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LIGHT AND HEAVY QUARK MASSES, TEST OF PCAC AND FLAVOUR BREAKINGS OF CONDENSATES IN QCD *

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ABSTRACT

Results from current algebra and QCD spectral sum rules (QSSR) in different hadronic channels within τ or n (sum rule variable) and t_c (continuum threshold) stability criterions or (and) t_c–values fixed from FESR are combined to give improved estimates of the light (u,d,s) and heavy (c,b) quark masses. In particular, the weighted averages of the s and b quark mass values are (unavoidably) accurate : $\overline{m}_s(1GeV) = (159.5 \pm 8.8)MeV$ and $M_b(q^2 = M_b^2) = (4.58 \pm 0.05)GeV$. Pion (resp. kaon) PCAC violations of $(5 \pm 0.5)\%(resp.(36.5 \pm 7.5)\%)$ are observed while the values of the quark condensate ratios are: $<\bar{d}d>/<\bar{u}u> = 1. - 9\ 10^{-3}$ and $<\bar{s}s>/\bar{u}u> = (0.6 \pm 0.1)$. These confirm and improve earlier QSSR estimates and provide an indirect support of the $\bar{q}q$ nature of the $a_0(0.98)$ and $K*_0(1.35)$. Values of the "current" running masses of the c, b quarks are presented. Attempts to give the values of the light quark "perturbative constituent" masses are done.

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The role of the quark masses and condensates is certainly important for controlling the size of chiral and flavour symmetry breakings in QCD. Quark mass ratios are unambiguously fixed from current algebra approach[1] due to their renormalization and scale invariances. The values of these ratios are[2]:

$$\frac{m_d - m_u}{m_d + m_u} = (0.28 \pm 0.03), \quad \frac{m_s}{\frac{1}{2}(m_u + m_d) \equiv \bar{m}} = 25.7 \pm 2.6 \ , \qquad (1)$$
$$\frac{m_s - \bar{m}}{m_d - m_u} = 43.5 \pm 2.2, \quad \frac{m_c - \bar{m}}{m_s - \bar{m}} = 9 \pm 2 \ .$$

The absolute values of the quark masses are however much more difficult to estimate and needs a well defined theoretical renormalization framework. This is acheived in QCD within the so-called $\overline{\rm MS}$-scheme where on can define properly an invariant mass $\hat{m}$ associated to the running mass $\bar{m}$ defined to three-loops as[3] :

$$\bar{m} = \frac{\hat{m}}{(\log\frac{Q}{\Lambda})^{\gamma_1/-\beta_1}}\{1 + \frac{1}{-\beta_1\log(\frac{Q}{\Lambda})}(\gamma_1\frac{\beta_2}{\beta_1^2}\log\log(\frac{Q^2}{\Lambda^2}) - \frac{1}{\beta_1}(\gamma_2 - \gamma_1\frac{\beta_2}{\beta_1}))\} \qquad (2a)$$

with :

$$Q^2 \equiv -q^2, \quad \beta_1 = -\frac{1}{2}(11 - \frac{2}{3}n), \quad \beta_2 = -\frac{1}{4}(51 - \frac{19}{3}n), \quad \gamma_1 = 2, \quad \gamma_2 = \frac{1}{6}(\frac{101}{2} - \frac{5}{3}n) \qquad (2b)$$

for $SU(3)_c \times SU(n)_f$, while for heavy quarks one can also conveniently work with the so-called "physical" pole mass $M(q^2 = M^2)$ which is scale independent or with its "euclidian" analogue $m(-q^2 = M^2)$ which is however gauge dependent. The relations among these different masses have been already given in the literature and read to two-loops:

$$\bar{m}^{4)} = M(q^2 = M^2)(1 - \frac{4}{3}(\frac{\alpha_s}{\pi})) \ , \qquad (3)$$
$$m(-q^2 = M^2)^{5)} = M(q^2 = M^2)(1 - (\frac{\alpha_s}{\pi})4\log 2) \quad \text{:Landau gauge} .$$

Analogously, one can define a renormalization group improved condensate $<\bar{\psi}\psi>$ from the scale invariance of the product $m<\bar{\psi}\psi>$.

The purpose of this note is to present the best values of these QCD parameters using QCD spectral sum rules (QSSR) à la SVZ[5] applied to the light, heavy and heavy-light hadron systems and the values of the mass ratios given in Eq.1). In addition to the discussions which we shall give below, the reader can find the details of our calculations in Ref.6).

