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NONEXISTENCE THEOREMS FOR YANG-MILLS FIELDS OUTSIDE THE BLACK HOLE OF THE SCHWARZSCHILD SPACETIME *

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ABSTRACT

In this paper, the authors prove some nonexistence theorems for regular static Yang-Mills fields outside the black hole of the Schwarzschild spacetime under certain boundary conditions.

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1. Introduction

In [2], Hu Hesheng proves a nonexistence theorem for nontrivial regular static Yang-Mills field with finite or slowly divergent energy in the region outside r=5M in the Schwarzschild spacetime and she conjectures such a nonexistence theorem should be true for r=2M, where 2M is the Schwarzschild radius. Later, in [3], the region is improved from r>5M to r>3M together with a improved energy condition. The aim of this paper is to prove that the conjecture is true for some boundary conditions.

The idea is as follows. First, under the boundary condition on the boundary r=2M, we prove that there exists an ope domain in which the Yang-Mills field is flat. Since in a neighbourhood of each point the field is (time-independent) gauge equivalent to an analytic field, we conclude that the field is flat globally outside r=2M .

2. Preliminaries.

As known, the metric of the Schwarzschild spacetime M_4 is

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = -(1-\frac{2M}{r})dt^2 + \frac{1}{(1-2M/r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2),$$
$$(\alpha, \beta, = 0,1,2,3) \tag{1}$$

where r=2M is the Schwarzschild radius.

Throughout this paper, we shall use the summation convention and the ranges of the indices as follows.

$$\alpha, \beta, \gamma, \ldots, = 0,1,2,3, \quad \text{and} \quad i,j,k,\ldots, = 1,2,3. \tag{2}$$

For convenience we use the notation x^0, x^1, x^2, x^3 instead of t,r, θ, φ respectively.

On any hypersurface t=constant, the volume element is

$$dV = \frac{r^2 \sin\theta}{\sqrt{1-2M/r}}\, dr d\theta d\varphi , \tag{3}$$

and the area element of the sphere r=R is

$$dS = R^2 \sin\theta\, d\theta\, d\varphi . \tag{4}$$

Let G be a compact Lie group and   its Lie algebra. Let $\mathcal{P}$ be a principal G-bundle over M . On the space of connections in $\mathcal{P}$, $\mathcal{C}$, we consider the Yang-Mills functional $\mathcal{J} : \mathcal{C} \to R$ defined by

$$\mathcal{J}(b) = -\frac{1}{4}\int (f_{\lambda\mu},\ f^{\lambda\mu})\,dV\ , \qquad b \in \mathcal{C}\ , \tag{5}$$

where

$$b = b_\lambda(x)\ dx^\lambda \tag{6}$$

is the connection (gauge potential) and

$$F = \frac{1}{2}\ f_{\lambda\mu}\,dx^\lambda \wedge dx^\mu \tag{7}$$

is the curvature (field strength) of b . We have

$$f_{\lambda\mu} = \frac{\partial b_\lambda}{\partial x^\mu} - \frac{\partial b_\mu}{\partial x^\lambda} - [\ b_\lambda,\ b_\mu\ ]\ . \tag{8}$$

A Yang-Mills connection is a critical point of $\mathcal{J}$ and then its curvature is a Yang-Mills field. So a Yang-Mills field satisfies the Yang-Mills equations

$$J_\alpha = g^{\lambda\mu}(f_{\alpha\lambda;\mu} + [\ b_\mu,\ f_{\alpha\lambda}]) = 0, \tag{9}$$

where ";" denotes the covariant derivative with respect to the metric (1). If F=0, the field is called trivial or flat. If the potential b is independent of t, the field is called static.

