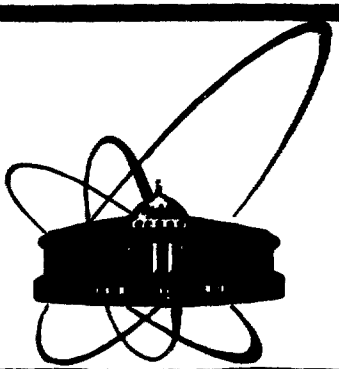

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HARD PROCESSES IN GENERAL FACTORISATION SCHEME

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1. INTRODUCTION

In most of physically interesting hard scattering processes we have to do beside the renormalised couplant $a=g^2/4\pi^2$ (in terminology and notation of [1]) also with various parton distribution and fragmentation functions (called densities in the following). These are not calculable in perturbative QCD and must therefore be extracted from experiment. In doing so we face the problem of their precise definition in terms of the bare parton densities, much in the same way as in the case of the definition of the renormalised couplant α in terms of the bare couplant α_B. As a consequence of this latter freedom the couplant becomes a function of free parameters $\{\mu, c_i, i\geq 2\}$ appearing in the definition.

$$\frac{da(\mu,c_i)}{d\ln\mu} = \beta(a) = -ba^2(\mu,c_i)(1 + ca(\mu,c_i) + c_2a^2(\mu,c_i) + \ldots) \quad (1)$$

where b as well as c are fixed once the number n_f of quark flavors is given (we stay in massless QCD throughout the paper). The parameters $\{\mu, c_i, i\geq 2\}$ specify the renormalisation scheme (RS) of the couplant α, while the subset $\{c_i, i\geq 2\}$ defines the renormalisation convention (RC). The whole theory is fixed by fixing some dimensionful quantity, as for instance the parameter Λ, specifying the solution of (1)

$$\tau \equiv b\ln\frac{\mu}{\Lambda} = \frac{1}{a} + c\ln\frac{ca}{1+ca} + \int_0^a \left(\frac{b}{\beta(a)} + \frac{1}{x^2(1+cx)}\right) dx. \quad (2)$$

According to [1], changing the RS of the couplant means varying the parameters μ, c_i at will, but holding Λ fixed. Internal consistency of the perturbation theory then implies that the coefficients r_k of perturbation expansion of some fully inclusive physical quantity $R(Q)$ (assumed for simplicity to depend on a single external momentum Q)

$$R(Q) = a^d(\mu,c_i)\left(1 + r_1(Q/\mu)a(\mu,c_i) + r_2(Q/\mu,c_i)a^2(\mu,c_i) + \ldots\right) \quad (3)$$

are unique functions of $Q/\mu, c_i, i\leq k$. For d=1 we have for instance

$$r_1(Q/\mu) = b\ln(\mu/Q) + r_1(\mu=Q) = b\ln(\mu/\Lambda) - \rho_1(Q/\Lambda), \quad (4)$$

where ρ_1 is RS invariant, i.e. is independent of the choice of μ, c_i. As μ and Λ enter in (2) always in the ratio μ/Λ, the change of the RS can equally well be accomplished by holding μ fixed by setting it equal to, say, Q and varying Λ instead. In this notation it is the dependence of $a(\mu=Q)$ and $r_k(\mu=Q)$ on Λ and c_i which expresses the RS ambiguity. The dependence of the couplant α and the coefficients r_k on μ and Λ are, however, only two different sides of the same coin and so it would be

redundant to vary both μ and Λ. For bookkeeping purposes we therefore single out one particular RS (by specifying $a(\mu=Q)$ and $r_k(\mu=Q)$ and let all the RG transformations be described by variations of the parameters μ, c_i. Although the choice of this referential renormalisation scheme (RRS) is completely arbitrary and has nothing to do with the RS ambiguity of finite order approximations to (3) (we would have to choose some RRS even if we were able to calculate the full sum (3)), some quantities, like r_k (Q/μ) will depend on it. On the other hand the invariant $\rho_i = b\ln(Q/\Lambda) - r_i(\mu=Q)$ is naturally independent also of the chosen RRS as the explicit dependence on the associated Λ of the logarithm $\ln(Q/\Lambda)$ is compensated by the implicit dependence of the coefficient $r_k(\mu=Q)$ on the RRS (higher order ρ_i's are Q-independent).

Having fixed the RRS as, say, $\overline{MS}$ (so for $\mu=Q$ we get $a(Q)$ and $r_k(\mu=Q)$ as defined by the usual $\overline{MS}$ counterterms), we may now choose any μ, c_i to evaluate R(Q) according to (3) because in the full sum (3) the dependences of the couplant and the coefficients r_k on these parameters fully compensate each other (we ignore here the complicated and pressing problem connected with the divergence of expansions like (3) [2,3]). The truncated approximations to (3) do, however, depend on this choice. Various ideas [1,4-7] have been proposed to resolve this finite order ambiguity. They stress different aspects of the problem, but there is usually little doubt as to the form of the N-th order approximant R^N: it is (3) truncated to that order. In principle one can imagine other forms of this approximant, like for instance

$$R^N(Q) = \sqrt{R^2(Q)^N} = a\sqrt{1+s_1 a+s_2 a^2+..s_N a^N} \quad , s_1=2r_1, s_2=r_1^2+2r_2, .. \qquad (5)$$

i.e. we first calculate $R^2(Q)$ to N-th order and then take the square root (5), but they are mostly rather artificial. Moreover, if we insist on the polynomial form of the N-th order approximant then only (3) truncated to that order is acceptable. Nevertheless in some cases there may really be good reasons to modify (3) and thus also its approximants R^N. This happens in the case of exponentiation of soft gluon emissions where, written schematically and apart from overall normalisation, we have instead of (3) [8,9]

$$R(Q) = e^{ra\left(a(1+\bar{r}_1 a+\bar{r}_2^2 a+...)\right)}, \quad \bar{r}_i = r_i - r \quad \text{etc.} \qquad (6)$$

(r is some number) and truncate then the series in the brackets.

