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SADDE (Scaled Absorbed Dose Distribution Evaluator)

A Code to Generate Input for VARSKIN

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DISCLAIMER

SUMMARY

The VARSKIN computer code has been limited to the isotopes for which the scaled absorbed dose distributions were provided by the Medical Internal Radiation Dose (MIRD) Committee or to data that could be interpolated from isotopes that had similar spectra. This document describes the methodology to calculate the scaled absorbed dose distribution data for any isotope (including emissions by the daughter isotopes) and its implementation by a computer code called SADDE (Scaled Absorbed Dose Distribution Evaluator). The SADDE source code is provided along with input examples and verification calculations.

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INTRODUCTION

Until now, the VARSKIN computer code (Traub et al. 1987) has been limited to those isotopes for which the scaled absorbed dose distributions were provided by Medical Internal Radiation Dose (MIRD) Pamphlet No. 7 (Berger 1971) or to data that could be estimated by interpolating between isotopes that had similar spectra (Williams 1987). This was an inconvenience to the users of VARSKIN and made calculations of absorbed dose distributions troublesome for isotopes not among the original list. There was a need, then, for a complementary code to VARSKIN that could calculate scaled absorbed dose distributions for those isotopes not published.

At the request of the Division of Accident Evaluation (Office of Nuclear Regulatory Research of the U.S. Nuclear Regulatory Commission), the Pacific Northwest Laboratory (PNL) undertook to create a code that could be used in calculating the scaled absorbed dose distribution for any isotope, including emissions by daughter isotopes. The result of this effort, the code SADDE (Scaled Absorbed Dose Distribution Evaluator) generates data files for VARSKIN from the calculated beta spectra, allowing users to more readily and accurately calculate beta skin dose. This document lists the code SADDE and discusses the methods used in developing it.

The following sections provide a description of the algorithms used in the SADDE code, instructions for using SADDE to compute scaled absorbed dose values on a personal computer, and verification of SADDE's accuracy. The SADDE source code is listed in Appendix A, along with input examples. Verification calculations are provided for six isotopes in Appendix B.

ALGORITHM DESCRIPTION

From the basic definitions of specific absorbed fraction for monoenergetic electrons given by Berger (1971), the absorbed dose rate is defined as follows:

$$R(x,E_0) = A\ n\ k\ E_0\ \Phi(x,E_0) \tag{1}$$

where $R(x,E_0)$ = the absorbed dose rate in rad/sec
x = the distance from the point source
A = the source activity
n = the number of electrons emitted at energy E_0 per unit of source activity
k = 1.6×10^{-8} g-rad/MeV
E_0 = the initial energy of the electron in MeV
$\Phi(x,E_0)$ = the specific absorbed fraction of the energy in g^{-1}.

For a spectrum of energies such as would be emitted from a source undergoing beta decay, the absorbed dose rate $R_\beta(x)$ would be

$$R_\beta(x) = A\ n_\beta\ k \int_0^{E_{max}} E\ S(E)\ \Phi(x,E)\ dE \tag{2}$$

where E_{max} is the endpoint energy of the beta spectrum, n_β is the number of beta particles emitted per disintegration, and $S(E)$ is the energy spectrum of the beta particles in particles/MeV. For convenience, Equation (2) can be cast into the same form as Equation (1):

$$R_\beta(x) = A\ n_\beta\ k\ E_{av}\ \Phi_\beta(x) \tag{3}$$

where E_{av}, the mean energy of the spectrum, is defined as

$$E_{av} = \int_0^{E_{max}} E\ \ S(E)\ dE \tag{4}$$

and

$$\Phi_\beta(x) = \int_0^{E_{max}} (E/E_{av})\ S(E)\ \Phi(x,E)\ dE \qquad (5)$$

The first step in determining the scaled absorbed dose distributions for a given isotope is calculating the emitted beta spectra. This is accomplished by a code called BETA, developed at Kansas State University by Richard Faw and Gale Simons which (with minor modifications) is installed as a subroutine in the SADDE code. The BETA code is used with their permission. The following information is needed to calculate the beta spectrum from the decay of a particular isotope:

1. whether to include beta radiations from the daughter isotope (y/n)
2. the number of decay modes for the parent isotope
3. the atomic number of the transition product nucleus
4. the atomic mass of the decaying isotope
5. the endpoint energy of the particular decay path
6. the probability of the particular decay path
7. the degree of forbiddenness for the particular decay
8. If the beta radiation from the daughter isotope is to be included, repeat items 2 through 7 for the daughter.

A compilation of decay schemes, endpoint energies, and degree of forbiddenness was published by Hogan, Zigman, and Macklin (1964). However, improved data on decay schemes has been developed since Hogan, Zigman, and Macklin's work. For updated endpoint energies, beta spectra, and decay schemes, several more recent compilations exist: Kocher (1981); Cross et al. (1982); and Lederer and Shirley (1978). Hogan, Zigman, and Macklin (1964) distinguish only allowed and forbidden transitions (second and third forbidden transitions are calculated the same as first forbidden); the degree of forbiddenness is not specified in the more recent compilations listed above. The forbiddenness of a transition can be determined using the following approach. First, the value of the Fermi integral function $f(Z,W_0)$, must be

determined from Figure 1 (Evans 1955; 1982) based on the maximum energy of the beta particle, E_{max}, and the atomic number, Z, of the decay product. This value of $f(Z,W_0)$ is then multiplied by the half-life, t (in seconds), of the decaying nucleus to obtain the value of ft. The product, ft, which depends only on Z, t, and E_{max}, can be used to determine the degree of forbiddenness. Since the product ft varies between 10 and 10^{18}, the $\log_{10}(ft)$ is used to determine the degree of forbiddenness according to Table 1. (Alternatively, the base-10 logarithm of the half-life can be summed with the base-10 logarithm of the f function to get the base-10 logarithm of the ft product.)

