
SINGLE PASS COLLIDER MEMO CN-369

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TITLE: ORTHOGONALITY OF FINAL WAIST CORRECTIONS
 AT THE IP OF THE SLC*

I. INTRODUCTION

Because the SLC final IP spot is produced by an aberration-dominated optical system¹, all components and couplings between dimensions of transverse phase-space must² be controlled in the experimental tuning algorithm. For equal emittances $\epsilon_x = \epsilon_y$, this amounts to *ten* linear optics adjustments^{2†}. These adjustments are coupled and depend non-linearly on phase-space parameters. A ten-dimensional non-linear fitting program³ is therefore used to match the lattice in the Final Focus to the input beam. Local orthogonal "knobs" are also defined for fine-tweaking around the initial solution, although this is not always practical because of steering from the lenses.

The three final waist corrections⁴ are however fully orthogonal to the other seven optical adjustments. This means that they do not cause any of the other seven optical distortions. We refer to this as *external orthogonality*.

They can also be made *internally orthogonal*. This means that each one of the three orthogonalized controls can be applied independently of the two others. It also allows one to simultaneously correct and determine the phase-space at the IP.

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† In the case of equal emittances these ten adjustments are grouped² in three sets:

1. Four corrections to minimize the spatial and angular dispersion in both planes,
2. Three corrections to the betatron angular spread at the IP, by controlling the magnitude of $\langle x'^2 \rangle$ and of $\langle y'^2 \rangle$, and by minimizing the $\langle x'y' \rangle$ correlation, and
3. Three adjustments to position the waists in both planes at the IP, by minimizing the correlations between the positions x, y , and the angles x', y' in both planes.

The ten variable quadrupoles used for these corrections are shown in Fig. 1. Because each correction is coupled to the ones downstream, they must be applied sequentially. A flow diagram illustrating this sequential application is shown in Fig. 2.

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Both have been demonstrated to work experimentally^{5,6}, and form the basis for the on-line tuning algorithm⁷. In this note, we show the orthogonality of the corrections and outline the experimental procedure and its limits.

II. EXTERNAL ORTHOGONALITY OF WAIST CORRECTIONS

The three final waist corrections are designed to cancel the correlations between the positions x , y , and the angles x' , y' in both planes at the IP. The $\langle xx' \rangle$ and $\langle yy' \rangle$ correlations* are minimized by combining trim windings in two regular quadrupoles of the Final Triplet, one defocusing (QD2B), and one focusing (QF3). We refer to them as the *in-plane* waist adjustments. The $\langle xy' \rangle$ correlation, which at the waist and for $\epsilon_x = \epsilon_y$ is equal⁸ to $\langle yx' \rangle$, is minimized with a single skew quadrupole (SQ3) just upstream of the Final Triplet. This one is referred to as the *out of plane* waist adjustment. The correction elements are indicated in Fig. 1.

These final adjustments are normally decoupled from *dispersion* corrections because the dispersion is nominally zero in the Final Triplet.

They are also close to decoupled from the *angular spread* corrections. To see this, we consider for simplicity an uncorrelated ($\langle xx' \rangle = 0$) beam focused to a waist by a thin lens in the horizontal plane. From linear optics, we compute (to first order) the variation of the spatial beam size σ_x and of the angular beam size $\sigma_{x'}$ as a result of varying the strength of the lens:

$$\begin{cases} \sigma_x^2(\delta_Q) = \sigma_x^2(0) + \delta_Q^2 \sigma_Q^2, \\ \frac{\Delta \sigma_{x'}}{\sigma_{x'}} \simeq \frac{-\delta_Q}{1 + (\frac{\sigma_Q}{\sigma_x})^2}. \end{cases} \quad (1)$$

In (1), σ_Q is the beam size at the lens and δ_Q is the fractional strength variation of the lens.

