PSI-Bericht Nr. 17
Oktober 1988

Paul Scherrer Institut

Labor für Thermohydraulik

Analytical Models for the Rewetting of Hot Surfaces

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Introduction

Some aspects concerning analytical models for the rewetting of hot surface are discussed. These include the problems with applying various forms of boundary conditions, compatibility of boundary conditions with the physics of the rewetting problems, recent analytical models, the use of the separation of variables method versus the Wiener-Hopf technique, and the use of transformations.
The report includes an updated list of rewetting models as well as benchmark solutions in tabular form for several models. It should be emphasized that this report is not meant to cover the topic of rewetting models. It merely discusses some points which are less commonly referred to in the literature.

1 Compatibility of models

When applying boundary conditions, or assuming a certain form of heat source in the solid, one must make sure that these are compatible with each other and yield accpetable physical behavior. Two common points are of special importance.

1.1 Constant dry side heat transfer coefficient

The assumption of regions with constant heat transfer coefficients on the dry side implies that a higher wall heat flux in each such region occurs the farther the location is downstream of the quench front (since the solid temperature increases downstream away from the front, while the reference temperature for heat convection is assumed to remain constant). This behavior is, of course, physically unrealistic. Examples of works which assumed one or more regions on the dry side with a constant heat transfer coefficient are: Andrèoni (1975) and Elias and Yadigaroglu (1977) with one-dimensional models, and Salcudean and Bui (1980), Salcudean and Rahman (1980-81), Sawan and Temraz (1981), Bonakdar and McAssey (1981), and Hsu et al. (1983) with two-dimensional models. It should be noted that while a model with a single region on the dry side having a constant non-zero heat transfer coefficient cannot yield accpetable results, a multi-region model with regions on the dry side having a constant heat transfer coefficient in each region can still yield acceptable results, provided the heat transfer coefficient in the last region downstream of the quench front is set to zero, as was done by Elias and Yadigaroglu (1977). An approach where the heat flux is specified is recommended, since for many models, a closed form solution for arbitrary wall heat flux may easily be obtained.

1.2 The form of the heat source in the solid

The form of the assumed heat generation has to be compatible with the other boundary conditions. For example, assuming constant heat generation in the solid is incompatible with the assumptions of zero dry side heat transfer coefficient and that the solid temperature far downstream of the quench front approaches a constant value.

2 Heat flux versus convective boundary conditions

Specifying heat flux boundary conditions has all the advantages with respect to the specification of convective boundary conditions, see e.g. Adiutori (1974). It isn't possible to have a closed form analytical solution for convective boundary conditions of general form (not even for a 1-D model), whereas it is possible to obtain such solutions for a position dependent heat flux of general form (provided it is compatible with the rest of the boundary conditions). The latter may be found even for a fuel-and-cladding configuration.

With a one dimensional quasi steady state model, specifying the heat flux uniquely determines the rewetting velocity since

$$P = \int_{-\infty}^{\infty} [q(x) - Q(x)] dx$$

where the dimensionless wall heat flux is q and the heat source is Q and

$$P = \frac{\rho c u \bar{\delta}}{k}, \qquad x = \frac{\bar{x}}{\bar{\delta}}, \qquad q = \frac{\bar{q}\bar{\delta}}{k(T_w - T_s)} \qquad Q = \frac{\bar{Q}\bar{\delta}^2}{k(T_w - T_s)}$$

with the dimensional quantities retaining their usual meaning.

Let us take an example where zero heat generation is assumed and the heat flux is assumed to be $Q_w e^{ax}$ on the wet side and $Q_d e^{-bx}$ on the dry side. Then from experimental results for P and the above relation, one can try to relate the four open parameters Q_w, Q_d, a, b to, e.g. liquid flow rate, system pressure, degree of subcooling etc. to find eventually a correlation between the rewetting velocity and system parameters.

A similar procedure may be applied to a 2-D rewetting model. This may be illustrated through the following example.

Suppose we solve a quasi steady state model for a slab where a wall heat flux $q(x)$ is assumed. Then the mathematical formulation is

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + 2s\frac{\partial \theta}{\partial x} = 0, \qquad 0 < y < 1 \qquad -\infty < x < \infty \tag{1}$$

where $s = P/2$. The solution of Eq. (1) is sought for the following boundary conditions

$$\frac{\partial \theta}{\partial y} = 0 \qquad y = 0, \quad -\infty < x < \infty \tag{2}$$

$$\frac{\partial \theta}{\partial y} = q(x) \quad y = 1, \quad -\infty < x < \infty \tag{3}$$

$$x \to -\infty, \quad \theta \to 0 \tag{4}$$

$$x \to \infty, \qquad \theta \to 1 \tag{5}$$

Define the transformation

$$\phi(x, y) = 1 + \theta(x, y)e^{sx} \tag{6}$$

to obtain the following new formulation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + 2s\frac{\partial \phi}{\partial x} = 0, \qquad 0 < y < 1 \qquad -\infty < x < \infty \tag{7}$$

where $s = P/2$. The solution of Eq. (1) is sought for the following boundary conditions

$$\frac{\partial \phi}{\partial y} = 0 \qquad\qquad y = 0, \quad -\infty < x < \infty \tag{8}$$

$$\frac{\partial \phi}{\partial y} = q(x)e^{-sx} \quad y = 1, \quad -\infty < x < \infty \tag{9}$$

$$\phi = o(e^{-sx}) \qquad \text{as} \quad x \to -\infty \tag{10}$$

$$\phi = O(e^{-sx}) \qquad \text{as} \quad x \to \infty \tag{11}$$

Define now the following complex Fourier Transform and its inversion formula

$$\Phi = (\alpha, y) = \int_{-\infty}^{\infty} \phi(x, y) e^{i\alpha x} dx \tag{12}$$

$$\phi(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\alpha, y) e^{-i\alpha x} d\alpha \tag{13}$$

The solution exists for Im $|\alpha| < s$. The transform of (7) gives

$$\frac{\partial^2 \Phi}{\partial y^2} - \gamma^2 \Phi = 0 \tag{14}$$

where $\gamma = (\alpha^2 + s^2)^{1/2}$, such that $\gamma = \alpha$ when $s = 0$.
The solution of (14) which satisfies the transform of conditions (8) and (9) is

$$\Phi(\alpha, y) = \frac{\cosh \gamma y}{\gamma \sinh \gamma} \int_{-\infty}^{\infty} q(x') e^{i\alpha x'} dx' \tag{15}$$

so that from (13) it follows that

$$\phi(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} q(x') \int_{-\infty}^{\infty} \frac{\cosh \gamma y}{\gamma \sinh \gamma} e^{i\alpha(x'-x)} d\alpha dx' \tag{16}$$

The second integral in (16) may easily be derived by using contour integration, e.g. by using the residue theorem for a semi-infinite circle in the upper half plane.
Setting $x = 0, y = 1$ in (16) gives a relation between the rewetting temperature $\theta(0, 1) = \phi(0, 1) - 1$ and the dimensionless rewetting velocity $2s$.

3 The separation of variables method and the Wiener-Hopf technique

The two commonly used methods for the solution of analytical rewetting models are separation of variables and the Wiener-Hopf technique. Each method has its advantages and shortcomings.

3.1 The separation of variables method

The solution of multi-region rewetting models with this method is quite straightforward for boundary conditions of the first, second, and third kind (homogeneous or inhomogeneous). First, a rewetting velocity is assumed. Next, a formal solution for the temperature distribution is obtained for each subregion, resulting in a series with a set of, yet, undetermined constants. Then requiring the continuity of the temperature and heat flux at the boundary between each two adjacent regions, and using the orthogonality properties of the eigenfunctions, the noted constant are determined. Finally, the temperature at the 'triple interline' is evaluated and compared to the prescribed rewetting temperature. If the assumption of the rewetting velocity is correct, the calculated temperature at the quench front should be close enough to the prescribed rewetting temperature. If not, different guesses are assumed in an iterative procedure.

It should be emphasized that a fuel-and-cladding configuration can easily be treated in the same manner by using Yeh's theorem (see e.g. Yeh (1980)). The theorem shows for regions with singularities inside (such as a jump in the temperature gradient at the interface between the fuel and the cladding), for which the Sturm-Liouville theorem cannot be applied, how to derive weight functions with respect to which the eigenfunctions are orthogonal.

Wherever the boundary conditions are discontinuous the temperature gradient suffers a logarithmic singularity, i.e. the axial temperature gradient is proportional to the lograrithm of the distance from the point of discontinuity in the boundary conditions, e.g. Blair (1975) and Olek (1988a). For example, in the two-region model with a constant wet side heat transfer coefficient and a zero dry-side one, the series solutions are slow to converge when the wet side Biot number or the Peclet number are large. Since commonly only a limited number of terms in the series is considered, the results for the rewetting velocity may be inaccurate. This problem can be solved by either using the Wiener-hopf technique whenever a solution by this method is possible (see next subsection), or by the way suggested by Casteglia et al. (1986), who derived an expression for the rewetting temperature in the form of a quotient of infinite products instead of a series, which yields accurate results.

3.2 The Wiener-Hopf technique

Employing the Wiener-hopf technique in the solution of rewetting models is less formal than the separation of variables method. Problems may be posed in a single or a double integral equation formulation, or by using Jones's direct method. In addition, the governing differential equation may be transformed into a wave equation through a known transformation. This yields kernels which are easier to decompose than those steming from the original heat equation. On top of that, different decomposition methods may be used like, e.g. using Cauchy's integral which yields the decomposed + and - functions in the form of an integral, or when dealing with meromorphic functions the decomposed functions may be put in the form of a quotient of infinite products. So under the name Wiener-Hopf technique there hide different methods of formulation and solution, which don't necessarily yield the same final form of results. Common to the different methods is that at some stage of the solution a decomposition is performed of some complex function into a + function that is regular in one half of the wave number plane and another - function which is regular in another half of this plane. Since these functions share a common region of regularity, by analytic continuation they must represent the same entire function. Using Liouville's theorem and observing the behaviour of the noted functions at infinity one finds this entire function. For details see appendix A.

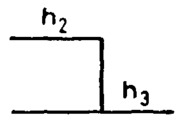
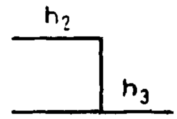
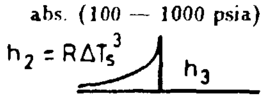
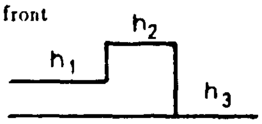
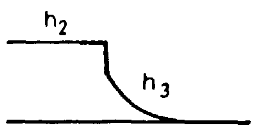
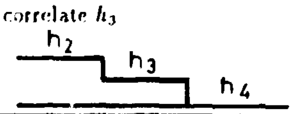
The advantage of the Wiener-Hopf technique with respect to separation of variables is that the accuracy of solution is not affected by the discontinuity in boundary conditions and usually a compact expression for the rewetting temperature is obtained, which involves the model parameters, including the rewetting velocity. This last feature is obtained according to the suggestion by Levine (1982), who showed that to obtain a relation between the rewetting temperature and the other model parameters an inversion of the transformed temperature is not necessary. However, multi-region models are not easy to address with the Wiener-Hopf technique.

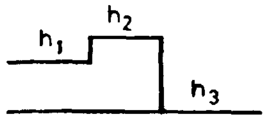
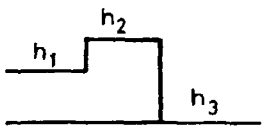
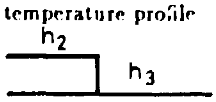
4 Rewetting models

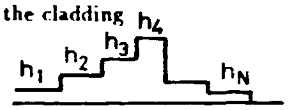
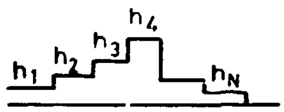
In this section a summary of rewetting models will be given in the form of the tables prepared by Elias and Yadigaroglu (1978). In addition, benchmark solutions for several rewetting models will be given.

4.1 A summary of rewetting models

Table 1: One Dimensional Models

Reference	Experimental data correlated	Heat-Transfer coefficients, W/(m²)(°C) [Btu/(hr)(ft²) (°F)]	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Semeria and Martinet (1965)	No experimental data correlated			
Yamanouchi (1968)	Yamanouchi (1968)	$h_2 = 2 \times 10^5 - 10^6$ $[4 \times 10^4 - 2 \times 10^5]$ $h_3 = 0$	150(302)	
Thompson (1972)	Bennett et al. (1966)	$h_1, h_2 \approx 7 \times 10^6$ $[1.2 \times 10^6]$ (peak values) $h_3 = 0$	T_s + 100°C (T_s = 180°F)	High-pressure data 6.9 — 69 bars abs. (100 — 1000 psia) 
Sun et al. (1974)	Yamanouchi Duffey and Porthouse (1973) (only low-flow-rate data)	$h_1 = 570[100]?$ $h_2 = 1.7 \times 10^4[3000]$ $h_3 = 0$	260(500)	Sputtering region between location of incipience of boiling and quench front 
Sun et al. (1975)	Yamanouchi (1968) Duffey and Porthouse (1973)	$h_2 = 1.7 \times 10^4$ [3000] $h_3 = \frac{h_2}{N} e^{-0.05z}$	260(500)	Precursory cooling included 
Chun and Chon (1975)	Case et al. (1973)	$h_2 = 2.56 \times 10^4[4500]$ $h_3 = 170[30]$ $h_4 = 0$	260(500)	Calculated length of dispersed flow region and mass carry-over to correlate h_3 

Reference	Experimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ $[Btu/(hr)(ft^2)(°F)]$	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Ishii (1975)	Bennett et al. (1966)	$h_1 = h_{CHF}$ $h_2 = 4 \times 10^5 [7 \times 10^4]$ $h_3 = 0$	260 — 390 (500 — 740)	1. Sputtering region between CHF and quench front 2. Correlated a pressure range of 6.9 — 69 bars abs.(100 — 1000 psia) 3. Defined thermal penetration length in sputtering region 
Andréoni (1975)	Andréoni (1975)	h_1, from Jens-Lottes correlation h_2, extracted from re-wetting data h_3, from experimental data	T_s + 200°C (~ T_s + 360°F)	
Chan and Grolmes (1975)	Guerrero et al. (1974)			Used heat flux approximation 1. Area under boiling curve to CHF for sputtering 2. Radiation model for dispersed flow 
Tan (1975)	Tan (1975)	Iloeje (1974)		1. Boiling curve from steady state correlations 2. Thermal non-equilibrium included 3. Internal heat generation 4. Finite difference wall segments with varying h
Yao (1976)	No experimental data correlated			Constant internal heat generation included; assumes parabolic radial temperature profile 

Reference	Experimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ $[Btu/(hr)(ft^2)(°F)]$	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Karyampudi and Chon (1976)				Radiation effects are analized 
Elias et al. (1976)	PWR FLECHT (Cadek and Dominics, 1971)	h_1 = 170 [30] $h_2, h_3, h_4, \cdots$ = boiling curve approximation	Not needed to be specified	1. 1-D in the axial direction of the cladding and 1-D in the radial direction of the Fuel 2. The model includes heat generation in the cladding 
Kirchner (1976)	FLECHT (Cermak, 1970) (Cadek, 1970)			1. Multiple region model 2. Boiling curves from steady state correlations
Elias and Yadigaroglu (1977)	Duffey and Porthouse(1973) (only low-flow-rate data)	h_1 = 170 [30] $h_2, h_3, h_4, \cdots$ = boiling-curve approximation	260(500)	
Chambré and Elias (1978)	Duffey and Porthouse (1972) Farmer (1972) Iloeje (1974) Rosal et al. (1975) Dua and Tien (1977)		Not needed to be specified	1. Taken from pool boiling correlations 2. The rewetting velocity is obtained as an eigenvalue of the heat conduction equation

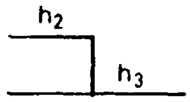
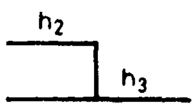
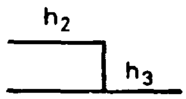
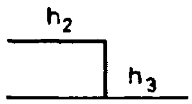
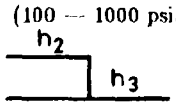
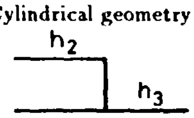
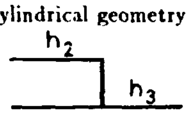
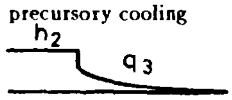
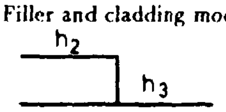
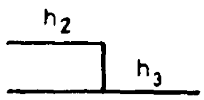
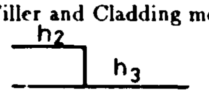
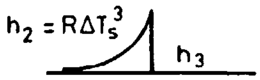
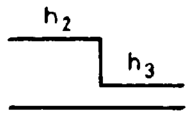
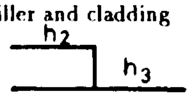
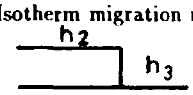
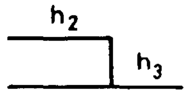
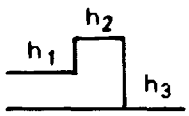
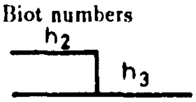
Reference	Experimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ $[Btu/(hr)(ft^2)(°F)]$	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Fischer et al. (1978)	FLECHT (Rosal,1975) FLECHT-SET (Waring and Hochreiter,1975) Semiscale (Peterson et al. 1976)			1. multiple region model 2. Boiling curves from steady state correlations
Elias and Chambré (1979)	Seban et al. (1978) Dua and Tien (1978)			1. Taken from pool boiling correlations for water and liquid nitrogen 2. Time dependent solution
Wendroff (1979)	No experimental data correlated			The solution is valid for high Biot numbers also 
Hirano and Asahi(1980)	Yamanouchi (1968) Duffey and Porthouse(1972) Duffey and Porthouse(1973) Piggott and Porthouse(1975) Dua and Tien (1978)	$Bi = 0.41 - 0.73$	For water 250(482) For Nitrogen -171(-276)	Heat transfer coefficients are taken from boiling correlations
Olek and Zvirin (1985)	No experimental data correlated		260(500)	Including temperature dependent properties 
Simopoulos (1986)	Elliott and Rose (1970), (1971)	$h_2 = 0.75 - 1.0 \times 10^6$ $h_3 = 1.2 - 3.5 \times 10^3$	0.3425 — 0.4464 (dimensionless)	1. Time dependent model with an arbitrary form of the heat transfer coefficient 2. Solution by finite differences

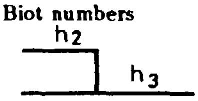
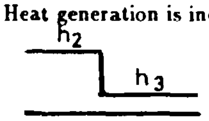
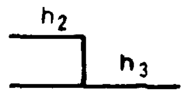
Table 1: Two Dimensional Models

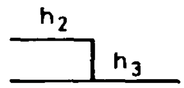
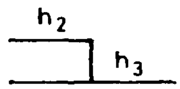
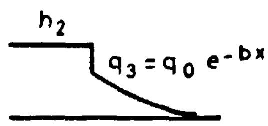
Reference	Experimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ [$Btu/(hr)(ft^2)(°F)$]	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Yoshioka and Hasewaga (1970, 1975)	Yoshioka and Hasewaga (1970) (1975)	h_2 = function of wall temperature and wet-front velocity $h_3 = 0$		Cartesian geometry 
Duffey and Porthouse (1973)	Yoshioka and Hasewaga(1970) Yamanouchi(1968) Duffey and Porthouse(1973) Andreoni and Countand(1972) Martini and Premoli(1973) Thompson(1972) Campanile and Pozzi (1972)	$h_2 = 10^4 - 2 \times 10^6$ [$1700 - 3.5 \times 10^5$] $h_3 = 0$	190 — 250 (375 — 480)	Cartesian geometry 
Edwards and Mather (1973)	No experimental data correlated	$h_{max} = 2 - 4 \times 10^5$ [$3.5 - 7 \times 10^4$]		Cartesian geometry 
Coney (1974)	Bennett et al. (1966)	$h_2 = 0.94 - 1.3 \times 10^6$ [$1.6 - 2.3 \times 10^5$] $h_3 = 0$	T_s + 68°C (T_s + 122°F)	Cartesian geometry Pressure range 6.9 — 69 bars abs (100 — 1000 psia) 
Thompson (1974)	Bennett et al. (1966)	$h_1, h_2 = 4 - 8 \times 10^5$ [$0.7 - 1.3 \times 10^5$] (peak values) $h_3 = 0$	T_s + 100°C (~ T_s + 180°F)	1. cylindrial geometry 2. Numerical solution 

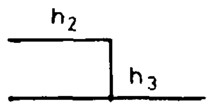
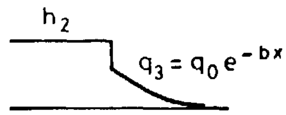
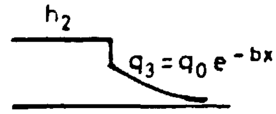
Reference	Exrerimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ $[Btu/(hr)(ft^2)(°F)]$	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Blair (1975)	Thompson (1974)	$h_2 = 1.7 \times 10^4$ [3000] $h_3 = 0$	260 (500)	Cylindrical geometry 
Yeh (1975)	No experimental data correlated			Cylindrical geometry 
Tien and Yao (1975)	No experimental data correlated			1. Cartesian geometry 2. Wiener-Hopf technique
Dua and Tien (1976)	Duffey and Porthouse (1973) Yamanouchi (1968)	$h_2 = 1.7 \times 10^4$ [3000] $q_3 = (q_o/N)e^{-az}$ (N, a: parameters)	260 (500)	1. Cartesian geometry 2. Wiener-Hopf technique with precursory cooling 
Pearson et al. (1977)	Piggott and Duffey (1975) Pearson et al. (1977)		150(302)	1. Cartesian geometry 2. Filler and cladding model 
Durack and Wendroff (1977)	No experimental data correlated			1. Cylindrical geometry 2. Isotherm migration method 

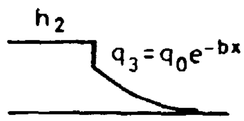
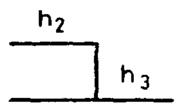
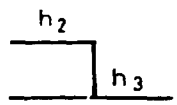
Reference	Experimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ $[Btu/(hr)(ft^2)(°F)]$	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Salcudean et al. (1978)	Lee and Chen (1978)		326(619)	1. Horizontal channels with a circumferentially varying heat transfer coefficient
Fairburn (1979)	Fairburn et al. (1979)	$h_{max} = 1.5 — 7.6 \times 10^3$ $[0.26 — 1.34 \times 10^3]$	250(482)	1. Cartesian geometry 2. Filler and Cladding model 
Linehan et al. (1979)	Linehan et al. (1979)		131 — 143 (267 — 290)	1. Cylindrical geometry 2. Stationary quench front. 3. Heat transfer coefficients are taken from correlations. 4. One and two-dimensional analyses
Wendroff (1979)	No experimental data correlated			Cartesian Geometry 
Salcudean and Bui (1980)	Salcudean and Bui (1980)	$h_2 = 2 \times 10^4 — 2.5 \times 10^5$ $[3.5 \times 10^3 — 4.4 \times 10^4]$ $h_3 = (1 — 10\%)h_2$		1. Cylindrical geometry 2. Horizontal channels 
Yeh (1980)	No experimental data correlated			1. Cylindrical geometry 2. Filler and cladding 

Reference	Experimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ $[Btu/(hr)(ft^2)(°F)]$	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Gurcak et al. (1980)	No experimental data correlated	$h_2 = 1.7 \times 10^4 [3000]$ $h_3 = 0$	267(512)	1. Cylindrical geometry 2. Isotherm migration method 
Salcudean and Rahman (1980-81)	Chen et al. (1979)	$h_1 = 10 - 280 \times 10^3$ $[1.8 - 49 \times 10^3]$ $h_2 = 0.01 - 0.05\ h_1$ $h_3 = 0$		1. Cylindrical geometry 2.Horizontal channels 
Laquer and Wendroff (1981)	No experimental data correlated			1. Cartesian geometry 2. Separation of variables and method of lines
Sawan and Temraz(1981)	No experimental		$T_{sat} + 3.6\ \Delta_{sub}$ + 20 °C $(T_{sat} + 6.48 \Delta T_{sub}$ + 280°F)	Cartesian geometry 
Bonakdar and McAssey(1981)	Yamanouchi (1968) Duffey and Porthouse (1973)	$h_2 = 1.7 \times 10^4 [3000]$		Cartesian geometry
Caflish and Keller(1981)	No experimental data correlated			1. Cartesian Geometry 2. Wiener-Hopf technique 3. An exact solution for all Biot numbers 

Reference	Experimental data correlated	Heat-Transfer coefficients, $W/(m^2)(°C)$ $[Btu/(hr)(ft^2)(°F)]$	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Chan and Banerjee (1981a, 1981b)	Chan and Banerjee (1981c)			1. Cylindrical geometry 2. A two fluid multi region model with steady state correlations to represent boiling curves 3. Horizontal channels
Levine (1982)	No experimental data correlated			1. Cartesian geometry 2. Wiener-Hopf technique 3. An exact solution for all Biot numbers 
Kimball and Roy(1982)	Piggott and Duffey(1975)		Taken from a correlation by Kimball and Roy(1983)	1. Cylindrical geometry 2. A four region model 3. Heat transfer coefficients are taken from correlations 4. Solution by finite integral transform
Hsu et al. (1983)	No experimental data correlated			1. Cylindrical geometry 2. Heat generation is included 
Evans (1984)	No experimental data correlated			1. Cylindrical geometry 2. Wiener-Hopf technique 3. An exact solution for all Biot numbers
Castiglia et al. (1986)	No experimental data correlated			1. Cartesian geometry 2. An exact solution by separation of variables 

Reference	Experimental data correlated	Heat-Transfer coefficients, W/(m²)(°C) [Btu/(hr)(ft²) (°F)]	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Chakrabati (1986a)	No experimental data correlated			1. 1. Cylinder with an insulated inner core 2. Wiener-Hopf technique 3. An exact solution for all Biot numbers 
Chakrabati (1986b)	No experimental data correlated			1. 1. Cartesian geometry 2. Composite slabs 3. Wiener-Hopf technique 4. An exact solution for all Biot numbers 
Carbajo (1986)	No experimental data correlated	For a Two-region model: h_2 = 62.260 — 516.000 [10.990 — 91.060] h_3 = 0	139 — 321 (379 — 610)	1. Cylindrical geometry 2. Filler and cladding, including heat generation 3. Multiple region model 4. Finite differences solution
Olek et al. (1988a)	Yamanouchi (1968) Duffey and Porthouse (1972) Yu et al. (1977) Dua and Tien (1989) Ueda and Inone (1984)		For water: 150 — 260 (302-500) For nitrogen: -171(-276)	1. Both cartesian and cylindrical geometry 2. Conjugate heat transfer model in which the temperatures in the solid and the liquid are solved simultaneously 3. Heat transfer coefficients are not needed as input 4. Wet side heat transfer coefficient can be calculated as part of the solution
Olek (1987)	No experimental data correlated			1. Cylindrical geometry 2. Solution by separation of variables 

Reference	Experimental data correlated	Heat-Transfer coefficients, W/(m^2)(°C) [Btu/(hr)(ft^2) (°F)]	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Olek et al. (1988b)	No experimental data correlated		260 (500)	1. Both cartesian and cylindrical geometry 2. Conjugate heat transfer model where the temperature distribution in the solid and the liquid are solved simultaneously 3. Includes heat generation 4. Heat transfer coefficients are not needed as input 5. Wet side heat transfer coefficient can be calculated as part of the solution
Olek (1988a)	No experimental data correlated			1. Cartesian geometry 2. Both separation of variables and the Wiener-Hopf technique are used for the solution 
Olek (1988b)	Dua and Tien (1978)	Bi = 0.0073 b=0.0085	-171 (-276) (Liquid Nitrogen)	1. Cartesian geometry Solution by separation of variables 
Olek (1988c)	No experimental data correlated			1. Cartesian geometry 2. Solution by the Wiener-Hopf technique 

Reference	Experimental data correlated	Heat-Transfer coefficients, W/(m²)(°C) [Btu/(hr)(ft²) (°F)]	Quench temperature, °C (°F)	Comments and heat-transfer coefficient profile
Olek (1988d)	No experimental data correlated			1. Cylindrical geometry 2. Solution by the Wiener-Hopf technique 
Olek (1988e)	No experimental data correlated			1. Fuel-and-cladding 2. Solution by the Wiener-Hopf technique 
Thomas (1988)	No experimental data correlated			1. Fuel-and-cladding 2. Solution by the Wiener-Hopf technique 

4.2 Benchmark solutions for several rewetting models

The main importance of the simplified rewetting models (like the two-region model with a step change in the heat transfer coefficient) is that they may serve to assess the solution capability of more complicated models, usually solved by numerical schemes. To this end, solutions in tabular form have been prepared for several rewetting models. The accuracy of the results is beleived to be of the order of the fourth significant digit. Results for the following models will be presented:

1. The two-region model for plane slab with a step change in the heat transfer coefficient, Olek (1988a).
2. The Dua and Tien (1976) model with precursory cooling for plane slab, Olek (1988b).
3. The Dua and Tien (1976) model with precursory cooling for solid cylinder, Olek (1988d).
4. Yeh's (1980) fuel-and-cladding model, Olek (1988e).

The two-region rewetting model for plane slab with a step change in the heat transfer coefficient

TABLE 1

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_w - T_s)$ for various Biot (B) and Peclet (P) numbers.

B \ P	0.01	0.05	0.10	0.50	1.00	5.00	10.00	50.00	100.00	500.00
0.01	0.09512	0.3902	0.6176	0.9615	0.9888	0.9987	0.9994	0.9999	1.000	1.000
0.05	0.04372	0.1998	0.3576	0.8492	0.9482	0.9937	0.9968	0.9994	0.9998	0.9999
0.10	0.03112	0.1459	0.2695	0.7585	0.9051	0.9875	0.9937	0.9988	0.9995	0.9999
0.50	0.01404	0.06809	0.1311	0.4887	0.7047	0.9422	0.9697	0.9931	0.9959	0.9994
1.00	0.009944	0.04861	0.09451	0.3785	0.5848	0.8945	0.9425	0.9874	0.9934	0.9987
5.00	0.004457	0.02199	0.04326	0.1896	0.3233	0.6770	0.7894	0.9426	0.9699	0.9937
10.00	0.003153	0.01559	0.03073	0.1371	0.2388	0.5516	0.6779	0.8951	0.9427	0.9875
50.00	0.001411	0.006987	0.01381	0.06279	0.1119	0.2929	0.3937	0.6780	0.7895	0.9426
100.00	0.0009977	0.004943	0.009772	0.04458	0.07977	0.2138	0.7935	0.5526	0.6781	0.8950
500.00	0.0004462	0.002211	0.004373	0.02001	0.03594	0.09868	0.1386	0.2935	0.3937	0.6779

The Dua and Tien (1976) model with precursory cooling for plane slab

TABLE 2

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_w - T_s)$ for various Biot (B) and Peclet (P) numbers, and for various precursory cooling magnitude parameters N. The influenced solid length parameter is $b = 0.005$.