For processes involving parton distribution and fragmentation functions perturbation theory leads to results which are more complicated than (3). I shall in the rest of this paper discuss mostly the simplest case, namely that of nonsinglet nucleon structure functions as exempli-

fied by the combinations

$$F_2^{NS}(x,Q) = \frac{1}{x}\left[F^{\nu p}_2(x,Q) - F^{\bar{\nu} p}_2(x,Q)\right] \qquad (7)$$

$$F_3^{NS}(x,Q) = F_3^{\nu p}(x,Q) + F_3^{\bar{\nu} p}(x,Q). \qquad (8)$$

I drop the superscript NS and denote by F(x,Q) in the rest of this paper generically all the NS structure functions like (7-8).

QCD predictions for F(x,Q) are burdened, beyond the RS ambiguity discussed above, also with the so called factorisation scheme (FS) ambiguity [10-11]. But before attempting to resolve, in one way or another, this ambiguity we must again first of all agree on the form of the N-th order approximant. There are two different, but equally plausible alternatives. The first starts with QCD predictions for the moments

$$F_N(Q) \equiv \int_0^1 x^{N-4} F(x,Q)dx \qquad (9)$$

of the structure function F(x,Q):

$$F_N(Q) = A_N\left[\frac{ca(M)}{1+ca(M)}\right]^{-d^N/b}\left(1+ca(M)\right)^{-d_1^N/bc}\left[1 + r_1^N(Q/M)a(\mu) + ..\right], \quad (10)$$

where $a(M)$, $a(\mu)$ are renormalised couplants, taken at generally different scales M and μ, A_N are numerical constants and d^N, d_1^N are first two coefficients in the expansion of the anomalous dimension

$$\gamma^N \equiv \frac{d\ln\langle p|O^N|p\rangle}{d\ln M} = d^N a(M) + d_1^N a^2(M) + \ldots \qquad (11)$$

describing the dependence of the matrix element of relevant Wilson operator O^N (in the proton state) on the factorisation mass M. The last bracket in (10) corresponds to "hard scattering" part of the structure function and is closely reminiscent of (3). While d^N are, similarly to b, c in (1) uniquely specified by n_f, all the remaining coeficients d_i^N, $i\geq 1$ are again completely arbitrary and define what is usually called a factorisation convention (FC): $FC=\{d_i^N\}$. This, together with the factorisation mass M, define the factorisation scheme (FS): $FS=\{M,FC\}$. Naturally, also the coefficients r_k in (10) do depend on this FC. To NLO, to which I restrict myself in this paper, only the first two terms, explicitely written out in (10-11) are taken into account and so in this approximation $FC=\{d_1^N\}$ and $FS=\{M,d_1^N\}$. To the order considered the ambiguities inherent in (10) are therefore connected with the freedom in the choice of both the $FS=\{M,d_1^N\}$ and the $RS=\{\mu\}$. Let me call calculational scheme (CS) the set of all parameters μ, M, d_1^N. All the information on the long-distance properties of the proton is contained in the constants A_N, which cannot be calculated perturbatively, but must be considered as free parameters to be extracted from comparison of (10) with experimental data. Note, however, that A_N are not equal to matrix ele-

ments of Wilson operators O^N at some particular scale M_o, but specify the solution of (11) by means of its *asymptotic* behaviour as $M\to\infty$

$$\langle p|O^N|p\rangle \xrightarrow[M\to\infty]{} A_N \left[\frac{c}{b\ln M}\right]^{-d^N/b}. \qquad (12)$$

The internal consistency of perturbation expansion (10) dictates again the dependence of r_i^N on μ,M and the FC=$\{d_i^N\}$ [10]:

$$r_i^N(Q/M,d_i^N) = d^N \ln\frac{Q}{M} + \frac{d^N}{b^i} + \varkappa^N, \qquad (13)$$

where $\varkappa^N$ are FS-invariants, which, however, still depend on the RRS of the couplants $a(\mu)$,$a(M)$.

The form (10) is a direct consequence of the operator production expansion technique. Were the moments $F_N(Q)$ experimentally measurable, we could compare them directly to (10). In practice only structure functions over a limited range of x-values are available and so we must turn (10) into prediction for them. The other possibility of extrapolating the measured structure function to the whole interval $\langle 0,1\rangle$ in order to calculate the moments (9) mixes experiment with phenomenological assumptions and should better be avoided.It is definitely preferable to incorporate such assumptions into free parameters of the theoretical formulae. A number of methods for translating (10) into prediction for the structure functions does exist. One of them, based on the use of Jacobi polynomials will be described in more detail in Section 4. I shall call it the momentum based (MB) formulation of QCD predictions for structure functions.

The alternative way of formulating QCD predictions for structure functions, embedded naturally ly in the parton model language [12], is to write them as a convolution of the (nonsinglet) quark density q(x,M,FC) (taken at the general scale M) and the hard scattering cross-section $C(z,Q/M,\mu,FC)=\delta(1-z)+a(\mu)C^1(z,Q/M,FC)$:

$$F(x,Q) = \int_o^1 \frac{dz}{z}\, q(x/z,M,FC)\left[\delta(1-z)+a(\mu)C^1(z,Q/M,FC)\right]. \qquad (14)$$

The quark density itself is a solution of the evolution equation

$$\frac{dq(x,M,FC)}{d\ln M} = \int_o^1 \frac{dz}{z}\, q(x/z,M,FC)P(z,a(M),FC), \qquad (15)$$

where

$$P(z,a(M),FC) = a(M)P^o(z) + a^2(M)P^1(z,FC) \qquad (16)$$

and $r_i^N, d^N, d_i^N, \varkappa^N$ are moments (defined as in (9)) of C^1, P^o, P^1 and $\varkappa(z)$ respectively. In terms of these functions eq.(13) reads

$$C^1(z,Q/M,FC)= P^o(z)\ln(Q/M) + P^1(z)/b + \varkappa(z). \qquad (17)$$

Clearly, (11) is just the Mellin transformation of (15). In this alternative language, let me call it parton model based (PB) formulation, FC is specified by the funtion $P^k(z)$. Provided M in (10) is independent of N and equal to M in (14), which thus is independent of x and moreover $a(\mu)$ and $a(M)$ are exact solution of eq.(2) truncated to the NLO, these two expressions yield identical $F(x,Q)$.