The energy momentum tensor of Yang-Mills field is

$$T_{\alpha\beta} = (f_{\alpha\lambda}\ ,\ f_{\beta\tau})\,g^{\lambda\tau} - \frac{1}{4}\ g_{\alpha\beta}(f_{\mu\nu}\ ,\ f_{\gamma\tau})\,g^{\mu\gamma}g^{\nu\tau}\ . \tag{10}$$

For Yang-Mills field the following conservation law holds

$$T^{\beta}_{\alpha\,;\beta} = 0\ , \tag{11}$$

where

$$T^{\beta}_{\alpha} = T_{\alpha\delta}\,g^{\beta\delta}\ . \tag{12}$$

If

$$\int_{r>2M} T_{00}\,dV < +\infty\ , \tag{13}$$

we say the energy of the field on the outside of the black hole is finite . If

$$\int_{r>2M} T_{00}\,dV = +\infty\ , \tag{14}$$

and

$$\int_{r>2M} \frac{T_{00}}{\psi(r)}\ dV < +\infty\ , \tag{15}$$

for a certain positive continuous and unbounded function $\psi(r)$, satisfying

$$\int_{2M}^{\infty} \frac{dr}{r\,\psi(r)} = +\infty\ , \tag{16}$$

we say the energy of the field on the outside of the black hole is slowly divergent.

The Christoffel symbols of the metric (1) are all zero except the following terms:

$$\Gamma^0_{10} = \Gamma^0_{01} = -\Gamma^1_{11} = M/r(r-2M), \quad \Gamma^1_{00} = M(r-2M)/r^3, \quad \Gamma^1_{22} = -(r-2M),$$
$$\Gamma^1_{33} = -(r-2M)\sin^2\theta, \quad \Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = 1/r, \quad \Gamma^2_{33} = -\frac{1}{2}\sin 2\theta,$$
$$\Gamma^3_{23} = \Gamma^3_{32} = \operatorname{ctg}\theta . \tag{17}$$

Set

$$P = \frac{1}{2}\left[\frac{1}{2}(f_{ab}, f^{ab}) - (f_{01}, f^{01})\right],$$
$$Q = \frac{1}{2}\left[(f_{1a}, f^{1a}) - (f_{0a}, f^{0a})\right], \quad a,b = 2,3. \tag{18}$$

Then $P \geqslant 0$ and $Q \geqslant 0$. The eqalities hold **simultaneously** if and only if all $f_{\lambda\mu}$ vanish.

In the following , we establish in detail a formula which is the correct form of that used in [3].

<u>Lemma 1</u>. For any $R > R_0 > 2M$, it holds

$$\int_{r=R} \frac{f(r)T_1^1}{\sqrt{1-2M/r}}\,dS - \int_{r=R_0} \frac{f(r)T_1^1}{\sqrt{1-2M/r}}\,dS$$
$$= \int_{R_0 \le r \le R} \left[\left(\frac{2r-3M}{r(r-2M)} f(r) - f'(r)\right)P + \left(f'(r) - \frac{3M}{r(r-2M)} f(r)\right)Q\right] dV, \tag{19}$$

where f(r) is any smooth function of r .

Proof. We use " $\|$ " to denote the covariant derivative on the hypersurface t=constant . Note that such a hypersurface is totally geodesic in the Schwarzschild spacetime. The boundary of the region $R_0 \leqslant r \leqslant R$ consists of r=R and $r=R_0$ and the unit normal vector on the boundary is

$$\sqrt{1-2M/r}\,\partial/\partial r, \qquad r=R \text{ or } R_0 . \tag{20}$$

From the conservation law (11), we have , noting the field is static and (17),

$$0 = T^{\beta}_{i;\beta} = \frac{\partial T_i^{\beta}}{\partial x^{\beta}} + T_i^{\mu}\Gamma^{\beta}_{\mu\beta} - T_{\mu}^{\beta}\Gamma^{\mu}_{i\beta}$$
$$= \frac{\partial T_i^j}{\partial x^j} + T_i^k\Gamma^{\beta}_{k\beta} - T_c^{\beta}\Gamma^c_{i\beta} - T_j^k\Gamma^j_{ik}$$
$$= \frac{\partial T_i^j}{\partial x^j} + T_i^k\Gamma^j_{kj} - T_j^k\Gamma^j_{ik} + T_i^k\Gamma^c_{kc} - T_c^A\Gamma^c_{iA} . \tag{21}$$