B \ P		0.01	0.05	0.1	0.5	1.0	5.0	10.0	50.0	100.0
0.01	N = 1	$4.750\cdot10^{-3}$	$2.343\cdot10^{-2}$	$4.615\cdot10^{-2}$	$1.983\cdot10^{-1}$	$3.319\cdot10^{-1}$	$7.113\cdot10^{-1}$	$8.31[illegible]\cdot10^{-1}$	$9.525\cdot10^{-1}$	$9.733\cdot10^{-1}$
	N = 10	$3.277\cdot10^{-3}$	$1.514\cdot10^{-1}$	$2.763\cdot10^{-1}$	$6.943\cdot10^{-1}$	$8.254\cdot10^{-1}$	$9.601\cdot10^{-1}$	$9.795\cdot10^{-1}$	$9.956\cdot10^{-1}$	$9.977\cdot10^{-1}$
	N = 100	$7.994\cdot10^{-2}$	$3.373\cdot10^{-1}$	$5.497\cdot10^{-1}$	$9.259\cdot10^{-1}$	$9.636\cdot10^{-1}$	$9.947\cdot10^{-1}$	$9.973\cdot10^{-1}$	$9.995\cdot10^{-1}$	$9.998\cdot10^{-1}$
	N = ∞	$9.512\cdot10^{-2}$	$3.903\cdot10^{-1}$	$6.176\cdot10^{-1}$	$9.615\cdot10^{-1}$	$9.888\cdot10^{-1}$	$9.987\cdot10^{-1}$	$9.994\cdot10^{-1}$	$9.999\cdot10^{-1}$	$1.000\cdot10^{0}$
0.05	N = 1	$9.775\cdot10^{-4}$	$4.877\cdot10^{-3}$	$9.726\cdot10^{-3}$	$4.720\cdot10^{-2}$	$9.019\cdot10^{-2}$	$3.313\cdot10^{-1}$	$4.958\cdot10^{-1}$	$9.211\cdot10^{-1}$	$8.959\cdot10^{-1}$
	N = 10	$8.137\cdot10^{-3}$	$3.999\cdot10^{-2}$	$7.814\cdot10^{-2}$	$3.146\cdot10^{-1}$	$4.865\cdot10^{-1}$	$8.281\cdot10^{-1}$	$9.053\cdot10^{-1}$	$9.793\cdot10^{-1}$	$9.771\cdot10^{-1}$
	N = 100	$3.042\cdot10^{-2}$	$1.428\cdot10^{-1}$	$2.634\cdot10^{-1}$	$7.259\cdot10^{-1}$	$8.660\cdot10^{-1}$	$9.742\cdot10^{-1}$	$9.863\cdot10^{-1}$	$9.973\cdot10^{-1}$	$9.986\cdot10^{-1}$
	N = ∞	$4.372\cdot10^{-2}$	$1.996\cdot10^{-1}$	$3.[illegible]76\cdot10^{-1}$	$8.491\cdot10^{-1}$	$9.482\cdot10^{-1}$	$9.937\cdot10^{-1}$	$9.968\cdot10^{-1}$	$9.994\cdot10^{-1}$	$9.998\cdot10^{-1}$
0.1	N = 1	$4.980\cdot10^{-4}$	$2.457\cdot10^{-3}$	$4.908\cdot10^{-3}$	$2.419\cdot10^{-2}$	$4.734\cdot10^{-2}$	$1.985\cdot10^{-1}$	$3.236\cdot10^{-1}$	$6.974\cdot10^{-1}$	$8.115\cdot10^{-1}$
	N = 10	$4.307\cdot10^{-3}$	$2.134\cdot10^{-3}$	$4.217\cdot10^{-2}$	$1.874\cdot10^{-1}$	$3.219\cdot10^{-1}$	$7.066\cdot10^{-1}$	$8.231\cdot10^{-1}$	$9.575\cdot10^{-1}$	$9.763\cdot10^{-1}$
	N = 100	$1.918\cdot10^{-2}$	$9.213\cdot10^{-2}$	$1.751\cdot10^{-1}$	$5.818\cdot10^{-1}$	$7.663\cdot10^{-1}$	$9.497\cdot10^{-1}$	$9.741\cdot10^{-1}$	$9.945\cdot10^{-1}$	$9.971\cdot10^{-1}$
	N = ∞	$3.112\cdot10^{-2}$	$1.459\cdot10^{-1}$	$2.695\cdot10^{-1}$	$7.565\cdot10^{-1}$	$9.051\cdot10^{-1}$	$9.875\cdot10^{-1}$	$9.937\cdot10^{-1}$	$9.988\cdot10^{-1}$	$9.995\cdot10^{-1}$
0.5	N = 1	$9.924\cdot10^{-5}$	$4.961\cdot10^{-4}$	$9.919\cdot10^{-4}$	$4.945\cdot10^{-3}$	$9.849\cdot10^{-3}$	$4.720\cdot10^{-2}$	$8.952\cdot10^{-2}$	$3.160\cdot10^{-1}$	$4.627\cdot10^{-1}$
	N = 10	$9.331\cdot10^{-4}$	$4.656\cdot10^{-3}$	$9.287\cdot10^{-3}$	$4.533\cdot10^{-2}$	$8.748\cdot10^{-2}$	$3.153\cdot10^{-1}$	$4.889\cdot10^{-1}$	$8.173\cdot10^{-1}$	$8.930\cdot10^{-1}$
	N = 100	$5.839\cdot10^{-3}$	$2.982\cdot10^{-2}$	$5.671\cdot10^{-2}$	$2.470\cdot10^{-1}$	$4.132\cdot10^{-1}$	$7.920\cdot10^{-1}$	$8.829\cdot10^{-1}$	$9.723\cdot10^{-1}$	$9.846\cdot10^{-1}$
	N = ∞	$1.404\cdot10^{-2}$	$6.807\cdot10^{-2}$	$1.311\cdot10^{-1}$	$4.887\cdot10^{-1}$	$7.047\cdot10^{-1}$	$9.422\cdot10^{-1}$	$9.697\cdot10^{-1}$	$9.931\cdot10^{-1}$	$9.951\cdot10^{-1}$
1.0	N = 1	$4.971\cdot10^{-5}$	$2.486\cdot10^{-4}$	$4.970\cdot10^{-4}$	$2.481\cdot10^{-3}$	$4.951\cdot10^{-3}$	$2.417\cdot10^{-2}$	$4.676\cdot10^{-2}$	$1.877\cdot10^{-1}$	$3.009\cdot10^{-1}$
	N = 10	$4.758\cdot10^{-4}$	$2.376\cdot10^{-3}$	$4.746\cdot10^{-3}$	$2.343\cdot10^{-2}$	$4.601\cdot10^{-2}$	$1.944\cdot10^{-1}$	$3.337\cdot10^{-1}$	$6.924\cdot10^{-1}$	$8.076\cdot10^{-1}$
	N = 100	$3.336\cdot10^{-3}$	$1.650\cdot10^{-2}$	$3.269\cdot10^{-2}$	$1.508\cdot10^{-1}$	$2.694\cdot10^{-1}$	$6.577\cdot10^{-1}$	$7.913\cdot10^{-1}$	$9.470\cdot10^{-1}$	$9.720\cdot10^{-1}$
	N = ∞	$9.944\cdot10^{-3}$	$4.861\cdot10^{-2}$	$9.451\cdot10^{-2}$	$3.785\cdot10^{-1}$	$5.848\cdot10^{-1}$	$8.915\cdot10^{-1}$	$9.425\cdot10^{-1}$	$9.874\cdot10^{-1}$	$9.934\cdot10^{-1}$
5.0	N = 1	$9.967\cdot10^{-6}$	$4.983\cdot10^{-5}$	$9.966\cdot10^{-5}$	$4.980\cdot10^{-4}$	$9.951\cdot10^{-4}$	$4.930\cdot10^{-3}$	$9.737\cdot10^{-3}$	$4.417\cdot10^{-2}$	$7.928\cdot10^{-2}$
	N = 10	$9.770\cdot10^{-5}$	$4.884\cdot10^{-4}$	$9.763\cdot10^{-4}$	$4.865\cdot10^{-3}$	$9.683\cdot10^{-3}$	$4.627\cdot10^{-2}$	$8.764\cdot10^{-2}$	$3.107\cdot10^{-1}$	$4.578\cdot10^{-1}$
	N = 100	$8.160\cdot10^{-4}$	$4.070\cdot10^{-3}$	$8.115\cdot10^{-3}$	$3.952\cdot10^{-2}$	$7.617\cdot10^{-2}$	$2.865\cdot10^{-1}$	$4.394\cdot10^{-1}$	$7.833\cdot10^{-1}$	$8.720\cdot10^{-1}$
	N = ∞	$4.457\cdot10^{-3}$	$2.199\cdot10^{-2}$	$4.316\cdot10^{-2}$	$1.896\cdot10^{-1}$	$3.233\cdot10^{-1}$	$6.770\cdot10^{-1}$	$7.834\cdot10^{-1}$	$9.326\cdot10^{-1}$	$9.693\cdot10^{-1}$
10.0	N = 1	$4.986\cdot10^{-6}$	$2.493\cdot10^{-5}$	$4.985\cdot10^{-5}$	$2.491\cdot10^{-4}$	$4.979\cdot10^{-4}$	$2.474\cdot10^{-3}$	$4.891\cdot10^{-3}$	$2.357\cdot10^{-2}$	$4.127\cdot10^{-2}$
	N = 10	$4.946\cdot10^{-5}$	$2.457\cdot10^{-4}$	$4.913\cdot10^{-4}$	$2.451\cdot10^{-3}$	$4.868\cdot10^{-3}$	$2.376\cdot10^{-2}$	$4.574\cdot10^{-2}$	$1.840\cdot10^{-1}$	$2.961\cdot10^{-1}$
	N = 100	$4.311\cdot10^{-4}$	$2.152\cdot10^{-3}$	$4.295\cdot10^{-3}$	$2.111\cdot10^{-2}$	$4.117\cdot10^{-2}$	$1.742\cdot10^{-1}$	$2.854\cdot10^{-1}$	$6.456\cdot10^{-1}$	$7.737\cdot10^{-1}$
	N = ∞	$3.153\cdot10^{-3}$	$1.553\cdot10^{-2}$	$3.073\cdot10^{-2}$	$1.371\cdot10^{-1}$	$2.388\cdot10^{-1}$	$5.516\cdot10^{-1}$	$6.773\cdot10^{-1}$	$8.951\cdot10^{-1}$	$9.427\cdot10^{-1}$
50.0	N = 1	$9.976\cdot10^{-7}$	$4.988\cdot10^{-6}$	$9.975\cdot10^{-6}$	$4.986\cdot10^{-5}$	$9.966\cdot10^{-5}$	$4.973\cdot10^{-4}$	$9.824\cdot10^{-4}$	$4.600\cdot10^{-3}$	$8.537\cdot10^{-3}$
	N = 10	$9.917\cdot10^{-6}$	$4.956\cdot10^{-5}$	$9.911\cdot10^{-5}$	$4.950\cdot10^{-4}$	$9.897\cdot10^{-4}$	$4.879\cdot10^{-3}$	$9.609\cdot10^{-3}$	$4.335\cdot10^{-2}$	$7.780\cdot10^{-2}$
	N = 100	$9.333\cdot10^{-5}$	$4.659\cdot10^{-4}$	$9.309\cdot10^{-4}$	$4.612\cdot10^{-3}$	$9.159\cdot10^{-3}$	$4.243\cdot10^{-2}$	$7.879\cdot10^{-2}$	$2.752\cdot10^{-1}$	$4.163\cdot10^{-1}$
	N = ∞	$1.411\cdot10^{-3}$	$6.981\cdot10^{-3}$	$1.381\cdot10^{-2}$	$6.379\cdot10^{-2}$	$1.119\cdot10^{-1}$	$3.919\cdot10^{-1}$	$5.337\cdot10^{-1}$	$8.780\cdot10^{-1}$	$7.895\cdot10^{-1}$
100.0	N = 1	$4.988\cdot10^{-7}$	$2.494\cdot10^{-6}$	$4.988\cdot10^{-6}$	$2.493\cdot10^{-5}$	$4.977\cdot10^{-5}$	$2.477\cdot10^{-4}$	$4.914\cdot10^{-4}$	$2.305\cdot10^{-3}$	$4.287\cdot10^{-3}$
	N = 10	$4.966\cdot10^{-6}$	$2.483\cdot10^{-5}$	$4.965\cdot10^{-5}$	$2.481\cdot10^{-4}$	$4.958\cdot10^{-4}$	$2.452\cdot10^{-3}$	$4.842\cdot10^{-3}$	$2.222\cdot10^{-2}$	$4.056\cdot10^{-2}$
	N = 100	$4.753\cdot10^{-5}$	$2.375\cdot10^{-4}$	$4.748\cdot10^{-4}$	$2.362\cdot10^{-3}$	$4.693\cdot10^{-3}$	$2.213\cdot10^{-2}$	$4.216\cdot10^{-2}$	$1.631\cdot10^{-1}$	$2.637\cdot10^{-1}$
	N = ∞	$3.977\cdot10^{-4}$	$4.543\cdot10^{-3}$	$3.772\cdot10^{-3}$	$1.958\cdot10^{-2}$	$3.778\cdot10^{-2}$	$2.138\cdot10^{-1}$	$3.935\cdot10^{-1}$	$5.516\cdot10^{-1}$	$6.731\cdot10^{-1}$

TABLE 3

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_w - T_s)$ for various Biot (B) and Peclet (P) numbers, and for various precursory cooling magnitude parameters N. The influenced solid length parameter is $b = 0.05$.

B	N \ P	0.01	0.05	0.1	0.5	1.0	5.0	10.0	50.0	100.0
0.01	N = 1	3.275·10^-2	1.523·10^-1	2.761·10^-1	6.936·10^-1	8.243·10^-1	9.582·10^-1	9.772·10^-1	9.931·10^-1	9.954·10^-1
	N = 10	7.990·10^-2	3.375·10^-1	5.496·10^-1	9.258·10^-1	9.639·10^-1	9.945·10^-1	9.971·10^-1	9.992·10^-1	9.996·10^-1
	N = 100	9.334·10^-2	3.842·10^-1	6.100·10^-1	9.578·10^-1	9.768·10^-1	9.983·10^-1	9.991·10^-1	9.998·10^-1	9.998·10^-1
	N = ∞	9.512·10^-2	3.902·10^-1	6.176·10^-1	9.615·10^-1	9.888·10^-1	9.987·10^-1	9.994·10^-1	9.999·10^-1	1.000·10^0
0.05	N = 1	8.127·10^-3	3.993·10^-2	7.901·10^-2	3.137·10^-1	4.844·10^-1	8.208·10^-1	8.956·10^-1	9.667·10^-1	9.776·10^-1
	N = 10	3.041·10^-2	1.427·10^-1	2.633·10^-1	7.254·10^-1	8.653·10^-1	9.732·10^-1	9.857·10^-1	9.961·10^-1	9.973·10^-1
	N = 100	4.183·10^-2	1.921·10^-1	3.453·10^-1	8.350·10^-1	9.377·10^-1	9.916·10^-1	9.957·10^-1	9.991·10^-1	9.996·10^-1
	N = ∞	4.372·10^-2	1.998·10^-1	3.576·10^-1	8.492·10^-1	9.482·10^-1	9.937·10^-1	9.968·10^-1	9.997·10^-1	9.998·10^-1
0.1	N = 1	4.259·10^-3	2.130·10^-2	4.207·10^-2	1.872·10^-1	3.200·10^-1	6.960·10^-1	8.103·10^-1	9.356·10^-1	9.562·10^-1
	N = 10	1.916·10^-2	9.205·10^-2	1.749·10^-1	5.812·10^-1	7.652·10^-1	9.478·10^-1	9.718·10^-1	9.921·10^-1	9.950·10^-1
	N = 100	2.933·10^-2	1.379·10^-1	2.557·10^-1	7.360·10^-1	8.888·10^-1	9.834·10^-1	9.915·10^-1	9.981·10^-1	9.990·10^-1
	N = ∞	3.112·10^-2	1.459·10^-1	2.695·10^-1	7.585·10^-1	9.051·10^-1	9.875·10^-1	9.937·10^-1	9.988·10^-1	9.995·10^-1
0.5	N = 1	9.291·10^-4	4.636·10^-3	9.245·10^-3	4.504·10^-2	8.668·10^-2	3.142·10^-1	4.646·10^-1	7.438·10^-1	8.138·10^-1
	N = 10	5.893·10^-3	2.894·10^-2	5.656·10^-2	2.462·10^-1	4.114·10^-1	7.852·10^-1	8.736·10^-1	9.609·10^-1	9.741·10^-1
	N = 100	1.230·10^-2	5.983·10^-2	1.159·10^-1	4.443·10^-1	6.578·10^-1	9.237·10^-1	9.591·10^-1	9.838·10^-1	9.937·10^-1
	N = ∞	1.404·10^-2	6.809·10^-2	1.311·10^-1	4.883·10^-1	7.047·10^-1	9.412·10^-1	9.637·10^-1	9.931·10^-1	9.959·10^-1
1.0	N = 1	4.731·10^-4	2.363·10^-3	4.718·10^-3	2.325·10^-2	4.551·10^-2	1.864·10^-1	3.001·10^-1	5.921·10^-1	6.860·10^-1
	N = 10	3.313·10^-3	1.644·10^-2	3.255·10^-2	1.493·10^-1	2.676·10^-1	6.482·10^-1	7.763·10^-1	9.256·10^-1	9.508·10^-1
	N = 100	8.285·10^-3	4.065·10^-2	7.940·10^-2	3.283·10^-1	5.223·10^-1	8.617·10^-1	9.228·10^-1	9.808·10^-1	9.883·10^-1
	N = ∞	9.914·10^-3	4.861·10^-2	9.451·10^-2	3.785·10^-1	5.848·10^-1	8.945·10^-1	9.425·10^-1	9.877·10^-1	9.934·10^-1
5.0	N = 1	9.698·10^-5	4.836·10^-4	9.666·10^-4	4.806·10^-3	9.534·10^-3	4.387·10^-2	7.901·10^-2	2.350·10^-1	3.041·10^-1
	N = 10	8.096·10^-4	4.037·10^-3	8.048·10^-3	3.943·10^-2	7.534·10^-2	2.771·10^-1	4.157·10^-1	7.197·10^-1	7.957·10^-1
	N = 100	3.073·10^-3	1.521·10^-2	3.009·10^-2	1.369·10^-1	2.437·10^-1	5.916·10^-1	7.243·10^-1	9.155·10^-1	9.432·10^-1
	N = ∞	4.457·10^-3	2.195·10^-2	4.316·10^-2	1.896·10^-1	3.231·10^-1	6.770·10^-1	7.834·10^-1	9.316·10^-1	9.633·10^-1
10.0	N = 1	4.861·10^-5	2.429·10^-4	4.856·10^-4	2.417·10^-3	4.801·10^-3	2.344·10^-2	4.415·10^-2	1.168·10^-1	1.793·10^-1
	N = 10	4.268·10^-4	2.130·10^-3	4.251·10^-3	2.086·10^-2	4.066·10^-2	1.673·10^-1	2.661·10^-1	5.573·10^-1	6.612·10^-1
	N = 100	1.924·10^-3	9.553·10^-3	1.894·10^-2	8.803·10^-2	1.606·10^-1	4.463·10^-1	5.871·10^-1	8.439·10^-1	9.042·10^-1
	N = ∞	3.153·10^-3	1.553·10^-2	3.073·10^-2	1.371·10^-1	2.388·10^-1	5.516·10^-1	7.233·10^-1	8.951·10^-1	9.427·10^-1
50.0	N = 1	9.749·10^-6	4.883·10^-5	9.760·10^-5	4.861·10^-4	9.693·10^-4	4.777·10^-3	8.516·10^-3	2.331·10^-2	4.187·10^-2
	N = 10	9.199·10^-5	4.593·10^-4	9.176·10^-4	4.545·10^-3	8.991·10^-3	4.013·10^-2	7.139·10^-2	2.053·10^-1	2.833·10^-1
	N = 100	5.795·10^-4	2.886·10^-3	5.741·10^-3	2.777·10^-2	5.343·10^-2	1.997·10^-1	2.911·10^-1	5.511·10^-1	6.695·10^-1
	N = ∞	1.411·10^-3	6.957·10^-3	1.381·10^-2	6.279·10^-2	1.119·10^-1	3.929·10^-1	5.937·10^-1	6.980·10^-1	7.895·10^-1
100.0	N = 1	4.888·10^-6	2.443·10^-5	4.884·10^-5	2.433·10^-4	4.841·10^-4	2.394·10^-3	4.277·10^-3	1.431·10^-2	2.138·10^-2
	N = 10	4.684·10^-5	2.339·10^-4	4.674·10^-4	2.319·10^-3	4.530·10^-3	2.091·10^-2	3.781·10^-2	1.161·10^-1	1.666·10^-1
	N = 100	3.291·10^-4	1.640·10^-3	3.167·10^-3	1.580·10^-2	3.024·10^-2	1.113·10^-1	1.751·10^-1	4.016·10^-1	5.188·10^-1
	N = ∞	3.977·10^-4	4.943·10^-3	9.791·10^-3	4.458·10^-2	7.977·10^-2	2.138·10^-1	2.935·10^-1	5.515·10^-1	6.791·10^-1

TABLE 4

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_w - T_s)$ for various Biot (B) and Peclet (P) numbers, and for various precursory cooling magnitude parameters N. The influenced solid length parameter is $b = 0.5$.

B \ P	N	0.01	0.05	0.1	0.5	1.0	5.0	10.0	50.0	100.0
0.01	N = 1	$7.981 \cdot 10^{-2}$	$3.371 \cdot 10^{-1}$	$5.490 \cdot 10^{-1}$	$9.245 \cdot 10^{-1}$	$9.680 \cdot 10^{-1}$	$9.928 \cdot 10^{-1}$	$9.953 \cdot 10^{-1}$	$9.980 \cdot 10^{-1}$	$9.996 \cdot 10^{-1}$
	N = 10	$9.333 \cdot 10^{-2}$	$3.841 \cdot 10^{-1}$	$6.400 \cdot 10^{-1}$	$9.577 \cdot 10^{-1}$	$9.767 \cdot 10^{-1}$	$9.951 \cdot 10^{-1}$	$9.990 \cdot 10^{-1}$	$9.997 \cdot 10^{-1}$	$9.999 \cdot 10^{-1}$
	N = 100	$9.499 \cdot 10^{-2}$	$3.886 \cdot 10^{-1}$	$6.469 \cdot 10^{-1}$	$9.641 \cdot 10^{-1}$	$9.826 \cdot 10^{-1}$	$9.987 \cdot 10^{-1}$	$9.993 \cdot 10^{-1}$	$9.999 \cdot 10^{-1}$	$1.000 \cdot 10^{0}$
	N = ∞	$9.512 \cdot 10^{-2}$	$3.902 \cdot 10^{-1}$	$6.476 \cdot 10^{-1}$	$9.645 \cdot 10^{-1}$	$9.881 \cdot 10^{-1}$	$9.987 \cdot 10^{-1}$	$9.999 \cdot 10^{-1}$	$9.999 \cdot 10^{-1}$	$1.000 \cdot 10^{0}$
0.05	N = 1	$2.017 \cdot 10^{-2}$	$1.420 \cdot 10^{-1}$	$2.620 \cdot 10^{-1}$	$7.141 \cdot 10^{-1}$	$8.537 \cdot 10^{-1}$	$9.649 \cdot 10^{-1}$	$9.771 \cdot 10^{-1}$	$9.901 \cdot 10^{-1}$	$9.930 \cdot 10^{-1}$
	N = 10	$4.186 \cdot 10^{-2}$	$1.910 \cdot 10^{-1}$	$3.451 \cdot 10^{-1}$	$8.344 \cdot 10^{-1}$	$9.385 \cdot 10^{-1}$	$9.907 \cdot 10^{-1}$	$9.948 \cdot 10^{-1}$	$9.985 \cdot 10^{-1}$	$9.991 \cdot 10^{-1}$
	N = 100	$4.353 \cdot 10^{-2}$	$1.930 \cdot 10^{-1}$	$3.563 \cdot 10^{-1}$	$8.483 \cdot 10^{-1}$	$9.471 \cdot 10^{-1}$	$9.93 \cdot 10^{-1}$	$9.96 \cdot 10^{-1}$	$9.995 \cdot 10^{-1}$	$9.997 \cdot 10^{-1}$
	N = ∞	$4.371 \cdot 10^{-2}$	$1.938 \cdot 10^{-1}$	$3.576 \cdot 10^{-1}$	$8.496 \cdot 10^{-1}$	$9.481 \cdot 10^{-1}$	$9.937 \cdot 10^{-1}$	$9.968 \cdot 10^{-1}$	$9.998 \cdot 10^{-1}$	$9.998 \cdot 10^{-1}$
0.1	N = 1	$1.301 \cdot 10^{-2}$	$9.133 \cdot 10^{-2}$	$1.735 \cdot 10^{-1}$	$5.752 \cdot 10^{-1}$	$7.561 \cdot 10^{-1}$	$9.323 \cdot 10^{-1}$	$9.553 \cdot 10^{-1}$	$9.803 \cdot 10^{-1}$	$9.860 \cdot 10^{-1}$
	N = 10	$2.936 \cdot 10^{-2}$	$1.377 \cdot 10^{-1}$	$2.554 \cdot 10^{-1}$	$7.350 \cdot 10^{-1}$	$8.876 \cdot 10^{-1}$	$9.817 \cdot 10^{-1}$	$9.897 \cdot 10^{-1}$	$9.969 \cdot 10^{-1}$	$9.981 \cdot 10^{-1}$
	N = 100	$3.092 \cdot 10^{-2}$	$1.451 \cdot 10^{-1}$	$2.680 \cdot 10^{-1}$	$7.560 \cdot 10^{-1}$	$9.013 \cdot 10^{-1}$	$9.863 \cdot 10^{-1}$	$9.933 \cdot 10^{-1}$	$9.986 \cdot 10^{-1}$	$9.993 \cdot 10^{-1}$
	N = ∞	$3.112 \cdot 10^{-2}$	$1.459 \cdot 10^{-1}$	$2.695 \cdot 10^{-1}$	$7.585 \cdot 10^{-1}$	$9.051 \cdot 10^{-1}$	$9.875 \cdot 10^{-1}$	$9.937 \cdot 10^{-1}$	$9.988 \cdot 10^{-1}$	$9.995 \cdot 10^{-1}$
0.5	N = 1	$5.690 \cdot 10^{-3}$	$2.807 \cdot 10^{-2}$	$5.520 \cdot 10^{-2}$	$2.388 \cdot 10^{-1}$	$3.962 \cdot 10^{-1}$	$7.339 \cdot 10^{-1}$	$8.403 \cdot 10^{-1}$	$9.087 \cdot 10^{-1}$	$9.338 \cdot 10^{-1}$
	N = 10	$1.224 \cdot 10^{-2}$	$5.957 \cdot 10^{-2}$	$1.153 \cdot 10^{-1}$	$4.424 \cdot 10^{-1}$	$6.537 \cdot 10^{-1}$	$9.162 \cdot 10^{-1}$	$9.540 \cdot 10^{-1}$	$9.840 \cdot 10^{-1}$	$9.893 \cdot 10^{-1}$
	N = 100	$1.383 \cdot 10^{-2}$	$6.713 \cdot 10^{-2}$	$1.293 \cdot 10^{-1}$	$4.836 \cdot 10^{-1}$	$6.993 \cdot 10^{-1}$	$9.395 \cdot 10^{-1}$	$9.678 \cdot 10^{-1}$	$9.922 \cdot 10^{-1}$	$9.952 \cdot 10^{-1}$
	N = ∞	$1.404 \cdot 10^{-2}$	$6.809 \cdot 10^{-2}$	$1.311 \cdot 10^{-1}$	$4.882 \cdot 10^{-1}$	$7.043 \cdot 10^{-1}$	$9.422 \cdot 10^{-1}$	$9.653 \cdot 10^{-1}$	$9.931 \cdot 10^{-1}$	$9.959 \cdot 10^{-1}$
1.0	N = 1	$3.204 \cdot 10^{-3}$	$1.587 \cdot 10^{-2}$	$3.140 \cdot 10^{-2}$	$1.431 \cdot 10^{-1}$	$2.534 \cdot 10^{-1}$	$5.801 \cdot 10^{-1}$	$6.843 \cdot 10^{-1}$	$8.727 \cdot 10^{-1}$	$9.459 \cdot 10^{-1}$
	N = 10	$8.114 \cdot 10^{-3}$	$4.030 \cdot 10^{-2}$	$7.970 \cdot 10^{-2}$	$3.251 \cdot 10^{-1}$	$5.172 \cdot 10^{-1}$	$8.485 \cdot 10^{-1}$	$9.077 \cdot 10^{-1}$	$9.694 \cdot 10^{-1}$	$9.802 \cdot 10^{-1}$
	N = 100	$9.335 \cdot 10^{-3}$	$4.765 \cdot 10^{-2}$	$9.365 \cdot 10^{-2}$	$3.723 \cdot 10^{-1}$	$5.773 \cdot 10^{-1}$	$8.896 \cdot 10^{-1}$	$9.353 \cdot 10^{-1}$	$9.855 \cdot 10^{-1}$	$9.920 \cdot 10^{-1}$
	N = ∞	$9.544 \cdot 10^{-3}$	$4.861 \cdot 10^{-2}$	$9.451 \cdot 10^{-2}$	$3.786 \cdot 10^{-1}$	$5.848 \cdot 10^{-1}$	$8.935 \cdot 10^{-1}$	$9.425 \cdot 10^{-1}$	$9.879 \cdot 10^{-1}$	$9.934 \cdot 10^{-1}$
5.0	N = 1	$7.553 \cdot 10^{-4}$	$3.762 \cdot 10^{-3}$	$7.488 \cdot 10^{-3}$	$3.593 \cdot 10^{-2}$	$6.795 \cdot 10^{-2}$	$2.484 \cdot 10^{-1}$	$3.003 \cdot 10^{-1}$	$4.989 \cdot 10^{-1}$	$5.852 \cdot 10^{-1}$
	N = 10	$3.991 \cdot 10^{-3}$	$1.981 \cdot 10^{-2}$	$3.917 \cdot 10^{-2}$	$1.839 \cdot 10^{-1}$	$3.350 \cdot 10^{-1}$	$5.535 \cdot 10^{-1}$	$6.789 \cdot 10^{-1}$	$8.656 \cdot 10^{-1}$	$9.151 \cdot 10^{-1}$
	N = 100	$4.293 \cdot 10^{-3}$	$2.033 \cdot 10^{-2}$	$4.129 \cdot 10^{-2}$	$1.848 \cdot 10^{-1}$	$3.116 \cdot 10^{-1}$	$6.630 \cdot 10^{-1}$	$7.767 \cdot 10^{-1}$	$9.343 \cdot 10^{-1}$	$9.636 \cdot 10^{-1}$
	N = ∞	$4.457 \cdot 10^{-3}$	$2.199 \cdot 10^{-2}$	$4.316 \cdot 10^{-2}$	$1.896 \cdot 10^{-1}$	$3.331 \cdot 10^{-1}$	$6.770 \cdot 10^{-1}$	$7.854 \cdot 10^{-1}$	$9.316 \cdot 10^{-1}$	$9.651 \cdot 10^{-1}$
10.0	N = 1	$3.317 \cdot 10^{-4}$	$1.952 \cdot 10^{-3}$	$3.889 \cdot 10^{-3}$	$1.873 \cdot 10^{-2}$	$3.589 \cdot 10^{-2}$	$1.231 \cdot 10^{-1}$	$1.771 \cdot 10^{-1}$	$3.314 \cdot 10^{-1}$	$4.137 \cdot 10^{-1}$
	N = 10	$1.949 \cdot 10^{-3}$	$9.177 \cdot 10^{-3}$	$1.813 \cdot 10^{-2}$	$8.413 \cdot 10^{-2}$	$1.526 \cdot 10^{-1}$	$4.093 \cdot 10^{-1}$	$5.285 \cdot 10^{-1}$	$7.655 \cdot 10^{-1}$	$8.358 \cdot 10^{-1}$
	N = 100	$2.396 \cdot 10^{-3}$	$1.457 \cdot 10^{-2}$	$2.874 \cdot 10^{-2}$	$1.230 \cdot 10^{-1}$	$2.261 \cdot 10^{-1}$	$5.334 \cdot 10^{-1}$	$6.533 \cdot 10^{-1}$	$8.801 \cdot 10^{-1}$	$9.308 \cdot 10^{-1}$
	N = ∞	$3.153 \cdot 10^{-3}$	$1.553 \cdot 10^{-2}$	$3.073 \cdot 10^{-2}$	$1.371 \cdot 10^{-1}$	$2.381 \cdot 10^{-1}$	$5.516 \cdot 10^{-1}$	$6.775 \cdot 10^{-1}$	$8.951 \cdot 10^{-1}$	$9.417 \cdot 10^{-1}$
50.0	N = 1	$8.157 \cdot 10^{-5}$	$4.068 \cdot 10^{-4}$	$8.109 \cdot 10^{-4}$	$3.993 \cdot 10^{-3}$	$7.531 \cdot 10^{-3}$	$3.763 \cdot 10^{-2}$	$7.153 \cdot 10^{-2}$	$3.067 \cdot 10^{-2}$	$1.233 \cdot 10^{-1}$
	N = 10	$5.365 \cdot 10^{-4}$	$2.669 \cdot 10^{-3}$	$5.305 \cdot 10^{-3}$	$2.610 \cdot 10^{-2}$	$4.714 \cdot 10^{-2}$	$1.434 \cdot 10^{-1}$	$2.131 \cdot 10^{-1}$	$4.115 \cdot 10^{-1}$	$5.133 \cdot 10^{-1}$
	N = 100	$1.213 \cdot 10^{-3}$	$6.014 \cdot 10^{-3}$	$1.190 \cdot 10^{-2}$	$5.463 \cdot 10^{-2}$	$9.840 \cdot 10^{-2}$	$2.672 \cdot 10^{-1}$	$3.630 \cdot 10^{-1}$	$6.367 \cdot 10^{-1}$	$7.413 \cdot 10^{-1}$
	N = ∞	$1.411 \cdot 10^{-3}$	$6.987 \cdot 10^{-3}$	$1.381 \cdot 10^{-2}$	$6.179 \cdot 10^{-2}$	$1.119 \cdot 10^{-1}$	$2.919 \cdot 10^{-1}$	$3.937 \cdot 10^{-1}$	$6.780 \cdot 10^{-1}$	$7.895 \cdot 10^{-1}$
100.0	N = 1	$4.108 \cdot 10^{-5}$	$2.049 \cdot 10^{-4}$	$4.084 \cdot 10^{-4}$	$1.987 \cdot 10^{-3}$	$3.830 \cdot 10^{-3}$	$1.905 \cdot 10^{-2}$	$3.135 \cdot 10^{-2}$	$4.751 \cdot 10^{-2}$	$6.535 \cdot 10^{-2}$
	N = 10	$8.997 \cdot 10^{-4}$	$1.491 \cdot 10^{-3}$	$2.962 \cdot 10^{-3}$	$1.418 \cdot 10^{-2}$	$2.674 \cdot 10^{-2}$	$8.827 \cdot 10^{-2}$	$1.287 \cdot 10^{-1}$	$2.699 \cdot 10^{-1}$	$3.517 \cdot 10^{-1}$
	N = 100	$3.032 \cdot 10^{-4}$	$4.014 \cdot 10^{-3}$	$7.999 \cdot 10^{-3}$	$3.671 \cdot 10^{-2}$	$6.657 \cdot 10^{-2}$	$1.873 \cdot 10^{-1}$	$2.602 \cdot 10^{-1}$	$4.995 \cdot 10^{-1}$	$6.205 \cdot 10^{-1}$
	N = ∞	$9.977 \cdot 10^{-4}$	$4.943 \cdot 10^{-3}$	$9.773 \cdot 10^{-3}$	$4.458 \cdot 10^{-2}$	$7.977 \cdot 10^{-2}$	$2.158 \cdot 10^{-1}$	$2.935 \cdot 10^{-1}$	$5.516 \cdot 10^{-1}$	$6.721 \cdot 10^{-1}$

The Dua and Tien (1976) model with precursory cooling for a solid cylinder

TABLE 5

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_w - T_s)$ for various Biot (B) and Peclet (P) numbers, and for various precursory cooling magnitude parameters N. The influenced solid length parameter is $b = 0.005$.