There is, nevertheless no reasons why M in (10) could not depend on M and/or M in (14) on x. In such case (10) and (14) are no longer equivavalent and so express two different forms of the NLO QCD predictions for $F(x,Q)$, *each* of them still burdened with the CS ambiguity. Before trying to resolve this ambiguity, we must therefore first of all decide which of the two discussed formulations to adopt. Although from the point of view of eventual applications to more complicated processes like Drell-Yan production of massive dileptons [13], large p_T photoproduction or hadroproduction of photons with large p_T [14,15],the PB formulation is definitely preferable on technical grounds, there is in fact no serious reason to prefer it in principle. At the end of Section 4 we shall see that indeed both formulations lead to similar formulae, namely the sums of exponentially improved expressions like (6), weighted by some functions of the parameters A_N.

In the following I shall first discuss, for both formulations and in a quantitative manner, the question of the choice of the CS.Then a formalism will be constructed which allows an easy transformation from one FS to another (changing the RS=$\{\mu\}$ is in eq. (1,4)). The whole problem of the appropriate choice of the FS has only very recently obtained some attention in the phenomenological analyses [13-15], but merely as far as the change of the scales μ and M is concerned. I shall demonstrate that the proper choice of the FC is probably even more important.

The rest of this paper is organised as follows. In the next Section I shall commnent on some of the popular choices of FS=$\{M,d_1^N\}$, including the one based on the extension *[10,11]* of the Principle of Minimal Sensitivity to quantities like (7,8). Their quantitative comparison in the case of the moments (9) can be found in Section 3. The explicit expression for the structure function $F(x,Q)$ in a general FS and using both the MB and PB formulations are constructed in Section 4. The generalisation of the results to more complicated processes is sketched in Section 5. Summary and conclusions are reserved for the last Section.

2. REVIEW OF CURRENTLY USED FACTORISATION SCHEMES

Of the more or less ad hoc chosen FS the following two have been used most frequently

2.1 The "universal" factorisation scheme

This is the rather unfortunate and misleading denomination for the FS in which M=Q and d_i^N are given by expressions first derived within the OPE technique in [16]. The corresponding branching function $P^i(z)$ was obtained in [16] by means of inverse Mellin transformation and in [12] using directly the PB formulation . As in both techniques the results (i.e. d_i^N or $P^i(z)$) are calculated (in dimensional regularisation) from certain renormalisation factors, retaining at each order only the pole terms, the denomination "MS" would be much more appropriate. The word "universal" is misleading as it gives rise to incorrect impression that only this FS can be used in all hard scattering processes. This, however, is not the case. Any FS, that is, any choice of M, μ and d_i^N (or P^i) can in principle be used in any hard scattering process, much in the same way as any RS=$\{\mu\}$ can be used in (3). In complete analogy to the couplant (1), quark density q(x,M,FC) is not a physical quantity and we are therefore free to define it in any way consistent with (15). Physics is not contained exclusively in the quark density, but rather in its convolution (14) with the hard scattering cross-section.

2.2 The "physical" factorisation scheme

In this FS, suggested first in [18], M is again set equal to Q but the function $P^i(z)$ is chosen in such a way that

$$C^i(z,M,FC) = 0 \quad \Rightarrow \quad F(x,Q) = q(x,Q) \qquad (18)$$

identically for the structure function (7). This structure function is singled out owing to the fact that the associated FS-invariant $\varkappa(z)$ has the following important property

$$\varkappa^i = \int_0^1 \varkappa(z)dz = 0 , \qquad (19)$$

In "physical" FS

$$P^i(z) = -b\varkappa(z) \quad \text{i.e.} \quad d_i^N = -b\varkappa^N \qquad (20)$$

and consequently (19) implies, for the structure function (7), fermion number conservation sum rule

$$\int_0^1 q(x,Q)dx = u_v(x,Q) - d_v(x,Q) = 1 , \qquad (21)$$

where u_v, d_v are valence quark densities as defined in the "physical" FS. Although the validity of (21) is not obligatory, it is certainly preferable to preserve this basic parton model property of quark density even in QCD, if only to maintain, as far as possible the intuitive connection of the latter with the former.

It is, however, obvious that although (18) combined with M=Q implies, for (7) the sum rule (21), the opposite is not true. Indeed for structure function (7) the property (19) requires $r_i^i = d_i^i/b$ and thus the vali-

dity of quark number conservation sum rule (i.e.$d_1^i=0$) implies $r_1^i=0$, but this does not mean that $r_1^N=0$ for all N, or equivalently, that $C^i=0$ identically! Even assuming M=Q, any partition (17) of the FS-invariant $\varkappa(z)$ into $P^i(z)$ and $C^i(z,Q/M)$ which meets the condition $d_1^i=0$ (i.e. P^i is a "+" distribution) guarantees the sum rule (21), not only that defined in (18). One of them is just the "universal" FS discussed above!

The "physical" FS is not a direct consequence of the physically well motivated condition (21) but is based in an essential way on the rather ad hoc assumption $d_1^N=0$ for all N, though only $d_1^i=0$ is required by (21). In fact it is very close to the "effective charges" criterion of [4], developed for resolving the RS ambiguity of expansions like (3).

Moreover, the "physical" FC, i.e. assuming (20) but leaving M still free, has a rather unwelcome feature. Due to the fact that the FS invariant $\varkappa(z)$ still depends on the RRS of the couplant the choice (20) (20) means that in this FC the branching function $P^i(z)$ is also RRS-dependent. In Subsection 2.4 this feature will be shown to lead to unsatisfactory results when optimisation with respect to M is performed in the "physical" FC.