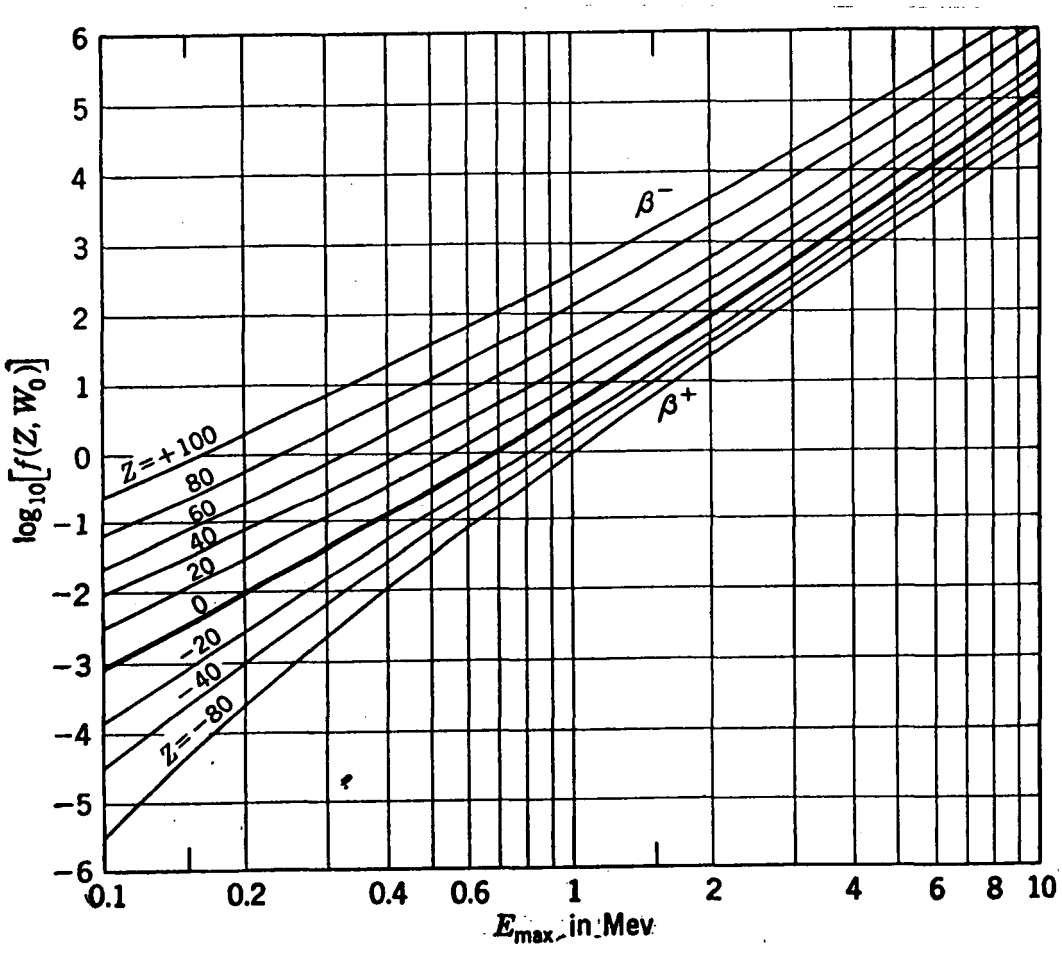

FIGURE 1. The Fermi Integral Function, $f(Z,W_0)$. Z is the atomic number of the decay product and is positive for β^- emitters and negative for β^+ emitters.

TABLE 1. Degree of Forbiddenness as a Function of $\log_{10}(ft)$

$\log_{10}(ft)$	Degree of Forbiddenness
2.7 - 3.7	Favored allowed transition (F = 0)
4 - 5.8	Normal allowed transition (F = 0)
6 - 12	First forbidden transition (F = 1)
12.2 - 13.5	Second forbidden transition (F = 2)
13.7 - 17.6	More highly forbidden transition (F = 3)

For those isotopes that have several decay modes, the half-life of the particular decay can be calculated by using the overall half-life (the half-life listed in the compilations) divided by its probability for decay by the particular path. For example, consider ^{41}Ar which has the decay scheme listed in Table 2. The effective half-life of mode 1 is 6588/0.9910, or 6648 seconds. The effective half-life of mode 2 is 6588/0.0090, or 732,000 seconds. From Figure 1, the $\log_{10}f(Z,W_0)$ of mode 1 is about 1.0, and about 2.5 for mode 2. Adding the base-10 logarithm for half-lives, we get a value of 4.8 for mode 1 (normal allowed transition from Table 1) and a value of 8.4 for mode 2 (first forbidden transition from Table 1).