Since the hypersurface is totally geodesic, we have from (21)

$$T^j_{i\|j} = T^\beta_c \Gamma^c_{i\beta} - T^k_i \Gamma^c_{kc} \quad , \tag{22}$$

particularly,

$$\begin{aligned} T^j_{1\|j} &= T^c_0 \Gamma^c_{1c} - T^1_1 \Gamma^c_{1c} \\ &= \frac{M}{r(r-2M)} T^0_c - \frac{M}{r(r-2M)} T^1_1 \ . \end{aligned} \tag{23}$$

For any vector field on the hypersurface t=constant , $V=v^i\frac{\partial}{\partial x^i}$, we have

$$\int_{R_c\le r\le R} (v^i T^j_{i\|j})\,dV = \int_{R_c\le r\le R} (v^i T^j_i)_{\|j}\,dV - \int_{R_c\le r\le R} T^j_i \left(\frac{\partial v^i}{\partial x^j} + v^k \Gamma^i_{kj}\right) dV \ . \tag{24}$$

Taking $V=f(r)\frac{\partial}{\partial r}$, i.e., $v^1 = f(r)$, $v^2 = v^3 = 0$, from (24) and Stokes formula we have

$$\int_{R_c\le r\le R} f(r) T^j_{1\|j}\,dV = \int_{r=R} \frac{f(r)T^1_1}{\sqrt{1-2M/r}}\,dS - \int_{r=R_c} \frac{f(r)T^1_1}{\sqrt{1-2M/r}}\,dS$$

$$- \int_{R_c\le r\le R} [T^1_1 f'(r) + (T^1_1\Gamma^1_{11} + T^2_2\Gamma^2_{12} + T^3_3\Gamma^3_{13}) f(r)]\,dV. \tag{25}$$

By means of (17),(23) and $T^2_2 + T^3_3 = -T^c_c - T^1_1$, (25) can be written as

$$\int_{r=R} \frac{f(r)T^1_1}{\sqrt{1-2M/r}}\,dS - \int_{r=R_c} \frac{f(r)T^1_1}{\sqrt{1-2M/r}}\,dS$$

$$= \int_{R_c\le r\le R} \left[\frac{3M-r}{r(r-2M)} f(r) T^c_c + \left(f'(r) - \frac{f(r)}{r-2M}\right) T^1_1 \right] dV \ . \tag{26}$$

Noting $T^c_c = -(P+Q)$ and $T^1_1 = -P + Q$, we get (19).

3. Main Results.

First we prove the following nonexistence theorem for any regular static solution for Yang-Mills equation outside the black hole.

Theorem 1 . Let G be a compact Lie group. The Yang-Mills equation on the outside of the black hole of the Schwarzschild spacetime does not admit any regular static solution which satisfies the following boundary condition:

$$\lim_{R\to 2M} \int_{r=R} (1 - 2M/r)^\beta T^1_1\,dS > 0 \ , \tag{27}$$

where $\beta \in [0 , 1)$.

Proof. Because $1 > \beta \geq 0$, we can find a positive α such that

$$\frac{\alpha}{1+\alpha} > \beta \ . \tag{28}$$

Set

$$f(r) = r^{\frac{3(1-\alpha)}{2+2\alpha}} \cdot (r-2M)^{\frac{1+3\alpha}{2+2\alpha}} \quad . \tag{29}$$

By a straightforward computation , from Lemma 1 we get, for any $R_2 > R_1 > 2M$,

$$\begin{aligned} &\int_{r=R_2} r^{\frac{2-\alpha}{1+\alpha}} \cdot (r-2M)^{\frac{\alpha}{1+\alpha}} T_1^1 \, dS - \int_{r=R_1} r^{\frac{2-\alpha}{1+\alpha}} (r-2M)^{\frac{\alpha}{1+\alpha}} T_1^1 \, dS \\ &= \int_{R_1 \le r \le R_2} \frac{2}{1+\alpha} r^{\frac{1-5\alpha}{2+2\alpha}} (r-2M)^{\frac{\alpha-1}{2+2\alpha}} (r-3M) (\alpha P + Q) \, dV. \end{aligned} \tag{30}$$