2.3 The "zero" factorisation convention

In some sense opposite to the "physical" FC is the FC in which $P^i(z)=0$ by definition. While in the former FC all of the NLO corrections to structure function (7) were included ("exponentiated") in the definition of the Q-evolved quark density, none is in the latter, as they are all shifted into the hard scattering cross-section, which in the "zero" FC has therefore the form

$$C^i(z,Q/M) = d^N\ln(Q/M)) + \varkappa(z)\cdot \qquad (22)$$

The evolution equation for the quark density q(x,M) is the same as in the LO. This FC has so far not been used in phenomenological analyses, though as we shall see later it is very close to the one prefered by the PMS criterion. It is also not far from the results of the conventional "nonexponentiated" formula

$$F_N(Q) = A_N\ (ca(M))^{-d^N/b}(1+a(M)[d^N\ln(Q/M)+\varkappa(z)+d^Nc/b]) \quad (23)$$

which results from expanding, wherever possible, the r.h.s. of (10) in powers of a(M) and retaining the first two terms only [19].

2.4 Defining factorisation scheme through optimisation

The idea [10,11] of choosing the RS=$\{\mu\}$ as well as the FS=$\{M,d_1^N\}$ at the stationary point of the function $F_N(Q)$ as given in (10) is a direct extension of the original PMS criterion of [1]. In [10] each of the moments $F_N(Q)$ is optimised separately and consequently the optimal FS and

RRS turn out to be N-dependent: $FS^{opt}=\{\bar{M}(N),\bar{d}_i^N(N)\}$, $RS^{opt}=\{\bar{\mu}(N)\}$. The optimisation with respect to μ yields the condition [11]

$$r_i^N = 0 \qquad (24)$$

which when combined with equations resulting from optimisation of (10) with respect to M and d_i^N gives (in the approximation $ca(\bar{M})\ll 1$)

$$\bar{M}(N)=\Lambda\exp\left[\frac{\varepsilon^N}{d^N} - \frac{c}{2b}\right] = Q\exp\left[\frac{\varkappa^N}{d^N} - \frac{c}{2b}\right] \qquad (25)$$

$$\bar{\mu}(N)=\Lambda\exp(\varepsilon^N/d^N)=\bar{M}(N)\exp(c/2b) \qquad (26)$$

$$d_i^N=-d^N c/2 \qquad (27)$$

so that

$$F_N^{FMS}(Q)= A_N\left[\frac{ca(\bar{M})}{1+ca(\bar{M})}\right]^{-d^N/b}(1+ca(\bar{M}))^{d^N/2b}. \qquad (28)$$

As the quantity $\varepsilon^N\equiv\varkappa^N+d^N\ln(Q/\Lambda)$ in (25-27) is both FS and RRS invariant (the dependence of $\varkappa^N$ on the RRS is compensated by the explicit dependence on Λ of the second term), the ratio $\bar{M}(N)/\Lambda$ is manifestly RRS-independent and consequently (28) unique.

Two features of the optimised result (28) are noteworthy. First, for c=0 the optimal $FC^{opt}=\{\bar{d}_i^N\}$ is just the "zero" FC of the previous Subsection and in fact even for realistic values of c these two FC are, for the same M, numerically practically indistinguishable. Secondly, the optimal $\bar{M}$, though for fixed N proportional to Q, is rapidly decreasing function of N for fixed Q, roughly like $QN^{-2/3}$. Although in both the optimised and "physical" FS $r_i^N=0$, the ways in which this is achieved are vastly different. In the optimised formula (28) most (for c=0 all) of the NLO corrections are incorporated in the dependence of the factorisation mass M on N, while in the "physical" FS they are fully shifted into the NLO anomalous dimensions d_i^N, or equivalently into the NLO branching function $P^i(z)$. The quantitative difference between these two realisations of the condition $r_i^N=0$ is significant as will be discussed in the next Subsection.

Closer examination of the formula (28) shows that the stationary point determined by eqs. (25-27) is not a local extreme but rather a saddle point. This is clear already from the fact that for M and d_i^N at the stationary point $F_N(Q)$ does not depend on μ, ($r_i^N=0$ there) which is a property typical for a saddle point. It is also evident that full optimisation, i.e. optimisation of all moments $F_N(Q)$ is possible only for structure function like (7) for which the FS-invariants $\varkappa^N$ have the property $\varkappa^i=0$. Recall that although $\varkappa^N$ do, for general M, depend on the RRS, $\varkappa^i$ does not as $\varkappa^N$ change by a term proportional to d^N when the RRS is varied and $d^i=0$.

Although the optimisation of (10) with respect to FC=$\{d_i^N\}$ is an integral part of the optimisation procedure and may in fact be the most important part thereof, we may for some reasons wish to fix it and optimise with respect to the scales μ and M only. In this case we again get the condition (24), but instead of (25) we find for the optimised factorisation mass $M^{fix}(N)$ the formula

$$M^{fix}(N)=\Lambda\exp\left[\frac{e^N}{d^N}+\frac{d_1^N}{bd^N}\right]=Q\exp\left[\frac{\varkappa^N}{d^N}+\frac{d_1^N}{bd^N}\right] \qquad (29)$$

and consequently

$$F_N^{fix}(Q)=A_N\left[\frac{ca(M^{fix})}{1+ca(M^{fix})}\right]^{-d^N/b}\left(1+ca(M^{fix})\right)^{-d_1^N/bc}. \qquad (30)$$

Provided d_1^N is fixed independently of the RRS, the ratio M^{fix}/Λ is RRS-invariant and so is therefore also (30). As mentioned at the end of the Subsection 2.2, this provision is violated in the "physical" FC. So in this FC the result (30) of optimisation with respect to μ and M remains still ambiguous as it depends through $d_1^N=-b\varkappa^N$ on the RRS of the couplant a(Q) ((29) implies $M^{fix}=Q$ in this FC). Of course, the dependences of a(Q) and $\varkappa^N$ on the RRS mutually cancell to the NLO, but numerically (30) does depend on it. This in itself is nothing wrong, but in (30) we have already optimised with respect to both μ and M. As, however, changing M or the RRS are merely two different ways of realising the same renormalisation group transformation, this is clearly unsatisfactory. The source of this unwelcome feature is clearly the very definition of the "physical" FC, namely the fact that by setting $d_1^N=-b\varkappa^N$ we force these parameters to depend on RRS.