Unfortunately, the log(ft) value does not tell the whole story and many exceptions to the rules exist. Figure 2 shows the frequency distribution of allowed and first forbidden beta decays among the isotopes listed in the 1963 Nuclear Data Sheets (Gleit, Tang, and Coryell 1963). Note the large overlap of log(ft) values for allowed and first forbidden transitions. To determine correctly the degree of forbiddenness, one must consult a compilation of

TABLE 2. Decay Scheme of ^{41}Ar

Overall half-life: 1.83 h (6588 sec)

Mode	Endpoint Energy	Probability	Forbiddenness
1	1.232	0.9910	0
2	2.515	0.0090	1

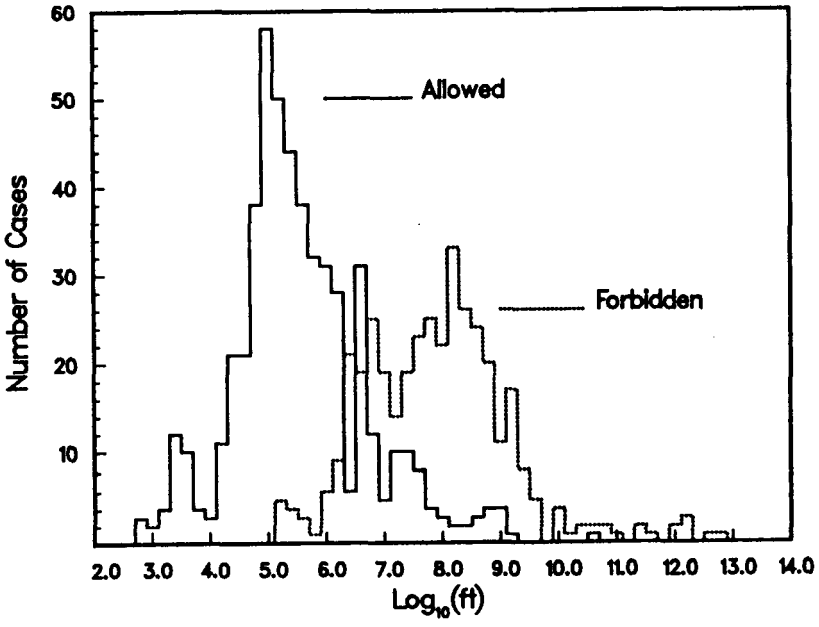

FIGURE 2. Frequency Distribution of Log(ft) Values for Allowed and First-Forbidden Beta Decays

nuclear decay schemes, such as Lederer and Shirley (1978), and find the total angular momentum and parity of the isotope before and after beta decay. The isotope may decay by Gammow-Teller Rules or by Fermi Rules. The rules for two classes of beta decay are summarized in Table 3. The degree of forbiddenness may be found by using the change in angular momentum (ΔI), the parity

TABLE 3. Selection Rules for Beta Decay

Class of Forbiddenness	Parity Change	ΔI		Approximate log (ft)
		Gammow-Teller	Fermi	
Allowed	No	0, ±1 (but not 0 → 0)	0	3 - 6
First	Yes	0, ±1, ±2 (not 0→0, 1/2→1/2)	0, ±1 (not 0 → 0)	6 - 10
Second	No	±2, ±3, 0 → 0 (not 0 ↔ 2)	±1, ±2 (not 0 ↔ 1)	> 10

change, and the log(ft) value. The rule for parity change given in Table 3 is never violated. By use of the log(ft) value, one can distinguish between Fermi or Gammow-Teller decay and, based on parity and angular momentum changes, assign the correct degree of forbiddenness.

For example, Figure 3 shows the decay scheme for ^{60}Co. From the decay scheme, we see that 99% of the time the ^{60}Co nucleus, which has a positive parity and five units of angular momentum, decays to an excited state of ^{60}Ni, which has positive parity and four units of angular momentum. Thus, for this decay there is no parity change and 1 unit of change for the angular momentum. The log(ft) value for this decay is about 7.5. Because there is no parity change, from Table 3 the decay must be either an allowed or a second forbidden transition. Because the log(ft) value is less than 10, the transition must be an allowed Gammow-Teller transition.

Using the input data (items 1 through 7 enumerated previously for parent and daughter, if desired), the routine BETA calculates a normalized beta spectrum, n(E), such that

$$\int_0^{E_{max}} n(E)\, dE = 1 \tag{6}$$

The calculated spectrum is divided into 150 energy bins and then interpolated to 1500 values using a spline interpolation routine. These interpolated values are stored in an array called COMP (see Appendix A for a listing of the FORTRAN programs). The average energy of the composite spectrum is calculated using the following algorithm:

$$EAV = \int_0^{E_{max}} COMP(E)\, E\, dE \tag{7}$$

where E_{max} is the maximum endpoint energy for the composite spectrum.