Letting $R_1 \longrightarrow 2M$ in (30), due to (27) and (28), we see the second term on the left-hand side of (30) tends zero , and we get

$$\begin{aligned} &\int_{r=R_2} r^{\frac{2-\alpha}{1+\alpha}} (r-2M)^{\frac{\alpha}{1+\alpha}} T_1^1 \, dS \\ &= \int_{2M < r \le R_2} \frac{2}{1+\alpha} r^{\frac{1-5\alpha}{2+2\alpha}} (r-2M)^{\frac{\alpha-1}{2+2\alpha}} (r-3M) (\alpha P + Q) \, dV , \end{aligned} \tag{31}$$

for any $R_2 > 2M$.

But if R_2 is sufficiently close to 2M , due to (27), the left-hand side of (31) is positive , while the right-hand side of (31) is nonpositive . This contradiction proves our theorem.

Remark 1 . When $\beta = 0$,the boundary condition here is much similar to that in [2], but now we need no condition concerning the energy .

The example in [2] shows there does exist a solution for the Yang-Mills equation outside the black hole of the Schwarzschild spacetime which satisfies the boundary condition

$$\lim_{R \to 2M} \int_{r=R} T_1^1 \, dS < 0 \quad .$$

Refering to the Theorem 1 , it is natural for us to study the case under the boundary condition

$$\lim_{R \to 2M} \int_{r=R} T_1^1 \, dS = 0 \ .$$

We have the following

Theorem 2 . Let G be a compact Lie group. The pure Yang-Mills equation on the outside of the black hole of the Schwarzschild spacetime does not admit any regular static solution which has slowly divergent energy and satisfies the boundary condition

$$\lim_{R \to 2M} \int_{r=R} T_1^1 \, dS = 0 \ . \tag{32}$$

Proof Set

$$f(r) = r - 2M \ . \tag{33}$$

By a straightforward computation from Lemma 1, we get

$$\int_{r=R_2} r\sqrt{1-2M/r}\, T_1^1\, dS - \int_{r=R_1} r\sqrt{1-2M/r}\, T_1^1\, dS = \int_{R_1\le r\le R_2} W\, dV, \tag{34}$$

where

$$W = \frac{r-3M}{r}(P+Q) \qquad . \tag{35}$$

Noting $T_{00} = (1-2M/r)(P+Q)$, we see

$$\lim_{r\to\infty}\frac{T_{00}}{W} = 1, \tag{36}$$

and

$$\lim_{r\to 2M}\frac{T_{00}}{W} = 0 . \tag{37}$$

Thus from (34) and (37), we see

$$\lim_{R_1\to 2M}\int_{R_1\le r\le R_2} T_{00}\, dV < +\infty , \tag{38}$$

and the assumption on energy says

$$\lim_{R\to\infty}\int_{2M\le r\le R} T_{00}\, dV = +\infty \qquad . \tag{39}$$

From (36), we have then

$$\lim_{R\to\infty}\int_{2M<r\le R} W\, dV = +\infty \qquad . \tag{40}$$

Since W is nonpositive inside r=3M, we deduce from (40) that there is a certain $R_1 > 3M$ such that

$$\int_{2M<r\le R_1} W\, dV = 0 . \tag{41}$$

Letting $R \to 2M$ and setting $R_2 = R > R_1$ in (34), we have

$$\int_{r=R} r\sqrt{1-\frac{2M}{r}}\, T_1^1\, dS = \int_{R_1\le r\le R} W\, dV , \tag{42}$$

for any $R > R_1$.

We claim that there exists a certain $R > R_1$ (>3M) such that $P = Q = 0$ outside r=R. For otherwise there would be two positive constants $\bar{R}$ and ε such that $\bar{R} > R_1$ and for any $R_2 > \bar{R}$

$$\int_{R_1\le r\le R_2} W\, dV > \varepsilon . \tag{43}$$

Set

$$\omega(R) = \begin{cases} 0 , & R_1 \le R < \bar{R} , \\ \frac{1}{R\psi(R)} , & \bar{R} \le R \le R_2 , \\ 0 , & R_2 < R , \end{cases} \tag{44}$$

where $\psi(R)$ is the same function in (15) and (16).