1. LIGHT QUARK MASSES AND TEST OF PCAC FROM THE PSEUDOSCALARS:

Since the pionneer work of Becchi et al[7]),it has been realized that QSSR analysis of the two-point correlator $\Psi_5(q^2)^i_j$ associated to the divergence of the axial current:

$$\partial_\mu A^\mu(x)^i_j = (m_i + m_j)\bar{\psi}_i(i\gamma_5)\psi_j \quad , \tag{4}$$

can provide a good determination[2]) of the light quark masses due to their leading role into the sum rules. FESR results[8]) have been recently improved[9]) by using a chiral lagrangian parametrization of the final state interactions to the spectral function. This procedure is necessary due to the high-t sensitivity of the FESR and improves the t_c (continuum threshold) stability of the predictions.In the case of the Laplace sum rule (LSR),the "modified" duality ansatz with two narrow resonances (π and π') plus a QCD continuum gives already a good parametrization of the spectral function where unlike the case of the FESR , the exponential factor suppresses higher states and finite width effects. In a narrow width approximation (NWA),the relevant quantity is:

$$r_\pi \equiv \frac{M^4_{\pi'} f^2_{\pi'}}{m^4_\pi f^2_\pi} \simeq 6 \sim 8 \simeq r_K \quad , \tag{5}$$

as fixed from different approaches[2,10]). The advantage of Eq.5) is that the strong dependence on $M_{\pi'}$ which is the main source of errors in a dual- like parametrization of the spectral function is absorbed into r_π. Constraints on the invariant $\hat{m}_u + \hat{m}_d$ using $\Psi_5(q^2)$ to three-loops and the effects of dimension-six condensates are given in Ref.11). The improvment of the previous results of Refs.11) and 2) is the study in Ref.6) of the t_c-stability of the predictions.In this range of t_c-values, the changes in r_π only affect slightly the results. The t_c-inflexion point is reached at 2.4 GeV^2, a value which coincides with the FESR result[9]). The τ-stability is reached for $\tau > .2GeV^{-2}$ where direct instanton effects estimated in Ref.12) are negligible. In a NWA , we obtain[6]):

$$(\hat{m}_u + \hat{m}_d) \simeq (27.5 \pm 5.8)MeV \quad , \tag{6a}$$

while in a dual-like model plus a finite width parametrization:

$$(\hat{m}_u + \hat{m}_d) \simeq (33.0 \pm 11.6)MeV \quad , \tag{6b}$$

where the error is mainly due to the value of the π' mass which ranges from 1.1 to 1.3 GeV depending on the parameters of the model. FESR result with wide ranges for the values of the gluon and four-quark condensates is[9]):

$$(\hat{m}_u + \hat{m}_d) \simeq (22.5 \pm 2.9)MeV \quad , \tag{6c}$$

A weighted average of the FESR and the above LSR results gives :

$$(\hat{m}_u + \hat{m}_d) = (24.0 \pm 2.5)MeV \quad , \tag{6d}$$

which with the help of Eq.1) leads to the values of the invariant and running 1 GeV masses to three-loops :

$$\begin{aligned} \hat{m}_u &= (8.7 \pm 0.8)MeV \; , \quad \bar{m}_u = (5.2 \pm 0.5)MeV \; , \\ \hat{m}_d &= (15.4 \pm 0.8)MeV \; , \quad \bar{m}_d = (9.2 \pm 0.5)MeV \; , \end{aligned} \tag{7}$$

for $\Lambda = (150 \pm 50)MeV$[13]). One can consider these results as the best ones from the QSSR. The extension of the above analysis to the $\bar{s}u$ channel is technically straightforward but we can no longer neglect the mass terms in the QSSR . We also allow a 50% deviation of the $<\bar{s}s>$ condensate value from the the $SU(3)_f$ expectations. However, for the LSR, the result is not sensitive either to this change or to the one of r_K in a reasonnable range in the t_c -stability regime. τ-stability is reached for $\tau > .2GeV^{-2}$ and t_c-stability for $t_c > 4GeV^2$. We obtain[6]) :