For σ_Q large compared to the minimum beam size $\sigma_x(0)$, small adjustments are sufficient to change the size at the waist significantly, with only a small perturbation to the angular spread. At the Final Triplet, σ_Q is typically a thousand times larger than $\sigma_x(0)$. The quadrupole adjustments need therefore be only a few percent. Over that range, the change in angular spread is negligible.

Thus the three final waist adjustments are essentially orthogonal to the other seven optical corrections. The opposite is not true: the other seven corrections

* In Transport⁹ notation, $\langle xx' \rangle = \sigma_{21}$, $\langle yy' \rangle = \sigma_{43}$, and so on...

are strongly coupled to the three final waist adjustments. The experimental tuning algorithm is therefore *sequential*. It is summarized in the flow diagram in Fig. 2.

III. INTERNAL ORTHOGONALITY OF WAIST CORRECTIONS

We begin by considering the *in-plane* waist adjustments (i.e. the minimization of the $\langle xx' \rangle$ and $\langle yy' \rangle$ correlations). A regular lens perturbs each plane proportionally to the beam size at the lens in that plane. Since this beam size is naturally larger in each lens in the plane in which it is focusing, QF3 and QD2B can be combined to control the horizontal and vertical waists independently. The coupling coefficients, found using TRANSPORT⁹, are:

$$\begin{pmatrix} \Delta f_x \\ \Delta f_y \end{pmatrix} = C \begin{pmatrix} \delta_{Q3} \\ \delta_{Q2B} \end{pmatrix}, \quad \text{with } C = \begin{pmatrix} -1.89 & 0.70 \\ 0.80 & -1.37 \end{pmatrix} \quad (2)$$

In (2), the fractional quadrupole strengths δ_{Q3} and δ_{Q2B} are in parts per thousand, and the longitudinal waist motions $\Delta f_{x,y}$ are in centimeters.

The couplings in C are close to independent of input mismatch. This is evident for a mismatch of the IP angular spreads from the form of (2). It is also true in the case of a largely correlated input phase-space into the Final Triplet (*in-plane* correlations), which the waist corrections are designed to cancel. This can be seen by considering the nominal IP phase-space obtained after the correction, and by back-tracking the beam into the lenses: since the changes in the strengths of QF3 and QD2B required by the correction are small, the beam sizes are perturbed negligibly in the lenses. Since their effect is proportional to this beam size, the couplings in C do not change significantly. The linear combinations defined in (2) can thus be used for orthogonal control *independent* of the mismatch of the input beam phase-space*.

In the case of the *out of plane* waist-adjustment, (the $\langle xy' \rangle$ and $\langle yx' \rangle$ correlations) using the skew lens SQ3, orthogonality to the *in-plane* corrections is obtained by requiring an upright beam shape near the Final Triplet (i.e. \langle

* This is not the case for the other optical adjustments in the Final Focus, where the relative settings must be calculated through a non-linear fitting program⁹ and depend thus on the initial condition. Since this initial condition must be obtained from measurements with possibly large errors, the calculated solution can be significantly off as a result. The correction can in this case require several iterations. This is especially the case for the angular spread corrections. Operationally, some improvement is obtained by basing the calculations on time-averaged quantities.

$xy \geq 0$). Operationally, this condition is obtained by observing the beam on a profile monitor (ST4) near the Final Triplet, and by adjusting a second skew lens, SQ17.5, located in the First Telescope, which is part of the angular spread corrections.

We show this by the following thin lens argument¹⁰. Consider two lenses, one regular and one skew, of strengths K and S respectively, with parallel to point focusing to the waist. The change in angular spread at the waist is in this case strictly zero. The combined effect of the two lenses on the beam is:

$$\sigma^{out} = R\sigma^{in}R^t, \text{ with } R = \begin{pmatrix} -K & 1 & -S & 0 \\ -1 & 0 & 0 & 0 \\ -S & 0 & K & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad (3)$$

where $\sigma_{out,in}$ are the beam matrices⁹ describing the four-dimensional phase-space at the waist and in the lenses. From (3), the beam-size at the waist is, in the horizontal plane:

$$(\sigma_x^{out})^2 = \sigma_x^2 + \sigma_x^2 K^2 + \sigma_y^2 S^2 - 2 \langle xx' \rangle K - 2 \langle yx' \rangle S + 2 \langle xy \rangle KS, \quad (4)$$

where the subscript *in* has been omitted on the right hand side. The coupling between the two corrections cancels if the correlation $\langle xy \rangle = 0$. This has been verified independently in two simulations^{11,12} of the optical corrections for the Final Focus System.