So far all the optimisation concerned exclusively the moments $F_N(Q)$ of structure functions. We can take the results (28) or (30) and invert them to find the corresponding structure function itself. However, as the optimisation procedure does not commute with the inverse Mellin transformation, the in this way obtained F(x,Q) will in general not be the same as if optimisation is applied directly to formula (14). Unfortunately, this is technically rather involved and practically impossible to do. But in any case we must first of all find an analogue of expression (10), which would explicitly exhibit what should be held fixed when varying the FS=$\{M,P^i(z)\}$. From (14-15) this is not obvious. But even after constructing in Section 4 such an expression, we shall see that it is practically hopeless to try to optimise it with respect to FC=$\{P^i(z)\}$. So some experience gained from optimisation of the moments will be invaluable.

3. NUMERICAL COMPARISON

In this Section the results corresponding to various options discussed in the previous Subsections are quantitatively compared. First, Fig.1 displays the dependence of $F_N(Q)$, as given in (10) and normalised to the common LO expression $A_N(ca(Q))^{-d^N/b}$, on the ratio Q/A for several lowest moments of the structure function (7). In all cases the RRS of the couplant is chosen to be $\overline{MS}$. The curves in Fig.1 separate into two distinct groups. The first contains those of the "physical","universal" and "zero" FC, all supplemented with the choice M=Q, together with the conventional nonexponentiated formula (23). Within this group the "physical" FC leads consistently to highest values of $F_N(Q)$ and exhibits also the steepest dependence on Q/A, while the lowest and least steep curve is that of the "zero" FC. The differences are, however, rather small and of little phenomenological significance. Once we let μ and M vary and optimise with respect to them the situation changes as is demonstrated by the three upper curves, corresponding to optimisation in the "universal" and "zero" FC as well as the fully optimised result (28). The optimisation with respect to M in the "physical" FC gives $M^{fix}=Q$ and so leads to the same results as already shown there.

Several conclusions can be drawn from Fig.1. First, the relevance of the optimisation with respect to μ and M depends sensitively on the chosen fixed FC. While no change (relative to the case M=Q) occurs in the "physical" FC, there is a significant jump in the "zero" FC, leading to results which are also practically indistinguishable from those of the full PMS procedure. But also when optimisation is performed in the "universal" FC do we come much closer to the latter than in the "physical" FC. Secondly, the upper three curves are also much steeper than the lower ones. This is further demonstrated in Fig.2, which displays the ratio $F_N(Q)/A_N$ for N=3,4,5. While for M=Q the difference between curves corresponding to the "physical" and "zero" FC can, for each moment N, be approximated by a uniform shift along the x-axis equivalent to the change of A by an N-dependent factor in the interval (1.05, 1.2>), no such simple change of A is capable to describe the relation between the fully optimised $F_N(Q)$ and any of the lower four curves. Qualitatively, this steeper increase of (28) at low Q/A has similar effect as the addition of higher twist terms to the latter curves.

The results presented in Figs.1,2 are instructive, especially if we prefer the PMS approach, but strictly speaking they concern the moments of structure function and cannot be straightforwardly generalised to other processes or even to the direct analysis of structure functions themselves. Nevertheless the results displayed in Figs.1,2 show that

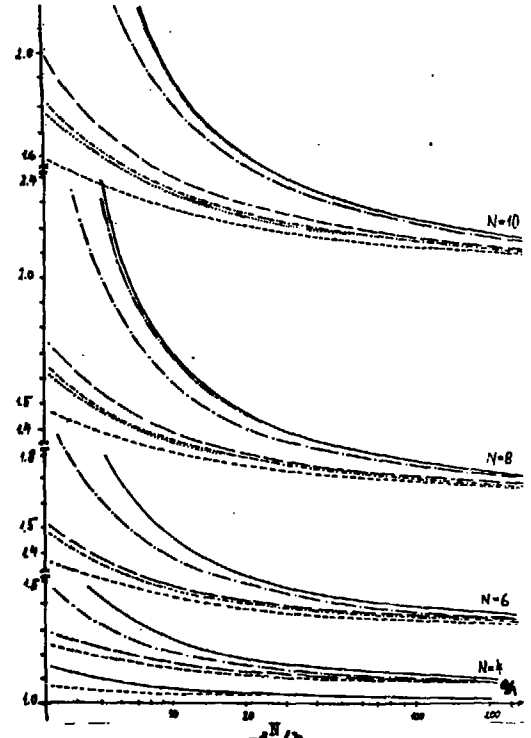

Fig.1. $F_N/(A_N(\alpha_s(Q))^{-d^N/b})$ for: PMS formula (———); optimisation in "zero"(— — —) and "universal"(— · —) PC; "physical"(— — —), "zero"(– – – –) and "universal" (– · – · –) FS and nonexponentiated expression (83) (·····).

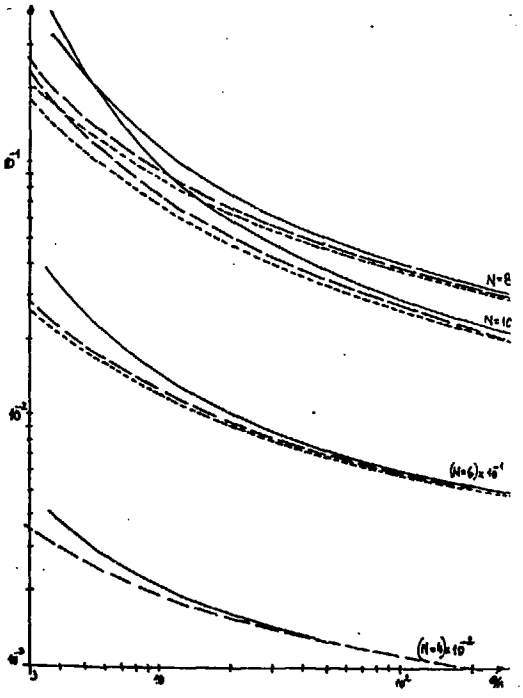

Fig.2. F_N/A_N in the notation of Fig.1. The curves for N= 6,8 scaled as indicated.

the "physical" FC is certainly not the only plausible choice of the FC.