$$(\hat{m}_u + \hat{m}_s) \simeq (288 \pm 25)MeV \quad , \tag{8a}$$

with a reduced error compared to the one quoted in Ref.2) both for the u,d and the s-quarks. As already mentioned before, this improvment is achieved thanks to the study of the t_c-stability. One can also compare this result to the "hybrid" determination deduced from Eqs.7 and 1 (see also Ref.9)):

$$(\hat{m}_u + \hat{m}_s) \simeq (308 \pm 45)MeV \quad , \tag{8b}$$

while FESR within a NWA gives[8]) :

$$(\hat{m}_u + \hat{m}_s) \simeq (290 \pm 26)MeV \quad . \tag{8c}$$

The three determinations agree very well within the errors and lead to the weighted average for the invariant mass:

$$\hat{m}_s = (283.0 \pm 16.7)MeV \quad . \tag{9}$$

Let us now test the validity of pion PCAC. As firstly emphasized in Ref.14), pion (kaon) PCAC can be tested from a QSSR estimate of the value of the pseudoscalar correlator at zero momentum transfer. This can be done by working with the LSR of the subtracted $(\Psi_5(q^2)^i_j - \Psi_5(0)^i_j)/q^2$ quantity or with its modified form[10a]). τ-stability of the prediction has already been studied in Ref.11) and the t_c one done in Ref.6) appears to give a real improvment of the prediction which is :

$$\Psi_5(0)^u_d = 2m^2_\pi f^2_\pi(1 - \rho_\pi) \quad , \tag{10a}$$

where :

$$\rho_\pi = (5 \pm 0.5)10^{-2} \quad , \tag{10b}$$

in agreement with previous results in Refs.11),9) but more accurate. The result indicates a deviation from the pion PCAC estimate which is mainly due to the m^2-term in the QSSR analysis. In the case of the kaon , we obtain[6]:

$$\Psi_5(0)^u_s = 2m_K^2 f_K^2(1-\rho_K) \quad , \tag{11a}$$

with:

$$\rho_K = (36.5 \pm 7.5)\% \quad . \tag{11b}$$

Eqs.10) and 11) indicate a large violation of the PCAC prediction and confirm previuos QSSR results.

2. FLAVOUR BREAKINGS OF CONDENSATES FROM THE SCALARS:

The true nature of the scalar $a_0(.98\ )$ and $K_0^*(1.35)$ is still a subject of speculations. If we adopt the conventional picture that they are associated to the divergence of the vector current as :

$$< 0|\partial_\mu V^\mu(x)|a_0 >= \sqrt{2}M_a^2 f_a \quad , \tag{12a}$$

we can try to determine the mass and decay constant and check a posteriori the consistency of the assumption. The determination of these parameters from QSSR within τ and t_c-stability criterions has been done in Ref.6) where the optimal predictions are reached for $\tau \simeq 0.4 \sim 0.8 GeV^{-2}$ and $t_c \simeq 1.3 \sim 1.9 GeV^2$ consistent with the FESR constraint:

$$M_a \simeq (1.0 \sim 1.05)GeV \quad \text{and} \quad f_a \simeq (0.6 \sim 1.3)MeV \quad , \tag{12b}$$

where the f_a values include the one from a two-parametr fit[11] and from a chiral lagrangian approach[15]. In the case of the K_0^* , optimal results are obtained for $\tau \simeq 0.2 \sim 0.6 GeV^{-2}$ and $t_c \simeq 3.5 \sim 4.5 GeV^2$. The latter being consistent with the one from a two-parameter fit[11] $(f_{K_0^*}, t_c)$. The predicted values are :

$$M_{K_0^*} \simeq (1.46 \sim 1.51)GeV \quad \text{and} \quad f_{K_0^*} \simeq (28.3 \pm 4.5)MeV \quad , \tag{12c}$$

which exhibit a $SU(3)_f$ splitting between the K_0^* and a_0 and rule out a K_0^*-mass below 1GeV. We use these values in Eq.12) into the LSR and deduce the value of the correlator at zero momentum transfer. Within τ and t_c-stability criterions, one obtains[6]:

$$\Psi(0)^u_d = -(0.4 \sim 0.5)10^{-6} GeV^4 \quad , \quad \Psi(0)^u_s = -(9.5 \pm 1.2)10^{-4} GeV^4 \quad . \tag{13}$$