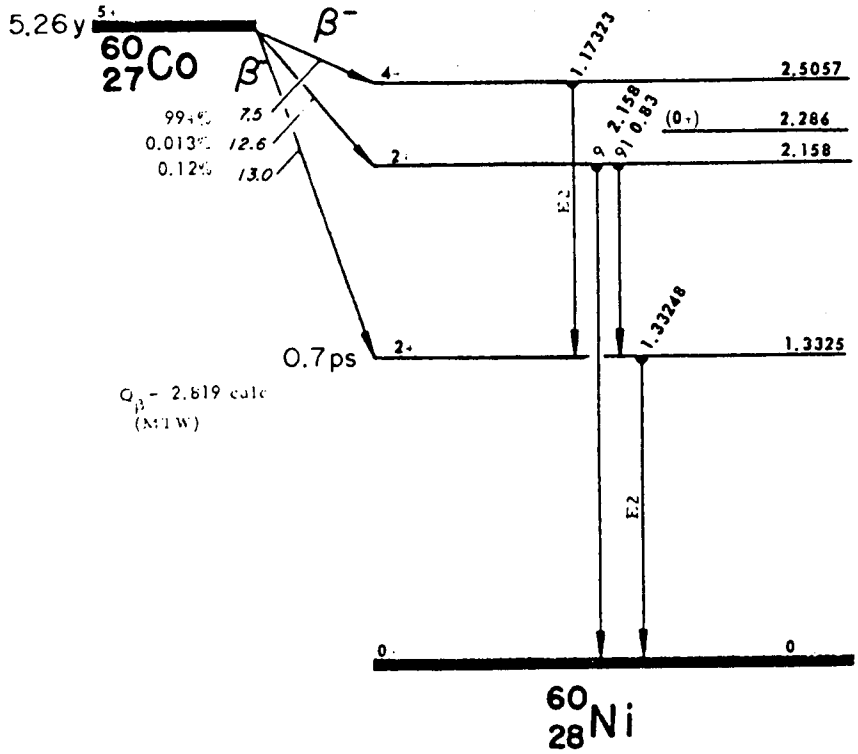

FIGURE 3. Decay Scheme for ^{60}Co

The subroutine SADD uses the average energy and COMP to calculate the scaled absorbed dose function, $\Phi_\beta(x)$. The SADD routine determines the maximum beta range and sets the range step size to 1/150 of this maximum range. The scaled absorbed dose function for the beta spectrum is calculated for each distance step by integrating through energy in steps of 1/150th of the maximum energy using the following equation:

$$\Phi_\beta = \frac{1}{E_{av}} \int_0^{max} E\, n(E)\, \Phi(x,E)\, dE \qquad (8)$$

where $\Phi(x,E)$ is the scaled absorbed dose function for monoenergetic electrons.

The function $\Phi(x,E)$ is determined within the SADD routine by calling a routine named SPENS, an algorithm for interpolating Spencer's data (1959) for scaled absorbed dose distribution as a function of distance and electron

energy through energy and distance (from Berger 1971). When SADD picks an energy (E) and a distance (x), $\Phi(x,E)$ is calculated by the following algorithm:

$$\Phi(x,E) = F(\xi,E)/4\ \pi\ \rho\ x^2\ x_{90}(E) \tag{9}$$

where ξ is $x/x_{90}(E)$ and ρ is the density of the absorbing medium.

$F(\xi,E)$ is a tabulated function of ξ and E that is interpolated to calculate the function value. Output of the routine SADD is the function $\Phi_\beta(x)$, the array PHI in the FORTRAN program, which has 150 values.

The x_{90} distance is calculated using the array PHI and EAV. This is accomplished by summing up the values of dE/dx over distance and arriving at the total energy in the beta spectrum:

$$\frac{dE}{dx} = 4\ \pi\ \rho\ x^2\ E_{av}\ \Phi_\beta(x) \tag{10}$$

therefore,

$$E_{Tot} = \sum_i \Delta x_i\ 4\ \pi\ \rho\ x^2\ E_{av}\ \Phi_\beta(x) \tag{11}$$

Once the total energy in the spectrum is found, a second sum is computed as follows:

$$FRAC = \sum_i \Delta x_i\ 4\ \pi\ \rho\ x^2\ E_{av}\ \Phi_\beta(x)/\ E_{tot} \tag{12}$$

FRAC is evaluated for successively larger values of x until FRAC is 0.9. The current x value becomes the x_{90} value and is stored in the FORTRAN variable X90.

The routine FBETA calculates the function $F_\beta(\xi)$. The routine uses X90 and PHI to perform the following calculation:

$$F_\beta(\xi) = 4\ \pi\ \rho\ x^2\ x_{90}\ \Phi_\beta(x) \tag{13}$$

The values of x are divided by x_{90} and then $F_\beta(\xi)$ is tabulated against the ξ values. The resulting values of $F_\beta(\xi)$ are stored in the array F. The final routine FIT uses linear interpolation to extract the 30 specific values of $F_\beta(\xi)$ needed by VARSKIN and writes these values to the output file.

INSTRUCTIONS FOR USING CODES

If the user has a personal computer (IBM or compatible) with an 8087 coprocessor, the generation of the scaled absorbed dose values can be calculated by running SADDE and answering the questions presented on the screen. To run SADDE, change the default directory to the drive or subdirectory containing the executable code named SADDE.EXE and type "SADDE" (don't type the quote marks). To generate scaled absorbed dose values without an 8087 coprocessor, you must use the SN087 code. Change drives or directories as above, and type "SN087" (don't type quote marks). If you don't have an 8087 coprocessor and try to use SADDE, the error message 'Floating Point Not Loaded' will appear.

Because the input is not printed out in the output files and all computer users are prone to errors, the creation of an input file is highly recommended. The input file simply contains the answers to the questions asked by the SADDE or SN087 codes. The input file can be created using EDLIN, which is included in the DOS package (see the DOS manual for information on creating files), or other word processors. The input file should contain the following information in the order listed:

1. a yes/no (y/n) decision whether to include beta radiations from the daughter isotope
2. the number of decay modes for the parent isotope, the atomic number of the transition product nucleus, and the atomic mass of the decaying isotope
3. the endpoint energy of the particular decay path, the probability of the particular decay path, and the degree of forbiddenness for the particular decay
4. information in step 3 (above) repeated for each decay path.

If the daughter's radiations are to be included, items 2 through 4 must be repeated for the daughter. Figure 4 contains example input files that show the necessary information listed in proper order and syntax.

Input Deck for ^{49}Ca

```
n
6,21.,49.
.530,0.0021,0
.775,0.0063,0
1.196,0.0710,0
1.751,0.0018,1
2.184,0.915,0
2.8964,0.0041,1
```

Input Deck for ^{60}Co

```
n
1  28.  60.
0.31790,1.0,0
```

Input Deck for ^{90}Y

```
n
1  40.  90.
2.2839,0.99988,1
```

Input Deck for ^{106}Rh

```
n
6,46.,106.
1.540,0.00427,0
1.979,0.0192,0
2.407,0.098,0
2.413,0.0058,1
3.029,0.082,0
3.541,0.787,0
```

Input Deck for $^{90}Sr/^{90}Y$

```
y
1, 39., 90.
0.546,1.000,1
1  40.  90.
2.2839,0.99988,1
```

FIGURE 4. Examples of Input Decks for the SADDE code

If the input file is named INPUT, after changing directories or drives as discussed above, run the code by typing "SADDE < INPUT" (don't type quote marks). See the IBM-DOS manual for further information on redirecting standard input.

