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TITLE: PULSE TO PULSE BEAM TRAJECTORY DETERMINATION AT THE IP**

It has long been known that a precise measurement of the SLC beam trajectory through the IP region is vital both from a machine and a detector point of view. One of the primary techniques used to maximize luminosity is the measurement of the deflection angle of one beam produced by the electromagnetic interaction with the other beam. In order to implement this procedure a pair of precision BPMs* were installed within the Final Triplet of quadrapoles on each side of the IP (the so called "long" BPMs). They were equipped with electronics that allowed them to measure both the incoming and outgoing beams on a single pulse of the machine, with the hope that they could be used to measure the aforementioned deflections. Since it was unclear whether these BPMs would provide a precise measurement of the beam position at the IP, which is important for certain physics studies such as heavy quark lifetime measurements, a pair of "vertex" BPMs were installed about 42 cm on either side of the IP. These were to provide a single pulse position resolution on the order of 25 microns. However, since they were so near the interaction point, they could only be used in single beam mode.

Before the IP BPMs could be used to measure beam-beam deflection, a series of measurements were made of the coefficients which relate the setting of an orbit correction magnet to the position of the beam at a particular BPM (R_{12}s). The purpose was to expose any problems such as misconnected cables, etc., by comparing the measured R_{12}s with theoretical predictions from the model of the Final Focus region. In addition to this function, these experiments also exposed the fact that the beam trajectory fluctuated up to several hundred microns in the long BPMs on a pulse to pulse basis. Since the beam position at the IP was known to be stable at the few micron level from measurements with the wire scanner, these fluctuations in the long BPMs were understood to mean that the beam angle into the IP was fluctuating. Since the angular fluctuation was not a readily correctable problem, and the spatial fluctuations in the long BPMs were about the same or greater than the expected signal from beam deflections, it was realized that a more sophisticated method of measuring deflections would have to be devised. The technique developed uses position measurements in the long BPMs to determine a three parameter fit to the beam trajectory at the IP. The three parameters in the fit are the beam postion, the incoming angle and the deflection angle. The result was very successful and allows the

* J.-C. Denard et. al., Monitoring the Beam Position in the SLC Interaction Region, SLAC PUB 4267, March 1987

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observation of beam-beam deflection even under marginal conditions. However, several technical points remained unclear concerning the optimal utilization of the procedure. In addition, the trajectory fitting routine uses a general fitting package installed in the main control computer, a VAX 8800. The purpose of this memo is to clear up the technical points and to demonstrate that the simplicity of the fit should allow the implementation of this procedure at the micro level, where it could provide trajectory information in real time which would go a long way toward making it useful as a fast feedback tool and in addition allow easy real time access to beam position data by the Mark II. It has already been demonstrated that this method of position determination is superior to the vertex BPMs and therefore makes them redundant.†

The beam trajectory in a single plane through the IP is determined by a three parameter fit using the position measurements of the IP BPMs as input. The fit is obtained by minimizing

$$\chi^2 = \sum_j \frac{1}{\sigma_j^2}(B_j - a_j x - b_j\theta - c_j\phi)^2 \tag{1}$$

where:

1. B_j is the position of the beam measured in the j_{th} BPM,
2. x is the beam position at the IP,
3. a_j relates the position of the beam at the IP to a position at the j_{th} BPM,
4. θ is the incoming angle of the beam at the IP,
5. b_j relates the incoming angle to a position at the j_{th} BPM,
6. ϕ is the deflection angle produced by the beam-beam interaction,
7. c_j relates the deflection angle to a position at the j_{th} BPM (Note that $c_j = 0$ for the incoming BPMs and $c_j = b_j$ for the outgoing BPMs), and
8. σ_j is the error on the j_{th} BPM measurement.

To find x, θ and ϕ the following set of equations must be solved:*

$$\frac{\partial\chi^2}{\partial x} = 0 \tag{2a}$$

$$\frac{\partial\chi^2}{\partial\theta} = 0 \tag{2b}$$

† S. Wagner and W. Koska, Can We Live Without the Vertex BPMs?, SLAC Memorandum of July 12, 1988

* We follow the procedure outlined in chapter 6 of Data Reduction and Error Analysis for the Physical Sciences by P. Bevington.