The relation of $\Psi_{(5)}(0)^i_j$ to the quark condensates :

$$\Psi_{(5)}(0)^i_j = -(m_i - (+)m_j) < \bar\psi_i\psi_i - (+)\bar\psi_j\psi_j > \quad , \tag{14a}$$

allows us to derive the value of the condensate ratios:

$$\frac{<\bar dd>}{<\bar uu>} = 1 - 9\ 10^{-3} \quad , \quad \frac{<\bar ss>}{<\bar uu>} = (0.6 \pm 0.1) \quad , \tag{14b}$$

which show a large departure from the $SU(3)_f$ expectations. The former being consistent with the earlier[16] and recent[11,9] results. The latter being consistent and improvment of previous results quoted in Ref.11). It is informative to compare these results with the ones obtained from chiral lagrangian to lowest order in m_s[2]:

$$\frac{<\bar dd>}{<\bar uu>} = 1 - (\frac{m_d - m_u}{m_s - \bar m})(1 - \frac{<\bar ss>}{<\bar uu>} + \frac{1}{16\pi^2 f_\pi^2}[M_K^2 - m_\pi^2(\log\frac{M_K^2}{m_\pi^2} + 1)]) \quad , \tag{14c}$$

which for the value of $<\bar dd> / <\bar uu>$ in Eq.14b) predicts :

$$\frac{<\bar ss>}{<\bar uu>} = 0.72 \sim 0.76 \quad . \tag{14d}$$

We might expect that better agreements between Eqs.14b) and 14d) could be reached once one is able to include the m_s^2-term into Eq.14c). In fact as m_s is of the order of Λ QCD , this effect can be important in the chiral lagrangian approach. Eqs.7, 10 and 14b) can be used for the obtention of the values of the invariant $\hat\mu$ and running condensates defined as:

$$<\bar\psi_i\psi_i>(Q) = -\hat\mu_i^3(\log\frac{Q}{\Lambda})^{\frac{\gamma_1}{-\beta_1}}/\{1 + \frac{1}{-\beta_1\log(\frac{Q}{\Lambda})}(\gamma_1\frac{\beta_2}{\beta_1^2}\log\log(\frac{Q^2}{\Lambda^2}) - \frac{1}{\beta_1}(\gamma_2 - \gamma_1\frac{\beta_2}{\beta_1}))\} \quad , \tag{15a}$$

from which we deduce $\hat\mu_i$ and $<\bar\psi_i\psi_i>$ at 1GeV:

$$\begin{aligned}
\hat\mu_u &= (188.9 \pm 6.6)MeV \quad , \quad (-<\bar uu>)^{\frac{1}{3}} = (223.9 \pm 7.8)MeV \quad , \\
\hat\mu_d &= (188.3 \pm 6.6)MeV \quad , \quad (-<\bar dd>)^{\frac{1}{3}} = (223.3 \pm 7.8)MeV \quad , \\
\hat\mu_s &= (159.3 \pm 10.5)MeV \quad , \quad (-<\bar ss>)^{\frac{1}{3}} = (188.8 \pm 12.5)MeV \quad .
\end{aligned} \tag{15b}$$

These results are in good agreement with the ones in Ref.11) but the accuracy has been improved. The values in Eq. 15b) show the tendency for the $<\bar\psi\psi>$ condensate to decrease for heavier quarks, a behaviour which is well understood for heavy ($h \equiv c, b...$) quark systems from its relation with the gluon condensate[5] ($<\bar hh> \simeq -\frac{1}{12\pi M_h} <\alpha_s G^2>$) but less for the light ones.

3. THE STRANGE QUARK MASS AND CONDENSATE FROM SOME OTHER CHANNELS:

Let us return to the estimate of the s-quark mass and condensate from some other channels. A careful analysis of the baryon systems shows that one cannot accurately extract the $SU(3)_f$ breaking parameters from this channel due to the leading role of the QCD continuum into the analysis[17]. Tensor $f_2 - f_2'$ meson splitting cannot be resolved by a low value of the strange quark mass as again the QCD continuum is too important and masks the $SU(3)_f$ breaking effects[18]. A much more promising channel is the φ-vector meson one. In this case, progress has also been done for improving the QCD expressions[19]. We use a FESR-like ratio of LSR in order to get the φ mass squared. FESR constraint fixes t_c to be $2 \sim 2.2 GeV^2$. The m_s^2 and $<\bar{s}s>$ terms are left as free parameters while we assume that the size of the ratio $<\bar{s}\Gamma s\bar{s}\Gamma s> / <\bar{u}\Gamma u\bar{u}\Gamma u>$ moves from $(<\bar{s}s>/<\bar{u}u>)^2$ to 1. A fit of the φ-mass within 5% accuracy gives[6]:

