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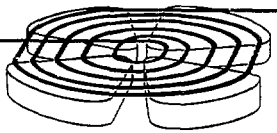

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FLUCTUATIONS OF SINGLE-PARTICLE DENSITY IN NUCLEAR COLLISIONS

S. AYIK *, C. GREGOIRE

GANIL, BP. 5027 F-14021 CAEN CEDEX; FRANCE

* Permanent address : Tennessee Technological University, Cookeville, TN 38505, USA.

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S. AYIK

Tennessee Technological University, Cookeville, TN 38505, U.S.A. *

C. GREGOIRE

GANIL, BP 5027, F-14021 Caen-Cedex, France

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Abstract

In order to incorporate fluctuations into the extended TDHF, a new approach is proposed. The evolution of the single-particle density is considered as a "generalized Langevin process" in which the correlated part of the two-body collisions acts as a "random force". In the semi-classical approximation, the correlation function of the random force is calculated. A possible algorithm for the numerical solution is discussed.

*Permanent address

The extended time-dependent Hartree-Fock (TDHF) in the semi-classical limit with the Uehling-Uhlenbeck (VUU) form of a collision term provides a numerically tractable model for the single-particle density f(r,p) in nuclear collisions [1]. The model has been successfully applied for describing the one-body observables such as the inclusive particle spectra and linear momentum transfer [2,3,4]. However, the model can not provide a description for the fluctuation phenomena responsible for multifragmentation processes and correlations in light particle emission. The obvious reason is that, the extended TDHF or any approximation to it determines the single-particle density which is averaged over all of the accessible channels (ensemble averaging) in a collision process. Consequently, *all* of fluctuations are washed out and a *smooth single-particle density* is produced.

In a recent work [5,6], a prescription is proposed in order to incorportate fluctuations into the VUU equation by treating the collision term as a quantum mechanical transition rate between Slater determinants rather than treating it as a continuous source term for describing the evolution of the single-particle momentum distribution. The fluctuation and dissipation properties of the single-particle density are not independent properties, but they must be related to each other -as for any relaxation process like in a Brownian motion problem- through a "generalized fluctuation-dissipation theorem" operating locally in phase-space. In the prescription given in refs. [5,6], fluctuations are introduced by hand, and are not treated consistently with the relaxation properties of the single-particle density.

Here, we propose a different approach for incorporating fluctuations into the extended TDHF. We consider the evolution of the single-particle density as a "generalized

Langevin process" similar to the motion of a Brownian particle in a heat bath, except that the Brownian coordinate is not the momentum of a particle but the whole single-particle density itself. The effects of the two-body collisions are two fold :

i) To produce dissipation by randomizing the single-particle momentum distribution which can be described by a collision term of the VUU form. ii) To induce fluctuations by propagating correlations from one point to the other in phase-space. In the present approach, the correlated part of the collisions are regarded as a "generalized random force" acting on the single-particle density. This "random force" is characterized by its correlation function. Once the correlation function of the random force is known, we have a well-defined Langevin problem for the fluctuating single-particle density.

The separation of the collision term into the uncorrelated part and the correlations can, be done formally by using the projection method of Nakajima-Zwanzig [7,8,9]. This yields an exact equation for the single-particle density in the semi-classical limit having the following form,

$$\frac{\partial}{\partial t} f(r,p) - \{h(f), f(r,p)\} = K(f) + I(r,p,t) \tag{1}$$

where, {...} indicates the Poisson bracket, K(f) is the "collision term" and I(r,p,t) is usually referred to as the "correlation term". The separation of the right-hand-side of the eq.(1) into a smoothly varying collision term and a rapidly fluctuating correlation term is done by introducing a projection-averaging procedure in such a way that when the ensemble averaging is performed, the average of the correlation term in eq.(1) vanishes and we recover the usual transport equation for the averaged single-particle density. However, before averaging *I(r,p,t)* never vanishes, even at an "initial time" corresponding to the initial condition of a collision problem, and it describes the prop-

agation of the fluctuations in the single-particle density. The correlation term varies rapidly in time with a characteristic time, τ_d, of the order of the duration time of a two-body collision, and it is almost impossible to calculate it microscopically. Therefore, we give up a detailed microscopic description and consider only the gross properties of the fluctuations. For this purpose, we assume that the correlation term in the eq.(1) acts like a "random force" on the single-particle density, and, in analogy to the Brownian motion problem, is characterized by the correlation function,

$$\overline{I(r,p,t)I(r',p',t')} = C(r,p;r',p')\delta(t-t') \qquad (2)$$

where bar indicates the ensemble averaging, with a correlation time of the order of τ_d -taken to be zero here-. With a given correlation function, we regard eq.(1) as a generalized Langevin problem for the fluctuating single-particle density. The "collision term" describes dissipation, that is the approach toward equilibrium, and the "random force" determines the fluctuations. During short time intervals, the entire single-particle density receives a random "kick". However, the "kicks" are not random at different parts of the phase space, but must be consistent with the correlation function of the random force.