4. FACTORISATION SCHEME AMBIGUITY IN PARTON MODEL BASED FORMULATION

In the case of the deep inelastic scattering, and so long as we do not attempt to optimise, there is neither a principal nor a practical reason for prefering the PB formulation to the MB one or vice versa. If optimisation is performed then, however, the PB formulation is definitely preferred. The PB formulation is also prefered, although now on on practical grounds, if other, more complicated processes (DY dilepton production, photoproduction of large p_T hadrons or hadroproduction of large p_T real photons) are considered in the NLO approximation. It is therefore vital to have at our disposal general formalism for analysing within the PB formulation, any hard scattering process in arbitrary FS. To my knowledge such a formalism is not available in the literature. In the rest of this paper a simple construction of such a formalism will be discussed, starting with the case of the nucleon structure functions (7-8).

The PB formulation of QCD predictions for the structure function $F(x,Q)$ is embodied in eq. (14-15). The moments of the quark density $q(x,M)$ are given explicitly as

$$q_N(M)=A_N\left[\frac{ca(M)}{1+ca(M)}\right]^{-d_1^N/b}\left[1+ca(M)\right]^{-d_2^N/bc} \qquad (31)$$

and the result of the convolution (14) is equivalent to (10), provided in both (10) and (14) M is a constant, independent of either N or x. Nevertheless this provision is not mandatory and so we allow for possible dependence of M in (14) on x.

As we want to vary all the parameters $\mu,M,P^i(z)$ specifying the CS, we must first of all decide what should be held fixed in the process. Starting from eqs.(14-15) the answer is not obvious as in the case of $F_N(Q)$ in (10). Indeed, in this respect the moments of structure functions are more primary quantities than structure functions themselves, as they are directly related to the corresponding Wilson operators. This connection tells us that it is the constant A_N in (31) which must be fixed when we vary M and the FC=$\{P^i(z)\}$. However, as already stressed these constants do not characterise quark densities at any fixed M_o, but rather specify their asymptotic behaviour as $M\to\infty$. On the other hand, it is the quark density $q(x,M_o)$ at some initial M_o which is usually used to specify the solution of (15). But it is obvious from (33) that by changing M and/or $P^i(z)$ (and thus d_i^N) while holding A_N fixed, we change $F_N(Q)$ for all Q. This means that variation of the FS in (14-15) does not only change $P^i(z)$ in (15) and M and $C^i(z,Q/M)$ in (14), but implies also the change of the initial condition $q(x,M_o)$!

So if we want to use the results (i.e. A and $q(x,M_o)$) of a particular analysis of some hard scattering process, performed in a given FS, for working out QCD predictions (for the same or other processes) in a different FS, we must inevitably know also the associated change of the boundary condition on (15). This information is, however, essentially equivalent to the explicit knowledge of the solution q(x,M) of (15) as a function of both M and $P^4(z)$. We can write down the evolution equation, including the boundary condition, for the quark density in the new FS only provided we know its explicit expression in terms of the moments (31) because only for these moments do we know what must be held fixed when M and $P^4(z)$ change. It is, however, clear that once we have such an expression at our disposal there is no further need to solve the evolution equation in the new FS as this expression itself represents the most general solution of it.

We therefore seek an explicit solution of (15) in terms of moments (31), which moreover allows an easy transformation of the initial condition $q(x,M_o)$, employed in all existing analyses, into the information on A_M. An efficient way of doing this is based on the use of orthogonal Jacobi polynomials as suggested in [20]. The numerical accuracy of this way of solving the evolution equation (15) has been studied in detail in [21] and turns out to be very satisfactory (better than 1% for all x and Q of interest). Following [20] we write the solution of (15) in the form

$$q(x,M)=x^{\alpha}(1-x)^{\beta}\sum_{k=0}^{\infty}\Theta_k^{\beta\alpha}(x)a_k^{\beta\alpha}(M,FC) \qquad (32)$$

and so obtain for the structure function F(x,Q) the expression

$$F(x,Q)=\int_x^1\frac{dz}{z}\,z^{\alpha}(1-z)^{\beta}\sum_{k=0}^{\infty}\Theta_k^{\beta\alpha}(z)a_k^{\beta\alpha}(M,FC)\left[\delta(1-\frac{x}{z})+a(\mu)\left[P^0(\frac{x}{z})\ln\frac{Q}{\mu}+w(\frac{x}{z})+\frac{1}{b}P^4(\frac{x}{z})\right]\right]. \qquad (33)$$

In (32-33) $\Theta_k^{\beta\alpha}(z)$ are the Jacobi polynomials and the "Jacobi" moments, $a_k^{\beta\alpha}(M,FC)$ are defined by means of the moments $q_N(M,FC)$ (31) as

$$a_k^{\beta\alpha}(M,FC)=\sum_{j=0}^{k}c_{kj}^{\alpha\beta}q_j(M,FC=\{d_i^N\}), \qquad (34)$$

where $c_{kj}^{\alpha\beta}$ are numerical coefficients (for their explicit values as well as for the exact definition of $\Theta_k^{\beta\alpha}(z)$ see [20-21]). To fix (33) unambiguously, α and β must first be specified. This is done in such a way so as to approximate the basic shape of the structure function F(x,Q) already by the lowest term in the series (32). For the nonsinglet structure functions it is quite sufficient [21] to take $\alpha=-0.5, \beta=3$.

Formula (33) represents the basic form of QCD prediction for $F(x,Q)$ which allows for the variation of the FS=$\{M,P^A(z)\}$ and specifies what - namely the constants A_N - must be held fixed. These constants must then be, together with Λ, extracted from phenomenological analyses of experimental data. They represent the most natural parametrisation of non-perturbative, long-distance properties of the nucleon. Furthermore, in contrast to the quark density $q(x,M_o)$ at some initial M_o, which is not a physical quantity and is thus ambiguously defined, the constants A_N are unique, independent of the CS used. Employing (33) as the theoretical formula, it is then straightforward to compare any two (or more) phenomenological analyses of any given structure function, performed in different, completely arbitrary FS. Recall, how it is sometimes difficult to compare results of published analyses, when these are done (as is usually the case) in different FS, using different parametrisations of $q(x,M_o)$ at different M_o.