After successful completion of SADDE or SN087, the output of these codes (the input data for BETADATA.DAT file for VARSKIN) is written to the file OUTPUT. The users should rename the file to an appropriate name (i.e., KR-85) before running any other cases, to prevent the OUTPUT file from being overwritten and the data lost.

The data in the OUTPUT file can be added to the BETADATA.DAT file used by VARSKIN using EDLIN or any other word processor. When adding the isotope, make the first line 4 spaces, followed by the isotope name chosen by the user (up to 7 characters). The 4 lines that follow are the output as generated by SADDE. (The last "isotope" must be 'ENDOFI' so that VARSKIN knows that it has reached the end of file on the BETADATA.DAT file.) The BETADATA.DAT file can contain up to 100 isotopes.

VERIFICATION

The beta spectra produced by the BETA routine were checked extensively against published spectra and agreed to within the accuracy that the values could be read from the graphs of the spectra. To ensure that the programs generated scaled absorbed dose distributions correctly, many of the isotopes listed in Berger (1971) were run to regenerate the published values. Using the decay scheme values given in Kocher (1981), the scaled absorbed dose distributions, x_{90} values, average energies, and yields were calculated. These values were then used in VARSKIN to calculate dose profiles from point particles on the skin. This also checks the doses calculated by disks since the doses along the entire beta range are calculated. The results of these calculations are shown in the tables in Appendix B. The results agree to within the error of the VARSKIN calculation (about 5%).

