF-term, i.e.,  $Kv^2[G(v) + R_1(v)]$  in eq. (15), will increase when the distribution function becomes anisotropic. (ii) The anisotropy introduces an extra "friction" term,  $KvR_2(v)$ , in eq. (14). These two effects tend to cancel. The first effect will dominate over the second in the high energy tail, i.e., the local temperature in the tail will be higher than predicted by the isotropic model. However, the extra "friction" term,  $R_2(v)$ , can be important in the transition region between bulk and tail distribution. In particular, it can lead to a decrease in the local temperature in this region, cf [9]. The success of the isotropic model in calculating velocity space averages can, at least partly, be understood from the fact that these two effects work against each other. Thus, although the isotropic model does not describe the detailed form of the anisotropic part of the distribution correctly, it should be a good approximation when the distribution is averaged over velocity space.

#### 5. Conclusions

An approximate model that provides a short computational procedure has been derived for the pitch angle averaged in velocity distribution, F(v), in the presence of ICRH. The model includes losses caused by particle and energy transport. Effects of these losses on F(v) have been analysed, and they are shown to mainly affect the high energy tail. Furthermore, criteria for when these losses are important have been given.

The confinement of high energy particles can be expected to be poor in the presence of ICRH. Effects of a velocity dependent particle confinement time have therefore been analysed. It is found that the effects of a velocity dependent particle confinement time can be important, in particular, if the particle confinement time decreases as  $1/v^2$  or faster.

The approximate model for F(v) is based on the assumption that the influence of anisotropy can be neglected. The neglect of anisotropy has previously, [8], been shown to provide useful and accurate results for most physically meaningful velocity space averages calculated from F(v). However, this approximation cannot be expected to describe the detailed form of the anisotropic high energy part of the distribution correctly. The influence of anisotropy has therefore been discussed qualitatively, and an expression for F(v) has been given in the limit of strong anisotropy.

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## Figure captions

- Fig. 1. Relative difference in power transfer to ions and electrons for finite  $\tau_E$  as compared to  $\tau_E = \infty$  with  $\tau_p = \infty$  and  $P_M = 0.2$  W/cm, where  $P_M$  denotes the RF-power absorbed by an equivalent Maxwellian with density,  $n_{3He}$ , and temperature  $T_e$ . The "isotropic" model has been used in this calculation, i.e.  $R_1(v) = R_2(v) = 0$ . (---)  $\Delta P_1/P_1 = [P_1(\tau_p = \infty) - \tau_1]/P_1$  (----) $P_e/P_e [P_e(\tau_p = \infty) - P_e]/P_e$ .
- Fig. 2. Relative difference in fusion reactivity for  $\tau_p \approx t_s$  as compared to  $\tau_p = \infty$  with  $v_s = 0.5 V_T$  and  $\tau_E = \infty$ . The "isotropic" model has been used in this calculation. (----)  $\Delta \langle \sigma v \rangle / \langle \sigma v \rangle = [\langle \sigma v \rangle (\tau_p = \infty) - \langle \sigma v \rangle] / \langle \sigma v \rangle.$
- Fig. 3. Relative dirrerence in power transfer to ions and electrons for finite  $\tau_p$  as compared to  $\tau_p = \infty$  with  $\tau_E = \infty$ ,  $v_s = 0.5 V_T$  and  $P_M = 0.2 \text{ W/cm}^3$ . The "iasotropic" model has been used in this calculation.

$$(---) \Delta P_i / P_i = \left[ P_i (\tau = \infty) - P_i \right] / P_i$$

$$(---) \Delta P_e / P_e = \left[ P_e (\tau_p = \infty) - P_e \right] P_e$$

Fig. 4. Relative difference in fusion reactivity for  $\tau_p$  given by eq. (34) as compared to  $\tau_p = \tau_p = \text{const with } \tau_E = \infty$ ,  $v_s = 0.5 v_T$ ,  $\tau_p = 1s$  ( $t_s = 0.32$ ) and  $P_M = 0.2 \text{ W/cm}^3$ . The "isotropic" model has been used in this calculation.  $\Lambda(\alpha v)/(\alpha v)(\tau = \omega) - \langle \alpha v \rangle^2/\langle \alpha v \rangle$ 

$$\Delta \langle \sigma v \rangle / \langle \sigma v \rangle (\tau = \infty) - \langle \sigma v \rangle ] / \langle \sigma v \rangle$$
  
(---) n=1, (---) n=2, (....) n=3, (-.-.) n=4.





Fig. 2