IV. EXPERIMENTAL PROCEDURE AND PHASE-SPACE DETERMINATION AT THE IP

After the $\langle xy \rangle$ correlation has been minimized at ST4, by adjusting the skew lens SQ3, the *out of plane* waist adjustment SQ3 and *in plane* controls $\Delta f_{x,y}$ defined in (2) form an orthogonal set of correctors. For minimization of the IP spot, they can therefore be applied independently and in any order.

From (4) or from (1), we see that the beam sizes depend parabolically on the controls. This enables one to find the optimal corrections by symmetry even if the minimum of the parabola is not resolved instrumentally. This minimum is not resolved for beam sizes smaller than the carbon filament target¹² used for diagnostic purposes (there are three wires, with diameters of 4, 7, and 20 microns).

In order to simultaneously measure the phase-space parameters at the IP, we first apply the SQ3 correction, to minimize the *out of plane* correlations. After this, the beam sizes at the IP can be written as a function of the *in-plane* Final Triplet orthogonal controls $\Delta f_{x,y}$ as:

$$\sigma_{x,y}^2 = \epsilon_{x,y}\beta_{x,y} + \frac{\epsilon_{x,y}}{\beta_{x,y}} \Delta f_{x,y}^2. \quad (5)$$

where $\epsilon_{x,y}$ and $\beta_{x,y}$ are the emittances and the β -functions in each plane respectively. Fitting a parabola to each measurement gives $\epsilon_{x,y}$ and $\beta_{x,y}$. Although the angular spreads $\sigma_{x',y'} = \sqrt{\frac{\epsilon_{x,y}}{\beta_{x,y}}}$ are well determined from the branches of the parabolas, the minimum linear beam sizes $\sigma_{x,y} = \sqrt{\epsilon_{x,y}\beta_{x,y}}$ are not if the minimum of the parabola is not resolved. This lack of resolution can occur because of the finite wire-target size mentioned above, and because of the following optical reasons:

1. For unequal emittances, because the correlations $\langle xy' \rangle$ and $\langle x'y \rangle$ are not⁸ necessarily equal, there may be residual uncorrected cross-plane coupling terms in the spot.
2. Before full implementation³ of the seven other optical corrections, the size of the third order chromatic aberrations can dominate⁵ the linear component of the beam size at the IP* .

In both cases, the linear variables $\beta_{x,y}$ and $\epsilon_{x,y}$ will be over-estimated. In order to relieve the effects from 2., a detuned optical configuration, with purposely small angular spreads, can be used, such that the third order aberrations are negligible compared to the linear beam size. For the reason, such a configuration, with $\beta_{x,y} = 3$ centimeters, instead of the nominal $\beta = 0.75$ centimeters, was used in the initial commissioning phase.

Effects from 1. cannot be handled in the Final Focus with the present correction scheme, which is designed for equal emittances in both planes. Operationally, it is therefore desirable to maintain equal emittances throughout the upstream parts of the SLC.

* The second order aberrations must also be cancelled by fitting the chromatic correction sextupoles after each significant optical adjustment, to take into account the deviations in the lattice caused by the optical matching. This results from the fact that the six quadrupoles used to correct the betatron phase-space straddle the chromatic correction.