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APPENDIX A

LISTING OF THE COMPUTER CODE SADDE

```
$NOFLOATCALLS
$DEBUG
      program sadde
      dimension eb(150),spec(150),comp(1500),x(1000),phi(1000)
      real xx90(1000),f(1000),b(150),c(150),d(150)
      dimension xr(30),fit(30),temp(150)
      character*1 answ
      character*80 deck1(20),deck2(20),dtemp
c
      data xr/0.,.01,.02,.03,.04,.05,.10,.15,.20,.25,.30,.35,.40,.45,
     1  .50,.55,.60,.65,.70,.75,.80,.85,.90,.95,1.0,1.1,1.2,1.4,1.6,1.8/
c
      n = 150
      iwt = 1
      nz = 8
      byield = 0.
      by = 0.
      by1 = 0.
      emax = 0.
      emax1 = 0.
      write(*,*)' are daughter isotopes to be included? [y/n]'
      read(*,'(a)')answ
      write(*,*)' enter number of decay paths, atomic number of the'
      write(*,*)' transition nucleus, and atomic weight'
      read(*,*)mod,z,a
      write(deck1(1),'(i2,f7.1,f9.3,3i6)')mod,z,a,n,iwt,nz
      do 50 i = 2,mod+1
      write(*,*)'  enter endpoint energy, yield, and forbiddenness'
           read(*,*)em,yld,nf
           byield = byield + yld
           if(em.gt.emax)emax=em
           write(deck1(i),'(2f8.4,i6)')em,yld,nf
   50 continue
      if(answ.eq.'y'.or.answ.eq.'Y')then
           by = byield
      write(*,*)' enter number of decay paths, atomic number of the'
      write(*,*)' transition nucleus, and atomic weight'
           read(*,*)mod1,z1,a1
           write(deck2(1),'(i2,f7.1,f9.3,3i6)')mod1,z1,a1,n,iwt,nz
           do 51 i = 2,mod1+1
      write(*,*)'  enter endpoint energy, yield, and forbiddenness'
                read(*,*)em,yld,nf
                by1 = by1 + yld
                if(em.gt.emax1)emax1=em
                write(deck2(i),'(2f8.4,i6)')em,yld,nf
   51      continue
           byield = byield + by1
           if(emax.gt.emax1)then
c alter deck2 to allow for the higher energy endpoint of the parent
                mod1 = mod1+1
      write(deck2(1),'(i2,f7.1,f9.3,3i6)')mod1,z1,a1,n,iwt,nz
                do 55 i=mod1+1,3,-1
```

```
                deck2(i)=deck2(i-1)
   55           continue
                write(deck2(2),'(2f8.4,i6)')emax,0.0,0
           else
c daughter beta has higher endpoint energy
                mod = mod + 1
      write(deck1(1),'(i2,f7.1,f9.3,3i6)')mod,z,a,n,iwt,nz
                do 56 i=mod+1,3,-1
                deck1(i)=deck1(i-1)
   56           continue
                write(deck1(2),'(2f8.4,i6)')emax1,0.0,0
           endif
c
c rank decks in order of decreasing endpoint energies
c
      if(mod.ge.2)then
 561  nswap = 0
      do 562 i = 2,mod
      read(deck1(i),'(2f8.4,i6)')e1,y1,i1
      read(deck1(i+1),'(2f8.4,i6)')e2,y2,i2
           if(e1.lt.e2)then
           dtemp = deck1(i+1)
           deck1(i+1) = deck1(i)
           deck1(i) = dtemp
           nswap = 1
           endif
 562  continue
      if(nswap.ne.0)goto561
      endif
c
      if(mod1.ge.2)then
 563  nswap = 0
      do 564 i = 2,mod1
      read(deck2(i),'(2f8.4,i6)')e1,y1,i1
      read(deck2(i+1),'(2f8.4,i6)')e2,y2,i2
           if(e1.lt.e2)then
           dtemp = deck2(i+1)
           deck2(i+1) = deck2(i)
           deck2(i) = dtemp
           nswap = 1
           endif
 564  continue
      if(nswap.ne.0)goto563
      endif
c
      call beta(eb,spec,deck1)
      call beta(eb,temp,deck2)
           do 57 i = 1,n
           spec(i) = (by*spec(i)+by1*temp(i))/byield
   57      continue
      else
c
```

```
c reorder deck1
c
      if(mod.ge.2)then
 571  nswap = 0
      do 572 i = 2,mod
      read(deck1(i),'(2f8.4,i6)')e1,y1,i1
      read(deck1(i+1),'(2f8.4,i6)')e2,y2,i2
           if(e1.lt.e2)then
           dtemp = deck1(i+1)
           deck1(i+1) = deck1(i)
           deck1(i) = dtemp
           nswap = 1
           endif
 572  continue
      if(nswap.ne.0)goto571
      endif
c
      call beta(eb,spec,deck1)
      endif
c
      write(*,*)' generating beta spectrum'
c
c to save beta spectrum for verification
c
c     open(4,file='spectrum',status='new')
c     do 199 i=1,150
c     write(4,*)eb(i),spec(i)
c 199 continue
c     close(4)
c
      m = 149
c
  200 call spline(m,eb,spec,b,c,d)
c
      do 250 i = 1,1500
      comp(i) = 0.0
  250 continue
c
      emax = amax1(emax,emax1)
      de = emax/1500.
      eav = 0.0
c
      do 300 i=1,1500
           a=de*(float(i)-0.5)
           ans=slvsp(m,a,eb,spec,b,c,d)
      if (ans.lt.0.)then
           ans=0.
      else
           eav = eav + ans*a*de
      endif
  300 comp(i) = ans
c
```

```
  400 write(*,*)' starting to generate distribution functions'
c
      iter = 150
c
      call sadd(iter,comp,emax,eav,x,phi)
c
      etot=0.
c
      delx = x(iter)/float(iter)
c
      do 500 i=1,iter
           etot=etot+phi(i)*x(i)*x(i)
  500 continue
      etot = etot*eav*4.*3.14159*delx
c
      sfrac=0.
c
      do 600 i=1,iter
      sfrac=sfrac+eav*phi(i)*4.*3.1415*x(i)*x(i)*delx/etot