B \ P	N	0.01	0.05	0.1	0.5	1.0	5.0	10.0	50.0	100.0
0.01	$N=1$	$2.441\cdot10^{-3}$	$1.199\cdot10^{-2}$	$2.381\cdot10^{-2}$	$1.102\cdot10^{-1}$	$1.991\cdot10^{-1}$	$5.545\cdot10^{-1}$	$7.129\cdot10^{-1}$	$9.238\cdot10^{-1}$	$9.593\cdot10^{-1}$
	$N=10$	$1.830\cdot10^{-3}$	$8.792\cdot10^{-2}$	$1.666\cdot10^{-1}$	$5.331\cdot10^{-1}$	$7.039\cdot10^{-1}$	$9.344\cdot10^{-1}$	$9.607\cdot10^{-1}$	$9.917\cdot10^{-1}$	$9.957\cdot10^{-1}$
	$N=100$	$5.361\cdot10^{-2}$	$2.396\cdot10^{-1}$	$4.165\cdot10^{-1}$	$8.654\cdot10^{-1}$	$9.427\cdot10^{-1}$	$9.905\cdot10^{-1}$	$9.953\cdot10^{-1}$	$9.990\cdot10^{-1}$	$9.995\cdot10^{-1}$
	$N=\infty$	$6.825\cdot10^{-2}$	$2.965\cdot10^{-1}$	$4.998\cdot10^{-1}$	$9.297\cdot10^{-1}$	$9.797\cdot10^{-1}$	$9.984\cdot10^{-1}$	$9.993\cdot10^{-1}$	$9.999\cdot10^{-1}$	$9.999\cdot10^{-1}$
0.05	$N=1$	$4.920\cdot10^{-4}$	$2.458\cdot10^{-3}$	$4.909\cdot10^{-3}$	$2.410\cdot10^{-2}$	$4.738\cdot10^{-2}$	$1.993\cdot10^{-1}$	$3.318\cdot10^{-1}$	$7.080\cdot10^{-1}$	$8.249\cdot10^{-1}$
	$N=10$	$4.308\cdot10^{-3}$	$2.134\cdot10^{-2}$	$4.218\cdot10^{-2}$	$1.883\cdot10^{-1}$	$3.228\cdot10^{-1}$	$7.099\cdot10^{-1}$	$8.303\cdot10^{-1}$	$9.598\cdot10^{-1}$	$9.789\cdot10^{-1}$
	$N=100$	$1.918\cdot10^{-2}$	$9.218\cdot10^{-2}$	$1.753\cdot10^{-1}$	$5.845\cdot10^{-1}$	$7.711\cdot10^{-1}$	$9.543\cdot10^{-1}$	$9.770\cdot10^{-1}$	$9.952\cdot10^{-1}$	$9.975\cdot10^{-1}$
	$N=\infty$	$3.112\cdot10^{-2}$	$1.460\cdot10^{-1}$	$2.699\cdot10^{-1}$	$7.638\cdot10^{-1}$	$9.118\cdot10^{-1}$	$9.923\cdot10^{-1}$	$9.965\cdot10^{-1}$	$9.994\cdot10^{-1}$	$9.997\cdot10^{-1}$
0.1	$N=1$	$2.472\cdot10^{-4}$	$1.235\cdot10^{-3}$	$2.469\cdot10^{-3}$	$1.226\cdot10^{-2}$	$2.427\cdot10^{-2}$	$1.107\cdot10^{-1}$	$1.989\cdot10^{-1}$	$5.480\cdot10^{-1}$	$7.010\cdot10^{-1}$
	$N=10$	$2.246\cdot10^{-3}$	$1.118\cdot10^{-2}$	$2.223\cdot10^{-2}$	$1.049\cdot10^{-1}$	$1.930\cdot10^{-1}$	$5.503\cdot10^{-1}$	$7.098\cdot10^{-1}$	$9.228\cdot10^{-1}$	$9.587\cdot10^{-1}$
	$N=100$	$1.173\cdot10^{-2}$	$5.726\cdot10^{-2}$	$1.110\cdot10^{-1}$	$4.286\cdot10^{-1}$	$6.327\cdot10^{-1}$	$9.128\cdot10^{-1}$	$9.550\cdot10^{-1}$	$9.906\cdot10^{-1}$	$9.951\cdot10^{-1}$
	$N=\infty$	$2.211\cdot10^{-2}$	$1.057\cdot10^{-1}$	$1.997\cdot10^{-1}$	$6.571\cdot10^{-1}$	$8.471\cdot10^{-1}$	$9.848\cdot10^{-1}$	$9.931\cdot10^{-1}$	$9.987\cdot10^{-1}$	$9.994\cdot10^{-1}$
0.5	$N=1$	$4.974\cdot10^{-5}$	$2.487\cdot10^{-4}$	$4.972\cdot10^{-4}$	$2.481\cdot10^{-3}$	$4.957\cdot10^{-3}$	$2.429\cdot10^{-2}$	$4.732\cdot10^{-2}$	$1.952\cdot10^{-1}$	$3.202\cdot10^{-1}$
	$N=10$	$4.760\cdot10^{-4}$	$2.377\cdot10^{-3}$	$4.748\cdot10^{-3}$	$2.347\cdot10^{-2}$	$4.616\cdot10^{-2}$	$1.967\cdot10^{-1}$	$3.285\cdot10^{-1}$	$7.051\cdot10^{-1}$	$8.229\cdot10^{-1}$
	$N=100$	$3.327\cdot10^{-3}$	$1.652\cdot10^{-2}$	$3.274\cdot10^{-2}$	$1.546\cdot10^{-1}$	$2.735\cdot10^{-1}$	$6.782\cdot10^{-1}$	$8.098\cdot10^{-1}$	$9.545\cdot10^{-1}$	$9.762\cdot10^{-1}$
	$N=\infty$	$9.948\cdot10^{-3}$	$4.870\cdot10^{-2}$	$9.498\cdot10^{-2}$	$3.855\cdot10^{-1}$	$6.043\cdot10^{-1}$	$9.314\cdot10^{-1}$	$9.671\cdot10^{-1}$	$9.936\cdot10^{-1}$	$9.968\cdot10^{-1}$
1.0	$N=1$	$2.490\cdot10^{-5}$	$1.245\cdot10^{-4}$	$2.490\cdot10^{-4}$	$1.244\cdot10^{-3}$	$2.486\cdot10^{-3}$	$1.230\cdot10^{-2}$	$2.423\cdot10^{-2}$	$1.081\cdot10^{-1}$	$1.907\cdot10^{-1}$
	$N=10$	$2.414\cdot10^{-4}$	$1.206\cdot10^{-3}$	$2.411\cdot10^{-3}$	$1.198\cdot10^{-2}$	$2.376\cdot10^{-2}$	$1.093\cdot10^{-1}$	$1.966\cdot10^{-1}$	$5.446\cdot10^{-1}$	$6.992\cdot10^{-1}$
	$N=100$	$1.845\cdot10^{-3}$	$9.186\cdot10^{-3}$	$1.828\cdot10^{-2}$	$8.747\cdot10^{-2}$	$1.648\cdot10^{-1}$	$5.148\cdot10^{-1}$	$6.811\cdot10^{-1}$	$9.131\cdot10^{-1}$	$9.535\cdot10^{-1}$
	$N=\infty$	$7.044\cdot10^{-3}$	$3.468\cdot10^{-2}$	$6.801\cdot10^{-2}$	$2.916\cdot10^{-1}$	$4.845\cdot10^{-1}$	$8.748\cdot10^{-1}$	$9.375\cdot10^{-1}$	$9.873\cdot10^{-1}$	$9.937\cdot10^{-1}$
5.0	$N=1$	$4.989\cdot10^{-6}$	$2.495\cdot10^{-5}$	$4.989\cdot10^{-5}$	$2.494\cdot10^{-4}$	$4.986\cdot10^{-4}$	$2.484\cdot10^{-3}$	$4.941\cdot10^{-3}$	$2.367\cdot10^{-2}$	$4.499\cdot10^{-2}$
	$N=10$	$4.918\cdot10^{-5}$	$2.459\cdot10^{-4}$	$4.918\cdot10^{-4}$	$2.455\cdot10^{-3}$	$4.900\cdot10^{-3}$	$2.401\cdot10^{-2}$	$4.675\cdot10^{-2}$	$1.931\cdot10^{-1}$	$3.174\cdot10^{-1}$
	$N=100$	$4.314\cdot10^{-4}$	$2.154\cdot10^{-3}$	$4.301\cdot10^{-3}$	$2.124\cdot10^{-2}$	$4.178\cdot10^{-2}$	$1.799\cdot10^{-1}$	$3.033\cdot10^{-1}$	$6.786\cdot10^{-1}$	$8.041\cdot10^{-1}$
	$N=\infty$	$3.155\cdot10^{-3}$	$1.564\cdot10^{-2}$	$3.095\cdot10^{-2}$	$1.419\cdot10^{-1}$	$2.557\cdot10^{-1}$	$6.451\cdot10^{-1}$	$7.773\cdot10^{-1}$	$9.417\cdot10^{-1}$	$9.696\cdot10^{-1}$
10.0	$N=1$	$2.495\cdot10^{-6}$	$1.248\cdot10^{-5}$	$2.495\cdot10^{-5}$	$1.247\cdot10^{-4}$	$2.494\cdot10^{-4}$	$1.244\cdot10^{-3}$	$2.477\cdot10^{-3}$	$1.198\cdot10^{-2}$	$2.301\cdot10^{-2}$
	$N=10$	$4.910\cdot10^{-5}$	$2.459\cdot10^{-4}$	$4.918\cdot10^{-4}$	$2.455\cdot10^{-3}$	$4.900\cdot10^{-3}$	$2.401\cdot10^{-2}$	$4.675\cdot10^{-2}$	$1.931\cdot10^{-1}$	$3.174\cdot10^{-1}$
	$N=100$	$2.247\cdot10^{-4}$	$1.123\cdot10^{-3}$	$2.243\cdot10^{-3}$	$1.113\cdot10^{-2}$	$2.203\cdot10^{-2}$	$1.006\cdot10^{-1}$	$1.809\cdot10^{-1}$	$5.148\cdot10^{-1}$	$6.733\cdot10^{-1}$
	$N=\infty$	$2.231\cdot10^{-3}$	$1.108\cdot10^{-2}$	$2.196\cdot10^{-2}$	$1.011\cdot10^{-1}$	$1.868\cdot10^{-1}$	$5.200\cdot10^{-1}$	$6.637\cdot10^{-1}$	$8.936\cdot10^{-1}$	$9.421\cdot10^{-1}$
50.0	$N=1$	$4.993\cdot10^{-7}$	$2.496\cdot10^{-6}$	$4.993\cdot10^{-6}$	$2.496\cdot10^{-5}$	$4.991\cdot10^{-5}$	$2.490\cdot10^{-4}$	$4.965\cdot10^{-4}$	$2.419\cdot10^{-3}$	$4.688\cdot10^{-3}$
	$N=10$	$4.970\cdot10^{-6}$	$2.485\cdot10^{-5}$	$4.970\cdot10^{-5}$	$2.484\cdot10^{-4}$	$4.965\cdot10$	$2.470\cdot10^{-3}$	$4.901\cdot10^{-3}$	$2.343\cdot10^{-2}$	$4.451\cdot10^{-2}$
	$N=100$	$4.757\cdot10^{-5}$	$2.378\cdot10^{-4}$	$4.754\cdot10^{-4}$	$2.370\cdot10^{-3}$	$4.722\cdot10^{-3}$	$2.284\cdot10^{-2}$	$4.399\cdot10^{-2}$	$1.786\cdot10^{-1}$	$2.951\cdot10^{-1}$
	$N=\infty$	$9.985\cdot10^{-4}$	$4.964\cdot10^{-3}$	$9.858\cdot10^{-3}$	$4.655\cdot10^{-2}$	$8.677\cdot10^{-2}$	$2.724\cdot10^{-1}$	$3.819\cdot10^{-1}$	$6.753\cdot10^{-1}$	$7.893\cdot10^{-1}$
100.0	$N=1$	$2.497\cdot10^{-7}$	$1.248\cdot10^{-6}$	$2.497\cdot10^{-6}$	$1.248\cdot10^{-5}$	$2.496\cdot10^{-5}$	$1.245\cdot10^{-4}$	$2.483\cdot10^{-4}$	$1.211\cdot10^{-3}$	$2.350\cdot10^{-3}$
	$N=10$	$2.489\cdot10^{-6}$	$1.244\cdot10^{-5}$	$2.488\cdot10^{-5}$	$1.244\cdot10^{-4}$	$2.488\cdot10^{-4}$	$1.238\cdot10^{-3}$	$2.464\cdot10^{-3}$	$1.187\cdot10^{-2}$	$2.279\cdot10^{-2}$
	$N=100$	$2.412\cdot10^{-5}$	$1.206\cdot10^{-4}$	$2.411\cdot10^{-4}$	$1.203\cdot10^{-3}$	$2.400\cdot10^{-3}$	$1.172\cdot10^{-2}$	$2.285\cdot10^{-2}$	$9.942\cdot10^{-2}$	$1.743\cdot10^{-1}$
	$N=\infty$	$7.661\cdot10^{-4}$	$3.811\cdot10^{-3}$	$7.575\cdot10^{-3}$	$3.703\cdot10^{-2}$	$6.726\cdot10^{-2}$	$1.984\cdot10^{-1}$	$2.841\cdot10^{-1}$	$5.495\cdot10^{-1}$	$6.766\cdot10^{-1}$

TABLE 6

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_w - T_s)$ for various Biot (B) and Peclet (P) numbers, and for various precursory cooling magnitude parameters N. The influenced solid length parameter is $b = 0.05$.

B \ P		0.01	0.05	0.1	0.5	1.0	5.0	10.0	50.0	100.0
0.01	N = 1	$1.829 \cdot 10^{-2}$	$9.790 \cdot 10^{-2}$	$1.666 \cdot 10^{-1}$	$5.318 \cdot 10^{-1}$	$7.032 \cdot 10^{-1}$	$9.231 \cdot 10^{-1}$	$9.591 \cdot 10^{-1}$	$9.898 \cdot 10^{-1}$	$9.939 \cdot 10^{-1}$
	N = 10	$5.361 \cdot 10^{-2}$	$2.396 \cdot 10^{-1}$	$4.165 \cdot 10^{-1}$	$8.653 \cdot 10^{-1}$	$9.466 \cdot 10^{-1}$	$9.904 \cdot 10^{-1}$	$9.951 \cdot 10^{-1}$	$9.988 \cdot 10^{-1}$	$9.993 \cdot 10^{-1}$
	N = 100	$6.644 \cdot 10^{-2}$	$2.896 \cdot 10^{-1}$	$4.900 \cdot 10^{-1}$	$9.229 \cdot 10^{-1}$	$9.759 \cdot 10^{-1}$	$9.976 \cdot 10^{-1}$	$9.989 \cdot 10^{-1}$	$9.998 \cdot 10^{-1}$	$9.999 \cdot 10^{-1}$
	N = ∞	$6.825 \cdot 10^{-2}$	$2.965 \cdot 10^{-1}$	$4.998 \cdot 10^{-1}$	$9.297 \cdot 10^{-1}$	$9.717 \cdot 10^{-1}$	$9.984 \cdot 10^{-1}$	$9.993 \cdot 10^{-1}$	$9.993 \cdot 10^{-1}$	$9.997 \cdot 10^{-1}$
0.05	N = 1	$4.304 \cdot 10^{-3}$	$2.133 \cdot 10^{-2}$	$4.215 \cdot 10^{-2}$	$1.880 \cdot 10^{-1}$	$3.222 \cdot 10^{-1}$	$7.060 \cdot 10^{-1}$	$8.242 \cdot 10^{-1}$	$9.511 \cdot 10^{-1}$	$9.704 \cdot 10^{-1}$
	N = 10	$1.918 \cdot 10^{-2}$	$9.215 \cdot 10^{-2}$	$1.752 \cdot 10^{-1}$	$5.843 \cdot 10^{-1}$	$7.707 \cdot 10^{-1}$	$9.536 \cdot 10^{-1}$	$9.761 \cdot 10^{-1}$	$9.943 \cdot 10^{-1}$	$9.967 \cdot 10^{-1}$
	N = 100	$2.330 \cdot 10^{-2}$	$1.380 \cdot 10^{-1}$	$2.561 \cdot 10^{-1}$	$7.402 \cdot 10^{-1}$	$8.954 \cdot 10^{-1}$	$9.883 \cdot 10^{-1}$	$9.945 \cdot 10^{-1}$	$9.988 \cdot 10^{-1}$	$9.994 \cdot 10^{-1}$
	N = ∞	$3.122 \cdot 10^{-2}$	$1.460 \cdot 10^{-1}$	$2.653 \cdot 10^{-1}$	$7.628 \cdot 10^{-1}$	$9.118 \cdot 10^{-1}$	$9.923 \cdot 10^{-1}$	$9.965 \cdot 10^{-1}$	$9.994 \cdot 10^{-1}$	$9.997 \cdot 10^{-1}$
0.1	N = 1	$2.743 \cdot 10^{-3}$	$1.116 \cdot 10^{-2}$	$2.219 \cdot 10^{-2}$	$1.047 \cdot 10^{-1}$	$1.925 \cdot 10^{-1}$	$5.457 \cdot 10^{-1}$	$7.010 \cdot 10^{-1}$	$9.068 \cdot 10^{-1}$	$9.425 \cdot 10^{-1}$
	N = 10	$1.173 \cdot 10^{-2}$	$5.722 \cdot 10^{-2}$	$1.110 \cdot 10^{-1}$	$4.283 \cdot 10^{-1}$	$6.311 \cdot 10^{-1}$	$9.115 \cdot 10^{-1}$	$9.534 \cdot 10^{-1}$	$9.887 \cdot 10^{-1}$	$9.931 \cdot 10^{-1}$
	N = 100	$2.031 \cdot 10^{-2}$	$9.741 \cdot 10^{-2}$	$1.849 \cdot 10^{-1}$	$6.198 \cdot 10^{-1}$	$8.173 \cdot 10^{-1}$	$9.770 \cdot 10^{-1}$	$9.890 \cdot 10^{-1}$	$9.977 \cdot 10^{-1}$	$9.987 \cdot 10^{-1}$
	N = ∞	$2.311 \cdot 10^{-2}$	$1.057 \cdot 10^{-1}$	$1.997 \cdot 10^{-1}$	$6.522 \cdot 10^{-1}$	$8.471 \cdot 10^{-1}$	$9.848 \cdot 10^{-1}$	$9.931 \cdot 10^{-1}$	$9.977 \cdot 10^{-1}$	$9.994 \cdot 10^{-1}$
0.5	N = 1	$4.749 \cdot 10^{-4}$	$2.372 \cdot 10^{-3}$	$4.737 \cdot 10^{-3}$	$2.340 \cdot 10^{-2}$	$4.597 \cdot 10^{-2}$	$1.937 \cdot 10^{-1}$	$3.193 \cdot 10^{-1}$	$6.604 \cdot 10^{-1}$	$7.662 \cdot 10^{-1}$
	N = 10	$3.322 \cdot 10^{-3}$	$1.649 \cdot 10^{-2}$	$3.268 \cdot 10^{-2}$	$1.513 \cdot 10^{-1}$	$2.729 \cdot 10^{-1}$	$6.745 \cdot 10^{-1}$	$8.041 \cdot 10^{-1}$	$9.459 \cdot 10^{-1}$	$9.677 \cdot 10^{-1}$
	N = 100	$8.293 \cdot 10^{-3}$	$4.074 \cdot 10^{-2}$	$7.971 \cdot 10^{-2}$	$3.339 \cdot 10^{-1}$	$5.388 \cdot 10^{-1}$	$8.972 \cdot 10^{-1}$	$9.480 \cdot 10^{-1}$	$9.886 \cdot 10^{-1}$	$9.938 \cdot 10^{-1}$
	N = ∞	$9.948 \cdot 10^{-3}$	$4.870 \cdot 10^{-2}$	$9.488 \cdot 10^{-2}$	$3.855 \cdot 10^{-1}$	$6.042 \cdot 10^{-1}$	$9.314 \cdot 10^{-1}$	$9.672 \cdot 10^{-1}$	$9.936 \cdot 10^{-1}$	$9.968 \cdot 10^{-1}$
1.0	N = 1	$2.406 \cdot 10^{-4}$	$1.202 \cdot 10^{-3}$	$2.403 \cdot 10^{-3}$	$1.194 \cdot 10^{-2}$	$2.364 \cdot 10^{-2}$	$1.073 \cdot 10^{-1}$	$1.900 \cdot 10^{-1}$	$4.930 \cdot 10^{-1}$	$6.210 \cdot 10^{-1}$
	N = 10	$1.840 \cdot 10^{-3}$	$9.164 \cdot 10^{-3}$	$1.823 \cdot 10^{-2}$	$8.722 \cdot 10^{-2}$	$1.643 \cdot 10^{-1}$	$5.106 \cdot 10^{-1}$	$6.730 \cdot 10^{-1}$	$8.974 \cdot 10^{-1}$	$9.374 \cdot 10^{-1}$
	N = 100	$5.491 \cdot 10^{-3}$	$2.713 \cdot 10^{-2}$	$5.343 \cdot 10^{-2}$	$2.562 \cdot 10^{-1}$	$4.055 \cdot 10^{-1}$	$8.111 \cdot 10^{-1}$	$9.017 \cdot 10^{-1}$	$9.775 \cdot 10^{-1}$	$9.877 \cdot 10^{-1}$
	N = ∞	$7.044 \cdot 10^{-3}$	$3.468 \cdot 10^{-2}$	$6.802 \cdot 10^{-2}$	$3.216 \cdot 10^{-1}$	$4.885 \cdot 10^{-1}$	$8.749 \cdot 10^{-1}$	$9.379 \cdot 10^{-1}$	$9.873 \cdot 10^{-1}$	$9.937 \cdot 10^{-1}$
5.0	N = 1	$4.894 \cdot 10^{-5}$	$2.446 \cdot 10^{-4}$	$4.893 \cdot 10^{-4}$	$2.440 \cdot 10^{-3}$	$4.864 \cdot 10^{-3}$	$2.351 \cdot 10^{-2}$	$4.483 \cdot 10^{-2}$	$1.628 \cdot 10^{-1}$	$2.468 \cdot 10^{-1}$
	N = 10	$4.295 \cdot 10^{-4}$	$2.148 \cdot 10^{-3}$	$4.282 \cdot 10^{-3}$	$2.113 \cdot 10^{-2}$	$4.151 \cdot 10^{-2}$	$1.770 \cdot 10^{-1}$	$2.951 \cdot 10^{-1}$	$6.370 \cdot 10^{-1}$	$7.500 \cdot 10^{-1}$
	N = 100	$1.930 \cdot 10^{-3}$	$9.600 \cdot 10^{-3}$	$1.907 \cdot 10^{-2}$	$9.031 \cdot 10^{-2}$	$1.685 \cdot 10^{-1}$	$5.101 \cdot 10^{-1}$	$6.681 \cdot 10^{-1}$	$8.987 \cdot 10^{-1}$	$9.420 \cdot 10^{-1}$
	N = ∞	$3.155 \cdot 10^{-3}$	$1.564 \cdot 10^{-2}$	$3.095 \cdot 10^{-2}$	$1.419 \cdot 10^{-1}$	$2.553 \cdot 10^{-1}$	$6.451 \cdot 10^{-1}$	$7.773 \cdot 10^{-1}$	$9.417 \cdot 10^{-1}$	$9.696 \cdot 10^{-1}$
10.0	N = 1	$2.455 \cdot 10^{-5}$	$1.228 \cdot 10^{-4}$	$2.445 \cdot 10^{-4}$	$1.225 \cdot 10^{-3}$	$2.444 \cdot 10^{-3}$	$1.190 \cdot 10^{-2}$	$2.293 \cdot 10^{-2}$	$8.864 \cdot 10^{-2}$	$1.408 \cdot 10^{-1}$
	N = 10	$2.234 \cdot 10^{-4}$	$1.111 \cdot 10^{-3}$	$2.230 \cdot 10^{-3}$	$1.106 \cdot 10^{-2}$	$1.196 \cdot 10^{-2}$	$9.871 \cdot 10^{-2}$	$1.749 \cdot 10^{-1}$	$4.683 \cdot 10^{-1}$	$6.004 \cdot 10^{-1}$
	N = 100	$1.175 \cdot 10^{-3}$	$5.854 \cdot 10^{-3}$	$1.165 \cdot 10^{-2}$	$5.600 \cdot 10^{-2}$	$1.065 \cdot 10^{-1}$	$3.644 \cdot 10^{-1}$	$5.188 \cdot 10^{-1}$	$8.192 \cdot 10^{-1}$	$8.914 \cdot 10^{-1}$
	N = ∞	$2.231 \cdot 10^{-3}$	$1.108 \cdot 10^{-2}$	$2.191 \cdot 10^{-2}$	$1.011 \cdot 10^{-1}$	$1.868 \cdot 10^{-1}$	$5.200 \cdot 10^{-1}$	$6.637 \cdot 10^{-1}$	$8.936 \cdot 10^{-1}$	$9.421 \cdot 10^{-1}$
50.0	N = 1	$4.930 \cdot 10^{-6}$	$2.464 \cdot 10^{-5}$	$4.928 \cdot 10^{-5}$	$2.461 \cdot 10^{-4}$	$4.912 \cdot 10^{-4}$	$2.407 \cdot 10^{-3}$	$4.676 \cdot 10^{-3}$	$1.908 \cdot 10^{-2}$	$3.173 \cdot 10^{-2}$
	N = 10	$4.720 \cdot 10^{-5}$	$2.359 \cdot 10^{-4}$	$4.716 \cdot 10^{-4}$	$2.349 \cdot 10^{-3}$	$4.674 \cdot 10^{-3}$	$2.230 \cdot 10^{-2}$	$4.213 \cdot 10^{-2}$	$1.521 \cdot 10^{-1}$	$2.323 \cdot 10^{-1}$
	N = 100	$3.311 \cdot 10^{-4}$	$1.651 \cdot 10^{-3}$	$3.297 \cdot 10^{-3}$	$1.615 \cdot 10^{-2}$	$3.148 \cdot 10^{-2}$	$1.284 \cdot 10^{-1}$	$2.114 \cdot 10^{-1}$	$5.025 \cdot 10^{-1}$	$6.365 \cdot 10^{-1}$
	N = ∞	$9.985 \cdot 10^{-4}$	$4.964 \cdot 10^{-3}$	$9.858 \cdot 10^{-3}$	$4.655 \cdot 10^{-2}$	$8.677 \cdot 10^{-2}$	$2.714 \cdot 10^{-1}$	$3.819 \cdot 10^{-1}$	$6.757 \cdot 10^{-1}$	$7.593 \cdot 10^{-1}$
100.0	N = 1	$2.466 \cdot 10^{-6}$	$1.233 \cdot 10^{-5}$	$2.466 \cdot 10^{-5}$	$1.231 \cdot 10^{-4}$	$2.458 \cdot 10^{-4}$	$1.205 \cdot 10^{-3}$	$2.344 \cdot 10^{-3}$	$9.634 \cdot 10^{-3}$	$1.612 \cdot 10^{-2}$
	N = 10	$2.391 \cdot 10^{-5}$	$1.195 \cdot 10^{-4}$	$2.390 \cdot 10^{-4}$	$1.191 \cdot 10^{-3}$	$2.373 \cdot 10^{-3}$	$1.143 \cdot 10^{-2}$	$2.182 \cdot 10^{-2}$	$8.722 \cdot 10^{-2}$	$1.327 \cdot 10^{-1}$
	N = 100	$1.833 \cdot 10^{-4}$	$9.150 \cdot 10^{-4}$	$1.827 \cdot 10^{-3}$	$9.093 \cdot 10^{-3}$	$1.763 \cdot 10^{-2}$	$7.526 \cdot 10^{-2}$	$1.290 \cdot 10^{-1}$	$3.513 \cdot 10^{-1}$	$4.800 \cdot 10^{-1}$
	N = ∞	$7.061 \cdot 10^{-4}$	$3.511 \cdot 10^{-3}$	$6.975 \cdot 10^{-3}$	$3.303 \cdot 10^{-2}$	$6.176 \cdot 10^{-2}$	$1.984 \cdot 10^{-1}$	$2.941 \cdot 10^{-1}$	$5.499 \cdot 10^{-1}$	$6.766 \cdot 10^{-1}$

TABLE 7

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_w - T_s)$ for various Biot (B) and Peclet (P) numbers, and for various precursory cooling magnitude parameters N. The influenced solid length parameter is $b = 0.5$.

B \ P		0.01	0.05	0.1	0.5	1.0	5.0	10.0	50.0	100.0
0.01	$N=1$	$5.357\cdot10^{-2}$	$2.394\cdot10^{-1}$	$4.161\cdot10^{-1}$	$8.644\cdot10^{-1}$	$9.416\cdot10^{-1}$	$9.891\cdot10^{-1}$	$9.937\cdot10^{-1}$	$9.978\cdot10^{-1}$	$9.985\cdot10^{-1}$
	$N=10$	$6.643\cdot10^{-2}$	$2.896\cdot10^{-1}$	$4.893\cdot10^{-1}$	$9.218\cdot10^{-1}$	$9.757\cdot10^{-1}$	$9.975\cdot10^{-1}$	$9.987\cdot10^{-1}$	$9.996\cdot10^{-1}$	$9.998\cdot10^{-1}$
	$N=100$	$6.807\cdot10^{-2}$	$2.958\cdot10^{-1}$	$4.988\cdot10^{-1}$	$9.290\cdot10^{-1}$	$9.793\cdot10^{-1}$	$9.983\cdot10^{-1}$	$9.992\cdot10^{-1}$	$9.998\cdot10^{-1}$	$9.999\cdot10^{-1}$
	$N=\infty$	$6.825\cdot10^{-2}$	$2.965\cdot10^{-1}$	$4.998\cdot10^{-1}$	$9.297\cdot10^{-1}$	$9.797\cdot10^{-1}$	$9.984\cdot10^{-1}$	$9.993\cdot10^{-1}$	$9.999\cdot10^{-1}$	$9.999\cdot10^{-1}$
0.05	$N=1$	$1.912\cdot10^{-2}$	$9.187\cdot10^{-2}$	$1.747\cdot10^{-1}$	$5.819\cdot10^{-1}$	$7.672\cdot10^{-1}$	$9.476\cdot10^{-1}$	$9.695\cdot10^{-1}$	$9.889\cdot10^{-1}$	$9.924\cdot10^{-1}$
	$N=10$	$2.328\cdot10^{-2}$	$1.379\cdot10^{-1}$	$2.560\cdot10^{-1}$	$7.398\cdot10^{-1}$	$8.949\cdot10^{-1}$	$9.877\cdot10^{-1}$	$9.938\cdot10^{-1}$	$9.983\cdot10^{-1}$	$9.989\cdot10^{-1}$
	$N=100$	$3.093\cdot10^{-2}$	$1.452\cdot10^{-1}$	$2.684\cdot10^{-1}$	$7.605\cdot10^{-1}$	$9.101\cdot10^{-1}$	$9.918\cdot10^{-1}$	$9.963\cdot10^{-1}$	$9.992\cdot10^{-1}$	$9.996\cdot10^{-1}$
	$N=\infty$	$3.112\cdot10^{-2}$	$1.460\cdot10^{-1}$	$2.698\cdot10^{-1}$	$7.628\cdot10^{-1}$	$9.118\cdot10^{-1}$	$9.923\cdot10^{-1}$	$9.965\cdot10^{-1}$	$9.994\cdot10^{-1}$	$9.997\cdot10^{-1}$
0.1	$N=1$	$1.467\cdot10^{-2}$	$5.693\cdot10^{-2}$	$1.104\cdot10^{-1}$	$4.254\cdot10^{-1}$	$6.274\cdot10^{-1}$	$9.005\cdot10^{-1}$	$9.407\cdot10^{-1}$	$9.780\cdot10^{-1}$	$9.849\cdot10^{-1}$
	$N=10$	$2.079\cdot10^{-2}$	$9.733\cdot10^{-2}$	$1.848\cdot10^{-1}$	$6.497\cdot10^{-1}$	$8.184\cdot10^{-1}$	$9.757\cdot10^{-1}$	$9.876\cdot10^{-1}$	$9.966\cdot10^{-1}$	$9.979\cdot10^{-1}$
	$N=100$	$2.191\cdot10^{-2}$	$1.048\cdot10^{-1}$	$1.981\cdot10^{-1}$	$6.489\cdot10^{-1}$	$8.442\cdot10^{-1}$	$9.839\cdot10^{-1}$	$9.926\cdot10^{-1}$	$9.985\cdot10^{-1}$	$9.992\cdot10^{-1}$
	$N=\infty$	$2.211\cdot10^{-2}$	$1.057\cdot10^{-1}$	$1.997\cdot10^{-1}$	$6.521\cdot10^{-1}$	$8.474\cdot10^{-1}$	$9.848\cdot10^{-1}$	$9.931\cdot10^{-1}$	$9.987\cdot10^{-1}$	$9.994\cdot10^{-1}$
0.5	$N=1$	$3.275\cdot10^{-3}$	$1.625\cdot10^{-2}$	$3.220\cdot10^{-2}$	$1.486\cdot10^{-1}$	$2.669\cdot10^{-1}$	$6.449\cdot10^{-1}$	$7.606\cdot10^{-1}$	$8.989\cdot10^{-1}$	$9.285\cdot10^{-1}$
	$N=10$	$8.264\cdot10^{-3}$	$4.060\cdot10^{-2}$	$7.942\cdot10^{-2}$	$3.375\cdot10^{-1}$	$5.365\cdot10^{-1}$	$8.917\cdot10^{-1}$	$9.427\cdot10^{-1}$	$9.832\cdot10^{-1}$	$9.896\cdot10^{-1}$
	$N=100$	$9.749\cdot10^{-3}$	$4.775\cdot10^{-2}$	$9.307\cdot10^{-2}$	$3.795\cdot10^{-1}$	$5.967\cdot10^{-1}$	$9.272\cdot10^{-1}$	$9.646\cdot10^{-1}$	$9.925\cdot10^{-1}$	$9.964\cdot10^{-1}$
	$N=\infty$	$9.948\cdot10^{-3}$	$4.870\cdot10^{-2}$	$9.488\cdot10^{-2}$	$3.855\cdot10^{-1}$	$6.041\cdot10^{-1}$	$9.314\cdot10^{-1}$	$9.672\cdot10^{-1}$	$9.936\cdot10^{-1}$	$9.968\cdot10^{-1}$
1.0	$N=1$	$1.803\cdot10^{-3}$	$8.974\cdot10^{-3}$	$1.785\cdot10^{-2}$	$8.504\cdot10^{-2}$	$1.593\cdot10^{-1}$	$4.767\cdot10^{-1}$	$6.138\cdot10^{-1}$	$8.163\cdot10^{-1}$	$8.673\cdot10^{-1}$
	$N=10$	$5.457\cdot10^{-3}$	$2.696\cdot10^{-2}$	$5.309\cdot10^{-2}$	$2.346\cdot10^{-1}$	$4.024\cdot10^{-1}$	$8.089\cdot10^{-1}$	$8.909\cdot10^{-1}$	$9.671\cdot10^{-1}$	$9.794\cdot10^{-1}$
	$N=100$	$6.945\cdot10^{-3}$	$3.371\cdot10^{-2}$	$6.646\cdot10^{-2}$	$2.847\cdot10^{-1}$	$4.748\cdot10^{-1}$	$8.695\cdot10^{-1}$	$9.330\cdot10^{-1}$	$9.853\cdot10^{-1}$	$9.922\cdot10^{-1}$
	$N=\infty$	$7.041\cdot10^{-3}$	$3.468\cdot10^{-2}$	$6.802\cdot10^{-2}$	$2.916\cdot10^{-1}$	$4.845\cdot10^{-1}$	$8.768\cdot10^{-1}$	$9.379\cdot10^{-1}$	$9.875\cdot10^{-1}$	$9.927\cdot10^{-1}$
5.0	$N=1$	$4.123\cdot10^{-4}$	$2.058\cdot10^{-3}$	$4.106\cdot10^{-3}$	$2.015\cdot10^{-2}$	$3.928\cdot10^{-2}$	$1.559\cdot10^{-1}$	$2.434\cdot10^{-1}$	$4.706\cdot10^{-1}$	$5.665\cdot10^{-1}$
	$N=10$	$1.895\cdot10^{-3}$	$9.422\cdot10^{-3}$	$1.874\cdot10^{-2}$	$8.846\cdot10^{-2}$	$1.647\cdot10^{-1}$	$4.910\cdot10^{-1}$	$6.365\cdot10^{-1}$	$8.506\cdot10^{-1}$	$9.052\cdot10^{-1}$
	$N=100$	$2.959\cdot10^{-3}$	$1.467\cdot10^{-2}$	$2.905\cdot10^{-2}$	$1.338\cdot10^{-1}$	$2.420\cdot10^{-1}$	$6.254\cdot10^{-1}$	$7.604\cdot10^{-1}$	$9.324\cdot10^{-1}$	$9.623\cdot10^{-1}$
	$N=\infty$	$3.155\cdot10^{-3}$	$1.564\cdot10^{-2}$	$3.095\cdot10^{-2}$	$1.419\cdot10^{-1}$	$2.557\cdot10^{-1}$	$6.451\cdot10^{-1}$	$7.773\cdot10^{-1}$	$9.417\cdot10^{-1}$	$9.696\cdot10^{-1}$
10.0	$N=1$	$2.124\cdot10^{-4}$	$1.061\cdot10^{-3}$	$2.117\cdot10^{-3}$	$1.043\cdot10^{-2}$	$2.046\cdot10^{-2}$	$8.505\cdot10^{-2}$	$1.381\cdot10^{-1}$	$3.078\cdot10^{-1}$	$3.952\cdot10^{-1}$
	$N=10$	$1.144\cdot10^{-3}$	$5.697\cdot10^{-3}$	$1.134\cdot10^{-2}$	$5.455\cdot10^{-2}$	$1.030\cdot10^{-1}$	$3.441\cdot10^{-1}$	$4.807\cdot10^{-1}$	$7.507\cdot10^{-1}$	$8.276\cdot10^{-1}$
	$N=100$	$2.038\cdot10^{-3}$	$1.012\cdot10^{-2}$	$2.008\cdot10^{-2}$	$9.388\cdot10^{-2}$	$1.728\cdot10^{-1}$	$4.947\cdot10^{-1}$	$6.394\cdot10^{-1}$	$8.769\cdot10^{-1}$	$9.293\cdot10^{-1}$
	$N=\infty$	$2.231\cdot10^{-3}$	$1.108\cdot10^{-2}$	$2.196\cdot10^{-2}$	$1.021\cdot10^{-1}$	$1.868\cdot10^{-1}$	$5.300\cdot10^{-1}$	$6.637\cdot10^{-1}$	$8.936\cdot10^{-1}$	$9.421\cdot10^{-1}$
50.0	$N=1$	$4.396\cdot10^{-5}$	$2.195\cdot10^{-4}$	$4.385\cdot10^{-4}$	$2.167\cdot10^{-3}$	$4.268\cdot10^{-3}$	$1.847\cdot10^{-2}$	$3.126\cdot10^{-2}$	$8.176\cdot10^{-2}$	$1.156\cdot10^{-1}$
	$N=10$	$3.148\cdot10^{-4}$	$1.570\cdot10^{-3}$	$3.131\cdot10^{-3}$	$1.527\cdot10^{-2}$	$2.958\cdot10^{-2}$	$1.147\cdot10^{-1}$	$1.800\cdot10^{-1}$	$3.913\cdot10^{-1}$	$4.989\cdot10^{-1}$
	$N=100$	$8.204\cdot10^{-4}$	$4.082\cdot10^{-3}$	$8.115\cdot10^{-3}$	$3.964\cdot10^{-2}$	$7.334\cdot10^{-2}$	$2.395\cdot10^{-1}$	$3.433\cdot10^{-1}$	$6.296\cdot10^{-1}$	$7.449\cdot10^{-1}$
	$N=\infty$	$9.985\cdot10^{-4}$	$4.964\cdot10^{-3}$	$9.858\cdot10^{-3}$	$4.655\cdot10^{-2}$	$8.677\cdot10^{-2}$	$2.734\cdot10^{-1}$	$3.819\cdot10^{-1}$	$6.753\cdot10^{-1}$	$7.893\cdot10^{-1}$
100.0	$N=1$	$2.211\cdot10^{-5}$	$1.105\cdot10^{-4}$	$2.206\cdot10^{-4}$	$1.091\cdot10^{-3}$	$2.149\cdot10^{-3}$	$9.351\cdot10^{-3}$	$1.591\cdot10^{-2}$	$4.265\cdot10^{-2}$	$6.128\cdot10^{-2}$
	$N=10$	$1.735\cdot10^{-4}$	$8.608\cdot10^{-4}$	$1.717\cdot10^{-3}$	$8.403\cdot10^{-3}$	$1.637\cdot10^{-2}$	$6.566\cdot10^{-2}$	$1.058\cdot10^{-1}$	$2.512\cdot10^{-1}$	$3.379\cdot10^{-1}$
	$N=100$	$5.393\cdot10^{-3}$	$2.685\cdot10^{-3}$	$5.340\cdot10^{-3}$	$2.555\cdot10^{-2}$	$4.835\cdot10^{-2}$	$1.650\cdot10^{-1}$	$2.431\cdot10^{-1}$	$4.914\cdot10^{-1}$	$6.150\cdot10^{-1}$
	$N=\infty$	$7.061\cdot10^{-4}$	$3.511\cdot10^{-3}$	$6.975\cdot10^{-3}$	$3.303\cdot10^{-2}$	$6.176\cdot10^{-2}$	$1.984\cdot10^{-1}$	$2.841\cdot10^{-1}$	$5.493\cdot10^{-1}$	$6.766\cdot10^{-1}$

Results for the fuel-and-cladding model

The rewetting temperature $\theta_0 \equiv (T_0 - T_s)/(T_W - T_s)$ for various model parameters: Peclet number P, wet side Biot numbers B_0, gap Biot number B_g, dimensionless radius of the fuel R, cladding to fuel ratio of thermal conductivities Γ and fuel to cladding ratio of Peclet numbers Q/P.