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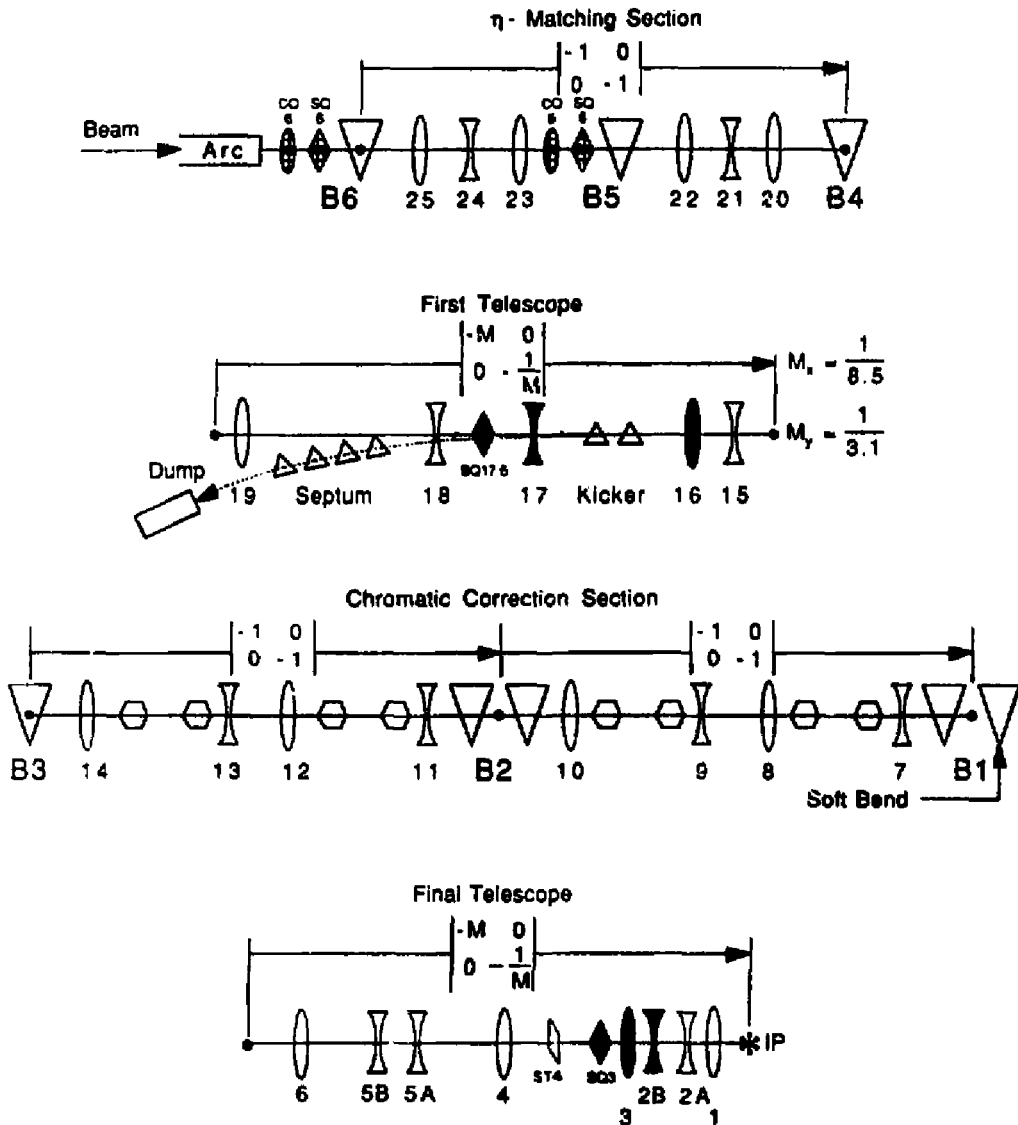


Fig. 1: Schematic of the SLC Final Focus System optics. The six variable quadrupoles used for adjusting the betatron phase-space are shown shaded. The four quadrupoles used to correct the dispersion function are shown cross-hatched. The profile monitor ST4 is located immediately upstream of the skew quadrupole SQ3. The sequential application of these ten adjustments is summarized in the flow-diagram in Fig. 2.