$$\hat{m}_s \simeq (180 \pm 100) MeV \quad , \tag{16a}$$

while an elaborated two-parameter fit $(m_s, <\bar{s}s>)$ leads to[11]:

$$\begin{aligned} \hat{m}_s &\simeq (205 \pm 95) MeV \quad , \\ m_s <\bar{s}s> &\simeq -(1 \sim 1.05) 10^{-3} GeV^4 \quad . \end{aligned} \tag{16b}$$

Our result though in agreement with previous estimate[20] is lesser accurate. A Gell-Mann-Okubo-like mass formula which involves the difference of mass squared of the φ and ρ which has been firstly used in Ref.21 for QCD shows that the difference or ratio of the FESR-like moments ratio is less sensitive to the QCD continuum effects than the individual moments ratio[6]. Therefore, we expect that the mass splitting can be better determined than the the absolute values of the meson masses. In the τ-stability regime,one obtains from the GMO-like sum rule :

$$\hat{m}_s \simeq (205 \pm 50) MeV \quad . \tag{16c}$$

Then, a weighted average of the vector meson result gives :

$$\hat{m}_s \simeq (200.9 \pm 40.5) MeV \quad . \tag{16d}$$

Finally,we use an "hybrid" determination of the strange quark mass by using the relation with m_c in Eq.1 and the value of m_c deduced from the ψ- sum rules (see Eq.25 below). We deduce :

$$\hat{m}_s \simeq (224 \pm 50) MeV \quad , \tag{17}$$

which exhausts the possible determination of m_s from QSSR plus the chiral lagrangian result in Eq.1). Taking the weighted average of the previous independent determinations in Eqs.9,16) and 17),we deduce the best estimate of $\hat{m}_s$:

$$\hat{m}_s = (266.7 \pm 14.7) MeV \quad , \tag{18a}$$

to which corresponds the running mass to three-loops :

$$\bar{m}_s(1GeV) = (159.5 \pm 8.8) MeV \quad . \tag{18b}$$

Eqs. 16b) and 18b) provide a value of $<\bar{s}s>$ in good agreement with the one obtained in Eq. 15b) and give an indirect support of the a priori $\bar{q}q$ nature of the $a_0(0.98)$ and $K_0^*(1.35)$ done for deriving the results in Eq. 15b). Another support to this conclusion comes from the values of the quark mass-difference obtained from the $\Psi(q^2)$-sum rule[6,11] which agree quite well with the one obtained from Eqs. 7) and 18). We are aware of the needed precise value of the strange quark mass in weak kaon decays and in some other low energy processes as well as for the estimate of the value of the topological charge of the $U(1)_A$ sector[22] or the η'-mass[22,23]. Our result for $(m_u + m_d)$ agrees with the one from an effective potential approach[24]. However, this method seems to underestimate the values of heavier (s,c) quark masses. Our values of the quark condensates are comparable with the ones [25] from the "dynamical" mass approach[26] but it is difficult to control the accuracy of the latter due to its sensitivity on the momentum integral cut-off and to the too low value of the scale $\hat{q} \simeq O(M_{dyn})$ with $M_{dyn} \simeq 300 MeV$, at which the quark dynamical mass and condensate are evaluated. Indeed, though low dimension condensates have been shown to preserve the OPE,it is difficult to exclude some violent (non)perturbative effects which might invalidate the lowest order expression :

$$M_{dyn} \simeq \left(-\frac{4\pi}{3}\alpha_s <\bar{\psi}\psi>|_{\hat{q}=M_{dyn}}\right)^{\frac{1}{3}} + \bar{m}(M_{dyn}) \quad . \tag{19a}$$

However, this expression gives the numerical values:

$$M_{dyn}^u \simeq M_{dyn}^d \simeq 300 MeV \quad \text{and} \quad M_{dyn}^s \simeq 560 MeV, \tag{19b}$$

which are in amazing agreement with the phenomenological expectations for the "constituent" quark masses:

$$M_{con}^u \simeq M_{con}^d \simeq \frac{1}{2} M_\rho \quad \text{and} \quad M_{con}^s \simeq \frac{1}{2} M_\varphi \quad . \tag{19c}$$