Multiplying (42) by $\omega(R)$ and substituting (44) into it and then integrating the both sides, we have

$$\int_{\bar{R}}^{R_2}\frac{dR}{R\psi(R)}\int_{r=R} r\sqrt{1-2M/r}\, T_1^1\, dS = \int_{\bar{R}}^{R_2}\frac{dR}{R\psi(R)}\int_{R_1\le r\le R} W\, dV . \tag{45}$$

It is easy to see that there exists a positive constant A such that , for $r > \bar{R}$,

$$|T^1_1| < AT_{oo} \quad . \tag{46}$$

Hence the left-hand side of (45) is less than

$$A\int_{\bar{R}\le r\le R_2} \frac{T_{oo}}{\gamma(r)}\, dV \quad . \tag{47}$$

On the other hand , the right-hand side of (45) is greater than

$$\varepsilon\int_{\bar{R}}^{R_2} \frac{dR}{R\gamma(R)} \quad . \tag{48}$$

Thus we have

$$A\int_{\bar{R}\le r\le R_2} \frac{T_{oo}}{\gamma(r)}\, dV \geqslant \varepsilon\int_{\bar{R}}^{R_2} \frac{dR}{R\gamma(R)} \quad . \tag{49}$$

Since the energy is slowly divergent , the left-hand side of (49) remains finite as $R_2 \longrightarrow +\infty$, but the **right-hand** side of (49) tends to $+\infty$ as $R_2 \longrightarrow +\infty$. This is a contradiction .

Thus we conclude that P=Q=0,i.e., $f_{\lambda\mu}= 0$, outside a certain R>3M . Consequently the energy would be finite . It is a contradiction . The theorem is proved.

Concerning the case of finite energy , we have the following.

Theorem 3 . Let G be a compact Lie group. The pure Yang-Mills equation on the outside of the black hole of the Schwarzschild spacetime does not admit any nontrivial regular static solution which has finite energy and satisfies the following boundary condition

$$\lim_{R\to 2M}\int_{r=R} T^1_1\, dS = 0 \ , \tag{50}$$

and

$$\lim_{R\to +\infty}\int_{r=R} rT^1_1\, dS = 0 \quad . \tag{51}$$

Proof. In this case , we see

$$E = \int_{2M<r} W\, dV \tag{52}$$

is finite too.

We have to consider three cases : E > 0, E = 0 and E < 0 .

(1) E > 0 . For this case we can go in the same way as the proof of the theorem 2 and conclude that outside a certain

$R>3M$, $P=Q=0$, i.e., $f_{\lambda\mu}=0$. Then , taking the last paragraph in section 1 into account, we deduce the global triviality of the solution.

(2) $E<0$. In this case, we see there exists a certain $R_1<3M$ such that

$$\int_{R_1\le r} W\, dV = 0 \ . \tag{53}$$

Now if

$$\int_{r=R_1} r\sqrt{1-2M/r}\; T\, dS > 0, \tag{54}$$

we can find a sufficiently small $\delta>0$ such that

$$\int_{r=R_1+\delta} r\sqrt{1-2M/r}\; T_1^1\, dS > 0 \ . \tag{55}$$

Thus we have

$$\int_{R_1+\delta\le r} W\, dV > 0 \ . \tag{56}$$

Then in the same way as above we come to the global triviality of the solution .

If

$$\int_{r=R_1} r\sqrt{1-2M/r}\; T_1^1\, dS < 0 \ , \tag{57}$$

we can find a sufficiently small $\delta>0$ such that

$$\int_{r=R_1-\delta} r\sqrt{1-2M/r}\; T_1^1\, dS < 0 \ . \tag{58}$$

Thus we have

$$\int_{R_1-\delta\le r} W\, dV = -\eta < 0 \ , \tag{59}$$

where $\eta>0$ and (34) says , for any $R>3M$,

$$\int_{r=R} r\sqrt{1-2M/r}\; T_1^1\, dS - \int_{r=R_1-\delta} r\sqrt{1-2M/r}\; T_1^1\, dS < -\eta \ . \tag{60}$$

Due to (58) we have , for any $R>3M$

$$\int_{r=R} r\sqrt{1-2M/r}\; T\, dS < -\eta \ . \tag{61}$$

It is easy to see there exists a $A>0$ such that , for $r>3M$,

$$(1-2M/r)\; T_1^1 > -A T_{00} \ . \tag{62}$$

Multiplying the both sides of (61) by $1/R$ and integrating , we get

$$-A\int_{3M\le r\le R} T_{00}\, dV < -\eta \int_{3M}^{R} \frac{dR}{R} \ . \tag{63}$$

Letting $R \to +\infty$, we again come to a contradiction .