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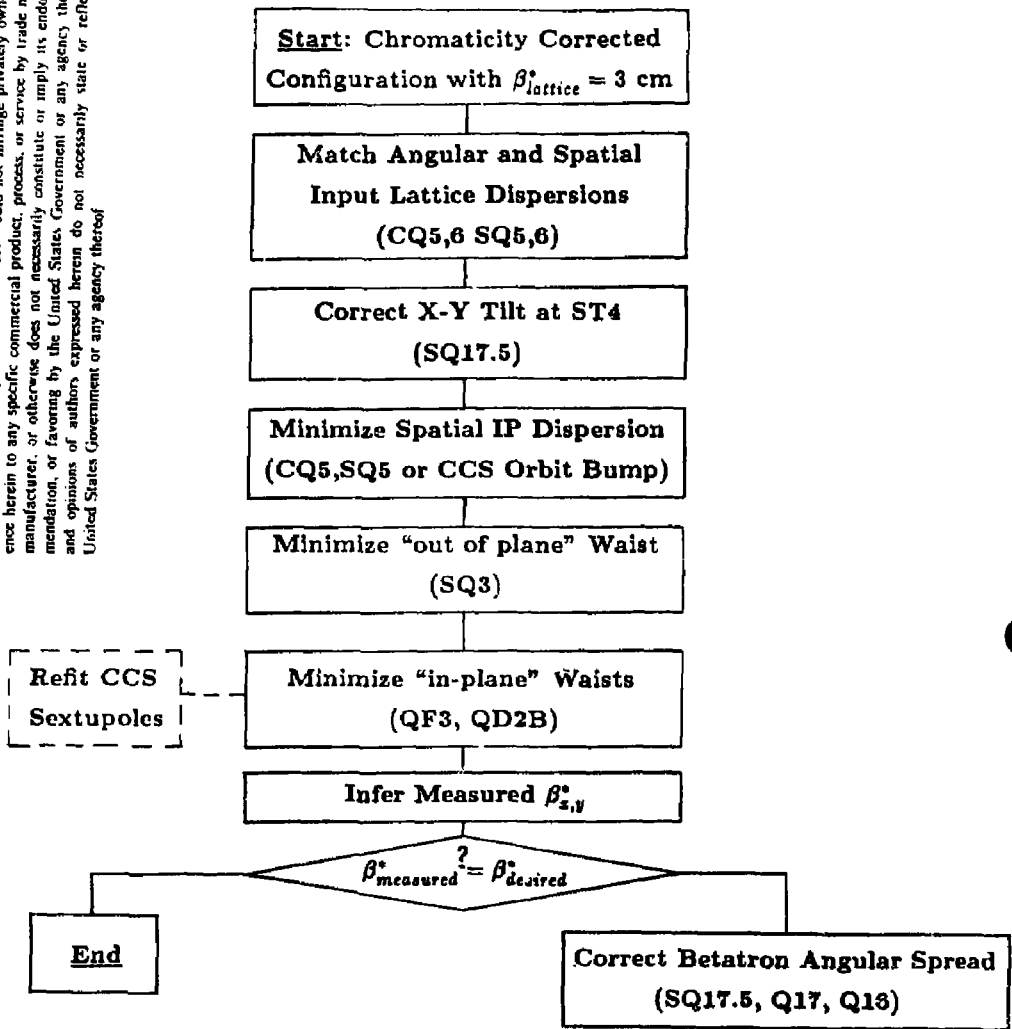


Fig. 2: Flow-diagram summarizing the ten linear optics adjustments required to minimize the beam size at the IP of the SLC. These adjustments are grouped and applied sequentially as shown in the boxes. Some iteration of the waist and IP dispersion minimization is usually performed. In addition, the sextupoles are refitted after each significant waist correction. The main correctors used at each step are indicated in parenthesis. The full system is shown in Fig. 1.