In order to define the "Langevin problem" completely, the correlation function, $C(r,p;r',p')$, must be evaluated explicitly. This can be done rather easily for a dilute gas, in the same approximation as for the derivation of the collision term in the VUU equation. The details of the calculations will be published elsewhere [10]. Here, we outline the derivation and give the result. Because of the short correlation time, it is sufficient to consider the propagation of the random force, $I(r,p,t)$, in eq.(2) over short time intervals, t'-t $= \tau << \tau_{mf}$, which are much smaller than the mean-free-time

between collisions. For simplicity, we first consider a homogeneous Fermi gas and at the end, translate the result into the phase space. During a small time interval τ, the correlations propagate according to [9,10],

$$I(k,t+\tau) = -i\sum_{ijl} v_{ki,lj}(\tau)[\hat{n}_{ji}\hat{n}_{lk} - \delta_{ij}\delta_{lk}n_i n_k] - c.c. \tag{3}$$

with

$$v_{ki,lj}(\tau) = v_{ki,lj}\ exp[-i\tau(\epsilon_k + \epsilon_i - \epsilon_j - \epsilon_l)] \tag{4}$$

where $\hat{n}_{ij}$ is the matrix element of the fluctuating single-particle density, n_j is the averaged occupation number, $v_{ki,lj}$ is the matrix elements of the residual two-body interaction and ϵ_j is the single-particle energy. All the quantities in eqs.(3) and (4) are evaluated at time t. Then, the correlation function of $I(k,t)$ can be evaluated by integrating over τ,

$$C(k,k') = \int_{-\infty}^{\infty} d\tau \overline{I(k,t)I(k',t+\tau)} \tag{5}$$

and assuming that the correlation function of the fluctuating single-particle density is given by

$$\overline{(\hat{n}_{ij} - \delta_{ij}n_i)(\hat{n}_{kl} - \delta_{kl}n_k)} = \delta_{ik}\delta_{lj}n_i(1-n_j) \tag{6}$$

The result for the correlation function of the random force in the phase space can be given as , $C(r,p;r',p') = C(p,p',r)\delta(r-r')$,

$$\begin{aligned} C(p,p',r) \quad &= \quad \sum_{3,4} W(pp';p_3p_4)[ff'(1-f_3)(1-f_4) + (1-f)(1-f')f_3f_4] \\ &-2 \quad \sum_{2,4} W(pp_2;p'p_4)[ff_2(1-f')(1-f_4) + (1-f)(1-f_2)f'f_4] \\ &+\delta_{p,p'} \sum_{2,3,4} W(pp_2;p_3p_4)[ff_2(1-f_3)(1-f_4) + (1-f)(1-f_2)f_3f_4] \end{aligned} \tag{7}$$

where $f_i = f(r_i p_i, t)$ and the transition probability is given by

$$W(p_1 p_2; p_3 p_4) = |v_1 - v_2| \frac{d\sigma}{d\Omega}(12;34)\delta(\vec{p_1} + \vec{p_2} - \vec{p_3} - \vec{p_4}) \tag{8}$$

with $\frac{d\sigma}{d\Omega}$ is the nucleon-nucleon scattering cross-section.

The correlation function is completely determined by the averaged properties of the single-particle density. This result can be regarded as a consequence of the "fluctuation-dissipation theorem" which relates the fluctuation and dissipation properties of the single-particle density locally in phase space. Using the symmetry properties of the transition probability in the eq.(8), i.e., $W(p_1 p_2; p_3 p_4) = W(p_3 p_4, p_1 p_2) = W(p_2 p_1; p_3 p_4)$, it is straightforward to show that the correlation function is symmetric, $C(p, p,', r) = C(p', p, r)$, and obeys the following sum rules,

$$\begin{aligned} \sum_p C(p, p', r) &= \sum_{p'} C(p, p', r) = 0 \\ \sum_p \vec{p} C(p, p', r) &= \sum_{p'} \vec{p'} C(p, p', r) = 0 \\ \sum_p \epsilon_p C(p, p', r) &= \sum_{p'} \epsilon_{p'} C(p, p', r) = 0 \end{aligned} \tag{9}$$

where $\epsilon_p = p^2/2m$. These sum rules correspond to the conservation of particle number, momentum and energy, respectively, and indicate the fact that the correlation function given by eq.(7) satisfies the conservation laws locally in the phase space, as it should be the case. The strength of the correlation function is determined by its diagonal elements, and is given by, $C(p, p, r) \equiv C(r, p)$,

$$C(r, p) = \sum_{2,3,4} W(pp_2; p_3 p_4)[f f_2(1 - f_3)(1 - f_4) + (1 - f)(1 - f_2) f_3 f_4] \tag{10}$$

This follows from the fact that the diagonal elements in the first two terms of eq.(7) vanish. It is interesting to observe that the strength of the correlation function given

by eq.(10) resembles the collision term in the VUU equation, but the sum of the gain and loss terms appear instead of the difference, as is also the case in the off-diagonal elements of eq.(7). This has a simple physical interpretation : the rate of change of the averaged occupation number is determined by the net effect of the gain and loss terms. On the other hand, the rate of change of the mean-square-fluctuations of the occupation numbers, which are described by the strength of the correlation function, are determined by the sum of the gain and loss terms, as in a random walk problem.