Now if

$$\int_{r=R_1} r\sqrt{1-2M/r}\; T_1^1\, dS = 0 \ , \tag{64}$$

we have , for any $R<R_1$,

$$-\int_{r=R} r\sqrt{1-2M/r}\; T_1^{*}\, dS = \int_{R\le r\le R_1} W\, dV \quad . \tag{65}$$

Letting $R \to 2M$, we get

$$\int_{2M< r\le R_1} W\, dV = 0 \quad . \tag{66}$$

Due to $2M < R_1 < 3M$, (66) forces P and Q to be zero for $r\le R_1$. This again means the global triviality of the solution.

(3) $E = 0$. We claim this case can not occur . Otherwise , from (34) and (50) we should have , for any $R>2M$

$$\int_{r=R} r\sqrt{1-2M/r}\; T\, dS = \int_{2M<r\le R} W\, dV \quad . \tag{67}$$

Letting $R \to +\infty$ in (67) , we should have

$$\lim_{R\to+\infty}\int_{r=R} rT\, dS = 0 \ . \tag{68}$$

this contradicts the condition (51) . The theorem is proved .

Remark 2 . We do not know whether the condition (51) is necessary or not.

Remark 3 . To justify the remark made in the last paragraph in section 1 , we proceed as following (refering to the section 4 in [1]). First one need to show that for each point P in the hypersurface t=constant , say t=0 , there is a neighbourhood U a a G-valued function $w(x^i)$, i=1,2,3, such that on U

$$\tilde{b}_\mu = \mathrm{ad}\, w\, b_\mu - \frac{\partial w}{\partial x^\mu}\, w^{-1}$$

satisfies

$$g^{ij}\frac{\partial \tilde{b}_i}{\partial x^j} = 0 \ . \tag{69}$$

To see this let u be a system of local coordinates of a neighbourhood of identity in G and $w = w(u^A)$. Then we have

$$\tilde{b}_i^A = \mathcal{B}^A_B\,(u(x^l))\,b_i^B + \mathcal{A}^A_B\,(u(x^l))\frac{\partial u^B}{\partial x^i} \ , \tag{70}$$

where $\mathcal{A}^A_B$ is nonsingular.

Differentiating and substituting in (68), we get

$$g^{ij}\frac{\partial^2 u^B}{\partial x^i\partial x^j} + \mathcal{A}^{-1\,B}_{\ \ A}\Big(\frac{\partial \mathcal{A}^A_D}{\partial u^C}\frac{\partial u^C}{\partial x^j}\frac{\partial u^D}{\partial x^i} + \frac{\partial}{\partial u^C}\big((\mathcal{B}^A_D b^D_i\big)\frac{\partial u^C}{\partial x^j}\Big)g^{ij} = 0 \ , \tag{71}$$

which is a quasilinear elliptic system of equations , and hence there exists functions $u^B(x^i)$ such that they are defined on some U around P and satisfy (71). Hence on U we have (69) . Now on U, $\tilde{b}$ satisfies the Yang-Mills equations so that

$$g^{ij}\frac{\partial^2 \tilde{b}_\alpha}{\partial x^i \partial x^j} = F\left(\tilde{b}_\beta, \frac{\partial \tilde{b}_\beta}{\partial x^i}, g^{\alpha\beta}, \frac{\partial g^{\alpha\beta}}{\partial x^i}\right), \tag{72}$$

where F is an analytic function of its arguments . Hence by elliptic regularity theory , $\tilde{b}_\alpha$ is analytic on U .

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