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$$\frac{\partial \chi^2}{\partial \phi} = 0 \tag{2c}$$

The calculation is straight forward but tedious. Details may be found in the Appendix 1. The result for x is:

$$x = X_1 \sum_j \frac{a_j}{\sigma_j^2} B_j + Y_1 \sum_j \frac{b_j}{\sigma_j^2} B_j + Z_1 \sum_j \frac{c_j}{\sigma_j^2} B_j \tag{3}$$

where X_1, Y_1, Z_1 are constants composed of various combinations of a_j, b_j and c_j.

The error on x is given by

$$\sigma_x^2 = \sum_i \sigma_i^2 \left(\frac{\partial x}{\partial B_i} \right)^2 \tag{4}$$

implying:

$$\sigma_x^2 = \frac{1}{\Delta} \left(\sum_j \frac{c_j^2}{\sigma_j^2} \sum_j \frac{b_j^2}{\sigma_j^2} - \left(\sum_j \frac{b_j c_j}{\sigma_j^2} \right)^2 \right). \tag{5}$$

Similar expressions hold for θ and ϕ (See Appendix 1).

These equations imply that we can find x, θ and ϕ via the matrix equation:

$$\begin{pmatrix} x \\ \theta \\ \phi \end{pmatrix} = \begin{pmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{pmatrix} \begin{pmatrix} \sum \frac{a_j}{\sigma_j^2} B_j \\ \sum \frac{b_j}{\sigma_j^2} B_j \\ \sum \frac{c_j}{\sigma_i^2} B_j \end{pmatrix} \tag{6}$$

where X_i, Y_i and Z_i are constants which may be precalculated and only change if the number of BPMs or the BPM resolutions change or if the coefficients a_j, b_j or c_j change. Note that the same comment applies to σ_x, σ_θ and σ_ϕ and that these errors may be calculated directly once the a_j, b_j and c_j are known. A typical example is given in Appendix 2.

In order to enhance signals from beam-beam deflection it was thought that some type of averaging procedure should be implemented. It was unclear as to the best method so two options were provided. These were sequential BPM measurements from which an average reading would be used in calculating the three beam trajectory parameters, or alternatively, a series of scans using single BPM readings from which the trajectory parameters could be determined and then averaged. If we assume that the σ_j are due to

BPM resolution we see that if we average over N scans:

$$\langle x \rangle = \frac{1}{N} \sum_i x_i \tag{7}$$

However this can be shown to be equivalent to (see Appendix 1):

$$\langle x \rangle = X_1 \sum_j \frac{a_j}{\sigma_j^2} \langle B_j \rangle + Y_1 \sum_j \frac{b_j}{\sigma_j^2} \langle B_j \rangle + Z_1 \sum_j \frac{c_j}{\sigma_j^2} \langle B_j \rangle \tag{8}$$

Similar results can be derived for $\langle \theta \rangle$ and $\langle \phi \rangle$. In other words, averaging a fitted parameter over N scans is equivalent to averaging N BPM readings and then solving for the parameter. It can also be shown that the error on the fitted parameter can be obtained by replacing the BPM resolution by the resolution divided by the square root of N in the error calculation when using the BPM averaging method. Since this is the faster and and simpler method, it is usually preferred.

It has been observed that the measured beam position in a BPM varies by a constant value depending on whether one beam or two beams are present. This is the result of some cross talk between the signals from the two beams. However it should be pointed out that constant offsets in the BPM readings produce constant offsets in the measurements of the desired parameters. These are unimportant when looking for relative variations, as is the case in beam-beam deflections. Hence it is unnecessary to determine offsets prior to measuring beam-beam deflections unless a comparison is to be made to data taken at some other time, in which case the change in offsets is the important parameter. This result is also derived in Appendix 1.