Eq.(1) with a correlation function of the "random force" given by eq.(7) is a stochastic, non-linear, integro-differential equation for the evolution of the fluctuating single-particle density. The solution of this equation is a very complex mathematical problem, and certainly requires power of the present-day supercomputer technology. Each solution of the stochastic eq.(1) generates an event and many solutions must be examined to provide a description of the collision process.

The main difficulty in solving the eq.(1) arises from the fact that the fluctuations in different parts of phase-space are correlated as expressed by the correlation function of the random force. However, an approximate algorithm may be developped for the solution by employing a "coarse-graining" procedure in the following way. One can consider that the phase space is divided into discrete cells with volume elements, $\Omega j = (2\pi\hbar)^3$, so that each cell represents one physical particle. This coarse-graining can be realized in two different but equivalent ways :

i) Euler's view. The phase space cells have fixed positions and shapes, but the number of particles in the cells is fluctuating. In this case, cells can be characterized by the mean occupation numbers averaged over the cells, $f_j =< f >_{\Omega j}$.

ii) Lagrange's view. The number of particles in the cells are fixed, but the position and the shape of the cells are fluctuating. This case, cells can be characterized by their mean positions and momentum, $r_j =< rf >_{nj}$ and $p_j =< pf >_{nj}$. In either case, the mean values evolve like an ordinary Langevin process provided that the time steps are sufficiently large, and the correlations between cells, aside from the local conservation laws, can be neglected. Then, during a time interval Δt, in the Euler's view, the mean occupation numbers of the cells change according to [10,11],

$$f_j(t+\Delta t) - f_j(t) = K(f_j)\Delta t + \sqrt{\frac{\Delta t}{2} C_j}\; X_{ji}(t) \tag{11}$$

where the first term on the right-hand-side determines the smooth evolution of the occupation numbers by the collision term, and the second term describes the random kick. The maximum kick is determined by the stregth of the correlation function averaged over the cell, and is given by $C_j =< C(r,p) >_{nj}$. In the Lagrange's view, which is physically more appealing and easier to implement to the numerical simulation, the mean momenta of the cells evolve like a Brownian motion in a heat bath according to [10,11]

$$p_j(t+\Delta t) - p_j(t) = F(r_j,p_j)\Delta t + \sqrt{\frac{\Delta t}{2} D_j}\; X_{ji}(t) \tag{12}$$

where the first term on the right-hand-side gives the smooth motion determined by the mean-field and the collision term which is indicated collectively by F and the second term describes the random kick on the mean momenta of the cell. In this case, the maximum kick is determined by $D_j =< p^2 C(r,p) >_{nj}$ which acts like a "diffusion coefficient" for describing the momentum fluctuations of the cells. In eqs.(11) and (12), Δt is a time interval which should be larger than the correlation time of the random force but small compared to the mean-free-time between collisions, and X_{ji} is a gaussian random

number with a second moment, $\overline{X_{ji}^2} = 2$. The kicks are not independent and involve four cells at a time, say $\Omega_i, \Omega_j, \Omega_i', \Omega_j'$, kicks are given either to the mean occupation numbers by the amounts of $\sqrt{\frac{\Delta t}{2} C_j} X_{ji}$ and $\sqrt{\frac{\Delta t}{2} C_i} X_{ji}$ or to the mean momenta by the amounts of $\sqrt{\frac{\Delta t}{2} D_j} X_{ji}$ and $\sqrt{\frac{\Delta t}{2} D_i} X_{ji}$, then the fluctuations in the cells Ω_i' and Ω_j', are determined by the energy and momentum conservation, as in a two-body collision. It can be shown that the kicks given to the cells in the eqs.(11) and (12) do not violate the Pauli principle provided that the time interval Δt is smaller than the mean-free-time between collisions. However, when the cells or the particles are moved to the new positions, it must be checked that the Pauli principle is not violated. This algorithm may be incorporated, without much difficulty, into the existing computer codes for solving VUU equation by test particle simulation, but the details of the calculations remain to be worked out.

In summary, we propose a new approach, in order to incorporate the fluctuations into the extended TDHF. The evolution of the single-particle density is considered as a "generalized Langevin process" in which the correlated part of the collision term acts as a "random force" generating fluctuations. The correlation function of the random force is calculated in the semi-classical approximation. The result is a "stochastic transport equation" for the fluctuating single-particle density. We discuss an approximate algorithm for the numerical solution.

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