TABLE 8

Hollow cylinder with an insulated inner core

$R = 0.9 \qquad B_g = 0$

P / B_0	0.10	0.50	1.00	5.00	10.00	50.00
0.10	0.09283	0.3827	0.6088	0.9597	0.9883	0.9987
0.50	0.04264	0.1953	0.3505	0.8437	0.9460	0.9936
1.00	0.03034	0.1425	0.2637	0.7511	0.9015	0.9873
5.00	0.01368	0.06643	0.1281	0.4807	0.6976	0.9417
10.00	0.009693	0.04741	0.09225	0.3715	0.5774	0.8936
50.00	0.004345	0.02145	0.04220	0.1856	0.3180	0.6753

TABLE 9

$R = 0.9 \qquad B_g = 0.1 \qquad Q/P = 0.1$

B_0	P	0.10	0.50	1.00	5.00	10.00	50.00
	Γ=0.1	0.3955	0.7235	0.7919	0.9632	0.9885	0.9987
0.10	Γ=1.0	0.1581	0.5915	0.7613	0.9632	0.9885	0.9987
	Γ=10.	0.09614	0.4056	0.6452	0.9623	0.9885	0.9987
	Γ=0.1	0.1891	0.4343	0.5176	0.8542	0.9471	0.9936
0.50	Γ=1.0	0.07265	0.3211	0.4825	0.8539	0.9470	0.9936
	Γ=10.	0.04410	0.2071	0.3764	0.8514	0.9469	0.9936
	Γ=0.1	0.1353	0.3266	0.4004	0.7646	0.9032	0.9873
1.00	Γ=1.0	0.05171	0.2367	0.3704	0.7642	0.9032	0.9873
	Γ=10.	0.03138	0.1511	0.2839	0.7610	0.9030	0.9873
	Γ=0.1	0.06130	0.1564	0.1997	0.4943	0.7008	0.9417
5.00	Γ=1.0	0.02332	0.1113	0.1833	0.4939	0.7007	0.9417
	Γ=10.	0.01415	0.07045	0.1382	0.4906	0.7003	0.9417
	Γ=0.1	0.04346	0.1120	0.1444	0.3829	0.5806	0.8936
10.00	Γ=1.0	0.01652	0.07952	0.1324	0.3826	0.5806	0.8936
	Γ=10.	0.01002	0.05029	0.09957	0.3798	0.5801	0.8936
	Γ=0.1	0.01949	0.05082	0.06627	0.1917	0.3201	0.6753
50.00	Γ=1.0	0.007407	0.03601	0.06072	0.1916	0.3201	0.6753
	Γ=10.	0.004492	0.02275	0.04556	0.1901	0.3198	0.6753

TABLE 10

$R = 0.9 \qquad B_g = 0.1 \qquad Q/P = 1$

B_0	P	0.10	0.50	1.00	5.00	10.00	50.00
	Γ=0.1	0.4294	0.7179	0.7904	0.9632	0.9885	0.9987
0.10	Γ=1.0	0.2017	0.5985	0.7552	0.9631	0.9885	0.9987
	Γ=10.	0.1112	0.4363	0.6608	0.9622	0.9885	0.9987
	Γ=0.1	0.2144	0.4297	0.5160	0.8542	0.9471	0.9936
0.50	Γ=1.0	0.09484	0.3329	0.4780	0.8538	0.9470	0.9936
	Γ=10.	0.05131	0.2271	0.3922	0.8512	0.9469	0.9936
	Γ=0.1	0.1546	0.3229	0.3990	0.7646	0.9032	0.9873
1.00	Γ=1.0	0.06774	0.2467	0.3670	0.7641	0.9032	0.9873
	Γ=10.	0.03654	0.1663	0.2970	0.7607	0.9030	0.9873
	Γ=0.1	0.07046	0.1546	0.1990	0.4943	0.7008	0.9417
5.00	Γ=1.0	0.03065	0.1166	0.1817	0.4938	0.7007	0.9417
	Γ=10.	0.01649	0.07781	0.1452	0.4904	0.7002	0.9417
	Γ=0.1	0.04999	0.1107	0.1439	0.3829	0.5806	0.8936
10.00	Γ=1.0	0.02172	0.08337	0.1313	0.3825	0.5805	0.8936
	Γ=10.	0.01168	0.05557	0.1047	0.3796	0.5801	0.8936
	Γ=0.1	0.02243	0.05023	0.06603	0.1917	0.3201	0.6753
50.00	Γ=1.0	0.009740	0.03777	0.06020	0.1915	0.3201	0.6753
	Γ=10.	0.005237	0.02514	0.04794	0.1900	0.3198	0.6753

TABLE 11

$R = 0.9 \quad B_g = 0.1 \quad Q/P = 10$

B_0	P	0.10	0.50	1.00	5.00	10.00	50.00
0.10	Γ=0.1	0.6882	0.7417	0.7952	0.9632	0.9885	0.9987
	Γ=1.0	0.6159	0.7332	0.7923	0.9632	0.9885	0.9987
	Γ=10.	0.3338	0.6636	0.7669	0.9630	0.9885	0.9987
0.50	Γ=0.1	0.3930	0.4528	0.5215	0.8542	0.9471	0.9936
	Γ=1.0	0.3355	0.4448	0.5182	0.8541	0.9471	0.9936
	Γ=10.	0.1629	0.3842	0.4907	0.8535	0.9470	0.9936
1.00	Γ=0.1	0.2902	0.3417	0.4038	0.7647	0.9032	0.9873
	Γ=1.0	0.2455	0.3353	0.4010	0.7646	0.9032	0.9873
	Γ=10.	0.1171	0.2871	0.3779	0.7638	0.9032	0.9873
5.00	Γ=0.1	0.1351	0.1642	0.2016	0.4943	0.7008	0.9417
	Γ=1.0	0.1134	0.1610	0.2001	0.4942	0.7007	0.9417
	Γ=10.	0.05330	0.1368	0.1877	0.4935	0.7006	0.9417
10.00	Γ=0.1	0.09610	0.1177	0.1458	0.3830	0.5806	0.8936
	Γ=1.0	0.08062	0.1153	0.1447	0.3829	0.5806	0.8936
	Γ=10.	0.03781	0.09792	0.1357	0.3822	0.5805	0.8936
50.00	Γ=0.1	0.04322	0.05343	0.06691	0.1918	0.3201	0.6753
	Γ=1.0	0.03623	0.05234	0.06640	0.1917	0.3201	0.6753
	Γ=10.	0.01696	0.04440	0.06223	0.1914	0.3201	0.6753

TABLE 12

$R = 0.9 \quad B_g = \infty \quad Q/P = 0.1$

B_0	P	0.10	0.50	1.00	5.00	10.00	50.00
0.10	Γ=0.1	0.4163	0.9300	0.9774	0.9941	0.9957	0.9987
	Γ=1.0	0.1568	0.6207	0.8507	0.9851	0.9931	0.9987
	Γ=10.	0.09587	0.4020	0.6405	0.9660	0.9893	0.9987
0.50	Γ=0.1	0.1982	0.7273	0.8969	0.9718	0.9792	0.9936
	Γ=1.0	0.07162	0.3310	0.5723	0.9317	0.9670	0.9936
	Γ=10.	0.04393	0.2042	0.3704	0.8626	0.9502	0.9936
1.00	Γ=0.1	0.1416	0.5876	0.8154	0.9459	0.9597	0.9873
	Γ=1.0	0.05088	0.2420	0.4414	0.8764	0.9375	0.9873
	Γ=10.	0.03124	0.1488	0.2785	0.7754	0.9084	0.9873
5.00	Γ=0.1	0.06403	0.2999	0.5153	0.7950	0.8385	0.9417
	Γ=1.0	0.02290	0.1126	0.2177	0.6379	0.7749	0.9417
	Γ=10.	0.01408	0.06923	0.1351	0.5047	0.7105	0.9417
10.00	Γ=0.1	0.04537	0.2165	0.3880	0.6815	0.7388	0.8936
	Γ=1.0	0.01622	0.08031	0.1570	0.5104	0.6602	0.8936
	Γ=10.	0.009975	0.04940	0.09728	0.3915	0.5904	0.8936
50.00	Γ=0.1	0.02034	0.09882	0.1842	0.3895	0.4485	0.6753
	Γ=1.0	0.007267	0.03630	0.07188	0.2640	0.3779	0.6753
	Γ=10.	0.004471	0.02234	0.04449	0.1962	0.3267	0.6753

TABLE 13

$R = 0.9 \quad B_y = \infty \quad Q/P = 1$

B_0	P	0.10	0.50	1.00	5.00	10.00	50.00
	Γ=0.1	0.4850	0.9205	0.9734	0.9941	0.9957	0.9987
0.10	Γ=1.0	0.1997	0.6522	0.8471	0.9848	0.9931	0.9987
	Γ=10.	0.1114	0.4403	0.6705	0.9668	0.9894	0.9987
	Γ=0.1	0.2595	0.7393	0.8875	0.9714	0.9791	0.9936
0.50	Γ=1.0	0.09488	0.3855	0.6042	0.9314	0.9672	0.9936
	Γ=10.	0.05146	0.2303	0.4021	0.8657	0.9507	0.9936
	Γ=0.1	0.1914	0.6238	0.8094	0.9453	0.9595	0.9873
1.00	Γ=1.0	0.06802	0.2916	0.4845	0.8768	0.9379	0.9873
	Γ=10.	0.03666	0.1690	0.3056	0.7802	0.9094	0.9873
	Γ=0.1	0.09023	0.3556	0.5405	0.7940	0.8382	0.9417
5.00	Γ=1.0	0.03095	0.1419	0.2553	0.6451	0.7773	0.9417
	Γ=10.	0.01655	0.07922	0.1501	0.5119	0.7129	0.9417
	Γ=0.1	0.06448	0.2646	0.4194	0.6809	0.7384	0.8936
10.00	Γ=1.0	0.02196	0.1021	0.1868	0.5200	0.6637	0.8936
	Γ=10.	0.01173	0.05660	0.1084	0.3982	0.5932	0.8936
	Γ=0.1	0.02915	0.1248	0.2072	0.3898	0.4484	0.6753
50.00	Γ=1.0	0.009858	0.04655	0.08677	0.2724	0.3819	0.6753
	Γ=10.	0.005258	0.02563	0.04967	0.2003	0.3289	0.6753

TABLE 14

$R = 0.9 \quad B_y = \infty \quad Q/P = 10$

B_0	P	0.10	0.50	1.00	5.00	10.00	50.00
	Γ=0.1	0.9722	0.9930	0.9940	0.9953	0.9961	0.9987
0.10	Γ=1.0	0.8127	0.9743	0.9849	0.9934	0.9954	0.9987
	Γ=10.	0.3580	0.7950	0.8920	0.9807	0.9918	0.9987
	Γ=0.1	0.8831	0.9664	0.9712	0.9771	0.9808	0.9936
0.50	Γ=1.0	0.5532	0.8900	0.9312	0.9686	0.9775	0.9936
	Γ=10.	0.1790	0.5264	0.6759	0.9151	0.9614	0.9936
	Γ=0.1	0.8029	0.9361	0.9448	0.9558	0.9628	0.9873
1.00	Γ=1.0	0.4353	0.8119	0.8757	0.9401	0.9566	0.9873
	Γ=10.	0.1296	0.4096	0.5531	0.8513	0.9277	0.9873
	Γ=0.1	0.5305	0.7664	0.7908	0.8252	0.8488	0.9417
5.00	Γ=1.0	0.2217	0.5380	0.6366	0.7801	0.8294	0.9417
	Γ=10.	0.05953	0.2053	0.2983	0.6043	0.7526	0.9417
	Γ=0.1	0.4096	0.6456	0.6751	0.7199	0.7525	0.8936
10.00	Γ=1.0	0.1606	0.4151	0.5075	0.6644	0.7271	0.8936
	Γ=10.	0.04230	0.1484	0.2191	0.4812	0.6359	0.8936
	Γ=0.1	0.2005	0.3549	0.3802	0.4254	0.4628	0.6753
50.00	Γ=1.0	0.07351	0.2029	0.2585	0.3763	0.4376	0.6753
	Γ=10.	0.01901	0.06791	0.1021	0.2485	0.3603	0.6753

5 The use of transformations for 1-D models

There are cases where the use of certain transformations is superior to the original formulation. For using the Isotherm Migration method see, e.g. Durack and Wendroff (1977) and Gurcak et al. (1980).
In the following, examples for 1-D models will be given. Suppose we would like to solve the following model

WET SIDE

$$\frac{d^2\theta}{dx^2} - P\frac{d\theta}{dx} - B\theta^n = 0, \qquad 0 < x < \infty \tag{17}$$

where here the wet side heat transfer coefficient is temperature dependent and

$$\theta = \frac{T - T_s}{T_w - T_s}, \qquad P = \frac{u\delta}{\alpha}$$

DRY SIDE

$$\frac{d^2\theta}{dx^2} - P\frac{d\theta}{dx} = 0, \qquad -\infty < x < 0 \tag{18}$$

BOUNDARY CONDITIONS

$$\theta \to 0 \quad \text{as} \quad x \to \infty \tag{19}$$

$$\theta = \theta_0 \quad \text{at} \quad x = 0 \tag{20}$$

$$\theta \to 1 \quad \text{as} \quad x \to -\infty \tag{21}$$

For $n = 1$ this is the model of Yamanouchi (1968). Let us outline the solution, since it will serve for demonstration purposes. The temperature distribution on the wet side is

$$\theta = \theta_0 e^{\alpha x} \tag{22}$$

with

$$\alpha = \frac{P}{2} - (\frac{P^2}{4} + B)^{1/2}$$

whereas the temperature distribution on the dry side is

$$\theta = 1 + (\theta_0 - 1)e^{Px} \tag{23}$$

Also we have

$$\left[\frac{d\theta(0)}{dx}\right]_{wet} = \alpha\theta \tag{24}$$

$$\left[\frac{d\theta(0)}{dx}\right]_{dry} = P(\theta_0 - 1) \tag{25}$$

Equating these gradients and eliminating P yields

$$P = B^{1/2}\frac{\theta_0}{(1-\theta_0)^{1/2}} \tag{26}$$

Suppose we would like to solve Yamanouchi's model numerically, with the formulation given by Eqs. (17)-(21). Let us pick an example where $B = 0.1$ and $\theta_0 = 0.270$. Then the analytical solution gives $P = 0.100$. The solution procedure was to assume a value for P and calculate the temperature gradient at $x = 0$, since from the dry side it is given by Eq. (25). With the given value of the temperature and its derivative at $x = 0$, Eq. (17) was integrated numerically for the wet side using the DGEAR subroutine from the IMSL library. The correct guess for P is the one which gives $\theta = 0$ at '$x = \infty$'. Table 15 shows what happens if we guess the correct (calculated analytically) value for P. Here Y denotes the temperature and YP its axial derivative. It can be seen that at first the temperature decreases, but later it increases and reaches a value of, e.g. 511 at $x = 100$. Tables 16 and 17 show how sensitive the solution is to the initial guess. The initial guess which is selected in these cases is $\pm 1\%$ of the correct value. It can be seen that the solution is very sensitive to the initial guess for P. Even if we decided that the correct solution gives a minimum for the temperature at a big value of x, say, at $x = 100$, the strategy will not succeed because of the noted sensitivity. We conclude that the original formulation given in Eqs. (17)-(21) is not suitable for a numerical solution.

TABLE 15

P= .10 B= .10 THO= .270

X	Y	YP
.0	.270D+00	-.730D-01
1.0	.206D+00	-.557D-01
2.0	.157D+00	-.425D-01
3.0	.120D+00	-.325D-01
4.0	.917D-01	-.248D-01
5.0	.700D-01	-.189D-01
6.0	.534D-01	-.144D-01
7.0	.408D-01	-.110D-01
8.0	.311D-01	-.841D-02
9.0	.237D-01	-.641D-02
10.0	.181D-01	-.490D-02
11.0	.138D-01	-.374D-02
12.0	.106D-01	-.285D-02
13.0	.806D-02	-.218D-02
14.0	.615D-02	-.166D-02
15.0	.469D-02	-.127D-02
16.0	.358D-02	-.968D-03
17.0	.273D-02	-.739D-03
18.0	.209D-02	-.564D-03
19.0	.159D-02	-.430D-03
20.0	.122D-02	-.329D-03
21.0	.929D-03	-.251D-03
22.0	.710D-03	-.192D-03
23.0	.543D-03	-.147D-03
24.0	.414D-03	-.112D-03
25.0	.316D-03	-.853D-04
26.0	.241D-03	-.650D-04
27.0	.183D-03	-.495D-04
28.0	.140D-03	-.378D-04
29.0	.107D-03	-.290D-04
30.0	.825D-04	-.223D-04
31.0	.637D-04	-.172D-04
32.0	.489D-04	-.132D-04
33.0	.371D-04	-.100D-04
34.0	.280D-04	-.755D-05
35.0	.209D-04	-.562D-05
36.0	.156D-04	-.417D-05
37.0	.118D-04	-.313D-05
38.0	.922D-05	-.239D-05
39.0	.727D-05	-.183D-05
40.0	.600D-05	-.142D-05
41.0	.520D-05	-.111D-05
42.0	.435D-05	-.756D-06
43.0	.365D-05	-.380D-06
44.0	.365D-05	-.109D-06
45.0	.373D-05	.264D-06
46.0	.440D-05	.657D-06
47.0	.545D-05	.121D-05
48.0	.717D-05	.195D-05

49.0	.962D-05	.304D-05
50.0	.132D-04	.457D-05
51.0	.187D-04	.667D-05
52.0	.267D-04	.965D-05
53.0	.380D-04	.140D-04
54.0	.545D-04	.203D-04
55.0	.788D-04	.295D-04
56.0	.115D-03	.428D-04
57.0	.166D-03	.619D-04
58.0	.242D-03	.898D-04
59.0	.351D-03	.130D-03
60.0	.509D-03	.189D-03
61.0	.738D-03	.273D-03
62.0	.107D-02	.396D-03
63.0	.155D-02	.574D-03
64.0	.225D-02	.832D-03
65.0	.325D-02	.120D-02
66.0	.471D-02	.175D-02
67.0	.683D-02	.253D-02
68.0	.989D-02	.366D-02
69.0	.143D-01	.530D-02
70.0	.207D-01	.768D-02
71.0	.300D-01	.111D-01
72.0	.435D-01	.161D-01
73.0	.630D-01	.233D-01
74.0	.912D-01	.338D-01
75.0	.132D+00	.489D-01
76.0	.191D+00	.708D-01
77.0	.277D+00	.102D+00
78.0	.401D+00	.148D+00
79.0	.580D+00	.215D+00
80.0	.840D+00	.311D+00
81.0	.122D+01	.450D+00
82.0	.176D+01	.652D+00
83.0	.255D+01	.944D+00
84.0	.369D+01	.137D+01
85.0	.535D+01	.198D+01
86.0	.775D+01	.287D+01
87.0	.112D+02	.415D+01
88.0	.162D+02	.601D+01
89.0	.235D+02	.870D+01
90.0	.340D+02	.126D+02
91.0	.493D+02	.182D+02
92.0	.714D+02	.264D+02
93.0	.103D+03	.383D+02
94.0	.150D+03	.554D+02
95.0	.217D+03	.802D+02
96.0	.314D+03	.116D+03
97.0	.454D+03	.168D+03
98.0	.658D+03	.244D+03
99.0	.953D+03	.353D+03
100.0	.138D+04	.511D+03

TABLE 16

P= .101   B= .10   THO= .270

X	Y	YP
.0	.270D+00	-.737D-01
1.0	.205D+00	-.566D-01
2.0	.156D+00	-.437D-01
3.0	.117D+00	-.341D-01
4.0	.865D-01	-.270D-01
5.0	.621D-01	-.221D-01
6.0	.416D-01	.190D-01
7.0	.234D-01	-.176D-01
8.0	.578D-02	-.179D-01
9.0	-.131D-01	-.202D-01
10.0	-.354D-01	-.248D-01
11.0	-.639D-01	-.326D-01
12.0	-.102D+00	-.447D-01
13.0	-.155D+00	-.628D-01
14.0	-.230D+00	-.894D-01
15.0	-.338D+00	-.128D+00
16.0	-.493D+00	-.185D+00
17.0	-.717D+00	-.268D+00
18.0	-.104D+01	-.387D+00
19.0	-.151D+01	-.561D+00
20.0	-.219D+01	-.812D+00
21.0	-.317D+01	-.118D+01
22.0	-.459D+01	-.170D+01
23.0	-.666D+01	-.247D+01
24.0	-.964D+01	-.358D+01
25.0	-.140D+02	-.518D+01
26.0	-.202D+02	-.750D+01
27.0	-.293D+02	-.109D+02
28.0	-.425D+02	-.158D+02
29.0	-.616D+02	-.228D+02
30.0	-.892D+02	-.331D+02
31.0	-.129D+03	-.479D+02
32.0	-.187D+03	-.694D+02
33.0	-.271D+03	-.101D+03
34.0	-.393D+03	-.146D+03
35.0	-.569D+03	-.211D+03
36.0	-.825D+03	-.306D+03
37.0	-.120D+04	-.443D+03
38.0	-.173D+04	-.642D+03
39.0	-.251D+04	-.930D+03
40.0	-.363D+04	-.135D+04
41.0	-.527D+04	-.195D+04
42.0	-.763D+04	-.283D+04
43.0	-.111D+05	-.410D+04
44.0	-.160D+05	-.594D+04
45.0	-.232D+05	-.860D+04
46.0	-.336D+05	-.125D+05
47.0	-.487D+05	-.181D+05
48.0	-.705D+05	-.262D+05

49.0	-.102D+06	-.379D+05
50.0	-.148D+06	-.549D+05
51.0	-.215D+06	-.795D+05
52.0	-.311D+06	-.115D+06
53.0	-.450D+06	-.167D+06
54.0	-.652D+06	-.242D+06
55.0	-.945D+06	-.350D+06
56.0	-.137D+07	-.508D+06
57.0	-.198D+07	-.736D+06
58.0	-.287D+07	-.107D+07
59.0	-.416D+07	-.154D+07
60.0	-.603D+07	-.224D+07
61.0	-.874D+07	-.324D+07
62.0	-.127D+08	-.470D+07
63.0	-.183D+08	-.680D+07
64.0	-.266D+08	-.986D+07
65.0	-.385D+08	-.143D+08
66.0	-.558D+08	-.207D+08
67.0	-.808D+08	-.300D+08
68.0	-.117D+09	-.434D+08
69.0	-.170D+09	-.629D+08
70.0	-.246D+09	-.911D+08
71.0	-.356D+09	-.132D+09
72.0	-.516D+09	-.191D+09
73.0	-.748D+09	-.277D+09
74.0	-.108D+10	-.402D+09
75.0	-.157D+10	-.582D+09
76.0	-.227D+10	-.843D+09
77.0	-.329D+10	-.122D+10
78.0	-.477D+10	-.177D+10
79.0	-.691D+10	-.256D+10
80.0	-.100D+11	-.371D+10
81.0	-.145D+11	-.538D+10
82.0	-.210D+11	-.779D+10
83.0	-.305D+11	-.113D+11
84.0	-.441D+11	-.164D+11
85.0	-.639D+11	-.237D+11
86.0	-.926D+11	-.343D+11
87.0	-.134D+12	-.498D+11
88.0	-.194D+12	-.721D+11
89.0	-.282D+12	-.104D+12
90.0	-.408D+12	-.151D+12
91.0	-.591D+12	-.219D+12
92.0	-.857D+12	-.318D+12
93.0	-.124D+13	-.460D+12
94.0	-.180D+13	-.667D+12
95.0	-.261D+13	-.966D+12
96.0	-.377D+13	-.140D+13
97.0	-.547D+13	-.203D+13
98.0	-.792D+13	-.294D+13
99.0	-.115D+14	-.426D+13
100.0	-.166D+14	-.617D+13

TABLE 17

P= .099 B= .10 THO= .270

X	Y	YP
.0	.270D+00	-.723D-01
1.0	.207D+00	-.548D-01
2.0	.159D+00	-.413D-01
3.0	.123D+00	-.308D-01
4.0	.969D-01	-.225D-01
5.0	.779D-01	-.157D-01
6.0	.651D-01	-.987D-02
7.0	.580D-01	-.447D-02
8.0	.562D-01	.103D-02
9.0	.603D-01	.720D-02
10.0	.711D-01	.148D-01
11.0	.906D-01	.247D-01
12.0	.122D+00	.383D-01
13.0	.169D+00	.574D-01
14.0	.239D+00	.845D-01
15.0	.342D+00	.123D+00
16.0	.492D+00	.179D+00
17.0	.709D+00	.260D+00
18.0	.102D+01	.377D+00
19.0	.148D+01	.546D+00
20.0	.214D+01	.791D+00
21.0	.310D+01	.114D+01
22.0	.448D+01	.166D+01
23.0	.649D+01	.240D+01
24.0	.939D+01	.347D+01
25.0	.136D+02	.502D+01
26.0	.197D+02	.727D+01
27.0	.285D+02	.105D+02
28.0	.412D+02	.152D+02
29.0	.596D+02	.220D+02
30.0	.862D+02	.319D+02
31.0	.125D+03	.461D+02
32.0	.181D+03	.667D+02
33.0	.261D+03	.966D+02
34.0	.378D+03	.140D+03
35.0	.547D+03	.202D+03
36.0	.792D+03	.293D+03
37.0	.115D+04	.424D+03
38.0	.166D+04	.613D+03
39.0	.240D+04	.887D+03
40.0	.347D+04	.128D+04
41.0	.503D+04	.186D+04
42.0	.728D+04	.269D+04
43.0	.105D+05	.389D+04
44.0	.152D+05	.563D+04
45.0	.221D+05	.815D+04
46.0	.319D+05	.118D+05
47.0	.462D+05	.171D+05
48.0	.668D+05	.247D+05

49.0	.967D+05	.357D+05
50.0	.140D+06	.517D+05
51.0	.203D+06	.749D+05
52.0	.293D+06	.108D+06
53.0	.424D+06	.157D+06
54.0	.614D+06	.227D+06
55.0	.888D+06	.328D+06
56.0	.129D+07	.475D+06
57.0	.186D+07	.688D+06
58.0	.269D+07	.995D+06
59.0	.390D+07	.144D+07
60.0	.564D+07	.208D+07
61.0	.816D+07	.302D+07
62.0	.118D+08	.436D+07
63.0	.171D+08	.632D+07
64.0	.247D+08	.914D+07
65.0	.358D+08	.132D+08
66.0	.518D+08	.191D+08
67.0	.750D+08	.277D+08
68.0	.108D+09	.401D+08
69.0	.157D+09	.580D+08
70.0	.227D+09	.840D+08
71.0	.329D+09	.121D+09
72.0	.476D+09	.176D+09
73.0	.688D+09	.254D+09
74.0	.996D+09	.368D+09
75.0	.144D+10	.533D+09
76.0	.209D+10	.771D+09
77.0	.302D+10	.112D+10
78.0	.437D+10	.162D+10
79.0	.632D+10	.234D+10
80.0	.915D+10	.338D+10
81.0	.132D+11	.489D+10
82.0	.192D+11	.708D+10
83.0	.277D+11	.103D+11
84.0	.401D+11	.148D+11
85.0	.581D+11	.215D+11
86.0	.841D+11	.311D+11
87.0	.122D+12	.450D+11
88.0	.176D+12	.651D+11
89.0	.255D+12	.942D+11
90.0	.369D+12	.136D+12
91.0	.534D+12	.197D+12
92.0	.772D+12	.285D+12
93.0	.112D+13	.413D+12
94.0	.162D+13	.598D+12
95.0	.234D+13	.865D+12
96.0	.339D+13	.125D+13
97.0	.490D+13	.181D+13
98.0	.709D+13	.262D+13
99.0	.103D+14	.379D+13
100.0	.149D+14	.549D+13

An alternative way is to use a different formulation which is amenable to numerical computation. This will be accomplished through the following transformation.
Making use of the relation

$$\frac{d\theta}{dx}\frac{dx}{d\theta} = \theta_x x_\theta = 1$$

gives

$$\frac{d\theta}{dx} = \frac{1}{x_\theta} = y^{1/2} \qquad \text{say}$$

$$\frac{d^2\theta}{dx^2} = \frac{d}{dx}\left(\frac{d\theta}{dx}\right) = \frac{d}{d\theta}\left(\frac{d\theta}{dx}\right)\frac{d\theta}{dx} = \frac{1}{x_\theta}\left(\frac{1}{x_\theta}\right)_\theta = \frac{1}{2}\left(\frac{1}{x_\theta^2}\right)_\theta = \frac{1}{2}\frac{dy}{d\theta}$$

so that the original equation becomes:

$$\frac{1}{2}\frac{dy}{d\theta} = Py^{1/2} + B\theta^n$$

or

$$\frac{dy}{d\theta} = 2Py^{1/2} + 2B\theta^n \tag{27}$$

with

$$y(0) = 0 \tag{28}$$

With this transformation, a first order differential equation has been obtained for the square of the axial temperature derivative as a function of temperature. The problem is now solved as follows. A Peclet number P is assumed and Eq. (26) is integrated with initial condition (27) till the point $\theta = \theta_0$, i.e. the integration is performed over a finite interval. The resulting value of $y^{1/2}$ (which is the the axial temperature derivative) is compared with the derivative obtained from the equations related to the dry side, namely, to $P(1 - \theta_0)$. Of course, the correct solution for P is obtained when the two match to a desired accuracy. Note that this transformation can be used only when y has a monotoneous behaviour, e.g. on the wet side only, or on the dry side only, but not over the whole slab. Note also that instead of $B\theta^n$ a general function $f(\theta)$ (which is compatible with the boundary conditions) may be assumed.
As an example, let us take again Yamanouchi's model, i.e. $n = 1$. Table 18 shows the results obtained when the correct P is assumed and the resulting relative error in the calculated temperature derivative. Tables 19 and 20 show the results when the guess for P is off by $\pm 1\%$ from the correct value. One may realize that now the results are much less sensitive to the guess for P.

TABLE 18

P= .100 B= .10 THO= .270

X	Y	P(1-THO)	ERROR%
.0000D+00	.0000D+00		
.2702D-01	.8237D-02		
.5403D-01	.1498D-01		
.8105D-01	.2211D-01		
.1081D+00	.2934D-01		
.1351D+00	.3659D-01		
.1621D+00	.4386D-01		
.1891D+00	.5114D-01		
.2161D+00	.5843D-01		
.2431D+00	.6572D-01		
.2702D+00	.7301D-01	.7298D-01	.041

TABLE 19

P= .101 B= .10 THO= .270

X	Y		
.0000D+00	.0000D+00		
.2702D-01	.8223D-02		
.5403D-01	.1495D-01		
.8105D-01	.2207D-01		
.1081D+00	.2929D-01		
.1351D+00	.3653D-01		
.1621D+00	.4379D-01		
.1891D+00	.5106D-01		
.2161D+00	.5834D-01	P(1-THO)	ERROR%
.2431D+00	.6562D-01		
.2702D+00	.7290D-01	.7298D-01	-.116

TABLE 20

P= .099 B= .10 THO= .270

X	Y		
.0000D+00	.0000D+00		
.2702D-01	.8251D-02		
.5403D-01	.1500D-01		
.8105D-01	.2214D-01		
.1081D+00	.2938D-01		
.1351D+00	.3665D-01		
.1621D+00	.4393D-01		
.1891D+00	.5123D-01		
.2161D+00	.5852D-01	P(1-THO)	ERROR%
.2431D+00	.6583D-01		
.2702D+00	.7313D-01	.7298D-01	.198

For the formulation given in Eqs. (26)-(27), a more elegant solution can be derived for $n \neq 1$ in the following way.

In order to scale out P and B, let us introduce the following new variables

$$\zeta = \frac{y}{(2P)^{\alpha}(2B)^{\gamma}} \qquad \eta = \frac{\theta}{(2P)^{\beta}(2B)^{\delta}}$$

Inserting these variables into the differential equation gives

$$(2P)^{\alpha-\beta}(2B)^{\gamma-\delta}\frac{d\zeta}{d\eta} = (2P)^{1+\alpha/2}(2B)^{\gamma/2}\zeta^{1/2} + (2P)^{n\beta}(2B)^{1+n\delta}\eta^{n}$$

Equating like powers of P and B, the following relations are obtained:

$$\alpha - \beta = 1 + \frac{\alpha}{2} = n\beta$$

$$\gamma - \delta = \frac{\gamma}{2} = 1 + n\delta$$

which yield:

$$\alpha = \frac{2n+2}{n-1} \qquad \beta = \frac{2}{n-1} \qquad \gamma = \frac{2}{1-n} \qquad \delta = \frac{1}{1-n}$$

The transformation is therefore

$$\zeta = \frac{y}{(2P)^{\frac{2n+2}{n-1}}(2B)^{\frac{2}{n-1}}} \qquad \eta = \frac{\theta}{(2P)^{\frac{2}{1-n}}(2B)^{\frac{1}{1-n}}}$$

With this transformation the differential eqution + initial condition become:

$$\frac{d\zeta}{d\eta} = \zeta^{1/2} + \eta^{n} \tag{29}$$

$$\zeta(0) = 0$$

For $\theta = \theta_0$ one obtains

$$\eta = \eta_0 = \frac{\theta_0}{(2P)^{\frac{2}{1-n}}(2B)^{\frac{1}{1-n}}}$$

also

$$y(\theta_0) = P^2(1-\theta_0)^2$$

which gives

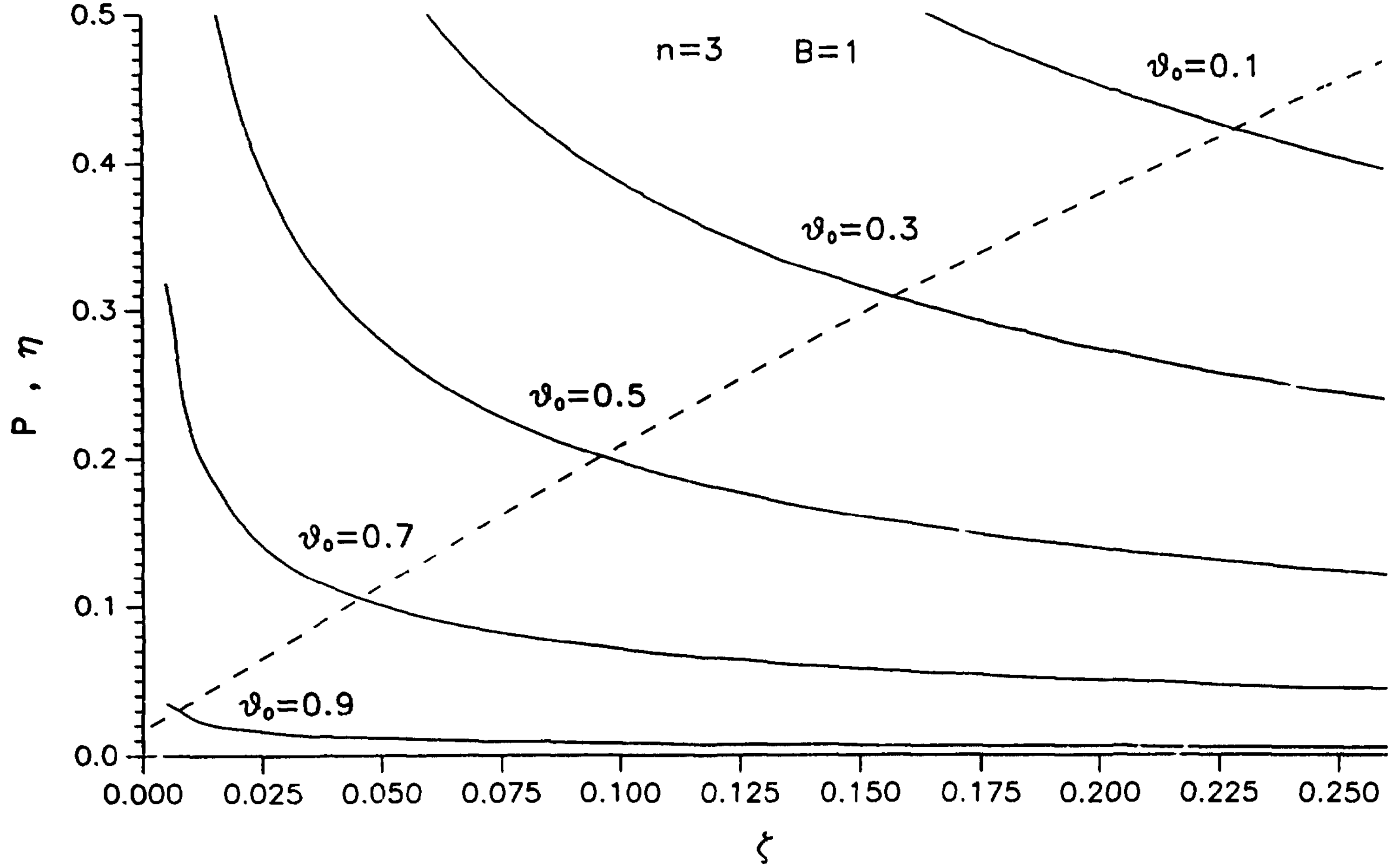
$$\zeta(\eta_0) = \frac{P^2(1-\theta_0)^2}{(2P)^{\frac{2n+2}{n-1}}(2B)^{\frac{2}{n-1}}}$$

An elimination of P from the last equation gives

$$P = \frac{1}{2}\frac{(1-\theta_0)^2}{(2B)^{\frac{2}{n-1}}}\zeta^{\frac{1-n}{4}} \tag{30}$$

Now plot η versus ζ and P versus ζ for various values of θ_0. Their point of intersection gives the desired value of P. Fig. 1 shows results for $n = 3$, $B = 1$ and different values of θ_0. It can be realized that for a given n, Eq. (29) has to be integrated only once, which is the advantage of this method of solution.