In contrast to M_{dyn}, the effective euclidian mass $M_{eff}(q^2)$ à la Politzer[27] can have a much better "perturbative" meaning if $-q^2 \gg \Lambda^2$ despite its gauge dependence. It reads to lowest order :

$$M_{eff}(q^2) = \bar{m}(q^2) + (3 + a)\frac{4\pi}{9q^2}\alpha_s <\bar{\psi}\psi> \quad , \tag{20a}$$

where $a = 0$ in the Landau gauge. At the resonance mass (say M_ρ) , we obtain:

$$M^u_{eff} \simeq 27MeV \quad , \quad M^d_{eff} \simeq 33MeV \quad \text{and} \quad M^s_{eff} \simeq 229MeV \quad , \tag{20b}$$

where we have included into the condensate contribution the radiative corrections obtained in Ref.19b$_1$). The difference between Eqs.19) and 20) are mainly due to the q^2-damping of the condensate effects.

4. HEAVY QUARK MASSES FROM QSSR:

Due to the importance of the "perturbative" quark mass in the QSSR for heavy quark systems[5,28]),one expects that the heavy quark sum rules provide a reliable prediction of the c and b quark masses. The value of the euclidian mass is better determined from the ψ andΥ sum rules due to the minimization of the radiative corrections into these sum rules for this choice of mass definition. One obtains :

$$\begin{aligned} m_c(q^2 = -M_c^2) &\simeq (1.26 \pm 0.02)GeV \quad , \\ m_b(q^2 = -M_b^2) &\simeq (4.23 \pm 0.05)GeV \quad . \end{aligned} \tag{21}$$

Using the relation between the euclidian and pole masses given in Eq.3),we deduce from the Table of Ref.4) for $\Lambda = (150 \pm 50)MeV$:

$$M_c(q^2 = M_c^2) \simeq (1.45 \pm 0.05)GeV \quad , \tag{22a}$$

$$M_b(q^2 = M_b^2) \simeq (4.67 \pm 0.10)GeV \quad . \tag{22b}$$

The value of the b-quark pole mass has also been deduced from a fit of the B and B^* masses with the result[29]):

$$M_b(q^2 = M_b^2) = (4.56 \pm 0.05)GeV \quad . \tag{23}$$

A weighted average of the heavy and heavy-light quark results gives the final estimate:

$$M_b(q^2 = M_b^2) = (4.58 \pm 0.05)GeV \quad . \tag{24}$$

One can deduce from Eqs.3) and 22a,24), the value of the invariant $\hat{m}$ and running $\bar{m}(1GeV)$ heavy quark masses . We have from the Table of Ref.4):

$$\begin{aligned} \hat{m}_c &= (1.92 \pm 0.18)GeV \quad , \quad \bar{m}_c(1GeV) = (1.40 \pm 0.06)GeV \quad , \\ \hat{m}_b &= (7.89 \pm 0.09)GeV \quad , \quad \bar{m}_b(1GeV) = (5.87 \pm 0.06)MeV \quad . \end{aligned} \tag{25}$$

The results in Eq.25) are consistent with the ones directly obtained from FESR within a m_c^2/q^2 expansion[30]):

$$\hat{m}_c = (2.08 \pm 0.35)GeV \quad \text{and} \quad \hat{m}_b \leq 8.2GeV \quad . \tag{26}$$

Eqs.25) and 26) give the weighted average:

$$\hat{m}_c = (1.95 \pm 0.16)GeV \quad . \tag{27}$$

These values are certainly useful for weak interaction and GUTS phenomenologies due to the uses of the "perturbative" mass in these approaches.

Non-perturbative effects to the perturbative heavy quark mass can be incorporated as in Ref. 2) and might shed light for connecting it with the quark mass used in a non-relativistic treatment of the bound state. However, the c and b quarks seem too light which renders this approach quite innacurate.