Furthermore it is frequently near to impossible to find out from published papers which FS has in fact been employed. Take for instance recent extensive analyses of nucleon structure functions, done by EMC [22], BDCMS [23] and CDHS [24] Collaborations. In none of them does one find a reliable information concerning the FS used. In [22] reference is made to a paper of Abbott et al. [25] which, however, contains only the LO analysis, while in [23] only a vague statement that "the program of Abbott and Barnett was used" can be found. In [24] the evolution equation for nonsinglet structure function is written in the form suggesting the use of "physical" FS, but when specifying the NLO branching function $P^A(z)$ the reader is referred to paper [12], the results of which correspond to the "universal" FS with μ=M. In order to avoid such unnecessary complications, I recommend the use of eq. (33) as the general form of the NLO QCD predictions (in the PB formulation) for any structure function and in arbitrary FS. Once Λ and A_N are fixed from an analysis of one particular process in a given CS, it is trivial to use them to write down predictions for other structure functions (or any other hard scattering process) in arbitrary calculational scheme.

From practical point of view it is essential that only 6-7 terms in the sum (32) are necessary for very accurate (typically better than 1%) approximation of $q(x,Q)$. That is more than the four parameters usually employed for the description of $q(x,M)$ at some initial M_o in the form

$$q(x,M_o)=Ax^{\alpha}(1-x)^{\beta}(1+\gamma x) \qquad (35)$$

but as in the case of (35), which of course is merely some ansatz, we may assume certain dependence of A_N on N and in this way still lower

the number of parameters required in the truncated form of (32). For instance we may use the results of the conventional parametrisation (35) to determine A_N in terms of $A,\alpha,\beta,\gamma,M_0,d_1^N$

$$A_N=A\,\frac{\Gamma(N+\alpha)\Gamma(\beta+1)}{\Gamma(N+\alpha+\beta+1)}\left[1+\gamma\,\frac{N+\alpha}{N+\alpha+\beta+1}\right]\left[\frac{ca(M_0)}{1+ca(M_0)}\right]^{d^N/b}\left(1+ca(M_0)\right)^{d_1^N/bc}. \quad (36)$$

Unfortunately it is presently impossible to use (36) for a reliable determination of the constants A_N. This is in part due to the mentioned lack of reliable information on the FS used in analyses like those in [22-24], but there is also another reason. In most of the phenomenological analyses using the evolution equation (15) in some FS, the boundary condition is specified not, as would be appropriate with respect to the equation (15), by $q(x,M_0)$, but rather by the full structure function $F(x,Q_0)$ at some Q_0. Consequently this parametrisation must first be transformed into the one for $q(x,Q_0)$ by unfolding the convolution (14) (except in the "physical" FS, where $F(x,Q)=q(x,Q)$ by definition). As the papers contain usually no information how this step was done, I preferred to use (36) for merely a semiquantitative estimate of A_N by plugging into it the LO parametrisation of ref. [22], corresponding to (35) with $A=0.97,\alpha=-0.65,\beta=3.16,\gamma=-0.13,M_0^2=5GeV^2$. Accordingly I set $d_1^N=0$ in (36). As d^N can be extended to any $N>0$, A_N can be calculated for any real $N>0$, not only integers [27]. The results, shown in Fig.3, indicate a smooth dependence of A_N on N which can be parametrised with sufficient accuracy as

$$A_N=\exp(h_0+h_1N+h_2N^2+h_3N^3). \quad (37)$$

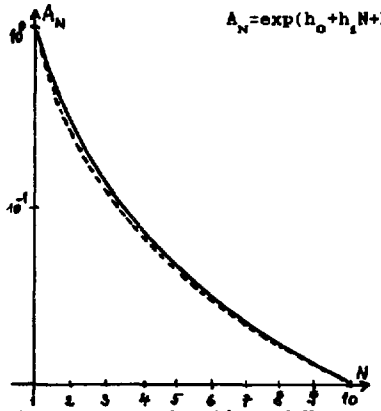

Fig.3. A_N as a function of N according to (36)(——), (37)(- - -).

Without any efforts to find the best values of h_is, we see that $h_0=1.4$, $h_1=-1.55,h_2=0.168,h_3=-0.007$ give a very satisfactory description of all $A_N,N\le10$. So again, as in the case of the initial condition (35), four parameters are sufficient for the parametrisation of nonperturbative properties of the nucleon. Phenomenological analyses would then result in the determination of A and $h_i,i\le3$ The above formula can also be used in another way. We do not have to employ (33) and may follow the conventional procedure of solving, in some FC=$\{P^i(z)\}$, (15) with the initial condition (35) at some M_0. Choo-

sing different M_0 in (35) and different FS={M,$P^i(z)$}in (14), we shall in general get different values of A,α,β,γ. Nevertheless, if these are substituted into (36), they must, for various approximations to be mutually consistent, yield the same A_N.

Starting with (33) we may now attempt to optimise it, for a given x, with respect to μ,M as well as FC={$P^i(z)$}. There are no serious problems with optimisation with respect to μ and M, but they turn up when we want to find the optimised $P^i(z)$. The reason for it is that even if we take only the first few terms in (33) and so only a few of lowest moments d_i^N of $P^i(z)$ are needed to calculate q(x,M) according to (32), the presence of $P^i(z)$ in the convolution (33) implies that we must vary $P^i(z)$ in the whole interval (0,1). To my knowledge there is no way of even formulating some kind of equation for the optimised $P^i(z)$ which would realise the formal condition $\delta F(x,Q)/\delta P^i(z)=0$.