FIG. 1



n=3 B=1
ϑ_0=0.1
ϑ_0=0.3
ϑ_0=0.5
ϑ_0=0.7
ϑ_0=0.9
P , η
ζ
0.0
0.1
0.2
0.3
0.4
0.5
0.000 0.025 0.050 0.075 0.100 0.125 0.150 0.175 0.200 0.225 0.250

6 Axial versus normal heat transfer

The question of the relative importance of axial conduction and normal heat transfer in a given situation cannot be answered by using theoretical models. The reason is that an answer based on a given theoretical model depends on the boundary conditions that are specified in the model. As an example, suppose we use the model by Olek et al. (1988a), which regards the rewetting problem as a conjugate heat transfer problem. In this model the temperature distributions in the solid and in the liquid are evaluated simultaneously with the requirement of the continuity of temperature and heat flux at the interface between the two. Thus, a heat transfer coefficient needs not be specified on the wet side and the dry side is assumed adiabatic. In such a model (as well as in other heat conduction models which assume an adiabatic dry side), heat from the high temperature dry side can flow only axially towards the wet side. At the quench front, part of this heat continues to flow axially and part is evacuated from the solid in the normal direction by convection to the liquid. Thus, one can apriori deduce that in this model axial conduction will be more important than normal conduction under all circumstances. A typical example is presented in Fig. 2 for the calculated axial and radial heat fluxes, which shows indeed the noted feature.
Obviously, if one assumes precursory cooling in such a model, the imortance of normal heat transfer will increase and may surpass axial conduction for some values of precursory cooling parameters.
Thus, a definite answer to the question of the relative importance of axial versus normal heat transfer cannot be given by using theoretical models. Intuitively one expects normal conduction to be of increasing importance with higher fluid flow rates.

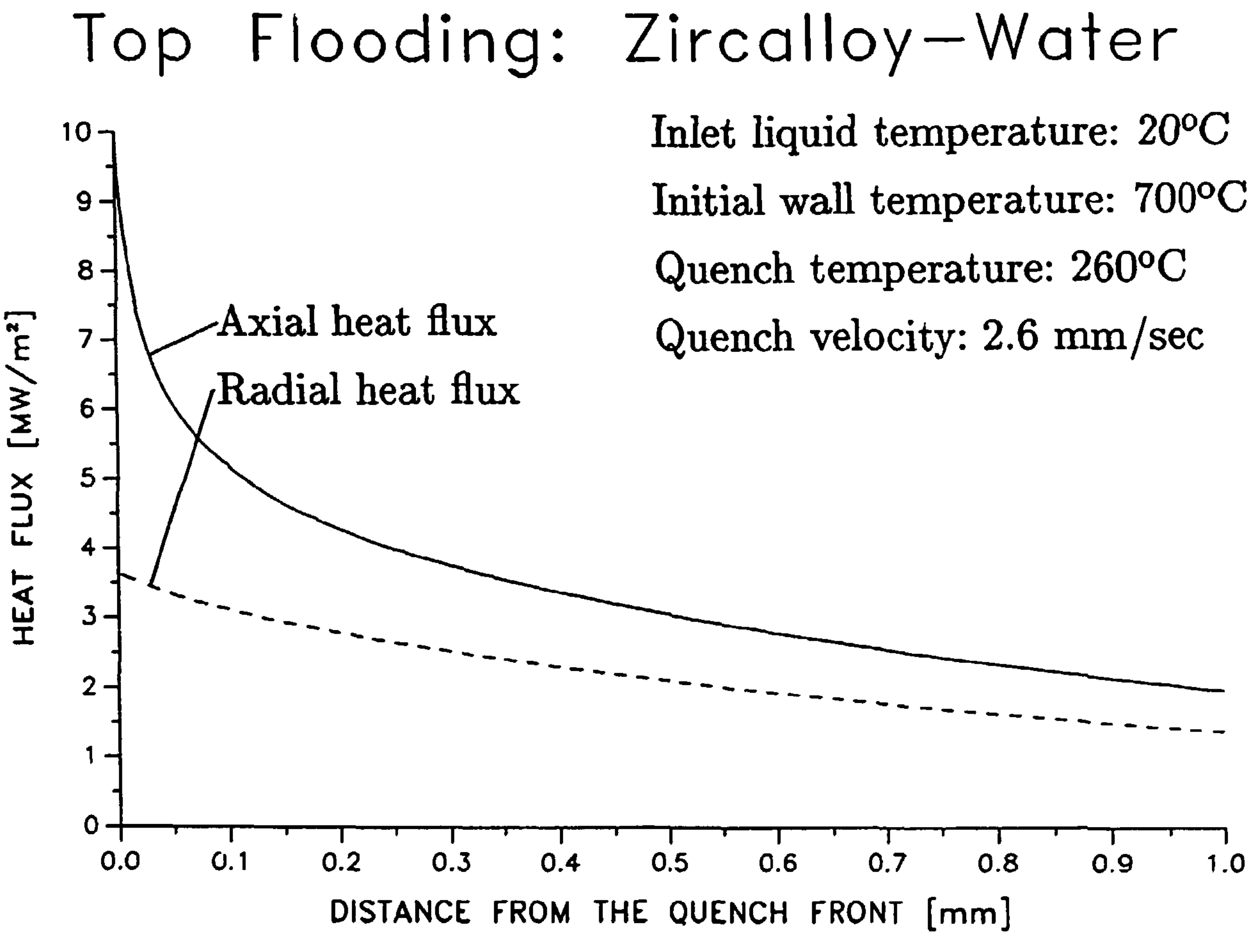


Fig. 2: The relative importance of axial versus radial heat transfer, as calculated from the conjugate heat transfer model by Olek et al. (1988a).

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APPENDIX A

The Wiener–Hopf Technique

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Appendix A: The Wiener-Hopf Technique

A.1 Introduction

In the following a short description of the Wiener-Hopf technique will be given, as well as its application to rewetting models. The aim is by no means to cover this method thoroughly, but to give some basic understanding of the method, which is one of the popular methods that were used to solve rewetting models. Most of the introductory part is taken from the book by Noble (1958).

The Wiener-Hopf technique provides a significant extension of the range of problems that can be solved by the Fourier, Laplace and Mellin integrals.
To fix ideas, let us consider three problems which are connected with the steady state wave equation. We use this equation, since later it will be shown that when employing the Wiener-Hopf technique to the solution of rewetting models, there is a big advantage in transforming the heat equation (through a well known transformation) into the steady state wave equation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + k^2 \phi = 0 \tag{A.1}$$

Suppose we wish to find a solution of this equation in the semi-infinite region $-\infty < x < \infty$, $y \geq 0$, such that ϕ represents an outgoing wave at infinity in each of the three separate cases

(i) $\quad \phi = f(x) \quad$ on $\quad y = 0, \quad -\infty < x < \infty;$

(ii) $\quad \partial\phi/\partial y = g(x) \quad$ on $\quad y = 0, \quad -\infty < x < \infty;$

(iii)
$$\begin{aligned} \phi &= f(x) \quad \text{on} \quad y = 0, \quad 0 < x < \infty; \\ \partial\phi/\partial y &= g(x) \quad \text{on} \quad y = 0, \quad -\infty < x < 0; \end{aligned} \tag{A.2}$$

Separation of variables solutions exist for (A.1) in the form $\phi = X(x)Y(y)$ with

$$X(x) = e^{\pm i\alpha x}, \qquad Y(y) = e^{\pm \gamma y}, \qquad \gamma = (\alpha^2 - k^2)^{1/2}$$

where α is a parameter. Together with the fact that the range of x is infinite this suggests the use of the Fourier integral in $-\infty < x < \infty$, and in fact the first two problems can be solved exactly by Fourier integrals, whereas the third one leads to equations which can be solved by the Wiener-Hopf technique.

Although it will appear that we must use the Fourier integral in the complex plane, consider in this section the ordinary (real variable) form

$$\Phi(\alpha, y) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \phi(x, y) e^{i\alpha x} dx$$

$$\phi(x, y) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \Phi(\alpha, y) e^{-i\alpha x} d\alpha, \tag{A.3}$$

where α is real. We use the method of Fourier transforms. Multiply both sides of (A.1) by $(2\pi)^{-1/2}\exp(i\alpha x)$ and integrate throughout with respect to x from $-\infty$ to ∞:

$$\frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \frac{\partial^2 \phi}{\partial x^2} e^{i\alpha x} dx + \frac{\partial^2 \Phi}{\partial y^2} + k^2 \Phi = 0 \tag{A.4}$$

Integrate the following expression by parts:

$$\int_{-A}^{A} \frac{\partial^2 \phi}{\partial x^2} e^{i\alpha x} dx = \left[\frac{\partial \phi}{\partial x} e^{i\alpha x} \right]_{-A}^{A} - i\alpha \left[\phi e^{i\alpha x} \right]_{-A}^{A} - \alpha^2 \int_{-A}^{A} \phi e^{i\alpha x} dx$$

Let $A \to \infty$ and assume that contributions from the bracketed terms at upper and lower limits tend to zero. This is connected with the condition that ϕ represents an outgoing wave at infinity. Eq. (A.4) then becomes

$$\frac{\partial \Phi}{\partial y^2} - \gamma^2 \Phi = 0, \quad \text{where} \quad \gamma^2 = (\alpha^2 - k^2) \tag{A.5}$$

Define

$$\gamma = +(\alpha^2 - k^2)^{1/2}, \qquad \alpha > k$$

where k is assumed here to be real. A difficulty arises since we need to define γ for $\alpha < k$ and it is not clear, for example, whether to take the upper or the lower sign in the formula $\gamma = \pm i(k^2 - \alpha^2)^{1/2}$, $|\alpha| < k$. To answer this question one uses analytic continuation arguments (see more in Noble's book, chapter 1.2). Assuming that the definition of γ has been settled, the solution of (A.5) which must be used is

$$\Phi = A(\alpha) e^{-\gamma y}, \tag{A.6}$$

since it appears that $\gamma = +(\alpha^2 - k^2)^{1/2}$ for $\alpha < -k$ so that this solution is bounded for all α as y tends to infinity whereas the solution in $\exp(+\gamma y)$ increases exponentially as y tends to infinity for $|\alpha| > k$. The function $A(\alpha)$ is an arbitrary function determined from the boundary condition on $y = 0$.
Now consider problems (i)-(iii) in turn. (i) Application of the boundary condition on $y = 0$

to (A.6) gives

$$(\Phi)_{y=0} = A(\alpha) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} f(\xi) e^{i\alpha\xi} d\xi$$

Substitute this value for $A(\alpha)$ in (A.6) and use the Fourier inversion formula (A.3). This gives the solution

$$\phi = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\alpha x - \gamma y} \int_{-\infty}^{\infty} f(\xi) e^{i\alpha\xi} d\xi d\alpha$$

(ii) In a similar way the second problem gives

$$\left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = -\gamma A(\alpha) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} g(\xi) e^{i\alpha\xi} d\xi$$

$$\phi = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \gamma^{-1} e^{i\alpha x - \gamma y} \int_{-\infty}^{\infty} g(\xi) e^{i\alpha\xi} d\xi d\alpha \tag{A.7}$$

(iii) In this case there are three methods of procedure which are basically identical but deserve separate mention.

A. In (A.2) extend $f(x)$ to denote the (unknown) value of $\phi(x,0) = f(x)$ on $y = 0$, $x < 0$, and $g(x)$ to denote the (unknown) value of $\partial\phi/\partial y$ on $y = 0$, $x > 0$. Define

$$\Phi_+(\alpha, y) = \frac{1}{(2\pi)^{1/2}} \int_0^{\infty} \phi(x, y) e^{i\alpha x} dx$$

$$\Phi_-(\alpha, y) = \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{0} \phi(x, y) e^{i\alpha x} dx$$

Then $\Phi_+(\alpha, 0)$, $\Phi_-(\alpha, 0)$ are the corresponding integrals of $\phi(x, 0) = f(x)$. Use a dash to denote differentiation with respect to y, so that

$$\Phi'_+(\alpha, 0) = \left(\frac{\partial \Phi}{\partial y}\right)_{y=0} = \frac{1}{(2\pi)^{1/2}} \int_0^{\infty} g(x) e^{i\alpha x} dx$$

with a corresponding definition for $\Phi'_-(\alpha, 0)$. The boundary conditions now yield

$$\Phi_+(\alpha, 0) + \Phi_-(\alpha, 0) = A(\alpha)$$

$$\Phi'_+(\alpha, 0) + \Phi'_-(\alpha, 0) = -\gamma A(\alpha)$$

Eliminate $A(\alpha)$ from these equations

$$\Phi'_+(\alpha,0) + \Phi'_-(\alpha,0) = -\gamma[\Phi_+(\alpha,0) + \Phi_-(\alpha,0)] \tag{A.8}$$

The functions Φ'_-, Φ_+ are known but there are two unknown functions, Φ'_+ and Φ_-. It will appear that if α is taken as a complex variable in the original Fourier transform (A.3), a process involving analytic continuation and Liouville's theorem can be used to determine the unknown functions in (A.8). This process is the 'Wiener-Hopf technique'. The approach is commonly referred to as Jones's direct method.

B. Next consider an integral equation formulation of the problem. In (A.7) interchange orders of integration, Let y tend to zero, introduce boundary condition (A.2) for $x > 0$, split the range of integration in ξ into $(-\infty, 0), (0, \infty)$ and rearange:

$$\int_0^\infty K(x-\xi)g(\xi)d\xi = f(x) - \int_{-\infty}^0 g(\xi)K(x-\xi)d\xi, \qquad (x > 0) \tag{A.9}$$

where

$$K(x-\xi) = -\frac{1}{2\pi}\int_{-\infty}^\infty \gamma^{-1}e^{i\alpha(\xi-x)}d\alpha$$

and the quantities on the right hand side of the equation are known. This is an integral equation for the unknown function $g(\xi)$, $\xi > 0$. The important feature from the present point of view is that the kernel $K(x-\xi)$ is a function of $(x-\xi)$. Such integral equations can be solved by the Wiener-Hopf technique.
In the literature the usual procedure is to obtain this type of equation by the Green function technique, and then to reduce the integral equation to (A.8) by Fourier transforms.

C. Finally, consider the formulation in terms of dual integral equations. From (A.6)

$$\phi = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^\infty A(\alpha)e^{-i\alpha x-\gamma y}d\alpha$$

The boundary conditions give

$$\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^\infty A(\alpha)e^{-i\alpha x}d\alpha = f(x), \qquad (x > 0)$$

$$-\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^\infty \gamma A(\alpha)e^{-i\alpha x}d\alpha = g(x), \qquad (x < 0) \tag{A.10}$$

These are dual integral equations for the unknown function $A(\alpha)$. These equations can be solved by a procedure depending on the essential step in the Wiener-Hopf technique.

This completes the introducory part. In order to solve problem (iii) above by any of the methods A, B, C, it is necessary to consider complex α. This requires a discussion of certain topics in complex variable and Fourier transform theory, which is given in the next chapter.

A.2 Complex variable theory

We start with a brief summary of complex variable theory required for succeeding developments. Greek letters will be used to denote complex variables, e.g. $\zeta = \xi + i\eta, \alpha = \sigma + i\tau$. When the complex variable is associated with the Fourier transform we invariably use $\alpha = \sigma + i\tau$. Latin letters a, b, k, etc., will be used for constatnts. It will be clear from the text whether these are to be regarded real or complex. We recall the following definitions and results.

Analytic and regular functions

If to each point ζ in a certain region R there correspond one or more complex numbers, denoted by χ, then we write $\chi = f(\zeta)$ and say that χ is a function of the complex variable ζ. If the function has a uniquely defined value at each point of the region R it is said to be single-valued in R. The crucial property possessed by useful functions is that at most points in R they are differentiable, i.e.

$$f'(\zeta) = \lim_{\delta \to 0} \frac{f(\zeta + \delta) - f(\zeta)}{\delta}$$

exists and is independent of the direction along which the complex number δ tends to zero. The function $\chi = f(\zeta)$ is said to be ***analytic*** at the point ζ when it is single-valued and differentiable at this point. The function f is said to be *regular in a region* R if it is analytic at every point of R. The phrase "$f(\zeta)$ is an analytic function in a region R" means that the function is analytic at every point of a region except for a certain number of exceptional points: this will be defined more precisely later in connexion with analytic continuation. Points at which the function is not analytic are called *singularities*. The singularities of a function are very important since they characterize the function.

The next idea required is that of a line integral. The central result concerning line integral is *Cauchy's theorem*.

Cauchy's theorem

If $f(\zeta)$ is an analytic function, continuous within and on the simple closed rectifiable curve C, and if $f'(\zeta)$ exists at each point within C, then

$$\int_C f(\zeta)d\zeta = 0$$

From this can be deduced ***Cauchy's integral formula***: If $f(\zeta)$ obeys the same conditions as for Cauchy's theorem and if α is any point within C, then

$$f(\alpha) = \frac{1}{2\pi i}\int_C \frac{f(\zeta)}{\zeta - \alpha}d\zeta$$

Some familiarity is assumed with the application of these theorems to evaluation of contour integrals by residues and shifting of contours in the complex plane, particularly when branch points are present.

An analytic function which is regular in every finite region of the ζ-plane is called an ***integral function***, e.g. a polyniomial in ζ is an integral fuction; also $\exp\zeta$ is an integral function.

Liouville's theorem

If $f(\zeta)$ is an integral function such that $|f(\zeta)| \leq M$ for all ζ, M being a constant, then $f(\zeta)$ is a constant.

It is easy to extend this result to the following: if $f(\zeta)$ is an integral function such that $|f(\zeta) \leq M|\zeta|^p$ as $|\zeta| \to \infty$ where M, p are constants, then $f(\zeta)$ is a polynomial of degree less then or equal to $[p]$, where $[p]$ is the integral part of p.

Taylor's theorem

If $f(\zeta)$ is an analytic function regular in the neighbourhood $|\zeta - a| < R$ of the point $\zeta = a$, it can be expressed in this neighbourhood as a convergent power series of the form

$$f(\zeta) = f(a) + \sum_{r=1}^{\infty} a_r \frac{(\zeta - a)^r}{r!}, \qquad a_r = \left[\frac{d^r f}{d\zeta^r}\right]_{\zeta=a}$$

A ***zero*** of an analytic function $f(\zeta)$ is a value of ζ such that $f(\zeta)$=0. It can be deduced from Taylor's theorem that the zeros of an analytic function are isolated points, i.e. if $f(\zeta)$ is regular in a region including $\zeta = a$ then there is a region $|\zeta - a| < \rho$, $(\rho > 0)$, inside which $f(\zeta)$ has no zeros except possibly $\zeta = a$ itself. If a singularity is isolated it is possible to deduce a ***Laurent expansion.*** If this expansion is of the form

$$f(\zeta) = \sum_{r=-n}^{\infty} a_r(\zeta - a)^r, \qquad (n > 0)$$

then the function is said to have a ***pole of order n*** at the point a.

Analytic continuation

It often happens that a representation of a function of a complex variable is valid only for restricted α. Thus the series

$$f(\alpha) = 1 + \alpha + \alpha^2 + \ldots$$

converges only for $|\alpha| < 1$. However for $|\alpha| < 1$ we have

$$f(\alpha) = (1 - \alpha)^{-1}$$

The extension of the definition of $f(\alpha)$ by identifying $f(\alpha)$ with $(1-\alpha)^{-1}$ for $|\alpha| > 1$ is called analytic continuation. It is possible to carry out analytic continuation systematically by means of power series but we do not go into details. We assume merely that the functions $f(\alpha)$ with which we deal are defined in such a way that if we start at any point $\alpha = a$ in the complex plane and draw a continuous curve to another point, say, $\alpha = b$ in such a way that no singularities of the function lie on the curve, then the values of $f(\alpha)$ vary continuously along the curve and can be determined from the definition of $f(\alpha)$. The expression "the analytic function $f(\alpha)$" can now be defined as the totality of all the values of $f(\alpha)$ which can be obtained by analytic continuation as just described, starting at a given point $\alpha = a$.

The natural question which arises is whether a function which is continued along two different curves from $\alpha = a$ to $\alpha = b$ will have the same final value for the two ways. This question is partly answered by the following theorem (Titchmarsh (1939)): If we continue an analytic function $f(\alpha)$ along two different routes from a to b and obtain two different values of $f(b)$ then $f(a)$ must have a singularity between the two routes. Of course the converse is not true, that if there is a singularity between the two routes we necessarily obtain two different values for $f(b)$; it needs to be a special type of singularity, namely a *branch-point* to produce a difference in value.

If the values of a function found by analytic continuation are unique, independent of the path of continuation, then the function is called *single-valued*. Otherwise the function is called *many-valued*.

A branch point a is a singular point such that there exists no neighbourhood $|\alpha - a| < \varepsilon$ in which $f(\alpha)$ is single-valued. By inserting certain lines in the complex plane and stating that paths of analytic continuation must not cross these lines it is possible to specify a *branch* of a many-valued function which is in itself single-valued. Such lines are called *branch-lines* or *cuts*. Branch points always occur in pairs, and branch lines join branch points.

A.3 Analytic functions defined by integrals

We often meet functions defined by integrals of the type

$$G(\alpha) = \int_C g(\alpha, \zeta) d\zeta \tag{A.11}$$

where $g(\alpha, \zeta)$ is a function of the complex variables α and ζ, and C is a contour in the complex ζ-plane. The variable α will be assumed to lie inside a region R, i.e. the boundary of R, if any, is excluded. The contour C is assumed to be smooth, i.e. it is possible to specify position on the contour by means of a parameter t such that $\zeta = \xi(t) + i\eta(t), t_0 \leq t \leq t_1$, and $\xi'(t), \eta'(t)$ exist and are continuous.
Before stating conditions for $G(\alpha)$ to be regular we remark that any line integral like (A.11) can be reduced to real integrals and we shall assume that these are Rimannn integrals.
We now state conditions under which $G(\alpha)$ is regular

Theorem A

Let $g(\alpha, \zeta) = f(\zeta)h(\alpha, \zeta)$ satisfy the conditions

(i) $h(\alpha, \zeta)$ is a continuous function of the complex variables α and ζ where α lies inside a region R and ζ lies on a contour C.

(ii) $h(\alpha, \zeta)$ is a regular function of α in R for every ζ on C.

(iii) $f(\zeta)$ has only a finite number of discontinuities on C and a finite number of maxima and minima on any finite part of C.

(iv) $f(\zeta)$ is bounded except at a finite number of points. If ζ_0 is such a point, so that $g(\alpha, \zeta) \to \infty$ as $\zeta \to \zeta_0$, then

$$\int_C g(\alpha, \zeta) d\zeta = \lim_{\delta \to 0} \int_{C-\delta} g(\alpha, \zeta) d\zeta$$

exists where the notation $(C - \delta)$ denotes the contour C apart from a small length δ surrounding ζ_0, and $\lim(\delta \to 0)$ denotes the limit as this excluded length tends to zero. The limit must be approached uniformly when α lies in any closed domain R' within R.

(v) If C goes to infinity then any bounded part of C must be smooth and conditions (i) and (ii) must be satisfied for any bounded part of C. The infinite integral defining $G(\alpha)$ must be uniformly convergent when α lies in any closed domain R' of R.

Then $G(\alpha)$ defined by (A.11) is a regular function of α in R.

As a special case ζ may be real, say $\zeta = \xi$, and the contour may consist of the portion $a \leq \xi \leq b$ of the real axis. Then (A.11) is an ordinary real integral:

$$G(\alpha) = \int_a^b g(\alpha, \xi) d\xi$$

Suppose that $g(\alpha, \zeta)$ satisfies the conditions of the above theorem, and $|g(\alpha, \zeta)| \leq M(t)$ for any α in R where position on the contour C is specified by the parameter t,

$\zeta = \xi(t) + i\eta(t)$, $a \leq t \leq b$, and $\int_a^b M(t)|\xi'(t) + \eta'(t)|dt$ converges, then $\int_C g(\alpha, \zeta) d\zeta$ is uniformly and absolutely convergent in R'.

We can now deduce some important results. In the following $\sigma_+, \sigma_-, \tau_+, \tau_-$ are real constants.

(1) If

$$F_+(\alpha) = \int_0^\infty f(x) e^{i\alpha x} dx,$$

where $\alpha = \sigma + i\tau$, $f(x)$ satisfies conditions (ii), (iv) above, and $|f(x)| < A\exp(\tau_- x)$ as $x \to \infty$, then $F(\alpha)$ is regular in the upper half-plan $\tau > \tau_-$. Similarly if $f(x)$ satisfies (iii), (iv) and $|f(x)| < B\exp(\tau_+ x)$ as $x \to -\infty$ then

$$F_-(\alpha) = \int_{-\infty}^0 f(x) e^{i\alpha x} dx,$$

is regular in the lowe.-half plane $\tau < \tau_+$. These statements follow immediately from the theorem since $\exp(i\alpha x)$ obviously satisfies (i), (ii) and the restrictions as $x \to \pm\infty$ ensure uniform convergence.

(2) If

$$F(\alpha) = \int_{-\infty+ic}^{\infty+ic} \frac{f(\zeta)}{\zeta - \alpha} d\zeta$$

where $\alpha = \sigma + i\tau$, $\zeta = \xi + ic$, c is a given constant, $f(\xi + ic)$ regarded as a function of ξ satisfies (iii), (iv) and $|f(\xi + ic| < C|\xi|^{-k}$, $k > 0$, for $|\xi| > M$, say, then $F(\alpha)$ is a regular function of α in $\tau > c$. Again the result follows from the theorem since under the stated conditions $(\zeta - \alpha)^{-1}$ satisfies (i), (ii). Also if we consider the region R' of (v) such that $c + \varepsilon \le \tau \le K$, $a \le \sigma \le b$, we have

$$|F(\alpha)| \le \int_{-\infty}^{\infty} \frac{|f(\xi + ic)|}{[(\xi - \sigma)^2 + (c - \tau)^2]^{1/2}} d\xi \le \int_{-\infty}^{\infty} \frac{|f(u + \sigma + ic)|}{(u^2 + \varepsilon^2)^{1/2}} du,$$

on introducing $u = \xi - \sigma$. Divide the range of u into $(-\infty, A)$, (A, B), (B, ∞) where the finite number of discontinuities that f is allowed from (iii), (iv) lie in (A, B), and $A < -(M + b)$, $B > (M - a)$. Under these conditions $|f(u + \sigma + ic)| < C|u + \sigma|^{-k}$, $k > 0$, in $(-\infty, A)$ and (B, ∞) and the integral is absolutely convergent, independent of the position of α in R'.

Theorem B

Let $f(\alpha)$ be an analytic function of $\alpha = \sigma + i\tau$, regular in the strip $\tau_- < \tau < \tau_+$, such that $|f(\sigma + i\tau)| < C|\sigma|^{-p}$, $p > 0$, for $|\sigma| \to \infty$, the inequality holding for all τ in the strip $\tau_- + \varepsilon \le \tau_+ - \varepsilon$, $\varepsilon > 0$. Then for $\tau_- < c < \tau < d < \tau_+$,

$$f(\alpha) = f_+(\alpha) + f_-(\alpha)$$

$$f_+(\alpha) = \frac{1}{2\pi i}\int_{-\infty+ic}^{\infty+ic}\frac{f(\zeta)}{\zeta-\alpha}d\zeta \quad ; \quad f_-(\alpha) = -\frac{1}{2\pi i}\int_{-\infty+id}^{\infty+id}\frac{f(\zeta)}{\zeta-\alpha}d\zeta \qquad \text{(A.12)}$$

where $f_+(\alpha)$ is regular for all $\tau > \tau_-$, and $f_-(\alpha)$ is regular for all $\tau < \tau_+$.
The statements regarding regularity are proved as in (2) above. To prove (A.12) apply Cauchy's integral theorem to the rectangle with the vertices $\pm a + ic$, $\pm a + id$. From our assumption as regards the behaviour of $f(\alpha)$ as $|\sigma| \to \infty$ in the strip, the integrals on $\sigma = \pm a$ tend to zero as $a \to \infty$ and we are left with the required equation.
We shall see later that theorem B enables the decomposition of certain kernels which is one of the key steps in the Wiener-Hopf technique.

Theorem C

If $\log K(\alpha)$ satisfies the conditions of theorem B, which implies in particular that $K(\alpha)$ is regular and non-zero in a strip $\tau_- < \tau < \tau_+$, $-\infty < \sigma < \infty$, and $K(\alpha) \to +1$ as $\sigma \to \pm\infty$ in the strip, then we can write $K(\alpha) = K_+(\alpha)K_-(\alpha)$ where $K_+(\alpha)$, $K_-(\alpha)$ are regular, bounded, and non-zero in $\tau > \tau_-$, $\tau < \tau_+$, respectively.
The conditions of the theorem are more restrictive thean necessary but cover the applications in this report. A more general theorem which takes into account more complicated cases, can be found in exercise 1.12 in Noble (1958). The more general theorem shows how to deal with zeros of $K(\alpha)$ in the strip, and with cases where $K(\alpha) \to \exp(i\mu)$, $\exp(i\nu)$ as $\sigma \to +\infty, -\infty$, respectively, or $|K(\alpha)| \sim |\sigma|^p$ as $|\sigma| \to \infty$, in the strip.

To prove theorem C, apply theorem B to $f(\alpha) = \log K(\alpha)$.

$$\log K(\alpha) = \frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty}\frac{\log K(\zeta)}{\zeta-\alpha}d\zeta - \frac{1}{2\pi i}\int_{id-\infty}^{id+\infty}\frac{\log K(\zeta)}{\zeta-\alpha}d\zeta$$

$$= f_+(\alpha) + f_-(\alpha), \quad \text{say,} \qquad \text{(A.13)}$$

where c, d are any numbers such that $\tau_- < c < \tau < d < \tau_+$. The integral for $f_+(\alpha)$ is convergent for all α such that $\tau > c$. Hence $f_+(\alpha)$ is bounded and regular in $\tau > \tau_-$ since we can choose c as near as we please to τ_-. Similarly $f_-(\alpha)$ is bounded and regular in $\tau < \tau_+$.
If we set

$$K_+(\alpha) = \exp[f_+(\alpha)] \quad ; \quad K_-(\alpha) = \exp[f_-(\alpha)] \qquad \text{(A.14)}$$

then

$$\log K_+(\alpha) + \log K_-(\alpha) = \log K(\alpha), \quad \text{i.e.} \quad K_+(\alpha)K_-(\alpha) = K(\alpha).$$

From the properties of $f_+(\alpha)$, it is seen that $K_+(\alpha)$ is regular, bounded, and non-zero in $\tau > \tau_-$. Similarly $K_-(\alpha)$ is regular, bounded, and non-zero in $\tau < \tau_+$. Therefore the theorem is proved since $K_+(\alpha)$, $K_-(\alpha)$ have been constructed which satisfy the necessary conditions. Obviously, decomposition of kernels into sums, quotients and products is carried out in much the same way.

A.4 The Wiener-Hopf procedure

The practical details of applying Fourier transforms in the examples considered later tend to obscure the essential simplicity of the complex variable procedure, which is therefore summarized in this section. The typical problem obtained by applying Fourier transforms to partial differential equations is the following. Find unknown functions $\Phi_+(\alpha)$, $\Psi_-(\alpha)$ satisfying

$$A(\alpha)\Phi_+(\alpha) + B(\alpha)\Psi_-(\alpha) + C(\alpha) = 0 \tag{A.15}$$

where this equation holds in a strip $\tau_- < \tau < \tau_+$, $-\infty < \sigma < \infty$ of the complex α-plane, $\Phi_+(\alpha)$ is regular in the half-plane $\tau > \tau_-$, $\Psi_-(\alpha)$ is regular in $\tau < \tau_+$, and certain information which will be specified later is available regarding the behaviour of these functions as α tends to infinity in the appropriate half-planes. The functions $A(\alpha)$, $B(\alpha)$, $C(\alpha)$ are given functions of α, regular in the strip. For simplicity we assume that A, B are also non-zero in the strip.