Finally, we would like to point out that it can be seen from Equation 5 that the number of arithmetic operations required to calculate the x, θ and ϕ parameters is 21 addition operations and 21 multiplication operations. If we extend this to both planes and both beams we have 84 additions and 84 multiplications. With an upgraded feedback micro it should be possible to perform these calculations in an interpulse period (8.3 ms at 120 pps), however additional work will have to be done to see how this job meshes with the other tasks which the micro will have to perform. A job operating in the Final Focus Feedback Micro (FB69) which would calculate x, θ and ϕ on a pulse to pulse basis would be very useful in monitoring changes in beam conditions. Depending on the severity of the change and the sophistication of the feedback algorithm the micro could either attempt to correct the problem itself or notify an operator. It would also be useful to pass the information to the Mark II, instead of or in addition to the raw BPM data. The fit x and y positions in particular might be very useful in monitoring the interaction point for the purpose of vertexing. One must keep in mind, however, that if offsets in BPMs are a function of beam intensities (which may not be the case for the long BPMs) then drifts in beam intensity will also show up as apparent drifts in beam position and deflection angle.

These would have to be disentangled from the real positional drifts, probably as part of the offline analysis.

APPENDIX 1

In order to determine the three parameters specifying the beam we must minimize the following χ^2 function for each.

$$\chi^2 = \sum_j \frac{1}{\sigma_j^2}(B_j - a_j x - b_j\theta - c_j\phi)^2$$

$$\frac{\partial\chi^2}{\partial x} = \sum_j \frac{a_j}{\sigma_j^2}(a_j x + b_j\theta + c_j\phi - B_j) = 0$$

$$\frac{\partial\chi^2}{\partial\theta} = \sum_j \frac{b_j}{\sigma_j^2}(a_j x + b_j\theta + c_j\phi - B_j) = 0$$

$$\frac{\partial\chi^2}{\partial\phi} = \sum_j \frac{c_j}{\sigma_j^2}(a_j x + b_j\theta + c_j\phi - B_j) = 0$$

Solving this system of three linear equations results in:

$$x = \frac{1}{\Delta}\begin{vmatrix} \sum \frac{a_jB_j}{\sigma_j^2} & \sum \frac{a_jb_j}{\sigma_j^2} & \sum \frac{a_jc_j}{\sigma_j^2} \\ \sum \frac{b_jB_j}{\sigma_j^2} & \sum \frac{b_jb_j}{\sigma_j^2} & \sum \frac{b_jc_j}{\sigma_j^2} \\ \sum \frac{c_jB_j}{\sigma_j^2} & \sum \frac{b_jc_j}{\sigma_j^2} & \sum \frac{c_jc_j}{\sigma_j^2} \end{vmatrix}$$

$$\theta = \frac{1}{\Delta}\begin{vmatrix} \sum \frac{a_ja_j}{\sigma_j^2} & \sum \frac{a_jB_j}{\sigma_j^2} & \sum \frac{a_jc_j}{\sigma_j^2} \\ \sum \frac{a_jb_j}{\sigma_j^2} & \sum \frac{b_jB_j}{\sigma_j^2} & \sum \frac{b_jc_j}{\sigma_j^2} \\ \sum \frac{a_jc_j}{\sigma_j^2} & \sum \frac{c_jB_j}{\sigma_j^2} & \sum \frac{c_jc_j}{\sigma_j^2} \end{vmatrix}$$

$$\phi = \frac{1}{\Delta}\begin{vmatrix} \sum \frac{a_ja_j}{\sigma_j^2} & \sum \frac{a_jc_j}{\sigma_j^2} & \sum \frac{a_jB_j}{\sigma_j^2} \\ \sum \frac{a_jb_j}{\sigma_j^2} & \sum \frac{b_jb_j}{\sigma_j^2} & \sum \frac{b_jB_j}{\sigma_j^2} \\ \sum \frac{a_jc_j}{\sigma_j^2} & \sum \frac{b_jc_j}{\sigma_j^2} & \sum \frac{c_jB_j}{\sigma_j^2} \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \sum \frac{a_ja_j}{\sigma_j^2} & \sum \frac{a_jb_j}{\sigma_j^2} & \sum \frac{a_jc_j}{\sigma_j^2} \\ \sum \frac{a_jb_j}{\sigma_j^2} & \sum \frac{b_jb_j}{\sigma_j^2} & \sum \frac{b_jc_j}{\sigma_j^2} \\ \sum \frac{a_jc_j}{\sigma_j^2} & \sum \frac{b_jc_j}{\sigma_j^2} & \sum \frac{c_jc_j}{\sigma_j^2} \end{vmatrix}$$