      if (sfrac.ge.0.9) goto 610
  600 continue
c
  610 x90 = x(i)
c
c
      do 700 i=1,iter
      f(i) = 4.*3.14159265*x(i)*x(i)*x90*phi(i)
      xx90(i) = x(i)/x90
  700 continue
c
      i=1
      j=1
c
  705 if (xr(i).lt.xx90(1)) then
      fit(i)=f(1)+(f(2)-f(1))*(xr(i)-xx90(1))/(xx90(2)-xx90(1))
      i=i+1
      goto 705
      endif
c
  710 if (xx90(j).eq.xr(i)) then
      fit(i)=f(j)
      i=i+1
      j=j+1
      goto 800
      endif
      if (xx90(j).gt.xr(i)) then
      fit(i)=f(j-1)+(f(j)-f(j-1))*(xr(i)-xx90(j-1))/(xx90(j)-xx90(j-1))
      i=i+1
      else
      j=j+1
      endif
  800 if (i.le.30) goto 710
```

```
c
      open(3,file = 'OUTPUT',status= 'NEW')
      write(3,900)eav,x90,byield
      write(*,*)' ave energy is ...',eav
      write(*,*)' x90 distance is..',x90
      write(*,*)' yield is ........',byield
      write(*,901)(fit(j),j=1,30)
      write(3,901)(fit(j),j=1,30)
  900 format(4x,3f8.4)
  901 format(4x,10f7.4,/,4x,10f7.4,/,4x,10f7.4)
      close(unit = 3)
      stop
      end
c ***********************************************************
      SUBROUTINE BETA(EB1,SPEC1,INPUT)

C ***********************************************************************
C
C This program generates beta-particle energy spectra based on the
C following input data:
C
C     MOD  = NUMBER OF MODES OF DECAY
C     EMAX = ENDPOINT ENERGY FOR BETA SPECTRA (MeV)
C     YLD  = YIELD (MODE PROBABILITY PER DISINTEGRATION
C     Z    = ATOMIC NUMBER OF RADIONUCLIDE
C     A    = ATOMIC MASS NUMBER OF RADIONUCLIDE
C     NF   = STATE OF FORBIDEDNESS (0,1,2,3)
C     NZ   = FACTOR N IN EXPRESSION ZETA = (1/2)**(1/NZ).  SUCCESSIVE
C            ENERGIES (LOG SPACING) VARY BY FACTOR ZETA. (SUGGEST NZ=6)
C     N    = NUMBER OF STEPS AT WHICH SPECTRUM IS EVALUATED
C     IWT  = 1 FOR EQUAL ENERGY SPACING, 2 FOR LOG SPACING
C
C
C The following auxiliary subprograms are required:
C
C     SUBROUTINES SPECT, NORMAL, INTWT
C     FUNCTIONS   GAMMA, FERMI,  SHAPE
C
C Output is in two columns:
C
C    1. Energy in units of mc^2
C    2. Energy spectrum in units of (mc^2)^-1
C
C Input data (NON-FORMATTED)
C
C     CARD 1:  MOD,Z,A,N,IWT,NZ
C     CARD 2:  EMAX,YLD,NF
C     CARD 3:                Additional cards if MOD > 1, in
C                            order of decreasing EMAX.
C
C ***********************************************************************
```

```
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /VALS/E(300),Y(300)
      COMMON /CONST/EMC2,PI,ALPH,VO,WO,Z,A,NF,N,EMAX
      COMMON /CMN1/IWT,EAVG
      DIMENSION SPEC(300)
      REAL*4 SPEC1(300),EB1(300)
      CHARACTER*80 INPUT(20)

C ***************************************************************************

   20 FORMAT(2E12.4)

C ***************************** READ FIRST TWO DATA CARDS

      READ(INPUT(1),'(I2,F7.1,F9.3,3I6)')MOD,Z,A,N,IWT,NZ
      READ(INPUT(2),'(2F8.4,I6)') EMAX,YLD,NF
      write(*,*)emax,yld,nf
C ***************************** INITIALIZE SPECTRUM

      DO 30 I=1,N
  30  SPEC(I) = 0.D0

C ***************************** INITIALIZE CONSTANTS

      IF (NZ.LT.3) NZ = 3
      ZN = REAL(NZ)
      ZT = (0.5D0)**(1.D0/ZN)
      PI = DACOS(-1.D0)
      EMC2 = .5110034D0
      ALPH=0.0072973503D0
      VO = 1.13D0*(ALPH**2)*(Z**(4.D0/3.D0))
      WO = EMAX/EMC2 + 1.D0

C ***************************** SET UP LOG ENERGY SCALE

      IF (IWT.EQ.2) THEN
   15    U = EMAX*ZT**(N-1)/EMC2
         IF (U.LT.VO) THEN
           N = N-1
           GO TO 15
         ENDIF
         NTOT = N
         DO 10 I = 1,N
         E(I) = EMAX*ZT**(N-I)/EMC2
   10 CONTINUE
      ENDIF

C ***************************** SET UP LINEAR ENERGY SCALE

      IF (IWT.EQ.1) THEN
         NTOT = N
         E(N) = EMAX/EMC2
```

```
        E(1) = V0
        CALL INTWT(1,N,E,Y)
      ENDIF

C ************************** EVALUATE NORMALIZED SPECTRUM

C ************************** CALCULATIONS FOR FIRST MODE

      CALL SPECT
      CALL NORMAL
      DO 31 I=1,N
      SPEC(I) = Y(I)*YLD
   31 CONTINUE
      IF(MOD.EQ.1) GO TO 41

C ************************** CALCULATIONS FOR ADDITIONAL MODES

      DO 40 J=2,MOD
      READ(INPUT(J+1),'(2F8.4,I6)') EMAX,YLD,NF
       write(*,*)emax,yld,nf
      W0 = EMAX/EMC2 + 1.D0
   50 IF(E(N).GT.EMAX/EMC2) THEN
        N = N-1
        GO TO 50
      ENDIF
      CALL SPECT
      CALL NORMAL
      DO 60 K = 1,N
      SPEC(K) = SPEC(K) + Y(K)*YLD
   60 CONTINUE
   40 CONTINUE
   41 IF(IWT.EQ.1) NSTRT = 2
      IF(IWT.EQ.2) NSTRT = 1
   80 DO 12 I=NSTRT,NTOT
      EB1(I-1)=E(I)*EMC2
      SPEC1(I-1)=SPEC(I)/EMC2
   12 CONTINUE
      RETURN
      END

C ********************************************************************************

      SUBROUTINE SPECT

      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /VALS/E(300),Y(300)
      COMMON /CONST/EMC2,PI,ALPH,V0,W0,Z,A,NF,N,EMAX
      DO 15 I=1,N
      W=E(I)+1.D0
      IF(E(I).GT.V0) THEN