The fundamental step in the Wiener-Hopf procedure for solution of this equation is to find $K_+(\alpha)$ regular and non-zero in $\tau > \tau_-$, $K_-(\alpha)$ regular and non-zero in $\tau < \tau_+$, such that

$$A(\alpha)/B(\alpha) = K_+(\alpha)/K_-(\alpha) \tag{A.16}$$

Sometimes K_+, K_- can be found by inspection but in any case, for the A, B which occur in our applications, they can always be found with the help of theorem C (the precise details will become clear in the next sections which deal with kernel decomposition). Use (A.16) to rearange (A.15) as

$$K_+(\alpha)\Phi_+(\alpha) + K_-(\alpha)\Psi_-(\alpha) + K_-(\alpha)C(\alpha)/B(\alpha) = 0 \tag{A.17}$$

Decompose $K_-(\alpha)C(\alpha)/B(\alpha)$ in the form

$$K_-(\alpha)C(\alpha)/B(\alpha) = C_+(\alpha) + C_-(\alpha) \tag{A.18}$$

where $C_+(\alpha)$ is regular in $\tau > \tau_-$, $C_-(\alpha)$ is regular in $\tau < \tau_+$. In the general case this can be done by using theorem B. With the help of (A.18) rearange (A.17) so as to define a function $J(\alpha)$ by

$$J(\alpha) = K_+(\alpha)\Phi_+(\alpha) + C_+(\alpha) = -K_-(\alpha)\Psi_-(\alpha) - C_-(\alpha) \tag{A.19}$$

So far this equation defines $J(\alpha)$ only in the strip $\tau_- < \tau < \tau_+$. But the second part of the equation is defined and is regular in $\tau > \tau_-$, and the third part is defined and is regular in $\tau < \tau_+$. Hence by analytic continuation we can define $J(\alpha)$ over the whole α-plane and $J(\alpha)$ is regular in the whole α-plane. Now suppose that it can be shown that

$$K_+(\alpha)\Phi_+(\alpha) + C_+(\alpha)| < |\alpha|^p \quad \text{as} \quad \alpha \to \infty, \quad \tau > \tau_-$$

$$K_-(\alpha)\Psi_-(\alpha) + C_-(\alpha)| < |\alpha|^q \quad \text{as} \quad \alpha \to \infty, \quad \tau < \tau_+ \tag{A.20}$$

Then by the extended form of Liouville's theorem $J(\alpha)$ is a polynomial $P(\alpha)$ of degree less than or equal to the integral part of min (p, q), i.e.

$$K_+(\alpha)\Phi_+(\alpha) + C_+(\alpha) = P(\alpha)$$

$$K_-(\alpha)\Phi_-(\alpha) + C_-(\alpha) = -P(\alpha) \tag{A.21}$$

These equations determine $\Phi_+(\alpha)$, $\Psi_-(\alpha)$ to within an arbitrary polynomial $P(\alpha)$, i.e. to within a finite number of arbitrary constants which must be determined otherwise.

The crucial step is the finding of $K_+(\alpha)$, $K_-(\alpha)$ to satisfy (A.16). The methods associated with the Wiener-Hopf technique for the solution of partial differential equations described in this report include an equation of the form (A.15).
If we say that a method is "based on the Wiener-Hopf technique" we imply that at some stage of the solution a decomposition of the form (A.16) is involved.

A.5 Kernel decomposition

The three common approaches to kernel decompositions are by inspection, by using theorem B, or for some kernels using infinite products. The last method will be explained in the following.

A.5.1 Expansion of meromorphic functions in partial fractions

A function is meromorphic in a region if it is regular in the region except for a finite number of poles. Let $f(\alpha)$ be a function whose only singularities except possibly at infinity are poles. For simplicity suppose that all poles are simple. Let them be $\alpha_1, \alpha_2 \ldots$, where $0 < |\alpha_1| \leq |\alpha_2| \leq \ldots$, and let the residues at the poles be $a_1, a_2, \ldots$, respectively. Suppose that there exists an increasing sequence of numbers R_m such that $R_m \to \infty$ and such that the circles C_m with equations $|\alpha| = R_m$ pass through no pole of $f(\alpha)$ for any m. Suppose that $f(\alpha)$ is bounded on C_m for all m. Then

$$f(\alpha) = f(0) + \sum_{n=1}^{\infty} a_n \left(\frac{1}{\alpha - \alpha_n} + \frac{1}{\alpha_n} \right)$$

for all α except the poles. As an example

$$\operatorname{cosec} \alpha - \frac{1}{\alpha} = \sum_{n=-\infty}^{\infty}{}' (-1)^n \left(\frac{1}{\alpha - n\pi} + \frac{1}{n\pi} \right)$$

where the dash means that the term for $n = 0$ is omitted.

A.5.2 The infinite product theorem

If $f(\alpha)$ is an integral function of α with simple zeros at $\alpha_1, \alpha_2, \ldots$, then it can be shown that $f'(\alpha)/f(\alpha)$ is a meromorphic function of α which can be expanded in partial fractions as shown in the previous section. On integrating this expansion the following infinite product representation of $f(\alpha)$ is found

$$f(\alpha) = f(0) \exp \left[\alpha \frac{f'(0)}{f(0)} \right] \sum_{n=1}^{\infty} \left(1 - \frac{\alpha}{\alpha_n} \right) e^{\alpha/\alpha_n}$$

In this form the exponential factors are necessary to ensure convergence, since $\alpha_n \sim an + b$ as $n \to \infty$ on the examples considered below. If $f(\alpha)$ is an even function of α, the roots occur in pairs, $\pm\alpha_n$, and $f'(0) = 0$, so that we can write

$$f(\alpha) = f(0) \sum_{n=-\infty}^{\infty}{}' \left(1 - \frac{\alpha}{\alpha_n}\right) e^{\alpha/\alpha_n} = f(0) \sum_{n=1}^{\infty} \left(1 - \frac{\alpha}{\alpha_n}\right)^2$$

where $\alpha_{-n} = -\alpha_n$ and the dash denotes that the term for $n = 0$ is omitted.
As examples of this type we have

$$(\alpha a)^{-1} \sin \alpha a \quad ; \quad \cos \alpha a$$

If $K(\alpha)$ is an integral function which can be expressed as an infinite product, the decomposition is immediate. The important case from our point of view occurs when $K(\alpha)$ is an even function. Then we can write

$$f(\alpha) = f(0) \sum_{n=1}^{\infty} [1 - (\alpha/\alpha_n)^2]$$

This can be decomposed in the form

$$K_{\pm}(\alpha) = [K(0)]^{1/2} e^{\mp\chi(\alpha)} \sum_{n=1}^{\infty} [1 \pm (\alpha/\alpha_n)] e^{\mp(\alpha/\beta_n)}$$

where the upper and lower signs go together and the terms have been arranged so that all the zeros of $K_+(\alpha)$ lie in the lower half-plane and vice versa. Hence $K_+(\alpha)$ is regular and non-zero in the upper half-plane (Im α) >-(Im α_1). The function $\chi(\alpha)$ is arbitrary and can be chosen to ensure that K_+, K_- have suitable behaviour as $\alpha \to \infty$ in appropriate half-planes. The infinit product will in general have exponential behaviour as $\alpha \to \infty$ whereas the functions which need to be decomposed later have algebraic behaviour at infinity due to additional terms multiplying the infinite products. These facilitate the choice of $\chi(\alpha)$ and it is convenient to postpone further discussion until we require the decomposition of concrete examples. It is emphsized that the correct choice of $\chi(\alpha)$ is crucial for the successful application of the Wiener-Hopf technique.
When the function has a branch point the infinite product method will break down. In this case one should use theorem B to perform the decomposition.

In the following sections, several rewetting models are treated. These include the formulation and solution part of several papers on rewetting models.

A.6 Rewetting of a slab with a step-like heat transfer coefficient

A complete description of this model and its solution by the Wiener-Hopf technique and by separation of variables can be found in Olek (1988a). Here we present the main features only.

A.6.1 Formulation

The nondimensional quasi steady state heat conduction equation in a coordinate system moving with the quench front at a constant velocity P is

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}-P\frac{\partial\theta}{\partial x}=0 \qquad 0<y<1, \quad -\infty<x<\infty \tag{A.22}$$

and the associated boundary conditions are:

$$\frac{\partial\theta}{\partial y}=0 \qquad y=0, \qquad -\infty<x<\infty \tag{A.23}$$

$$\frac{\partial\theta}{\partial y}=0 \qquad y=1, \qquad x<0 \tag{A.24}$$

$$\frac{\partial\theta}{\partial y}+B\theta=0 \qquad y=1, \qquad x>0 \tag{A.25}$$

$$\theta\longrightarrow 1 \qquad x\longrightarrow -\infty \tag{A.26}$$

$$\theta\longrightarrow 0 \qquad x\longrightarrow +\infty \tag{A.27}$$

$$\theta=\theta_0 \qquad x=0, \qquad y=1 \tag{A.28}$$

where

$$x\equiv\frac{\tilde{x}}{\delta}, \qquad y\equiv\frac{\tilde{y}}{\delta}, \qquad \theta\equiv\frac{T-T_s}{T_w-T_s}, \qquad B\equiv\frac{h\delta}{k}, \qquad P\equiv\frac{\rho c u\delta}{k}$$

The axial and normal coordinates are $\tilde{x}$ and $\tilde{y}$, respectively, and the slab thickness is δ. The solid is at an initial temperature T_w and it is cooled to a temperature T_s. The density, specific heat at constant pressure, and thermal conductivity are ρ, c and k, respectively. The rewetting velocity is denoted by u, the rewetting temperature is T_0, and h is the heat transfer coefficient.
The main interest is solving for the rewetting velocity (Peclet number), P, in terms of the Biot number, B, and the rewetting temperature $\theta_0\equiv(T_0-T_s)/(T_w-T_s)$.

A.6.2 Solution

Following Evans (1984), define

$$\theta(x,y) = 1 - \phi(x,y)e^{sx} \tag{A.29}$$

where $s = P/2$

and obtain

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - s^2\phi = 0 \qquad 0 < y < 1, \qquad -\infty < x < \infty \tag{A.30}$$

$$\frac{\partial\phi}{\partial y} = 0 \qquad y = 0, \qquad -\infty < x < \infty \tag{A.31}$$

$$\frac{\partial\phi}{\partial y} = 0 \qquad y = 1, \qquad x < 0 \tag{A.32}$$

$$\frac{\partial\phi}{\partial y} + B\phi = Be^{-sx}, \qquad y = 1, \qquad x > 0 \tag{A.33}$$

$$\phi(x,y) = O(e^{+sx}) \qquad \text{as} \qquad x \longrightarrow -\infty \tag{A.34}$$

$$\phi(x,y) = O(e^{-sx}) \qquad \text{as} \qquad x \longrightarrow +\infty \tag{A.35}$$

$$\phi = \phi_0, \qquad x = 0, \qquad y = 1 \tag{A.36}$$

Let us define the following Fourier transforms:

$$\Phi(\alpha,y) \equiv \Phi_+(\alpha,y) + \Phi_-(\alpha,y) = \int_{-\infty}^{\infty} \phi(x,y)e^{i\alpha x}dx \tag{A.37}$$

with

$$\Phi_+(\alpha,y) = \int_0^{\infty} \phi(x,y)e^{i\alpha x}dx$$

$$\Phi_-(\alpha,y) = \int_{\infty}^{0} \phi(x,y)e^{i\alpha x}dx$$

and note that from eq. (A.34) and eq. (A.35) it follows that the functions Φ_+ and Φ_- are analytic in the regions

$$D_- : \text{Im}\alpha < s$$

and

$$D_+ : \text{Im}\alpha > -s$$

respectively.

The transform of eq. (A.30) is

$$\frac{d^2\Phi}{dy^2} - \gamma^2\Phi = 0 \tag{A.38}$$

where $\gamma = (\alpha^2 + s^2)^{1/2}$, with the positive branch of the squareroot. The solution of (A.38) with boundary condition (A.31) gives

$$\Phi(\alpha, y) = C(\alpha)\cosh(\gamma y), \qquad \alpha \in D_+ \cup D_- \tag{A.39}$$

The transforms of conditions (A.32) and (A.33) are

$$\Phi'_- = 0 \qquad \alpha \in D_- \tag{A.40}$$

$$\Phi'_+ + B\Phi_+ = \frac{iB}{\alpha + is} \qquad \alpha \in D_+ \tag{A.41}$$

whilst from eq. (39)

$$\Phi' = \gamma \tanh\gamma(\Phi_+ + \Phi_-), \tag{A.42}$$

where primes denote transforms of y derivates of ϕ, and the arguments are $(\alpha, 1)$ throughout.

From equations (A.40)-(A.41) follows the Wiener-Hopf functional relation for the determination of the two unknown functions Φ_+ and Φ_- :

$$\Phi_+(1 + B\frac{\coth\gamma}{\gamma}) + \Phi_- = \frac{iB}{\alpha + is}\frac{\coth\gamma}{\gamma} \tag{A.43}$$

Let

$$K(\alpha) = K_+(\alpha)K_-(\alpha) = 1 + B\frac{\coth\gamma}{\gamma}$$

where the functions $K_+(\alpha), K_-(\alpha)$ are analytic above and below the lines $\mathrm{Im}\alpha = -s, +s$, respectively. Then eq. (A.43) can be re-arranged as follows

$$\begin{aligned} \Phi_+K_+(\alpha) + \Phi_-/K_-(\alpha) &= \frac{i}{\alpha + is}\left[K_+(\alpha) - 1/K_-(\alpha)\right] \\ &= \frac{i}{\alpha + is}\left[K_+(\alpha) - 1/K_-(-is)\right] \\ &\quad - \frac{i}{\alpha + is}\left[1/K_-(\alpha) - 1/K_-(-is)\right] \\ &= M_+(\alpha) + M_-(\alpha) \end{aligned} \tag{A.44}$$

where

$$M_+(\alpha) = \frac{i}{\alpha + is}\left[K_+(\alpha) - 1/K_-(-is)\right]$$

$$M_-(\alpha) = -\frac{i}{\alpha + is}\left[1/K_-(\alpha) - 1/K_-(-is)\right]$$

and $M_\pm(\alpha)$ is regular in $D_\pm$.

Thus, eq. (A.44) may be rewritten as

$$\Phi_+K_+(\alpha) - M_+(\alpha) = -\Phi_-/K_-(\alpha) + M_-(\alpha) \equiv E(\alpha), \text{say}, \tag{A.45}$$

which characterizes an entire function. $E(\alpha)$, through its representation in the upper and lower halves of the α-plane. Since $\Phi_+(\alpha)$ and $\Phi_-(\alpha)$ tend to zero at infinity in their half planes of regularity, while $K_+(\alpha)$ and $K_-(\alpha)$ remain bounded, the entire function vanishes according to Liouville's theorem.

Hence,

$$\Phi_+(\alpha,1) \equiv \Phi_+(\alpha) = M_+(\alpha)/K_+(\alpha) = \frac{i}{\alpha + is}\{1 - [K_+(\alpha)K_-(-is)]^{-1}\} \tag{A.46}$$

$$\Phi_-(\alpha,1) \equiv \Phi_-(\alpha) = M_-(\alpha)K_-(\alpha) = -\frac{i}{\alpha + is}\{1 - K_-(\alpha)/K_-(-is)\} \tag{A.47}$$

Adding eqs. (A.46) and (A.47) gives

$$\Phi(\alpha,1) = \frac{iB\coth\gamma}{(\alpha + is)\gamma K_+(\alpha)K_-(-is)} \tag{A.48}$$

$$= \frac{iBK_-(\alpha)}{(\alpha + is)(\gamma\tanh\gamma + B)K_-(-is)} \tag{A.49}$$

and the solution for the temperature distribution in the slab is obtained by inversion

$$\phi(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Phi(\alpha)\frac{\cosh(\gamma y)}{\cosh\gamma}e^{-i\alpha x}d\alpha \tag{A.50}$$

The problem is solved once $K_\pm(\alpha)$ is determined.

For $x > 0$ it is advantageous to use (A.49) in (A.50) since then the contour may be deformed into D_- picking up contributions from the residues due to the zeros of

$$\gamma\tanh\gamma + B = 0 \tag{A.51}$$

in D_-. For similar reasons, for $x < 0$ it is desirable to use (A.48) in (A.50).

The rewetting temperature needs not to be calculated from (A.50). Instead, following Levine (1982) and Evans (1984), it can be evaluated in a simple way as follows

$$\begin{aligned}\Phi_+(\alpha) &= \int_0^\infty \phi(x,1)e^{i\alpha x}dx \\ &= i\alpha^{-1}\phi(0,1) - (i\alpha)^{-1}\int_0^\infty \frac{\partial\phi}{\partial x}(x,1)e^{i\alpha x}dx\end{aligned} \tag{A.52}$$

It turns out that the integral term vanishes as $\alpha \to \infty$ in D_+, if it is assumed that $\partial\phi/\partial x$ is bounded.

From (A.46) and (A.52) it follows that (assuming $K_\pm(\alpha) \to 1$ as $\alpha \to \infty$ in $D_\pm$)

$$\theta_0 = 1 - \phi(0,1) = 1 + \lim_{\alpha\to\infty}[i\alpha\Phi_+(\alpha)] = [K_-(-is)]^{-1} \tag{A.53}$$

Now all it remains is to factorize

$$K(\alpha) = 1 + B\frac{\coth\gamma}{\gamma} \equiv K_+(\alpha)(K_-(\alpha)$$

where $K_\pm\alpha$ is regular in $D_\pm$.

On applying the Cauchy residue theorem within the strip $|\ \mathrm{Im}\alpha\ | < s$ and noting the asymptotic order of magnitude $\log K(\alpha) = O(1/\alpha), |\ \mathrm{Re}\alpha\ | \to \infty$, it follows that

$$K(\alpha) = \frac{1}{2\pi i}\int_{C_+}\frac{\log K(\xi)}{\xi-\alpha}d\xi - \frac{1}{2\pi i}\int_{C_-}\frac{\log K(\xi)}{\xi-\alpha}d\xi \tag{A.54}$$

where C_+/C_- is an infinite contour from $\mathrm{Re}\alpha = -\infty$ to $\mathrm{Re}\ \alpha = \infty$ lying in D and passing below/above the point $\alpha \in D$.

Thus,

$$\log K_\pm(\alpha) = \pm\frac{1}{2\pi i}\int_{C_\pm}\frac{\log K(\xi)}{\xi-\alpha}d\xi \tag{A.55}$$

from which it follows that $K_\pm(\alpha) = 1$ as $\alpha \to \infty$, as assumed earlier.

If $\alpha = \pm i\beta$, with $\beta \geq s$, the contours $C_\pm$ may be shifted to real axis yielding

$$\log K_\pm(\pm i\beta) = \frac{\beta}{\pi}\int_0^\infty\frac{\log K(\tau)}{\tau^2+\beta^2}d\tau = \frac{1}{\pi}\int_0^\infty \log K(\beta z)\frac{dz}{1+z^2} \tag{A.56}$$

Hence,

$$\theta_0 = \exp\left[-\frac{1}{\pi}\int_0^\infty \log K(sz)\frac{dz}{1+z^2}\right] \tag{A.57}$$

where

$$K(sz) = 1 + B\frac{\coth s(1+z^2)^{1/2}}{s(1+z^2)^{1/2}}$$

This is the same expression as the one obtained by Levine (1982), using a singular integral equation formulation of the problem. The advantage of the simplicity of Jones's direct method employed here, over the integral formulation of Levine (1982) which involves using Green functions, is obvious.

For computational purposes it is advisable to use the transformation $\sec^2\Omega = 1 + z^2$, to finally obtain

$$\theta_0 = \exp\{-\frac{1}{\pi}\int_0^{\pi/2}\log\left[1 + \frac{B}{s}\frac{\coth(s\cdot\sec\Omega)}{\sec\Omega}\right]d\Omega\} \tag{A.58}$$

An alternative decomposition is possible in terms of infinite products. $K(\alpha)$ can be represented in the form

$$K(\alpha) = \frac{\gamma \sinh\gamma + B\cosh\gamma}{\gamma\sinh\gamma}$$

so that it is a quotient of two even functions.
Each of these two functions can be decomposed as follows:

$$\begin{aligned}
\gamma\sinh\gamma + B\cosh\gamma &= L_+(\alpha)L_-(\alpha) \\
&= B\prod_{n=1}^{\infty}(1+\gamma^2/\rho_{n-1}^2) \\
&= B\prod_{n=1}^{\infty}\left[(1+s^2/\rho_{n-1}^2)^{1/2} + i\alpha/\rho_{n-1}\right]\exp(-i\alpha/n\pi) \\
&\times \prod_{n=1}^{\infty}\left[(1+s^2/\rho_{n-1}^2)^{1/2} - i\alpha/\rho_{n-1}\right]\exp(i\alpha/n\pi)
\end{aligned}$$

Hence

$$L_-(\alpha) = B^{1/2}\prod_{n=1}^{\infty}\left[(1+s^2/\rho_{n-1}^2)^{1/2} + i\alpha/\rho_{n-1}\right]\exp(-i\alpha/n\pi)$$

$$L_+(\alpha) = B^{1/2}\prod_{n=1}^{\infty}\left[(1+s^2/\rho_{n-1}^2)^{1/2} - i\alpha/\rho_{n-1}\right]\exp(i\alpha/n\pi)$$

Similarly

$$\begin{aligned}
\gamma\sinh\gamma = Q_+(\alpha)Q_-(\alpha) &= (s^2+\alpha^2)\prod_{n=1}^{\infty}(1+\gamma^2/n^2\pi^2) \\
&= (s+i\alpha)(s-i\alpha)\prod_{n=1}^{\infty}\left[(1+s^2/n^2\pi^2)^{1/2} + i\alpha/n\pi\right]\exp(-i\alpha/n\pi) \\
&\times\prod_{n=1}^{\infty}\left[(1+s^2/n^2\pi^2)^{1/2} - i\alpha/n\pi\right]\exp(i\alpha/n\pi)
\end{aligned}$$

so that

$$Q_-(\alpha) = (s+i\alpha)\prod_{n=1}^{\infty}\left[(1+s^2/n^2\pi^2)^{1/2} + i\alpha/n\pi\right]\exp(-i\alpha/n\pi)$$

$$Q_+(\alpha) = (s-i\alpha)\prod_{n=1}^{\infty}\left[(1+s^2/n^2\pi^2)^{1/2} - i\alpha/n\pi\right]\exp(i\alpha/n\pi)$$

where the exponential factors have been included in the products to assure their absolute convergence, and $i\rho_n, n = 0, 1, 2, \ldots$ are the zeros of $\gamma\sinh\gamma + B\coth\gamma = 0$, or equivalently ρ_n are the positive roots of

$$\rho \tan \rho = B$$

Now the factorization of $K(\alpha)$ is readily obtained as

$$K_-(\alpha) = \frac{L_-(\alpha)}{Q_-(\alpha)} = \frac{B^{1/2}}{s + i\alpha} \prod_{n=1}^{\infty} \frac{(1 + s^2/\rho_{n-1}^2)^{1/2} + i\alpha/\rho_{n-1}}{(1 + s^2/n^2\pi^2)^{1/2} + i\alpha/n\pi}$$

$$K_+(\alpha) = \frac{L_+(\alpha)}{Q_+(\alpha)} = \frac{B^{1/2}}{s - i\alpha} \prod_{n=1}^{\infty} \frac{1 + s^2/\rho_{n-1}^2)^{1/2} - i\alpha/\rho_{n-1}}{(1 + s^2/n^2\pi^2)^{1/2} - i\alpha/n\pi}$$

Observe that $K_+(\alpha) = K_-(-\alpha)$, and from the asymptotic nature of ρ_n,

$$\rho_n \sim n\pi + B/n\pi \qquad \text{as} \qquad n \to \infty$$

it follows that $K_\pm(\alpha) \to 1$ as $\alpha \to \infty$ in D.

Hence,

$$\theta_0 = \frac{2s}{B^{1/2}} \prod_{n=1}^{\infty} \frac{(1 + s^2/n^2\pi^2)^{1/2} + s/n\pi}{(1 + s^2/\rho_{n-1}^2)^{1/2} + s/\rho_{n-1}} \tag{A.59}$$

A.7 A model for the rewetting of a slab with precursory cooling

The model originally suggested by Dua and Tien (1976) is solved accurately by the Wiener-hopf technique. The solution which appears in the sequel is taken from Olek (1988c).

A.7.1 Formulation

Assuming a constant rewetting velocity u, the quasi steady state heat conduction equation can be written as

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} - 2s\frac{\partial\theta}{\partial x} = 0 \qquad 0 < y < 1, \qquad -\infty < x < \infty \tag{A.60}$$

where θ is a dimensionless temperature, and x and y are the longitudal and traversal coordinates, respectively, which are normalized with respect to the thickness of the slab, δ. The Peclet number, $2s$, represents the dimensionless rewetting velocity, and it is given by

$$2s \equiv \frac{u\delta}{\alpha}$$

with α denoting the thermal diffusivity of the slab.
Eq. (A.60) is subject to the following boundary conditions

$$\frac{\partial\theta}{\partial y} = 0 \quad y = 0, \quad -\infty < x < \infty \quad \text{(insulated boundary)} \tag{A.61}$$

$$\frac{\partial\theta}{\partial y} = Ae^{bx} \quad y = 1, \quad x < 0 \quad \text{(decaying heat flux)} \tag{A.62}$$

$$\frac{\partial\theta}{\partial y} + B\theta = 0 \quad y = 1, \quad x > 0 \quad (B = \text{constant cooling rate}) \tag{A.63}$$

$$\theta \to 1 \quad \text{as} \quad x \to -\infty \quad \text{(hot end of the slab)} \tag{A.64}$$

$$\theta \to 0 \quad \text{as} \quad x \to +\infty \quad \text{(cold end of the slab)} \tag{A.65}$$

With the additional condition, which establishes the relationship between the rewetting velocity and the rewetting temperature

$$\theta = \theta_0 \quad \text{at} \quad y = 1, x = 0 \quad \text{(rewetting temperature at the solid-liquid-vapor interline)} \tag{A.66}$$

A.7.2 Solution

Following Evans (1984) we define

$$\theta(x,y) = 1 - \phi(x,y)e^{sx} \tag{A.67}$$

and obtain

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - s^2\phi = 0 \tag{A.68}$$

$$\frac{\partial\phi}{\partial y} = 0 \quad , \qquad y = 0, \qquad -\infty < x < \infty \tag{A.69}$$

$$\frac{\partial\phi}{\partial y} = -Ae^{(b-s)x} \quad , \qquad y = 1, \qquad x < 0 \tag{A.70}$$

$$\frac{\partial\phi}{\partial y} + B\phi = Be^{-sx} \quad , \qquad y = 1, \qquad x > 0 \tag{A.71}$$

$$\phi = O(e^{-sx}) \quad \text{as} \quad x \to +\infty \tag{A.72}$$

About this behaviour of ϕ as $x \to -\infty$ we can learn from the solution of this problem by Olek (1987b), using separation of variables. For $x < 0$ it was shown that

$$\phi = \frac{1}{2}y^2Ae^{(b-s)x} + \sum_{n=1}^{\infty}\left\{c_n e^{[s^2+(n-1)^2\pi^2]^{1/2}x} + \frac{q_n Ae^{(b-s)x}}{(n-1)^2\pi^2 + 2sb - b^2}\right\} \cos[(n-1)\pi y]$$

where c_n and q_n are constants. From the above expression it follows that

$$\phi = O\left[e^{(b-s)x}\right] \text{ as } x \longrightarrow -\infty \tag{A.73}$$

Conditions (A.72) and (A.73) ensure that the Fourier transform

$$\Phi(\alpha,y) = \int_{-\infty}^{\infty} \phi(x,y)e^{i\alpha x}dx \quad \text{exists in} \quad D: -s < \text{Im}\alpha < -(s-b) \tag{A.74}$$

whilst

$$\Phi_+(\alpha,y) = \int_0^{\infty} \phi(x,y)e^{i\alpha x}dx \quad \text{exists in} \quad D_+ : \text{Im}\alpha > -s$$

$$\Phi_-(\alpha,y) = \int_{-\infty}^{0} \phi(x,y)e^{i\alpha x}dx \quad \text{exists in} \quad D_- : \text{Im}\alpha < -(s-b)$$

The transform of (A.68) is

$$\frac{d^2\Phi}{dy^2} - \gamma^2\Phi = 0 \tag{A.75}$$

where $\gamma = (\alpha^2 + s^2)^{1/2}$, with the positive branch of the squareroot.
The solution of (A.75) subject to the transform of condition (A.69) gives

$$\Phi(\alpha, y) = C(\alpha)\cosh(\gamma y) \quad , \quad \alpha \in D = D_+ \cup D_- \tag{A.76}$$

The transforms of (A.70) and (A.71) yield

$$\Phi'_-(\alpha, 1) = \frac{iA}{\alpha + i(s-b)} \quad , \quad \alpha \in D_- \tag{A.77}$$

and

$$\Phi'_+(\alpha, 1) + B\Phi_+(\alpha, 1) = \frac{iB}{\alpha + is} \quad , \quad \alpha \in D_+ \tag{A.78}$$

whereas from (A.76) it follows that

$$\Phi'(\alpha, 1) = \gamma \tanh\gamma \Big[\Phi_+(\alpha, 1) + \Phi_-(\alpha, 1)\Big] \quad , \quad \alpha \in D \tag{A.79}$$

where prime denotes transforms of y derivatives of Φ. Henceforth, the argument $(\alpha, 1)$ will be omitted for brevity.
From (A.77)-(A.79) follows the Wiener-Hopf equation

$$\Phi_+(1 + B\frac{\coth\gamma}{\gamma}) + \Phi_- = \Big[\frac{iB}{\alpha + is} + \frac{iA}{\alpha + i(s-b)}\Big]\frac{\coth\gamma}{\gamma}, \quad \alpha \in D \tag{A.80}$$

Suppose

$$K(\alpha) = 1 + B\frac{\coth\gamma}{\gamma} = K_+(\alpha)K_-(\alpha) \tag{A.81}$$

where $K_\pm(\alpha)$ is regular in $D_\pm, K_\pm(\alpha) \neq 0$. Then rearrangement of (A.80) gives

$$\Phi_+ K_+(\alpha) + \Phi_-/K_-(\alpha) = \Big[\frac{i}{\alpha + is} + \frac{iA/B}{\alpha + i(s-b)}\Big]\Big[K_+(\alpha) - \frac{1}{K_-(\alpha)}\Big] = M_+(\alpha) + M_-(\alpha) \tag{A.82}$$

where

$$M_+(\alpha) = \frac{i}{\alpha + is}\Big[K_+(\alpha) - \frac{1}{K_-(-is)}\Big] + \frac{iA/B}{\alpha + i(s-b)}\Big\{K_+(\alpha) - K_+[-i(s-b)]\Big\}$$

$$M_-(\alpha) = -\frac{i}{\alpha + is}\Big[\frac{1}{K_-(\alpha)} - \frac{1}{K_-(-is)}\Big] + \frac{iA/B}{\alpha + i(s-b)}\Big\{K_+[-i(s-b)] - \frac{1}{K_-(\alpha)}\Big\}$$

Eq. (A.82) can now be recast in the form

$$\Phi_+ K_+(\alpha) - M_+(\alpha) = M_-(\alpha) - \Phi_-/K_-(\alpha) = E(\alpha), \text{ say,} \tag{A.83}$$

which characterizes an entire function $E(\alpha)$, through its representation in the upper and lower halves of the α-plane.
Since it will be shown that

$$K_\pm(\alpha) = O(1) \quad \text{as} \quad \alpha \longrightarrow \infty \quad \text{in} \quad D_\pm$$

whilst

$$\Phi_\pm = o(1) \quad \text{as} \quad \alpha \longrightarrow \infty \quad \text{in} \quad D_\pm$$

it follows from Liouville's theorem that $E(\alpha) \equiv 0$, so that

$$\Phi_+(\alpha, 1) \equiv \Phi_+ = \frac{M_+(\alpha)}{K_+(\alpha)} = \frac{i}{\alpha + is}\left[1 - \frac{1}{K_+(\alpha)K_-(-is)}\right] + \frac{iA/B}{\alpha + i(s-b)}\left\{1 - \frac{K_+[-i(s-b)]}{K_+(\alpha)}\right\} \tag{A.84}$$

$$\Phi_-(\alpha, 1) \equiv \Phi_- = M_-(\alpha)K_-(\alpha) = -\frac{i}{\alpha + is}\left[1 - \frac{K_-(\alpha)}{K_-(-is)}\right] + \frac{iA/B}{\alpha + i(s-b)}\left\{K_-(\alpha)K_+[-i(s-b)] - 1\right\} \tag{A.85}$$

Adding gives

$$\Phi(\alpha, 1) \equiv \Phi(\alpha) = \frac{iB(\alpha + is)^{-1}}{K_-(-is)K_+(\alpha)}\frac{\coth\gamma}{\gamma} + \frac{(iA/B)K_+[-i(s-b)]}{\alpha + i(s-b)}\left[K_-(\alpha) - \frac{1}{K_+(\alpha)}\right] \tag{A.86}$$

Hence, the temperature distribution is given by

$$\phi(x, y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(\alpha)\frac{\cosh\gamma y}{\cosh\gamma}d\alpha \tag{A.87}$$

Following Levine (1982) and Evans (1984) we use the relation

$$\Phi_+(\alpha) \;=\; \int_0^\infty \phi(x,1)e^{i\alpha x}dx = i\alpha^{-1}\phi(0,1) - (i\alpha)^{-1}\int_0^\infty \frac{\partial\phi}{\partial x}(x,1)e^{i\alpha x}dx \tag{A.88}$$

and note that the integral term vanishes as $\alpha \to \infty$ in D_+, provided $\partial\phi/\partial x$ is bounded, to obtain the following expression for the rewetting temperature

$$\begin{aligned} \theta_0 &= 1 - \phi(0,1) = 1 + \lim_{\alpha\to\infty}[i\alpha\Phi_+(\alpha)] \\ &= [K_-(-is)]^{-1} - (A/B)\left\{1 - K_+[-i(s-b)]\right\} \end{aligned} \tag{A.89}$$

since $K_+(\alpha) \to 1$ as $\alpha \to \infty$ in D_+.
The factorization of $K(\alpha)$ in terms of infinite products is given by Olek (1987a):

$$K_{\mp}(\alpha) = \frac{B^{1/2}}{s \pm i\alpha} \prod_{n=1}^{\infty} \frac{(1 + s^2/\rho_{n-1}^2)^{1/2} \pm i\alpha/\rho_{n-1}}{(1 + s^2/n^2\pi^2)^{1/2} \pm i\alpha/n\pi} \tag{A.90}$$

where $i\rho_n, n = 0, 1, 2, \ldots$ are the zeros of $\gamma \sinh\gamma + B\cosh\gamma = 0$, or equivalently ρ_n are the positive roots of

$$\rho \tan\rho = B \tag{A.91}$$

A heat balance at the quench front yields

$$\frac{A}{B} = -\frac{\theta_0}{N} \tag{A.92}$$

where N is the magnitude of precursory cooling, which represents a fractional drop in the heat flux within an infinitely small distance ahead of the quench front, e.g. Dua and Tien (1976).

Inserting (A.92) in (A.89) gives the rewetting temperature in the form

$$\theta_0 = \frac{[K_-(-is)]^{-1}}{1 - \{1 - K_+[-i(s-b)]\}/N} \tag{A.93}$$

with

$$\left[K_-(-is)\right]^{-1} = \frac{2s}{B^{1/2}} \prod_{n=1}^{\infty} \frac{(1 + s^2/n^2\pi^2)^{1/2} + s/n\pi}{(1 + s^2/\rho_{n-1}^2)^{1/2} + s/\rho_{n-1}} \tag{A.94}$$

and

$$K_+\left[-i(s-b)\right] = \frac{B^{1/2}}{b} \prod_{n=1}^{\infty} \frac{(1 + s^2/\rho_{n-1}^2)^{1/2} - (s-b)/\rho_{n-1}}{(1 + s^2/n^2\pi^2)^{1/2} - (s-b)/n\pi} \tag{A.95}$$

An alternative direct decomposition of $K(\alpha)$ is possible by applying Cauchy's residue theorem within the strip $-s < \mathrm{Im}\alpha < -(s-b)$. Noting the asymtotic order of magnitude of $\log K(\xi) = O(1/\alpha)$ as $\alpha \to \infty$ in D, one obtains

$$K(\alpha) = \exp\left[\frac{1}{2\pi i}\int_{C_+} \frac{\log K(\xi)}{\xi - \alpha} d\xi - \frac{1}{2\pi i}\int_{C_-} \frac{\log K(\xi)}{\xi - \alpha} d\xi\right] \tag{A.96}$$

where the point ξ is located above/below the infinite contours C_+/C_- passing between $\mathrm{Re}\alpha = -\infty$ to $\mathrm{Re}\alpha = +\infty$.
Thus,

$$K_\pm(\alpha) = \exp\left[\pm\frac{1}{2\pi i}\int_{C_\pm} \frac{\log K(\xi)}{\xi - \alpha} d\xi\right] \tag{A.97}$$

If $\xi = i\beta$ with $\beta \geq s$, the contours $C_\pm$ may be shifted to the real axis, giving

$$K_\pm(\pm i\beta) = \exp\left[\frac{\beta}{\pi}\int_0^{\infty} \frac{\log K(\tau)}{\tau^2 + \beta^2} d\tau\right] \tag{A.98}$$

or in a more useful form for computicn

$$K_{\pm}(\pm i\beta) = \exp\left\{\frac{1}{\pi}\int_0^{\pi/2}\log\left\{1 + B\frac{\coth[s^2+\beta^2(sec^2\Omega - 1)]^{1/2}}{[s^2+\beta^2(sec^2\Omega - 1)]^{1/2}}\right\}d\Omega\right\} \tag{A.99}$$

It should be noted that in order to be able to use (A.99) for the calculation of $K_+[-i(s-b)]$, we must have $b > 2s$.