Expanding these expressions and rearranging terms leads to the matrix equation:

$$\begin{pmatrix} x \\ \theta \\ \phi \end{pmatrix} = \begin{pmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{pmatrix} \begin{pmatrix} \sum \frac{a_j}{\sigma_j^2} B_j \\ \sum \frac{b_j}{\sigma_j^2} B_j \\ \sum \frac{c_j}{\sigma_j^2} B_j \end{pmatrix}$$

where

$$X_1 = \frac{1}{\Delta}\left(\sum \frac{c_j c_j}{\sigma_j^2} \sum \frac{b_j b_j}{\sigma_j^2} - \sum \frac{b_j c_j}{\sigma_j^2} \sum \frac{b_j c_j}{\sigma_j^2}\right)$$

$$Y_1 = \frac{1}{\Delta}\left(\sum \frac{b_j c_j}{\sigma_j^2} \sum \frac{a_j c_j}{\sigma_j^2} - \sum \frac{a_j b_j}{\sigma_j^2} \sum \frac{c_j c_j}{\sigma_j^2}\right)$$

$$Z_1 = \frac{1}{\Delta}\left(\sum \frac{a_j b_j}{\sigma_j^2} \sum \frac{b_j c_j}{\sigma_j^2} - \sum \frac{a_j c_j}{\sigma_j^2} \sum \frac{b_j b_j}{\sigma_j^2}\right)$$

$$X_2 = \frac{1}{\Delta}\left(\sum \frac{b_j c_j}{\sigma_j^2} \sum \frac{a_j c_j}{\sigma_j^2} - \sum \frac{a_j b_j}{\sigma_j^2} \sum \frac{c_j c_j}{\sigma_j^2}\right)$$

$$Y_2 = \frac{1}{\Delta}\left(\sum \frac{a_j a_j}{\sigma_j^2} \sum \frac{c_j c_j}{\sigma_j^2} - \sum \frac{a_j c_j}{\sigma_j^2} \sum \frac{a_j c_j}{\sigma_j^2}\right)$$

$$Z_2 = \frac{1}{\Delta}\left(\sum \frac{a_j c_j}{\sigma_j^2} \sum \frac{a_j b_j}{\sigma_j^2} - \sum \frac{a_j a_j}{\sigma_j^2} \sum \frac{b_j c_j}{\sigma_j^2}\right)$$

$$X_3 = \frac{1}{\Delta}\left(\sum \frac{a_j b_j}{\sigma_j^2} \sum \frac{b_j c_j}{\sigma_j^2} - \sum \frac{b_j b_j}{\sigma_j^2} \sum \frac{a_j c_j}{\sigma_j^2}\right)$$

$$Y_3 = \frac{1}{\Delta}\left(\sum \frac{a_j b_j}{\sigma_j^2} \sum \frac{a_j c_j}{\sigma_j^2} - \sum \frac{a_j a_j}{\sigma_j^2} \sum \frac{b_j c_j}{\sigma_j^2}\right)$$

$$Z_3 = \frac{1}{\Delta}\left(\sum \frac{a_j a_j}{\sigma_j^2} \sum \frac{b_j b_j}{\sigma_j^2} - \sum \frac{a_j b_j}{\sigma_j^2} \sum \frac{a_j b_j}{\sigma_j^2}\right).$$

The error on x is given by:

$$\sigma_x^2 = \sum_i \sigma_i^2 \left(\frac{\partial x}{\partial B_i}\right)^2$$

$$\sigma_x^2 = \sum_i \frac{\sigma_i^2}{\Delta^2} \left\{ \frac{a_i}{\sigma_i^2} \left(\sum \frac{c_j c_j}{\sigma_j^2} \sum \frac{b_j b_j}{\sigma_j^2} - \sum \frac{b_j c_j}{\sigma_j^2} \sum \frac{b_j c_j}{\sigma_j^2} \right) + \right.$$

$$\frac{b_i}{\sigma_i^2} \left(\sum \frac{b_j c_j}{\sigma_j^2} \sum \frac{a_j c_j}{\sigma_j^2} - \sum \frac{c_j c_j}{\sigma_j^2} \sum \frac{a_j b_j}{\sigma_j^2} \right)$$

$$\left. \frac{c_i}{\sigma_i^2} \left(\sum \frac{a_j b_j}{\sigma_j^2} \sum \frac{b_j c_j}{\sigma_j^2} - \sum \frac{a_j c_j}{\sigma_j^2} \sum \frac{b_j b_j}{\sigma_j^2} \right) \right\}^2$$