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REFERENCES

1) For a review on current algebra,see e.g : V.D.Alfaro,S.Fubini,G.Furlan and C. Rossetti, Currents in hadron physics (North Holland 1973).
2) J.Gasser and H.Leutwyler, Phys. Reports 87C(1982)77; Ann.Phys. 158(1984)142; Nucl.Phys.250B(1985)465.
3) This relation has been first introduced by E.G.Floratos, S.Narison and E.de Rafael, Nucl.Phys.155B(1979)155; For a review see e.g: S.Narison , Phys. Reports. 84(1982)263.
4) S.Narison , Phys. Lett.197B(1987)405 and references therein.
5) M.A.Shifman, A.I.Vainshtein and V.I.Zakharov, Nucl.Phys.147B(1979),385,448.
6) For detailed derivations of these results see: S.Narison, QCD spectral sum rules: towards a theory of hadrons, World Scientific Book (in preparation) .
7) C.Becchi, S.Narison, E.de Rafael and F.J.Yndurain , Z.Phys. C8(1981)335;S.Narison and E.de.Rafael , Phys. Lett. 103B(1981)57.
8) S.G.Gorishny, A.L.Kataev and S.A.Larin, Phys. Lett. 135B(1984)457 and references therein.
9) C.A.Dominguez and E.de Rafael, Ann.Phys. 174(1987)372.
10) S.Narison, N.Paver and D.Treleani, Nuov.Cim 74A(1983)347;F.J.Yndurain, CERN preprint TH 4216/85 (1985) unpublished.
11) S.Narison, Riv.Nuov.Cim.2 Vol10(1987)1 and contribution to the LEAR workshop (1985) Tigne, Haute-Savoie, ed Gastaldi et al. and references therein.
12) E.V.Shuryak, Phys.Reports 115(1984)158 and references therein.
13) For a recent determination of Λ see e.g : A.Pich and S.Narison, CERN preprint TH 5061/88 (Phys.Lett.B in press);S.G.Gorishny,A.L.Kataev, and S.A.Larin, INR Moscow preprint (1988) and references therein.
14) S.Narison, Phys.Lett.104B(1981)485.
15) A.Pich, Phys.Lett.196B(1987)561.
16) E.Bagan, A.Bramon, S.Narison and N.Paver, Phys.Lett.135B(1984)463.
17) H.G.Dosch,M.Jamin and S.Narison , Heidelberg-Montpellier preprint (to appear) and references therein.
18) E.Bagan and S.Narison , Brookhaven preprint BNL41341(1988).
19a) S.G.Gorishny, A.L.Kataev and S.A.Larin, Nuov.Cim.92A(1986)119; D.J.Broadhurst and S.C.Generalis, Open Univ. preprint OUT-4102-8(1982) unpublished.
19b) P.Pascual and E.de Rafael, Z.Phys.C12(1982)127; G.T.Loladze, L.R.Surguladze and F.V.Tkachov, Phys.Lett.162B(1985)363; K.G.Chetyrkin, S.G.Gorishny, and V.P.Spiridinov, Phys.Lett.160B(1985)149;
20) L.J.Reinders and H.Rubinstein, Phys.Lett.145B(1984)108.

21) S.Narison, Z.Phys.C22(1984)161.
22) S.Narison, Z.Phys.C26(1984)209.
23) S.Narison, N.Pak and N.Paver, Phys.Lett.147B(1984)102.
24) A.Barducci,R.Casalbuoni,S.De Curtis,D.Dominici and R.Gatto , Phys.Lett. 193B (1987) 305; Phys.Rev. D38(1988)238.
25) N.Paver, Riazzudin and M.D.Scadron, Phys.Lett.197B(1987)430.
26) V.Elias, M.D.Scadron and R.Tarrach, Phys.Lett. 162B (1985) 176; V.Elias, M.D.Scadron, T.Steele and R.Tarrach, Phys.Rev.D 34 (1986) 3537.
27) H.D.Politzer, Nucl.Phys.117B(1976)397; see also Ref.$19b_1$.
28) J.S.Bell and R.A.Bertlmann, Nucl.Phys.177B(1981)218;R.A.Bertlmann, Nucl.Phys. 204B(1982)387; L.J.Reinders, H.Rubinstein and S.Yazaki, Phys.Reports 127(1985) 1; B.Guberina, R.Meckbach, R.D.Peccei and R.Rückl, Nucl.Phys. 184B(1981)476.
29) S.Narison, "Beautiful mesons from QSSR",Montpellier preprint PM 88-02 (Phys. Lett. B in press).
30) S.Narison and E.de Rafael, Nucl.Phys.155B(1979)155.

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