The solution to a model which neglects the precursory cooling (assuming an insulated slab ahead of the quench front), may be obtained as a special case, by setting $N = \infty$ in eq. (A.93).

A.8 Rewetting of a solid cylinder with precursory cooling

The following model and its solution are extracted from Olek (1988d).

A.8.1 Formulation

Consider an infinitly long solid cylinder. The heat conduction equation in a coordinate system which moves at a constant rewetting velocity, u, can be written in the form

$$\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\frac{\partial^2\theta}{\partial z^2}-2s\frac{\partial\theta}{\partial z}=0 \qquad 0<r<1, \qquad -\infty<z<\infty \tag{A.100}$$

where θ is a dimensionless temperature, and r and z are radial and axial coordinates, respectively, which are normalized with respect to the radius of the rod, R.
The Peclet number, $P\equiv 2s$, represents the dimensionless rewetting velocity, and it is given by

$$2s\equiv\frac{uR}{\alpha}$$

with the thermal diffusivity of the cylinder denoted by α.

The solution of eq. (A.100) must satisfy the following boundary conditions

$$\frac{\partial\theta}{\partial r}=0 \qquad \text{at} \quad r=0, \qquad -\infty<z<\infty \tag{A.101}$$

$$\frac{\partial\theta}{\partial r}=Ae^{bz} \qquad \text{at} \quad r=1, \qquad z<0 \qquad \text{(decaying heat flux)} \tag{A.102}$$

$$\frac{\partial\theta}{\partial r}+B\theta=0 \qquad \text{at} \quad r=1, \qquad z>0 \qquad (\text{B} = \text{constant cooling rate}) \tag{A.103}$$

$$\theta\to 1 \qquad \text{as} \quad z\to-\infty \qquad \text{(hot end of the rod)} \tag{A.104}$$

$$\theta\to 0 \qquad \text{as} \quad z\to\infty \qquad \text{(cold end of the rod)} \tag{A.105}$$

$$\theta=\theta_0 \qquad \text{at} \quad r=1, \qquad z=0 \qquad \text{(rewetting temperature)} \tag{A.106}$$

A.8.2 Solution

Define

$$\phi(r,z) = [1-\theta(r,z)]e^{-sz} \tag{A.107}$$

to obtain the following new formulation of the model

$$\frac{\partial^2\phi}{\partial r^2} + \frac{1}{r}\frac{\partial\phi}{\partial r} + \frac{\partial^2\phi}{\partial z^2} - s^2\phi = 0 \tag{A.108}$$

$$\frac{\partial\phi}{\partial r} = 0 \qquad \text{at} \quad r = 0, \quad -\infty < z < \infty \tag{A.109}$$

$$\frac{\partial\phi}{\partial r} = -Ae^{(b-s)z} \qquad \text{at} \quad r = 1, \qquad z < 0 \tag{A.110}$$

$$\frac{\partial\phi}{\partial r} + B\phi = Be^{-sz} \quad \text{at} \quad r = 1, \qquad z > 0 \tag{A.111}$$

$$\phi = O[e^{(b-s)z}] \qquad \text{as} \quad z \to -\infty \tag{A.112}$$

$$\phi = O(e^{-sz}) \qquad \text{as} \quad z \to \infty \tag{A.113}$$

$$\phi = \phi_0 \qquad \text{at} \quad r = 1, \qquad z = 0 \tag{A.114}$$

Conditions (A.112) and (A.113) ensure that the Fourier transform

$$\Phi(r,\alpha) = \int_{-\infty}^{\infty} \phi(r,z)e^{i\alpha z}dz \quad \text{exists in} \quad D \ : -s < \text{Im} < -(s-b) \tag{A.115}$$

while

$$\Phi_+(r,\alpha) = \int_0^{\infty} \phi(r,z)e^{i\alpha z}dz \quad \text{exists in} \quad D_+ : \text{Im}\alpha > -s$$

and

$$\Phi_-(r,\alpha) = \int_{-\infty}^{0} \phi(r,z)e^{i\alpha z}dz \quad \text{exists in} \quad D_- : \text{Im}\alpha < -(s-b)$$

The transform of (A.108) is

$$\frac{d^2\Phi}{dr^2} + \frac{1}{r}\frac{d\Phi}{dr} - \gamma^2\Phi = 0 \tag{A.116}$$

where $\gamma = (\alpha^2 + s^2)^{1/2}$, with the positive branch of the squareroot.

The solution of (A.116) which satisfies the transform of condition (10) is

$$\Phi(r,\alpha) = C(\alpha)I_0(\gamma r) \quad , \qquad \alpha \in D = D_+ \cup D_- \tag{A.117}$$

with I_0 denoting the zero order modified Bessel function of the first kind.

The transforms of (A.110) and (A.111) are

$$\Phi'_-(1,\alpha) = \frac{iA}{\alpha + i(s-b)} \quad , \quad \alpha \in D_- \tag{A.118}$$

and

$$\Phi'_+(1,\alpha) + B\Phi_+(1,\alpha) = \frac{iB}{\alpha + is} \quad , \quad \alpha \in D_+ \tag{A.119}$$

respectively, whereas from (A.117) it follows that

$$\Phi'(1,\alpha) = \frac{\gamma I_1(\gamma)}{I_0(\gamma)}[\Phi_+(1,\alpha) + \Phi_-(1,\alpha)] \tag{A.120}$$

where I_1 is the first order modified Bessel function of the first kind, and prime denotes transforms of r derivatives of ϕ. Henceforth, the argument $(1,\alpha)$ will be omitted for brevity.

From (A.118)-(A.120) follows the Wiener-Hopf equation

$$\Phi_+\left[1 + B\frac{I_0(\gamma)}{\gamma I_1(\gamma)}\right] + \Phi_- = \left[\frac{iB}{\alpha + is} + \frac{iA}{\alpha + i(s-b)}\right]\frac{I_0(\gamma)}{\gamma I_1(\gamma)} \tag{A.121}$$

Suppose

$$K(\alpha) = 1 + B\frac{I_0(\gamma)}{\gamma I_1(\gamma)} = K_+(\alpha)K_-(\alpha) \tag{A.122}$$

where $K_\pm(\alpha)$ is regular in $D_\pm, K_\pm(\alpha) \neq 0$. Then rearrangement of (A.121) gives

$$\Phi_+K_+(\alpha) + \Phi_-/K_-(\alpha) = \left[\frac{i}{\alpha + is} + \frac{iA/B}{\alpha + i(s-b)}\right]\left[K_+(\alpha) - \frac{1}{K_-(\alpha)}\right] = M_+(\alpha) + M_-(\alpha) \tag{A.123}$$

where

$$M_+(\alpha) = \frac{i}{\alpha + is}\left[K_+(\alpha) - \frac{1}{K_-(-is)}\right] + \frac{iA/B}{\alpha + i(s-b)}\left\{K_+(\alpha) - K_+[-i(s-b)]\right\}$$

$$M_-(\alpha) = -\frac{i}{\alpha + is}\left[\frac{1}{K_-(\alpha)} - \frac{1}{K_-(-is)}\right] + \frac{iA/B}{\alpha + i(s-b)}\left\{K_+[-i(s-b)] - \frac{1}{K_-(\alpha)}\right\}$$

and $M_\pm(\alpha)$ is regular in $D_\pm$.

Eq. (A.123) can be rewritten as follows

$$\Phi_+K_+(\alpha) - M_+(\alpha) = -\Phi_-/K_-(\alpha) + M_-(\alpha) \equiv E(\alpha) \tag{A.124}$$

where $E(\alpha)$ is an entire function which is regular everywhere in the α-plane. Since $K_\pm(\alpha) = O(1)$ as $\alpha \to \infty$ in $D_\pm$, and $\Phi_\pm(\alpha) = o(1)$ as $\alpha \to \infty$ in $D_\pm$, it follows from Liouville's theorem that $E(\alpha) \equiv 0$, giving

$$\Phi_+(1,\alpha) \equiv \Phi_+(\alpha) = \frac{M_+(\alpha)}{K_+(\alpha)} = \frac{i}{\alpha + is}\left[1 - \frac{1}{K_+(\alpha)K_-(-is)}\right] + \frac{iA/B}{\alpha + i(s-b)}\left\{1 - \frac{K_+[-i(s-b)]}{K_+(\alpha)}\right\} \tag{A.125}$$

$$\Phi_-(1,\alpha) \equiv \Phi_-(\alpha) = M_-(\alpha)K_-(\alpha) = -\frac{i}{\alpha + is}\left[1 - \frac{K_-(\alpha)}{K_-(-is)}\right] + \frac{iA/B}{\alpha + i(s-b)}\left\{K_-(\alpha)K_+[-i(s-b)] - 1\right\} \tag{A.126}$$

Adding gives

$$\Phi(1,\alpha) \equiv \Phi(\alpha) = \frac{iB(\alpha + is)^{-1}I_0(\gamma)}{K_-(-is)K_+(\alpha)\gamma I_1(\gamma)} + \frac{(iA/B)K_+[-i(s-b)]}{\alpha + i(s-b)}\left[K_-(\alpha) - \frac{1}{K_+(\alpha)}\right] \tag{A.127}$$

so that the temperature distribution in the rod is expressible by

$$\phi(r,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(\alpha)\frac{I_0(\gamma r)}{I_0(\gamma)}e^{-i\alpha z}d\alpha \tag{A.128}$$

The rewetting temperature can be found using the fact that

$$\Phi_+(\alpha) = \int_0^\infty \phi(1,z)e^{i\alpha z}dz = i\alpha^{-1}\phi(1,0) - (i\alpha)^{-1}\int_0^\infty \frac{\partial\phi}{\partial z}(1,z)e^{i\alpha z}dz$$

with the integral term vanishing as $\alpha \to \infty$, provided $\partial\phi/\partial z$ is bounded.

Hence,

$$\begin{aligned}\theta_0 = 1 - \phi(1,0) &= 1 + \lim_{\alpha\to\infty}\left[i\alpha\Phi_+(\alpha)\right] \\ &= \left[K_-(is)\right]^{-1} - \frac{A}{B}\left\{1 - K_+[-i(s-b)]\right\}\end{aligned} \tag{A.129}$$

since $K_\pm(\alpha) \to 1$ as $\alpha \to \infty$ in $D_\pm$.

A heat balance at the quench front yields

$$\frac{A}{B} = -\frac{\theta_0}{N} \tag{A.130}$$

where the dimensionless constant N is controlling the magnitude of the precursory cooling, e.g. Dua and Tien (1976).
Thus, inserting (A.130) in (A.129) and eliminating θ_0 gives

$$\theta_0 = \frac{[K_-(-is)]^{-1}}{1 - \{1 - K_+[-is(s-b)]\}/N} \tag{A.131}$$

It is interesting to note that instead of a relation between the rewetting temperature and the rewetting velocity of the form

$$\theta_0 = f(\theta_0, s)$$

(with f being some function) which is obtained by separation of variables, see for example, Olek (1988b), a relation of the form

$$\theta_0 = g(s)$$

(with g being another function) is obtained in eq. (A.131). This means that in order to map the rewetting velocity vs. the rewetting temperature with the Wiener-Hopf technique solution no iterative procedure, like in the case of the solution by separation of variables, is needed.

It remains to carry out the decomposition of

$$K(\alpha) = 1 + B\frac{I_0(\gamma)}{\gamma I_1(\gamma)} = K_+(\alpha)K_-(\alpha)$$

where $K_\pm(\alpha)$ is regular in $D_\pm$.
First present $K(\alpha)$ in the form

$$K(\alpha) = \frac{\gamma I_1(\gamma) + BI_0(\gamma)}{\gamma I_1(\gamma)} \tag{A.132}$$

and note that both numerator and denominator in (A.132) are even functions. Each of these two functions can be decomposed in a manner similar to the one in [A.113], giving

$$K_-(\alpha) = \frac{(2B)^{1/2}}{s + i\alpha} \prod_{n=1}^{\infty} \frac{(1 + s^2/\rho_{n-1}^2)^{1/2} + i\alpha/\rho_{n-1}}{(1 + s^2/\nu_n^2)^{1/2} + i\alpha/\nu_n} \tag{A.133}$$

$$K_+(\alpha) = \frac{(2B)^{1/2}}{s - i\alpha} \prod_{n=1}^{\infty} \frac{(1 + s^2/\rho_{n-1}^2)^{1/2} - i\alpha/\rho_{n-1}}{(1 + s^2/\nu_n^2)^{1/2} - i\alpha/\nu_n} \tag{A.134}$$

where $i\nu_n, \quad n = 1, 2, \ldots$ are the positive roots of $I_1(\gamma) = 0$, and $i\rho_n, \quad n = 0, 1, 2, \ldots$ are the positive roots of $\gamma I_1(\gamma) + BI_0(\gamma) = 0$.

From eqs. (A.133) and (A.134) it follows that the rewetting temperature is given by eq. (A.131) with

$$\left[K_-(-is)\right]^{-1} = \frac{2^{1/2}s}{B^{1/2}} \prod_{n=1}^{\infty} \frac{(1 + s^2/\nu_n^2)^{1/2} + s/\nu_n}{(1 + s^2/\rho_{n-1}^2)^{1/2} + s/\rho_{n-1}} \tag{A.135}$$

and

$$K_+\left[-i(s-b)\right] = \frac{(2B)^{1/2}}{b}\prod_{n=1}^{\infty}\frac{(1+s^2/\rho_{n-1}^2)^{1/2}-(s-b)/\rho_{n-1}}{(1+s^2/\nu_n^2)^{1/2}-(s-b)/\nu_n} \tag{A.136}$$

Another way to factorize $K(\alpha)$ is possible by applying Cauchy's residue theorem within the strip $-s <$ Im$\alpha < -(s-b)$.
Since $\log K(\xi) = O(1/\alpha)$ as $\alpha \to \infty$ in D, it follows that

$$K(\alpha) = \exp\left[\frac{1}{2\pi i}\int_{C_+}\frac{\log K(\xi)}{\xi-\alpha}d\xi - \frac{1}{2\pi i}\int_{C_-}\frac{\log K(\xi)}{\xi-\alpha}d\xi\right] \tag{A.137}$$

where the contours $C_\pm$ pass between Re$\alpha = -\infty$ to Re$\alpha = \infty$ above/below a point ξ, located within the aforementioned strip.

Thus,

$$K_\pm(\alpha) = \exp\left[\pm\frac{1}{2\pi i}\int_{C_\pm}\frac{\log K(\xi)}{\xi-\alpha}d\xi\right] \tag{A.138}$$

If $\xi = i\beta$ with $\beta \geq s$, the contours $C_\pm$ may be shifted to the real axis, giving

$$K_\pm(\pm i\beta) = \exp\left[\frac{1}{\pi}\int_0^\infty \frac{\log[K(\tau)]}{1+\tau^2}d\tau\right] \tag{A.139}$$

A.9 Solution to a fuel-and-cladding rewetting model

In the following a model of suggested by Yeh (1980) is solved by the Wiener-Hopf technique. Further details can be found in Olek (1988e).

A.9.1 Analysis

The model is schematically depicted in Fig. 1. Far upstream of the quench front (at $\tilde{z} \to -\infty$), the wet region is quenched to a temperature T_s, while the far pre-quenched zone (at $\tilde{z} \to \infty$) is still at the initial wall temperature T_w. The heat transfer coefficient h in the wetted region is assumed to be constant, whereas the dry portion of the rod is assumed to be adiabatic. The gap is modelled by a convection boundary condition.

WET REGION (A): $z < 0$

DRY REGION (B): $z > 0$

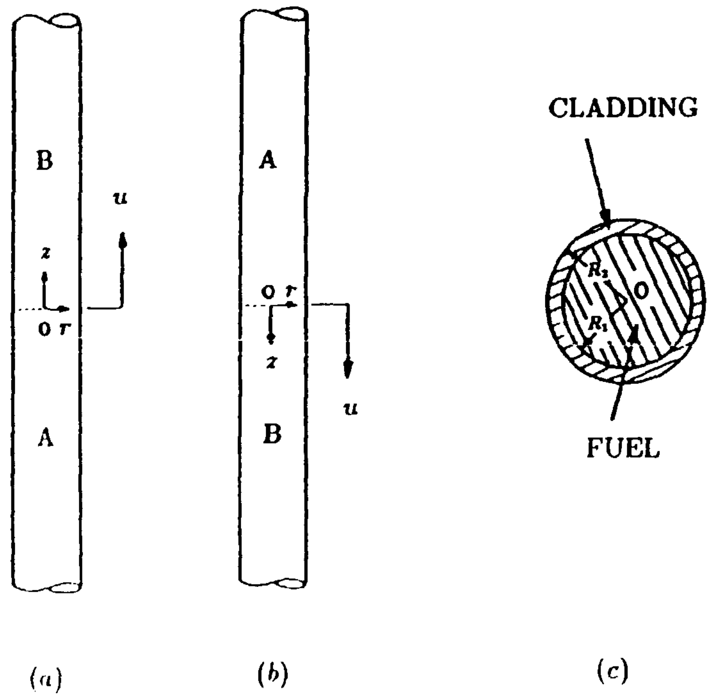

FIG. 1

Schematics of the rewetting model: (a) bottom-flooding, (b) top-flooding.

Assuming a constant rewetting velocity u, the quasi-steady-state heat conduction equation for a frame of reference $(\tilde{r},\tilde{z})$ moving along the solid at this velocity is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta_1}{\partial r}\right)+\frac{\partial^2\theta_1}{\partial z^2}+2s_1\frac{\partial\theta_1}{\partial z}=0 \qquad 0<r<R \;\;,\quad -\infty<z<\infty \tag{A.140}$$

for the fuel, and

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta_2}{\partial r}\right)+\frac{\partial^2\theta_2}{\partial z^2}+2s_2\frac{\partial\theta_2}{\partial z}=0 \qquad R<r<1 \;\;,\quad -\infty<z<\infty \tag{A.141}$$

for the cladding, where

$$\theta\equiv\frac{T-T_S}{T_w-T_S} \quad,\quad r\equiv\frac{\tilde{r}}{R_2} \quad,\quad z\equiv\frac{\tilde{z}}{R_2} \quad,\quad R\equiv\frac{R_1}{R_2}$$

R_1 and R_2 are the radii of the fuel and the outer surface of the cladding, repectively, θ the dimensionless temperature, and

$$s_1\equiv\frac{uR_2}{2\alpha_1} \quad,\quad s_2\equiv\frac{uR_2}{2\alpha_2}$$

with α_1 and α_2 denoting the thermal diffusivities of the fuel and the cladding, respectively. The solution of eqs.(A.140) and (A.142) is sought for the following boundary conditions

$$\frac{\partial\theta_1}{\partial r}=0 \qquad \text{at} \quad r=0, \quad -\infty<z<\infty \tag{A.142}$$

$$\frac{\partial\theta_2}{\partial r}+B_g(\theta_1-\theta_2)=0, \qquad \frac{\partial\theta_1}{\partial r}=\Gamma\frac{\partial\theta_2}{\partial r} \qquad \text{at} \quad r=R, \quad -\infty<z<\infty \tag{A.143a, b}$$

$$\frac{\partial\theta_2}{\partial r}+B_0\theta_2=0 \qquad \text{at} \quad r=1, \quad z<0 \tag{A.144}$$

$$\frac{\partial\theta_2}{\partial r}=0 \qquad \text{at} \quad r=1, \quad z>0 \tag{A.145}$$

$$\theta_1,\theta_2\rightarrow 0 \qquad \text{as} \quad z\rightarrow-\infty \tag{A.146}$$

$$\theta_1,\theta_2\rightarrow 1 \qquad \text{as} \quad z\rightarrow\infty \tag{A.147}$$

$$\theta_2,\frac{\partial\theta_2}{\partial r}\sim O(1) \qquad \text{at} \quad r=1, \quad z=0 \tag{A.148}$$

where

$$B_g\equiv\frac{h_gR_2}{k_g} \qquad\qquad B_0\equiv\frac{hR_2}{k_2}$$

Here B_g is the gap Biot number (the reciprocal of the gap resistance) , based on the gap heat transfer coefficient h_g and the thermal conductivity of the gap k_g, B_0 is the wet side Biot number. The ratio between the thermal conductivity of the cladding k_2 and the that of the fuel k_1 is $\Gamma = k_2/k_1$.

The additional condition which establishes the relation between the rewetting temperature and the other model parameters is

$$\theta = \theta_0 \qquad \text{at} \quad r = 1, \quad z = 0 \qquad (A.149)$$

Let new dependent variables be defined by

$$\theta_j(r,z) = 1 - \phi_j(r,z)e^{-S_j z}, \qquad (j = 1,2) \qquad (A.150)$$

With these new variables, eqs.(A.140)-(A.149) transform into the following equations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi_j}{\partial r}\right) + \frac{\partial^2 \phi_j}{\partial z^2} - s_j^2\phi_j = 0, \quad (j = 1,2), \quad r_{j-1} < r < r_j, \quad -\infty < z < \infty \qquad (A.151)$$

where $r_0 \equiv 0$, $r_1 \equiv R$ and $r_2 \equiv 1$.

$$\frac{\partial \phi_1}{\partial r} = 0 \qquad \text{at} \quad r = 0, \quad -\infty < z < \infty \qquad (A.152)$$

$$\frac{\partial \phi_2}{\partial r}e^{-S_2 z} + B_g(\phi_1 e^{-S_1 z} - \phi_2 e^{-S_2 z}) = 0 \qquad \text{at} \quad r = R, \quad -\infty < z < \infty \qquad (A.153a)$$

$$\frac{\partial \phi_1}{\partial r}e^{-S_1 z} = \Gamma\frac{\partial \phi_2}{\partial r}e^{-S_2 z} \qquad \text{at} \quad r = R, \quad -\infty < z < \infty \qquad A.153b)$$

$$\frac{\partial \phi_2}{\partial r} + B_0\phi_2 = B_0 e^{S_2 z} \qquad \text{at} \quad r = 1, \quad z < 0 \qquad (A.154)$$

$$\frac{\partial \phi_2}{\partial r} = 0 \qquad \text{at} \quad r = 1, \quad z > 0 \qquad (A.155)$$

$$\phi_1 \sim O(e^{S_1 z}) \quad \text{and} \quad \phi_2 \sim O(e^{S_2 z}) \qquad \text{as} \quad z \to -\infty \qquad (A.156)$$

$$\phi_1 \sim O(e^{-S_1 z}) \quad \text{and} \quad \phi_2 \sim O(e^{-S_2 z}) \qquad \text{as} \quad z \to +\infty \qquad (A.157)$$

$$\phi_2, \frac{\partial \theta_2}{\partial r} \sim O(1) \qquad \text{at} \quad r = 1, \quad z = 0 \qquad (A.158)$$

$$\phi_2 = \phi_0 = 1 - \theta_0 \qquad \text{at} \quad r = 1, \quad z = 0 \qquad (A.159)$$

Conditions (A.156) and (A.157) ensure that the Fourier transform

$$\Phi_j(r,\alpha) = \int_{-\infty}^{\infty} \phi_j(r,z)e^{i\alpha z}dz \quad \text{exists in} \quad D = D_1 \cup D_2 : \ \tau^- < \mathrm{Im}(\alpha) < \tau^+, (j = 1,2)$$

(A.160)

whilst

$$\Phi_j^+(r,\alpha) = \int_0^\infty \phi_j(r,z)e^{i\alpha z}dz \quad \text{exists in} \quad D_+ : \text{Im}\,\alpha > \tau^-, \quad (j=1,2)$$

$$\Phi_j^-(r,\alpha) = \int_{-\infty}^0 \phi(r,z)e^{i\alpha z}dz \quad \text{exists in} \quad D_- : \text{Im}\alpha < \tau^+, \quad (j=1,2)$$

where $\tau^+ = \min(s_1, s_2)$ and $\tau^- = \max(-s_1, -s_2)$.

Applying a Fourier transform to eq.(A.151) gives

$$\frac{d^2\Phi_j(r,\alpha)}{dr^2} + \frac{1}{r}\frac{d\Phi_j(r,\alpha)}{dr} - \gamma_j^2\Phi_j(r,\alpha) = 0 \qquad (j=1,2) \tag{A.161}$$

with $\gamma_j = (\alpha^2 + s_j^2)^{1/2}, \quad (j=1,2)$.
The solution of eq.(A.161) which satisfies the transform of e .(A.152) is

$$\Phi_1(r,\alpha) = A(\alpha)I_0(\gamma_1 r)$$

$$\Phi_2(r,\alpha) = B(\alpha)I_0(\gamma_2 r) + C(\alpha)K_0(\gamma_2 r) \tag{A.162}$$

Taking the Fourier transform of condition (A.153a) gives

$$\begin{aligned}&[(\alpha+is_2)^2+s_2^2]^{1/2}\{B(\alpha+is_2)I_1[R((\alpha+is_2)^2+s_2^2)^{1/2}]-\\&-C(\alpha+is_2)K_1[R((\alpha+is_2)^2+s_2^2)^{1/2}]\}+B_g\{A(\alpha+is_1)I_0[R((\alpha+is_1)^2+s_1^2)^{1/2}]-\\&-B(\alpha+is_2)I_0[R((\alpha+is_2)^2+s_2^2)^{1/2}]-C(\alpha+is_2)K_0[R((\alpha+is_2)^2+s_2^2)^{1/2}]\}=0\end{aligned} \tag{A.163}$$

whereas the transform of (A.153b) gives

$$\begin{aligned}&[(\alpha+is_1)^2+s_1^2]^{1/2}A(\alpha+is_1)I_1[R((\alpha+is_1+s_1^2)^{1/2}]=\Gamma[(\alpha+is_2)^2+s_2^2]^{1/2}\times\\&\times\{B(\alpha+is_2)I_1[R((\alpha+is_2)^2+s_2^2)^{1/2}]-C(\alpha+is_2)K_1[R((\alpha+is_2)^2+s_2^2)^{1/2}]\}\end{aligned} \tag{A.164}$$

Application of Fourier transforms to conditions (A.154) and (A.155) results in the following relations

$$\begin{aligned}B_0\Phi_2^+(1,\alpha) - \frac{iB_0}{\alpha - is_2} &= B(\alpha)[\gamma_2 I_1(\gamma_2) + B_0 I_0(\gamma_2)]\\&+C(\alpha)[-\gamma_2 K_1(\gamma_2) + B_0 K_0(\gamma_2)]\end{aligned} \tag{A.165}$$

$$\Phi_2'(1,\alpha) \equiv \Phi_2'^{-}(1,\alpha) = \gamma_2[B(\alpha)I_1(\gamma_2) - C(\alpha)K_1(\gamma_2)] \tag{A.166}$$

where prime denotes a derivative with respect to r.
Solving eqs.(A.163) and (A.164) for $B(\alpha + is_2)$ and $C(\alpha + is_2)$, with a formal replacement of α by $\alpha - is_2$ yields

$$B(\alpha) = A(\alpha + is_1 - is_2)f_1(\alpha) \qquad C(\alpha) = A(\alpha + is_1 - is_2)f_2(\alpha) \tag{A.167}$$

where

$$f_1(\alpha) = \frac{pI_1(Rp)[B_gK_0(Rq) + qK_1(Rq)] + \Gamma qB_gK_1(Rq)I_0(Rp)}{\Gamma q\{[B_gK_0(Rq) + qK_1(Rq)]I_1(Rq) - [B_gI_0(Rq) - qI_1(Rq)]K_1(Rq)\}}$$

$$f_2(\alpha) = \frac{B_g\Gamma qI_1(Rq)I_0(Rp) - pI_1(Rp)[B_gI_0(Rq) - qI_1(Rq)]}{\Gamma q\{[B_gK_0(Rq) + qK_1(Rq)]I_1(Rq) - [B_gI_0(Rq) - qI_1(Rq)]K_1(Rq)\}} \tag{A.168}$$

whence $p = [(\alpha - is_2)(\alpha + 2is_1 - is_2)]^{1/2}$, $\quad q \equiv \gamma_2 = (\alpha^2 + s_2^2)^{1/2}$.

Substituting from eq. (A.167) the values of $B(\alpha)$ and $C(\alpha)$ in the relations (A.165) and (A.166), and eliminating $A(\alpha + is_1 - is_2)$, finally gives the Wiener-Hopf relation

$$B_0\Phi_2^+(1,\alpha) - K(\alpha)\Phi_2'^{-}(1,\alpha) = \frac{iB_0}{\alpha - is_2} \tag{A.169}$$

where

$$K(\alpha) = 1 + \frac{B_0}{\gamma_2}\frac{I_0(\gamma_2)}{I_1(\gamma_2)}\frac{1 + \frac{f_2}{f_1}\frac{K_0(\gamma_2)}{I_0(\gamma_2)}}{1 - \frac{f_2}{f_1}\frac{K_1(\gamma_2)}{I_1(\gamma_2)}} \tag{A.170}$$

From eq.(A.170) the following particular cases may be recovered:

1. *Hollow cylinder with an insulated inner core.* For this case $B_g = 0$, and eq.(A.167) yields

$$\frac{f_2}{f_1} = \frac{I_1(R\gamma_2)}{K_1(R\gamma_2)}$$

so that

$$K(\alpha) = 1 + \frac{B_0}{\gamma_2}\frac{I_1(R\gamma_2)K_0(\gamma_2) + K_1(R\gamma_2)I_0(\gamma_2)}{K_1(R\gamma_2)I_1(\gamma_2) - I_1(R\gamma_2)K_1(\gamma_2)} \tag{A.171}$$

which is the form obtained by Chakrabarti (1986a).

2. *Homogeneous cylinder.* Set $R = 0$ in eq.(A.171) to obtain

$$K(\alpha) = 1 + \frac{B_0}{\gamma_2}\frac{I_0(\gamma_2)}{I_1(\gamma_2)} \tag{A.172}$$

which agrees with evans (1984).

3. *Plane slab.* Setting in eq.(A.171) $R = 1 - \varepsilon$ where $\varepsilon \to 0$ together with $R_1, R_2 \to \infty$ (so that the modified Bessel functions may be replaced by their asymptotic approximations) yields

$$K(\alpha) = 1 + \bar{B}_0 \frac{\coth(\bar{\gamma}_2)}{\bar{\gamma}_2} \tag{A.173}$$

where $\bar{B}_0 = \varepsilon B_0$, i.e. a Biot number based on the thickness of the slab and $\bar{\gamma}_2 = \varepsilon\gamma_2$, which yields the same expression as derived by Olek (1988a).

A.9.2 Solution

Setting $K(\alpha) = K_+(\alpha)K_-(\alpha)$ and utilizing it in eq.(A.169) gives, after some rearrangement, the following relation

$$\frac{\Phi_2'^{+}(1,\alpha)}{K_+(\alpha)} - \frac{i}{\alpha - is_2}\left[\frac{1}{K_+(\alpha)} - \frac{1}{K_+(is_2)}\right] = \frac{1}{B_0}\Phi_2'^{-}(1,\alpha)K_-(\alpha) - \frac{i}{\alpha - is_2}\frac{1}{K_+(is_2)} \tag{A.174}$$

Since the left-hand side of (A.174) is analytic for $\mathrm{Im}(\alpha) > \tau_-$ while the right-hand side is analytic for $\mathrm{Im}(\alpha) < \tau_+$, each side represents the same entire function $E(\alpha)$, say, $E(\alpha)$ which is found by analyzing the asymptotic behavior as $|\alpha| \to \infty$ of each side of (A.174) in the respective half-planes, and applying Liouville's theorem.
As $\Phi_2^+(1,\alpha)$ and $\Phi_2^-(1,\alpha)$ tend to zero at infinity in their half-planes of regularity, while $K_\pm(\alpha)$ remain bounded, it follows that $E(\alpha) \equiv 0$, giving

$$\Phi_2^+(1,\alpha) = \frac{i}{\alpha - is_2}\left[1 - \frac{K_+(\alpha)}{K_+(is_2)}\right] \tag{A.175}$$

$$\Phi_2^-(1,\alpha) = \frac{iB_0}{\alpha - is_2}\frac{1}{K_+(is_2)K_-(\alpha)} \tag{A.176}$$

Following Levin (1982) and Evans (1984), we use the relation

$$\Phi_2^+(1,\alpha) = \int_0^\infty \phi(1,z)e^{i\alpha z}dz = i\alpha^{-1}\phi(1,0) - (i\alpha)^{-1}\int_0^\infty \frac{\partial\phi}{\partial z}(1,z)e^{i\alpha z}dz \tag{A.177}$$

and note that the integral term vanishes as $\alpha \to \infty$ in D_+, as implied by (A.158). Combining (A.175) and (A.176) and assuming $K_\pm(\alpha) \to 1$ as $\alpha \to \infty$ in $D_\pm$, the following relation for the rewetting temperature is obtained

$$\theta_0 = 1 - \phi(1,0) = 1 + \lim_{\alpha\to\infty}[i\alpha\Phi_2^+(1,\alpha)] = [K_+(is_2)]^{-1} \tag{A.178}$$

It remains to factorize $K(\alpha)$. Following Chakrabarti (1986b), we choose a direct decomposition approach.
Since $\log[K(\zeta)] = O(1/\alpha)$ as $\alpha \to \infty$ in D, it follows from Cauchy's theorem that

$$\log K(\alpha) = \frac{1}{2\pi i}\int_{C_+}\frac{\log[K(\zeta)]}{\zeta - \alpha}d\zeta - \frac{1}{2\pi i}\int_{C_-}\frac{\log[K(\zeta)]}{\zeta - \alpha}d\zeta \tag{A.179}$$

where C_+/C_- is an infinite contour from $\mathrm{Re}(\alpha) = -\infty$ to $\mathrm{Re}(\alpha) = \infty$ located in D and passing below/above the point $\alpha \in D$.
Thus,

$$\log K_\pm(\alpha) = \pm\frac{1}{2\pi i}\int_{C_\pm}\frac{\log[K(\zeta)]}{\zeta - \alpha}d\zeta \tag{A.180}$$

In particular

$$\log[K_+(is_2)] = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\log[K(\sigma)]}{\sigma - is_2}d\sigma \tag{A.181}$$

where the contour has been shifted to the real axis. eq.(A.181) can be put in a different form

$$\log[K_+(is_2)] = \frac{1}{2\pi i}\left\{\int_0^\infty \frac{\log[K(\sigma)/K(-\sigma)]}{\sigma^2 + s_2^2}d\sigma + is_2\int_0^\infty \frac{\log[K(\sigma)K(-\sigma)]}{\sigma^2 + s_2^2}d\sigma\right\}$$

giving

$$[K_+(is_2)]^{-1} = \exp\left\{-\frac{1}{\pi}\int_0^\infty \frac{s_2\log[\rho(\sigma)] + \sigma\psi(\sigma)}{\sigma^2 + s_2^2}d\sigma\right\} \tag{A.182}$$

or in a form which is more convenient for numerical computation

$$[K_+(is_2)]^{-1} = \exp\left\{-\frac{1}{\pi}\int_0^{\pi/2}\{\log[\rho(s_2\tan\epsilon)] + \psi(s_2\tan\epsilon)\tan\epsilon\}d\epsilon\right\} \tag{A.183}$$

where the functions $\rho(\sigma)$ and $\psi(\sigma)$ are given by

$$\rho(\sigma)e^{i\psi(\sigma)} = K(\sigma) = 1 + \frac{B_0 I_0(\sigma^2+s_2^2)^{1/2}}{(\sigma^2+s_2^2)^{1/2} I_1(\sigma^2+s_2^2)^{1/2}} \frac{1+[a(\sigma)+ib(\sigma)]\frac{K_0(\sigma^2+s_2^2)^{1/2}}{I_0(\sigma^2+s_2^2)^{1/2}}}{1-[a(\sigma)+ib(\sigma)]\frac{K_1(\sigma^2+s_2^2)^{1/2}}{I_1(\sigma^2+s_2)^{1/2}}} \tag{A.184}$$

with $\quad a(\sigma)+ib(\sigma) = f_2(\sigma)/f_1(\sigma)$

The explicit expressions for the functions $\rho(s_2 \tan\epsilon)$ and $\psi(s_2 \tan\epsilon)$, which appear in eq.(A.183) are

$$\rho(s_2 \tan\epsilon) = (X^2+Y^2)^{1/2} \tag{A.185}$$

$$\psi(s_2 \tan\epsilon) = \tan^{-1}(Y/X) \tag{A.186}$$

where

$$X = 1 + Z_0 \frac{Z_1 Z_3 + Z_2 Z_4}{Z_1^2 + Z_2^2} \qquad Y = Z_0 \frac{Z_1 Z_4 - Z_2 Z_3}{Z_1^2 + Z_2^2}$$

The Z functions are defined through the following sequence of relations

$$Z_0 = \frac{B_0 I_0(s_2 \sec\epsilon)}{s_2 \sec\epsilon I_1(s_2 \sec\epsilon)}$$

$$Z_1 = 1 - a(s_2 \tan\epsilon)\frac{K_1(s_2 \sec\epsilon)}{I_1(s_2 \sec\epsilon)} \qquad Z_2 = -b(s_2 \tan\epsilon)\frac{K_1(s_2 \sec\epsilon)}{I_1(s_2 \sec\epsilon)}$$

$$Z_3 = 1 + a(s_2 \tan\epsilon)\frac{K_0(s_2 \sec\epsilon)}{I_0(s_2 \sec\epsilon)} \qquad Z_4 = b(s_2 \tan\epsilon)\frac{K_0(s_2 \sec\epsilon)}{I_0(s_2 \sec\epsilon)}$$

The explicit expressions for $a(s_2 \tan\epsilon)$ and $b(s_2 \tan\epsilon)$ are

$$a = \frac{(W_1 V_1 + W_2)(W_4 + W_3 V_1) + W_1 W_3 V_2^2}{(W_4 + W_3 V_1)^2 + W_3^2 V_2^2} \tag{A.187}$$

$$b = \frac{W_1 V_2 (W_4 + W_3 V_1) - W_3 V_2 (W_1 V_1 + W_2)}{(W_4 + W_3 V_1)^2 + W_3^2 V_2^2} \tag{A.188}$$

with

$$W_1 = B_g \Gamma q' I_1(Rq') \qquad W_2 = q' I_1(Rq') - B_g I_0(Rq')$$

$$W_3 = B_g \Gamma q' K_1(Rq') \qquad W_4 = q' K_1(Rq') + B_g K_0(Rq')$$

$$p' = p(s_2 \tan \epsilon) = [(s_2^2 \tan^2 \epsilon + 2s_1 s_2 - s_2^2) + i2s_2(s_1 - s_2)\tan \epsilon]^{1/2} = (\Delta + i\Lambda)^{1/2} = \xi + i\eta$$

$$q' = s_2 \sec \epsilon \qquad \xi = \mu \cos(\lambda/2) \qquad \eta = \mu \sin(\lambda/2)$$

$$\mu = (\Delta^2 + \Lambda^2)^{1/4} = s_2^{1/2} \sec \epsilon [s_1^2 + (s_1 - s_2)^2 + 2s_1(s_1 - s_2)\cos 2\epsilon]^{1/4}$$

$$\lambda = \tan^{-1}\left[\frac{(s_1 - s_2)\sin 2\epsilon}{s_1 + (s_1 - s_2)\cos 2\epsilon}\right] \qquad \Delta = s_2^2 \tan^2 \epsilon + 2s_1 s_2 - s_2^2 \qquad \Lambda = 2s_2(s_1 - s_2)\tan \epsilon$$

and

$$\frac{I_0(Rp')}{p' I_1(Rp')} = V_1 + iV_2$$

A.10 Integral equation formulations

In this section, solutions by integral equation formulations will be presented. These will be explained through concrete examples, which are the solutions to the model for the rewetting of a plane slab by Caflisch and Keller (1981) and Levine (1982). The former solve the model using the original formulation, whereas the latter uses a new dependent variable which transforms the differential equation into a steady state wave equation (similar to what we did when using Jones's direct method). The advantages of the second approach will be demonstrated. Finally, a comparison between Jones's direct approach and the single integral equation formulation is given. From the comparison it is concluded that the direct method is preferable to an integral equation formulation.