This leads, after considerable algebraic manipulation, to the simple expression:

$$\sigma_x^2 = \frac{1}{\Delta} \left(\sum \frac{c_j^2}{\sigma_j^2} \sum \frac{b_j^2}{\sigma_j^2} - \left(\sum \frac{b_j c_j}{\sigma_j^2} \right)^2 \right).$$

From the symmetry of the arguements, it is obvious that:

$$\sigma_\theta^2 = \frac{1}{\Delta} \left(\sum \frac{a_j^2}{\sigma_j^2} \sum \frac{c_j^2}{\sigma_j^2} - \left(\sum \frac{a_j c_j}{\sigma_j^2} \right)^2 \right)$$

and

$$\sigma_\phi^2 = \frac{1}{\Delta} \left(\sum \frac{a_j^2}{\sigma_j^2} \sum \frac{b_j^2}{\sigma_j^2} - \left(\sum \frac{a_j b_j}{\sigma_j^2} \right)^2 \right).$$

Now let us show that averaging over N measurements of a parameter is equivalent to first averaging over N BPM measurements and then calculating the parameter. We will show this explicitly for x. The expressions for the average value of x and its error are:

$$\langle x \rangle = \frac{1}{N} \sum x_i$$

and

$$\sigma_{\langle x \rangle} = \frac{\sigma_x}{\sqrt{N}}$$

where N is the number of scans averaged over. We see that

$$\frac{1}{N}\sum_i x_i = \frac{1}{N}\sum_i \left(X_1 \sum \frac{a_j}{\sigma_j^2} B_j^i + Y_1 \sum \frac{b_j}{\sigma_j^2} B_j^i + Z_1 \sum \frac{c_j}{\sigma_j^2} B_j^i \right)$$

$$= X_1 \sum \frac{a_j}{\sigma_j^2} \frac{1}{N} \sum_i B_j^i + Y_1 \sum \frac{b_j}{\sigma_j^2} \frac{1}{N} \sum_i B_j^i + Z_1 \sum \frac{c_j}{\sigma_j^2} \frac{1}{N} \sum_i B_j^i.$$

So

$$\langle x \rangle = X_1 \sum \frac{a_j}{\sigma_j^2} \langle B_j \rangle + Y_1 \sum \frac{b_j}{\sigma_j^2} \langle B_j \rangle + Z_1 \sum \frac{c_j}{\sigma_j^2} \langle B_j \rangle$$

We also see that

$$\frac{\sigma_x^2}{N} = \frac{1}{N\Delta} \left(\sum \frac{c_j^2}{\sigma_j^2} \sum \frac{b_j^2}{\sigma_j^2} - \left(\sum \frac{b_j c_j}{\sigma_j^2} \right)^2 \right)$$

$$= \frac{N^2}{N^3\Delta} \left(\sum \frac{c_j^2}{\sigma_j^2} \sum \frac{b_j^2}{\sigma_j^2} - \left(\sum \frac{b_j c_j}{\sigma_j^2} \right)^2 \right)$$

$$= \frac{1}{\Delta(\frac{\sigma_j^2}{N})} \left(\sum \frac{c_j^2}{(\frac{\sigma_j^2}{N})} \sum \frac{b_j^2}{(\frac{\sigma_j^2}{N})} - \left(\sum \frac{b_j c_j}{(\frac{\sigma_j^2}{N})} \right)^2 \right)$$

where $\Delta(\frac{\sigma_j^2}{N})$ indicates that $\sigma_j^2 \rightarrow \frac{\sigma_j^2}{N}$ everywhere in the parameter Δ. This calculation indicates that the correct uncertainty to associate with an averaged parameter (whether BPM averaged or scan averaged) is obtained if $\sigma_j^2 \rightarrow \frac{\sigma_j^2}{N}$ in the expression for σ_x^2 for a single measurement. Obviously similar results hold for σ_θ^2 and σ_ϕ^2. Note that we could also make this transformation in the expression where $\langle x \rangle$ is found using $\langle B_j \rangle$ however it leads to the previously obtained expression due to the net cancellations of N in the X_1, Y_1 and Z_1.