For the deep inelastic scattering we can, within the MB formulation, first optimise separately each of the moments $F_N(Q)$ according to (28) and then turn them into the corresponding structure functions using the general expansion

$$F(x,Q)=x^{\alpha}(1-x)^{\beta}\sum_{k=0}^{\infty}\Theta_k^{\beta\alpha}(x)F_k^{\beta\alpha}(Q), \qquad (38)$$

where analogously to (34)

$$F_k^{\beta\alpha}(Q)=\sum_{j=0}^{k}c_{kj}^{\beta\alpha}F_j(Q). \qquad (39)$$

Substituting (28) into (39) we see that the result of the PB formulation (33) has the same structure as (38): they are both given as a sum of exponentiated moments $q_j(Q)$ (31), weighted by certain functions of M and $P^i(z)$, depending in (33) on x and in (38) on N.

5. BEYOND THE NUCLEON STRUCTURE FUNCTIONS

For other, more complicated processes, like those mentioned at the beginning of the previous Section, only the PB formulation is of practical use. However, as the optimisation with respect to the FC of various parton distribution and fragmentation functions is hopeless, we must in practice choose some FC={$P^i(z)$} (for each parton leg in principle separately) and optimise, if we wish so, with respect to remaining parameters μ and M only. For instance, in the case of hadroproduction of photons with large p_T, the differential cross-section (in the nonsinglet channel) as a function of x_F, p_T of the produced photon reads

$$\frac{d\sigma}{dx_F dp_T}=q(x_1,M_1,FC_1)\otimes K(p_T,x_F,M_1,M_2,FC_1,FC_2,\mu)\otimes q(x_2,M_2,FC_2), \qquad (40)$$

where $q(x_i,M_i,FC_i)$ i=1,2 are the nonsinglet quark densities of the two

colliding hadrons, as defined in (15) in factorisation conventions FC_i, taken at the scales M_i, and the function K, the generalisation of $C(z,Q/M,FC)$ in (14), describes again the "hard scattering" of two partons. For exact definition of the convolution * see [13]. In practice we are forced to assume $M_1=M_2=M$ and choose some moreless ad hoc $FC_1=FC_2=FC$. This later choice is inevitably subjective, but at least some lesson can be drawn from the discussion of the previous Section. There the optimal FC was shown to practically coincide with the "zero" FC of Subsection 2.3 and even the "universal" FC lead, after optimisation with respect to μ,M, to results which were much closer to the fully optimised ones than those of the "physical" FC. Although this feature is very probably specific to structure functions (7-8), it seems reasonable to carry out any analysis (including the optimisation of (40) with respect to μ and M) at least for all the three aforementioned FC. The eventual discrepancy of in this way obtained results represents, in my view, a plausible measure of the theoretical uncertainty associated with (40).

In practice further simplification is forced upon us. The optimisation of (40) with respect to μ and M cannot be done analytically, but the stationary point must be found by mapping (40) as a function of μ and M. The optimised result is, however, also a function of the unknown constants A_N. To determine these constants as well as A by fitting (40) to experimental data would require on one hand much better accuracy of the data and on the other large amount of computer time. In practice, the constants A_N must therefore be taken from other processes, like the deep inelastic lepton-nucleon scattering. This is quite legal to do but we must be carefull to use in (40) the correct quark density $q(x,M)$ and hard scattering cross-section $K(x_F,p_T,M,FC)$, corresponding to the FS adopted. It is inconsistent to proceed as in [13-15] where the approximate (though phenomenologically quite succesfull) but essentially only LO parametrisation [27] of $q(x,M)$ was used in convolution of the type (40) with truly NLO hard scattering cross-section K.

6.SUMMARY AND CONCLUSIONS

In the previous Sections we have discussed various ambiguities appearing at the NLO in the theoretical description of hard scattering processes involving parton distribution and fragmentation functions. Two different formulations, one starting from the moments of structure functions, the other working directly with the evolution equations for parton densities, were shown to be in principle equally plausible representations of NLO QCD predictions for the nucleon structure functions. There

is no obvious way of resolving this ambiguity, if we do not insist on optimisation, which naturally prefers the latter formulation. In both of these formulations we are still faced with the ambiguities connected with the choice of the FS={M,FC} as well as the RS={μ}. Various currently used choices of the former were reviewed and quantitatively compared. The quark densities were argued to play a role quite similar to that of the renormalised couplant a(μ). In particular there is no natural definition of the FC={$P^k(z)$}, very much as there is in QCD (contrary to QED) no natural definition of the couplant a(μ). The "best" (in whatever sense we mean this) FC is furthermore expected to be process dependent. In the case of nucleon structure functions the fully optimised moments F_N turned out to practically coincide with those of the "zero" FC combined with optimisation with respect to μ and M and quite far from those of the "physical" FS.

The main aim of this paper was to construct a general expression for the NLO QCD predictions in arbitrary FS={M,d_i^N}. Such an expression allowing easy and straightforward transformation from one FS into another is necessary if we want to use the results of existing phenomenological analyses of data on deep inelastic scattering for working out QCD predictions for other, more complicated processes in general FS. Jacobi polynomials turned out to be very convenient for this purpose as they lead to simple but simultaneously rather accurate expressions. An important aspect of the whole construction is a new parametrisation of the nonperturbative properties of the hadrons. Contrary to the conventional way of parametrising the uncalculable properties of the hadrons by means of the quark desity q(x,M) at some referential M_o the use of constants A_N has an important advantage: they are independent of the chosen calculational scheme.

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Хила И. E2-88-293

Жесткие процессы в общей схеме факторизации

Рассматривается проблема определения функций распределения кварков в адронах в высших порядках теории возмущений /КХД/. Строится формализм, позволяющий простым образом переходить от одного такого определения /схемы факторизации/ к другому. Этот формализм приводит к новому виду параметризации непертурбативных свойств адронов, который в отличие от обычного вида, использующего значение функции распределения при заданном начальном Q_0, однозначен.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1988

Chýla J. E2-88-293

Hard Processes in General Factorisation Scheme

The problem of defining quark distribution functions in higher orders of perturbative QCD is reviewed. A formalism is constructed, which allows simple transformation from one such definition (factorisation scheme) into another. This formalism leads to a new way of parametrising the nonperturbative properties of hadrons, which in contrast to the conventional way, employing quark distribution functions at some referential Q_0, is uniqe.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1988

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