A.10.1 Solution with the original formulation

We begin with a formulation similar to (A.22)–(A.28).

$$\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial x^2} - P\frac{\partial\theta}{\partial x} = 0 \qquad 0 < y < 1, \qquad -\infty < x < \infty \tag{A.189}$$

where θ is a dimensionless temperature, and y and x are traversal and axial coordinates, respectively, which are normalized with respect to the radius of the rod, δ.
The Peclet number, P represents the dimensionless rewetting velocity and is given by

$$P \equiv \frac{u\delta}{\alpha}$$

with the rewetting velocity denoted by u and the thermal diffusivity of the slab denoted by α.

The solution of eq. (A.189) must satisfy the following boundary conditions

$$\frac{\partial\theta}{\partial y} = 0 \qquad \text{at} \quad y = 0, \qquad -\infty < x < \infty \tag{A.190}$$

$$\frac{\partial\theta}{\partial y} = 0 \qquad \text{at} \quad y = 1, \qquad x > 0 \tag{A.191}$$

$$\frac{\partial\theta}{\partial y} + B\theta = 0 \quad \text{at} \quad y = 1, \qquad x < 0 \tag{A.192}$$

$$\theta \to 0 \qquad \text{as} \quad x \to -\infty \tag{A.193}$$

$$\theta \to 1 \qquad \text{as} \quad x \to \infty \tag{A.194}$$

$$\theta = \theta_0 \qquad \text{at} \quad y = 1, \qquad x = 0 \tag{A.195}$$

with

$$\frac{T - T_s}{T_w - T_s}, \qquad B \equiv \frac{h\delta}{k}$$

where h is the wet side heat transfer coefficient and k is the thermal conductivity of the slab.

Consider the Green function $G(x - x_0, y, y_0)$. This function is defined as the solution of eq.(A.189) representing the potential at a point (x, y) caused by a line source at the point (x_0, y_0) in a region of any shape with given boundary conditions. In our case G satisfies the following equations:

$$\frac{\partial^2 G}{\partial y^2} + \frac{\partial^2 G}{\partial x^2} - P\frac{\partial G}{\partial x} = \delta(x - x_0)\delta(y - y_0) \tag{A.196}$$

$$\frac{\partial G}{\partial y} = 0 \qquad \text{on} \quad y = 0, 1 \tag{A.197}$$

We can express $\theta(x, y)$ in the strip $-\infty < x < \infty$, $0 < y < 1$ in terms of $\theta(x, 1)$, its normal derivative and the Green function. Green's theorem is applied over a rectangle whose boundaries are $y = 0$, $y = 1$, $x = -l$, $x = l_1$ $(l, l_1 >> 0)$ and we get

$$\theta(x, y) = \int \left[G(x, y, x_0, y_0)\frac{\partial \theta(x_0, y_0)}{\partial n} - \theta(x_0, y_0)\frac{\partial G(x, y, x_0, y_0)}{\partial n} \right] ds$$

where ds is the element of arc-length along the boundary of the rectangle which we have described. The operation $\partial/\partial n$ denotes the outer normal derivative. It is to be noted that this integral is treated as an improper one in the sense that we may allow l and l_1 to become infinitive after we discuss the magnitude of the integrals along these boundaries. We are permitted to do this because of the decay properties of $G(x, y, x_0, y_0)$ for $x >> x_0$ or $x << x_0$ and the assumption regarding $\theta(x, 1)$ for $|x| >> 0$.
The Fourier transform of G in $x_0 - x$ is given by (Morse and Feshbach (1953))

$$\hat{G}(\alpha, y, y_0) = \frac{(2\pi)^{1/2}}{\sigma \sin \sigma} \cos(\sigma y_<) \cos \sigma(y_> - 1) \tag{A.198}$$

Here

$$y_< = \min(y, y_0), \qquad y_> = \max(y, y_0)$$

and

$$\sigma = (-i\alpha P - \alpha^2)^{1/2} \tag{A.199}$$

In particular

$$\hat{G}(\alpha, 1, 1) = (2\pi)^{-1/2}\frac{\cot\sigma}{\sigma} \tag{A.200}$$

The only real pole of $\hat{G}$ is at $\alpha = 0$, and near it $\hat{G}$ is of the form $(2\pi)^{-1/2}i/P\alpha$. This is the transform of the discontinuous function with a value $1/2P$ for $x - x_0 > 0$ and $-1/2P$ for $x_0 - x < 0$. Therefore

$$G(x - x_0, y, y_0) \to \pm\frac{1}{2P}, \quad \text{as} \quad x_0 - x \to \pm\infty \tag{A.201}$$

In fact G approaches these constant values at a rate $e^{-P|x_0 - x|}$, and consequently

$$\theta(x_0, y_0) = \frac{1}{2} + B\int_{-\infty}^{0} \theta(x, 1)G(x_0 - x, y_0, 1)dx \tag{A.202}$$

When we set $y_0 = 1$ in (A.202), it becomes an integral equation for $\theta(x, 1)$.
To solve this equation, we introduce $\theta_+(x)$ defined by

$$\begin{aligned} \theta_+(x) &= \frac{1}{2}, & x < 0, \\ &= \theta(x) - \frac{1}{2}, & x > 0 \end{aligned} \tag{A.203}$$

Then we define the Fourier transforms $\Phi_\pm(\alpha)$ by

$$\begin{aligned} \Phi_+(\alpha) &= \frac{1}{(2\pi)^{1/2}}\int_0^\infty \theta_+(x, 1)e^{i\alpha x}dx \\ \Phi_-(\alpha) &= \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^0 \theta(x, 1)e^{i\alpha x}dx \end{aligned} \tag{A.204}$$

Upon taking Fourier transforms of both sides of eq.(A.202) with $y_0 = 1$, we get

$$\Phi_+ + \Phi_- = (2\pi)^{1/2}B\Phi_-\hat{G} \tag{A.205}$$

By using (A.200) for $\hat{G}$ we can rewrite (A.205) in the form

$$\frac{\Phi+}{\Phi_-} = -\frac{\sigma\sin\sigma - B\cos\sigma}{\sigma\sin\sigma} \tag{A.206}$$

We can determine Φ_+ and Φ_- from (A.206) by modifying slightly the result of Morse and Feshbach (1953), p. 1528 to yield

$$\Phi_+ = \frac{c}{\alpha}\prod_{m=0}^{\infty}\frac{\alpha + \frac{1}{2}iP(1+\gamma_m)}{\alpha + \frac{1}{2}iP(1+\beta_m)}$$

$$\Phi_- = -c\frac{\nu_0^2}{B}\left(\prod_{m=1}^{\infty}\frac{\nu_m^2}{m^2\pi^2}\right)\frac{1}{\alpha + \frac{1}{2}iP(1-\gamma_0)}\left[\prod_{m=1}^{\infty}\frac{\alpha + \frac{1}{2}iP(1-\beta_m)}{\alpha + \frac{1}{2}iP(1-\gamma_m)}\right] \qquad \text{(A.207)}$$

Here c is undetermined, but γ_m and β_m are defined by

$$\gamma_m = (1 + 4\nu_m^2/P^2)^{1/2},$$

$$\beta_m = (1 + 4\pi^2 m^2/P^2)^{1/2}, \qquad \text{(A.208)}$$

where ν_m is the $(m+1)$st positive root of

$$\nu_m \tan \nu_m = B \qquad \text{(A.209)}$$

We note that $m\pi < \nu_m < (m + \frac{1}{2})\pi$ and $\gamma_m > 1, \beta_m \geq 1$.

The function Φ_+ is analytic in $\alpha > 0$ with an infinite number of poles on the negative imaginary axis; Φ_- is analytic in $\text{Im}\,\alpha < \frac{1}{2}P(\gamma_0 - 1)$ with an infinite number of poles on the positive imaginary axis. The term $-\frac{1}{2}$ in θ_+ implies that $\theta_+(\pm\infty) = \pm\frac{1}{2}$ and leads to the pole in Φ_+ at $\alpha = 0$. We introduce it to eliminate the delta function which occurs in the transform

$$\frac{1}{(2\pi)^{1/2}}\int_0^{\infty}\theta(x,1)e^{i\alpha x}dx = \Phi_+ + \frac{1}{2}(2\pi)^{1/2}\delta(\alpha)$$

For the same reason we have chosen the arbitrary constant in G to make it antisymmetric at $x = \pm\infty$.

The temperature θ is obtained by inverting its transform to get

$$\theta = \frac{1}{2} + \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}[\Phi_+(\alpha) + \Phi_-(\alpha)]e^{-i\alpha x}d\alpha$$

$$= \frac{1}{2} + \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}\Phi_+(\alpha)e^{-i\alpha x}d\alpha, \qquad x > 0,$$

$$= \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}\Phi_-(\alpha)e^{-i\alpha x}d\alpha, \qquad x < 0 \qquad \text{(A.210)}$$

The second and third expressions result from deforming the contour to Im $\alpha = \mp\infty$ and using the analiticity of $\Phi_\mp$, respectively. By letting x tend to $+\infty$ in equation (A.210) and using eq. (A.194), we get the following equation for c:

$$1 = \frac{1}{2} + \lim_{x\to\infty} \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \Phi_+(\alpha)e^{-i\alpha x}d\alpha \tag{A.211}$$

The integral in (A.211) can be calculated by deforming the contour downward. Since $\mathrm{Re}(-i\alpha x) < 0$ if $x > 0$ and Im $\alpha < 0$, the deformed contour integral vanishes and the residues at all poles other than $\alpha = 0$ tend to zero as $x \to +\infty$. Since the original contour passes through $\alpha = 0$, this pole contributes only half of its residue. Thus

$$\frac{1}{2} = \lim_{x\to\infty} \left[-\frac{1}{(2\pi)^{1/2}}(2\pi i)\frac{1}{2}\mathrm{Res}_{\alpha=0}\Phi_+ \right] = -\frac{(2\pi)^{1/2}}{2} ic \prod_{m=0}^{\infty} \frac{1+\gamma_m}{1+\beta_m} \tag{A.212}$$

By solving (A.212) for c we obtain

$$c = \frac{i}{(2\pi)^{1/2}} \prod_{m=0}^{\infty} \frac{1+\beta_m}{1+\gamma_m} \tag{A.213}$$

This completes the determination of the solution for $\theta(x, y)$ in terms of B and P.

Calculation of θ_0

Finally we shall calculate $\theta_0 = \theta_0(B, P)$ in terms of B and P. From the second form of eq. (A.210) we obtain

$$\theta_0 = \frac{1}{2} + \lim_{x\to 0^+} \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \Phi_+ e^{-i\alpha x} d\alpha = \frac{1}{2} + \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \Phi_+ d\alpha$$
$$- \lim_{x\to 0^+} \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \Phi_+(1 - e^{-i\alpha x})d\alpha \tag{A.214}$$

To evaluate the first integral in eq. (A.214), we deform the contour to an upper semi-circle Γ_1 of infinite radius. This pics up half the singularities of the residue at $\alpha = 0$, and yields

$$\int_{-\infty}^{\infty} \Phi_+ d\alpha = 2\pi i \frac{1}{2}\mathrm{Res}_{\alpha=0}\Phi_+ - \frac{1}{(2\pi)^{1/2}} \int_{\Gamma_1} \Phi_+ d\alpha \tag{A.215}$$

To evaluate the second integral in eq. (A.214), we deform the contour to the lower semi-circle $\Gamma_2(R)$ of radius R. This pics up residues at poles along the imaginary axis. However as $x \to 0$, the factor $1 - e^{-i\alpha x}$ tends to zero, and all these residues tend to zero. All that remains is the integral along $\Gamma_2(R)$. But as $R \to \infty$, $e^{-i\alpha x} \to 0$. Thus

$$\int_{-\infty}^{\infty} \Phi_+(1 - e^{-i\alpha x})d\alpha = \lim_{R\to\infty} \int_{\Gamma_2(R)} \Phi_+ d\alpha \tag{A.216}$$

The poles of Φ_+ are at $\alpha_m = -\frac{1}{2}iP(1+\beta_m) = -i(\pi m + \frac{1}{2}P) + O(1/m)$. If $|\alpha| \to \infty$, with $|\alpha - \alpha_m| > \frac{1}{4}\pi$, then $\Phi_+ = c/\alpha + O(1/\alpha^2)$. Therefore

$$\lim_{R\to\infty} \int_{\Gamma_2(R)} \Phi_+ d\alpha = \int_{\Gamma_1} \Phi_+ d\alpha = c\pi i \tag{A.217}$$

Also $\mathrm{Res}_{k=0}\Phi_+ = \prod_{m=0}^{\infty}(1+\gamma_m)/(1+\beta_m)$. Substituting this with eq. (A.217) and (A.213) into eq. (A.214) yields

$$\theta_0(B,P) = \prod_{m=0}^{\infty} \frac{1+\beta_m}{1+\gamma_m} \tag{A.218}$$

We could also calculate

$$\theta_0 = \lim_{x\to 0^-} \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \Phi_- e^{-i\alpha x} d\alpha$$

by the same procedure to find

$$\theta_0(B,P) = \frac{\nu_0^2}{B} \prod_{m=1}^{\infty} \frac{\nu_m^2}{m^2\pi^2} \prod_{m=0}^{\infty} \frac{1+\beta_m}{1+\gamma_m} \tag{A.219}$$

From eqs. (A.218) and (A.219) we can see that ν_m must satisfy

$$\frac{\nu_0^2}{B} \prod_{m=1}^{\infty} \frac{\nu_m^2}{m^2\pi^2} = 1 \tag{A.220}$$

This is consistent with the inequalities listed after eq. (A.209). It follows from eq. (A.208) that $\theta_0(B,P)$ given by eq. (A.218) is a strictly decreasing function of P, so it can be inverted to yield $P(B,\theta_0)$.

Validity of the solution

We now show that $\theta(x,y)$ defined by eqs. (A.202) and (A.210) solve eq. (A.189). The mth term in either of the infinite products in (A.207) is $1+O(B/m^2)$ for m large and α not at one of the poles. It follows that these products converge absolutely and uniformly away from the poles and that Φ_+, Φ_-, and $\hat{G}$ have the following properties:

$$\Phi_+(\alpha) = i\frac{1}{(2\pi)^{1/2}\alpha} + O(1) \quad \text{for} \quad \alpha \sim 0,$$

$$= \frac{c}{\alpha} + O(1/\alpha^2) \quad \text{as} \quad \alpha \to \pm\infty \tag{A.221}$$

$$\Phi_-(\alpha) = O(1) \quad \text{for} \quad \alpha \sim 0,$$

$$= -c\frac{\nu_0^2}{B}\left(\prod_{m=1}^{\infty}\frac{\nu_m^2}{m^2\pi^2}\right)\frac{1}{\alpha} + O(1/\alpha^2) \quad \text{as} \quad \alpha \to \pm\infty, \tag{A.222}$$

$$\hat{G}(\alpha,1,1) = \frac{i}{(2\pi)^{1/2}\alpha P}, \quad \text{for} \quad \alpha \sim 0,$$

$$= \mp\frac{1}{(2\pi)^{1/2}\alpha} + O(1/\alpha^2) \quad \text{as} \quad \alpha \to \pm\infty \tag{A.223}$$

From eqs. (A.222), (A.223) and (A.205) we see that $\Phi_+ + \Phi_- = O(1/\alpha^2)$ as $\alpha \to \pm\infty$. Thus the coefficients of the $1/\alpha$ terms must cancel. This is an independent proof of the identity (A.220). A comparison of the residues at $\alpha = 0$ of the two sides of eq. (A.205) shows that

$$\Phi_-(0) = \frac{P}{(2\pi)^{1/2}B} \tag{A.224}$$

Except for the singularity of Φ_+ at $\alpha = 0$, Φ_+ and Φ_- have infinitely many derivatives which decay as $\alpha \to \pm\infty$ on the real axis.

We now define $\theta_1(x)$ by

$$\theta_1(x) = \frac{1}{2} + \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}[\Phi_+(\alpha) + \Phi_-(\alpha)]e^{i\alpha x}d\alpha \tag{A.225}$$

From the properties of Φ_+, Φ_- listed above, it follows that θ_1 decreases rapidly (i.e. faster than x^{-n} for any integer n) at $x = -\infty$, that $\theta_1 - 1$ is rapidly decreasing at $x = \infty$ and that θ_1 is continuous. The Fourier transform of eq. (A.225) is just the second of eqs. (A.204) with θ replaced by θ_1.

Next we define Ψ by

$$\Psi(\alpha, y_0) = (2\pi)^{1/2}B\Phi_-(\alpha)G(\alpha, y_0, 1) \quad \text{for} \quad 0 \leq y_0 \leq 1 \tag{A.226}$$

This function decays exponentially fast at $\alpha = \pm\infty$ for $0 \le y_0 < 1$. By using in eq. (A. 226) both eq. (A.224) and the fact that $G(\alpha, y_0, 1) = \frac{i}{(2\pi)^{1/2} P\alpha} + O(1)$ for $\alpha \sim 0$ we see that $\Psi(\alpha, y_0) = \frac{i\alpha}{(2\pi)^{1/2}} + O(1)$ for $\alpha \sim 0$.

Finally we define $\theta(x_0, y_0)$, which we will show to be the solution of eq. (A.189), by

$$\theta(x_0, y_0) = \frac{1}{2} + \frac{1}{(2\pi)^{1/2}} \int_{-\infty}^{\infty} \Psi(\alpha, y_0) e^{-i\alpha x} d\alpha \quad \text{for} \quad -\infty < x < \infty, \quad 0 \le y_0 < 1 \tag{A.227}$$

From the properties of Ψ it follows that θ is infiniteit differentiable in the open strip and that θ approaches 0 and 1 as x approaches $+\infty$ and $-\infty$, respectively, thus satisfying eqs. (A.193) and (A.194). Moreover, from eq. (A.198) it follows that θ solves the differential equation (A.189).

Next we check the flux conditions (A.190) and (A.191). A direct calculation shows that $(\partial/\partial y_0)\Psi$ is continuous on $0 \le y_0 \le 1$ and

$$\begin{aligned} \frac{\partial}{\partial y_0}\Psi(\alpha, y_0) &= 0 \qquad \text{for} \quad y_0 = 0 \\ &= B\Phi_-(\alpha) \qquad \text{for} \quad y_0 = 1 \end{aligned} \tag{A.228}$$

Also

$$G(\alpha, y_0, 1) \to G(\alpha, 1, 1) \quad \text{as} \quad y_0 \to 1$$

uniformly in α. Thus a comparison of eqs. (A.226) and (A.227) with eqs. (A.205) and (A.225) shows that

$$\theta(x_0, y_0) \to \theta_1(x_0) \quad \text{as} \quad y_0 \to 1 \tag{A.229}$$

The flux conditions (A.190) and (A.191) follow from eqs. (A.228) and (A.227) by using eqs. (A.229) and the second equation of (A.204), with θ replaced by θ_1. The final condition (A.195) then holds as a consequence of eq. (A.218). Thus the solution we have constructed does solve the problem.

A.10.2 Solution with a new dependent variable

We begin with a formulation similar to (A.189)–(A.195).

$$\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial x^2} + 2s\frac{\partial\theta}{\partial x} = 0 \qquad 0 < y < 1, \qquad -\infty < x < \infty \tag{A.230}$$

where θ is a dimensionless temperature, and y and x are traversal and axial coordinates, respectively, which are normalized with respect to the radius of the rod, δ.
The Peclet number, $2s$ represents the dimensionless rewetting velocity and is given by

$$s \equiv \frac{u\delta}{2\alpha}$$

with the rewetting velocity denoted by u and the thermal diffusivity of the slab denoted by α.

The solution of eq. (A.230) must satisfy the following boundary conditions

$$\frac{\partial\theta}{\partial y} = 0 \qquad \text{at} \quad y = 0, \qquad -\infty < x < \infty \tag{A.231}$$

$$\frac{\partial\theta}{\partial y} = 0 \qquad \text{at} \quad y = 1, \qquad x > 0 \tag{A.232}$$

$$\frac{\partial\theta}{\partial y} + B\theta = 0 \quad \text{at} \quad y = 1, \qquad x < 0 \tag{A.233}$$

$$\theta \to 0 \qquad \text{as} \quad x \to -\infty \tag{A.234}$$

$$\theta \to 1 \qquad \text{as} \quad x \to \infty \tag{A.235}$$

$$\theta = \theta_0 \qquad \text{at} \quad y = 1, \qquad x = 0 \tag{A.236}$$

with

$$\frac{T - T_s}{T_w - T_s}, \qquad B \equiv \frac{h\delta}{k}$$

where h is the wet side heat transfer coefficient and k is the thermal conductivity of the slab.

Let a new dependent variable be defined by

$$\phi(x, y) = e^{sx}\theta(x, y) \tag{A.237}$$

With this new variable eqs. (A.230-A.236) become

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - s^2\phi = 0 \qquad 0 < y < 1, \qquad -\infty < x < \infty \tag{A.238}$$

$$\frac{\partial\phi}{\partial y} = 0 \qquad \text{at} \quad y = 0, \quad -\infty < x < \infty \tag{A.239}$$

$$\frac{\partial\phi}{\partial y} = 0 \qquad \text{at} \quad y = 1, \quad x > 0 \tag{A.240}$$

$$\frac{\partial\phi}{\partial y} + B\phi = 0 \quad \text{at} \quad y = 1, \quad x < 0 \tag{A.241}$$

$$\phi \sim \epsilon e^{sx} \to 0, \quad \text{as} \quad x \to -\infty \tag{A.242}$$

$$\phi \to e^{sx}, \qquad \text{as} \quad x \to \infty \tag{A.243}$$

$$\phi = \phi_0 = \theta_0 \tag{A.244}$$

Introduce a Green function $G(x, y; x', y')$ which obeys the relations

$$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} - s^2 G = -\delta(x - x')\delta(y - y') \tag{A.245}$$

$$\frac{\partial G}{\partial y} = 0 \quad \text{at} \quad y = 0, 1 \tag{A.246}$$

and admits the series representation

$$G(x, y; x', y') = \frac{1}{2s} e^{-s|x-x'|} + \frac{1}{s} \sum_{n=1}^{\infty} \frac{\cos n\pi y \cos n\pi y'}{\beta_n} e^{-s\beta_n|x-x'|} \tag{A.247}$$

where

$$\beta_n = (1 + n^2\pi^2/s^2)^{1/2}$$

On the use of Green's theorem the expression

$$\phi(x,y) = e^{sx} - B\int_{-\infty}^{0} G(x,y;x',1)\phi(x,1)dx' \tag{A.248}$$

follows and thence an integral equation for

$$\phi_-(x) = \phi(x,1), \quad \text{for} \quad x < 0 \tag{A.249}$$

namely,

$$\phi_-(x) = e^{sx} - B\int_{-\infty}^{0} K(|x-x'|)\phi_-(x')dx' \tag{A.250}$$

wherein

$$K(|x-x'|) = G(x,1;x',1) \tag{A.251}$$

An extended form of this integral equation, viz.

$$\phi_-(x) + \phi_+(x) = e^{sx} - B\int_{-\infty}^{\infty} K(|x-x'|)\phi_-(x')dx', \quad x < 0$$

$$= -B\int_{-\infty}^{\infty} K(|x-x'|)\phi_-(x')dx', \qquad x > 0 \tag{A.252}$$

with the stipulations

$$\phi_-(x) = 0, \quad x > 0$$

$$\phi_+(x) = 0, \quad x < 0 \tag{A.253}$$

provide the requisite functional relation that lends itself to Wiener-Hopf analysis via complex Fourier transforms; thus, define the individual transforms

$$\Phi_-(\alpha) = \int_{-\infty}^{0} \phi_-(x)e^{-i\alpha x}dx$$

$$\Phi_+(\alpha) = \int_{0}^{\infty} \phi_+(x)e^{-i\alpha x}dx$$

$$K(\alpha) = \int_{-\infty}^{\infty} K(|x|)e^{-i\alpha x}dx \tag{A.254}$$

and a ready consequence of (A.252) is that

$$\Phi_-(\alpha) + \Phi_+(\alpha) = \frac{i}{\alpha + is} - BK(\alpha)\Phi_-(\alpha)$$

or

$$[1 + BK(\alpha)]\Phi_-(\alpha) + \Phi_+(\alpha) = \frac{i}{\alpha + is} \tag{A.255}$$

Once the transform of the Green's function has been found that of the function K follows immediately; the appropriate expressions are

$$G(\alpha, y, y') = \int_{-\infty}^{\infty} G(x, y; 0, y')dx$$

$$= \frac{\cosh \gamma y_< \cosh \gamma(1 - y_>)}{\gamma \sinh \gamma} \tag{A.256}$$

where $\gamma = (\alpha^2 + s^2)^{1/2}$ and

$$K(\alpha) = G(\alpha, 1, 1) = \frac{\coth \gamma}{\gamma} \tag{A.257}$$

which enables us to treat (A.255) in the form

$$\Psi(\alpha)\Phi_-(\alpha) + \Phi_+(\alpha) = \frac{i}{\alpha + is} \tag{A.258}$$

where

$$\Psi(\alpha) = 1 + B\frac{\coth \gamma}{\gamma} \tag{A.259}$$

The transform relation (A.258) is valid within the strip $|\text{Im}\ \alpha| < s$ of the complex α-plane; and $\Phi_-(\alpha)$[1] ,$\Phi_+(\alpha)$ are analytic functions in the overlapping half-planes Im $\alpha > -s(+)$, Im $\alpha > s(-)$, respectively. Both $K(\alpha)$ and $\Psi(\alpha)$ are analytic functions in the strip, the latter possessing simple poles at $\alpha = \pm is$, $\alpha = \pm is\beta_n$, $n = 1, 2, \ldots$ and zeros at the points $\alpha = \pm is\delta_n$ ($\gamma = \pm i\nu_n$), $n = 1, 2, \ldots$ outside the strip, where $\delta_n = (1 + \nu_n^2/s^2)^{1/2}$. We see that with the original formulation, the Green function chosen by Caflisch and Keller has a pole at the origin, a complicating feature which calls for particular attention.

[1] $\Phi_-(\alpha)$ is analytic in the half plane Im $\alpha > -(s^2 + \nu_0^2)^{1/2}$, where ν_0 is the first positive root of $\nu \tan \nu = B$ since $\phi_-(x) = O[\exp s(1 + \nu_0^2/s^2)^{1/2}x]$, $x \to \infty$

Let

$$\Psi(\alpha) = \frac{\Psi_+(\alpha)}{\Psi_-(\alpha)} \tag{A.260}$$

where the functions $\Psi_+(\alpha)$, $\Psi_-(\alpha)$ are analytic above and below the lines Im $\alpha = -s, +s$, respectively, and $\Psi_\pm(\alpha) \neq 0$; then (A.257) can be rewritten in the form

$$\Psi_+(\alpha)\Phi_-(\alpha) - i\frac{\Psi_-(-is)}{\alpha + is} = -\Psi_-(\alpha) - \Psi(\alpha)\Phi_+(\alpha) + \frac{i}{\alpha + is}[\Psi_-(\alpha) - \Psi_-(-is)]$$
$$= E(\alpha), \quad \text{say,} \tag{A.261}$$

which characterizes an entire function $E(\alpha)$, through its representation in the upper and lower halves of the α-plane. Since $\Phi_-(\alpha)$ and $\Phi_+(\alpha)$ tend to zero at infinity in their repective planes of regularity, while $\Psi_+(\alpha)$ and $\Psi_-(\alpha)$ remain bounded, the entire function vanishes and the determination

$$\Phi_-(\alpha) = \frac{i}{\alpha + is}\frac{\Psi_-(-is)}{\Phi_+(\alpha)} \tag{A.262}$$

results.

The decomposition (A.260) was already performed, and we can use, e.g. (A.99)

$$\Psi_\pm(\pm i\chi) = \exp\left\{ \pm\frac{1}{\pi}\int_0^{\pi/2} \log\left\{1 + B\frac{\coth[s^2 + \chi^2(\sec^2\Omega - 1)]^{1/2}}{[s^2 + \chi^2(\sec^2\Omega - 1)]^{1/2}}\right\}d\Omega\right\} \tag{A.263}$$

if $\alpha = i\chi$, with $\chi \geq s$. To evaluate the rewetting temperature, let us use the relation

$$\Phi_-(\alpha) = \int_{-\infty}^0 \phi_-(x)e^{-i\alpha x}dx = \frac{\phi_-(0)}{-i\alpha)} + \frac{1}{i\alpha}\int_{-\infty}^0 \frac{\partial\phi_-(x)}{\partial x}e^{-i\alpha x}dx \tag{A.264}$$

and noting that the latter integral vanishes when $\alpha \to \infty$ (Im $\alpha > 0$) since $\partial\phi_-(x)/\partial x$ is integrable, we arrive at the connection

$$\phi_-(0) = \lim_{\alpha\to\infty}[-i\alpha\Phi_-(\alpha)] \tag{A.265}$$

The latter implies, after recourse to the characterization (A.262) for $\Phi_-(\alpha)$

$$\theta_0 = \phi_0 = \phi_-(0) = \lim_{\alpha\to\infty}\frac{\alpha}{\alpha + is}\frac{\Psi_-(-is)}{\Psi_+(\alpha)} = \Psi_-(-is) \tag{A.266}$$

Therefore

$$\theta_0 = \Psi_-(-is) = \exp\left[-\frac{1}{\pi}\int_0^{\pi/2}\log\left(1 + B\frac{\coth s\sec\Omega}{s\sec\Omega}\right)d\Omega\right] \tag{A.267}$$

A.10.3. Comparison of methods

We compare here Jones's direct method with the integral equation method.

The integral equation method needs: choice of Green's functions, formulation of the integral equation, application of transforms. Jones's method is more direct since the Wiener-Hopf equation is obtained directly from transforms applied to the partial differential equation.

The Green's function method for formulating integral equations is cumbersome. It is sometimes not completely obvious which Green's function should be chosen; examples have occured in the literature where this has caused confusion or made problems seem more complicated than need be. Also complicated functions may be introduced, whose Fourier transforms are required; these complicated functions are avoided altogether in Jones's method-the required transforms are obtained in the process of solution. In each Wiener-Hopf equation of the type (A.15) there are two unknown functions. In Jones's method these appear in a completely symmetrical way and the physical significance of each of the unknown functions is immediately obvious. In the integral equation approach the symmetry is lost in the integral equation itself, though it reappears when the integral equation is transformed.

The main advantage of the integral equation method of approach seems that it is very easy to recognize problems that can be solved by the Wiener-Hopf technique since the corresponding integral equations have semi-infinite range and the kernels are of the form $k(x-\xi)$ only. Sometimes when using Jones's method in complicated problems it may not be so immediately obvious that the transform equation can be reduced to the Wiener-Hopf form.

It may be possible that a solution can be obtained by means of the Wiener-Hopf technique from an integral equation formulated by a Green's function prodedure, whereas it may not be possible to obtain the same solution by Jones's method. But no such case has so far occured in connection with partial differential equations, to our knowledge.

To summerize, a sufficient number of points have been shown to recommend the use of Joenes's direct method.

A.11. Summary of the Wiener-Hopf technique

The basics of the Wiener-Hopf technique which were presented in this appendix are like the tip of the iceberg. A large amount of material has not been covered, such as: the decomposition of complicated kernels, the approximate decomposition of complicated kernels, the use of the dual integral equations method, use of transforms other than the complex Fourier transform, limitations of the Wiener-Hopf technique, simultaneous Wiener-Hopf equations, the complex variable problem solved by the Wiener-Hopf tecgnique as a special case of the Hilbert problem, etc.

For example, there are some guidelines how to perform an approximate factorization. When ceratain rules are not obeyed, an erroneous decomposition may result like in , e.g. Tien and Yao (1975) and Dua and Tien (1976).

The two-sided Laplace transform is often used in the literature (not for rewetting models until now) instead of the complex Fourier transform. Sometimes Mellin's transform is used in problems of cylindrical geometry.

For additional reading the books of Noble (1958) and Carrier et al. (1966) are recommended.