We can also show that constant offsets in the BPM readings produce constant offsets in the measurements of the desired parameters, which are unimportant when we are looking for relative variations. For example, if we assume offsets for each BPM reading, O_j, then

$$x = X_1 \sum \frac{a_j}{\sigma_j^2}(B_j + O_j) + Y_1 \sum \frac{b_j}{\sigma_j^2}(B_j + O_j) + Z_1 \sum \frac{c_j}{\sigma_j^2}(B_j + O_j)$$

$$x = X_1 \sum \frac{a_j}{\sigma_j^2} B_j + Y_1 \sum \frac{b_j}{\sigma_j^2} B_j + Z_1 \sum \frac{c_j}{\sigma_j^2} B_j +$$

$$\left\{ X_1 \sum \frac{a_j}{\sigma_j^2} O_j + Y_1 \sum \frac{b_j}{\sigma_j^2} O_j + Z_1 \sum \frac{c_j}{\sigma_j^2} O_j \right\}$$

where the term in brackets is a constant for constant offsets. Hence we are back to our original expression plus a constant term which only changes if the BPM offsets change. Similar results obviously can be obtained for θ and ϕ. Since the errors on x, θ and ϕ have no dependence on the B_j, they are not affected in any way by constant offsets.

APPENDIX 2

In this appendix we list typical coefficient values associated with the four long BPMs. These coefficients are elements of the first order beam transfer matrix or "R" matrix.* They are a function of the Final Quadrapole Triplet settings and may vary up to 5% as the trims on these Quads are changed.

We can determine which R matrix elements correspond to which coefficients from the two equations:

$$\vec{X}_{BPM} = (R^{BPM \to IP})^{-1} \vec{X}_{IP}$$

for the BPMs upstream of the IP and

$$\vec{X}_{BPM} = R^{IP \to BPM} \vec{X}_{IP}$$

for the BPMs downstream of the IP. So for BPMs which come before the IP on a given beam, $a = R_{22}(R_{44}), b = -R_{12}(-R_{34})$, and $c = 0$ in the $x(y)$ plane, where the R matrix is calculated from the BPM to the IP. For BPMs which come after the IP on a given beam, $a = R_{11}(R_{33})$ and $b = c = R_{12}(R_{34})$ in the $x(y)$ plane where the R matrix is calculated from the IP to the BPM.

BPM	a	b	c
235 x or 865 y	0.29948	-7.53478	0.0
240 x or 860 y	0.94888	-3.26214	0.0
260 x or 840 y	1.05200	3.57350	3.57350
265 x or 835 y	0.75009	4.73309	4.73309
235 y or 865 x	0.75009	-4.73309	0.0
240 y or 860 x	1.05200	-3.57350	0.0
260 y or 840 x	0.94888	3.26214	3.26214
265 y or 835 x	0.29948	7.53478	7.53478

* K.L. Brown, F. Rothacker, D.C. Carey, Ch. Iselin: TRANSPORT-A Computer Program for Designing Charged Particle Beam Transport Systems, SLAC-91, Rev. 2 (May 1977)

When these coeficients are inserted into the expressions for the errors on the fit parameters, assuming 20μm BPM resolutions,† we obtain for positrons:

	x	y
σ_X	23.6μm	23.6μm
σ_θ	5.96μrad	3.07μrad
σ_ϕ	7.96μrad	7.96μrad

These values are consistent with the errors calculated using the general fitting program on the VAX. The results for electrons are the same with σ_θ reversed in x and y. Note that these errors can be modified to reflect any BPM resolution, (σ_{BPM}) by multiplying them by $\frac{\sigma_{BPM}}{20\mu m}$.

† 20μm is a reasonable estimate of the long BPM resolutions.