      Y(I) = FERMI(W-V0)*(((W-V0)**2-1.D0)**0.5D0)*(W-V0)*((W0-W)**2)
      Y(I) = Y(I)*SHAPE(W)
```

```
      ELSE
      Y(I) = 0.D0
      ENDIF
   15 CONTINUE
      RETURN
      END

C *********************************************************************

      SUBROUTINE NORMAL

      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /VALS/E(300),Y(300)
      COMMON /CONST/EMC2,PI,ALPH,V0,W0,Z,A,NF,N,EMAX
      COMMON /CMN1/IWT,EAVG
      DIMENSION V(300)
      CALL INTWT(IWT,N,E,V)
      FF = 0.D0
      GG = 0.D0
      DO 35 I=1,N
      FF = FF + Y(I)*V(I)
      GG = GG + Y(I)*V(I)*E(I)
   35 CONTINUE
      IF(IWT.EQ.2) THEN
        FF = FF + Y(1)*(E(1)-V0)/2.D0
        GG = GG + Y(1)*E(1)*(E(1)-V0)/2.D0
      ENDIF
        FF = FF + Y(N)*(EMAX/EMC2-E(N))/2.D0
        GG = GG + Y(N)*E(N)*(EMAX/EMC2-E(N))/2.D0
      DO 40 I = 1,N
      Y(I) = Y(I)/FF
   40 CONTINUE
      EAVG = EMC2*GG/FF
      RETURN
      END

C *********************************************************************

      DOUBLE PRECISION FUNCTION GAMMA(X)

      IMPLICIT REAL*8(A-H,O-Z)
      B1 =-.57719652D0
      B2 = .988205891D0
      B3 =-.897056937D0
      B4 = .918206857D0
      B5 =-.756704078D0
      B6 = .482199394D0
      B7 =-.19352781 8D0
      B8 = .035868343D0
      Z  = 1.D0
   10 IF(X.GE.2.D0) THEN
        Z = Z*(X-1.D0)
```

```
        X = X-1.D0
        GOTO 10
      ENDIF
      Y = X-1.D0
      Y2=Y*Y
      Y4=Y2*Y2
      GAMMA=Z*(1.D0+B1*Y+B2*Y2+B3*Y*Y2+B4*Y4+B5*Y*Y4+B6*Y4*Y2+B7*Y4*Y2*Y
     &+B8*Y4*Y4)
      END

C *********************************************************************

      DOUBLE PRECISION FUNCTION FERMI(W)

*     Revised 2 Jul 86
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /CONST/EMC2,PI,ALPH,V0,W0,Z,A,NF,N,EMAX
      S=(1.D0-(ALPH*Z)**2)**0.5D0-1.D0
      R=((1.123D0*A**(1.D0/3.D0))-(0.941D0*A**(-1.D0/3.D0)))
      P=(1.D0+S/2.D0)/((ABS(GAMMA(3.D0+2.D0*S)))**2)
      B=R**(2.D0*S)
      B1=(ALPH*Z)**2+0.25D0
      V=(ALPH*Z*W)/((W**2-1.D0)**0.5D0)
      G = 2.D0*V*PI
      IF(XP.LT.20.0) G = G/(1.D0-DEXP(-2.D0*V*PI))
      D=G*(((B1*(W**2))-0.25D0)**S)
      FERMI = P*B*D
      END

C *********************************************************************

      DOUBLE PRECISION FUNCTION SHAPE(W)

      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /CONST/EMC2,PI,ALPH,V0,W0,Z,A,NF,N,EMAX
      GOTO(1,2,3)NF
      GOTO 10
    3 SN=(W**2-1.D0)**3+(W0-W)**6+7.D0*(W**2-1.D0)*((W0-W)**2)
     &*((W**2-1.D0)+(W0-W)**2)
      GO TO 14
    2 SN=(W**2-1.D0)**2+(W0-W)**4+(10.D0/3.D0)*(W**2-1.D0)*((W0-W
     &)**2)
      GO TO 14
    1 SN=(W**2-1.D0)+(W0-W)**2
      GO TO 14
   10 SN=1.D00
   14 SHAPE = SN
      END

C *********************************************************************

      SUBROUTINE INTWT(IWT,NWT,WTAB,WATES)
```

```
C     IWT=1,EQUALLY SPACED POINTS
C     IWT=2,SUCCESSIVELY MULTIPLICATED SPACED POINTS
C     NWT----NUMBER OF ENTRIES
C     WTAB---ABSCISSA LIST FOR WHICH INTEGRATION WEIGHTS ARE DETERMINED
C     WATES--LIST OF INTEGRATION WEIGHTS TO BE DETERMINED
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION WTAB(300),WATES(300)
      WTA=NWT
      WTAB1 = WTAB(1)
      WTABN = WTAB(NWT)
      IF(NWT-2)10,20,20
   10 WATES(1)=0.D0
      GO TO 199
   20 IF(IWT-2)30,40,40
   30 WTDEL=DABS(WTABN-WTAB1)/(WTA-1.D0)
      GO TO 50
   40 WTDEL=DABS(DLOG(WTABN/WTAB1))/(WTA-1.D0)
      WTAB1=DLOG(WTAB1)
      DO 48 I=2,NWT
      WTAB(I)=DEXP(WTAB1+(I-1)*WTDEL)
   48 CONTINUE
      GO TO 60
   50 DO 55 I=1,NWT
      WTAB(I)=WTAB1+(I-1)*WTDEL
   55 CONTINUE
   60 NWTA=(WTA/2.+.1)
      NWTB=(WTA/2.-.1)
      NWTC=(WTA/4.+.1)
      NWTD=(WTA/4.-.1)
      IF (NWT-2) 199,65,70
   65 WATES(1) = WTDEL/2.D0
      WATES(2) = WATES(1)
      GO TO 199
   70 WATES(1)=WTDEL/3.D0
      WTC=WATES(1)
      WATES(NWT)=WATES(1)
      DO 80 I=1,NWTB
      WATES(I+1)=WTDEL+WTC
      INDX=NWT-I
      WATES(INDX)=WTDEL+WTC
      WTC=-WTC
   80 CONTINUE
      WTD=1.D0/24.D0
      IF(NWTC-NWTD)100,100,90
   90 WTD=-WTD
  100 IF(NWTA-NWTB)140,140,110
  110 WATES(NWTB)=WATES(NWTB)-WTD*WTDEL
      WATES(NWTB+1)=WATES(NWTB+1)+5.D0*WTD*WTDEL
      WATES(NWTB+3)=WATES(NWTB)
      WATES(NWTB+2)=WATES(NWTB+1)
  140 IF (IWT.EQ.2) THEN
```

```
        DO 160 I = 1,NWT
        WATES(I) = WATES(I)*WTAB(I)
  160   CONTINUE
      ENDIF
  199 RETURN
      END
C
C*******************************************************************************
      SUBROUTINE SPLINE(M,X,Y,B,C,D)
        INTEGER N,M
        REAL X(M),Y(M),B(M),C(M),D(M)
C
        N=M
        NM1=N-1
        IF(N.LT.2) RETURN
        IF(N.LT.3) GO TO 50
C
        D(1)=X(2)-X(1)
        C(2)=(Y(2)-Y(1))/D(1)
        DO 10 I=2,NM1
           D(I)=X(I+1)-X(I)
           B(I)=2.*(D(I-1)+D(I))
           C(I+1)=(Y(I+1)-Y(I))/D(I)
           C(I)=C(I+1)-C(I)
10      CONTINUE
C
        B(1)=-1.*D(1)
        B(N)=-1.*D(N-1)
        C(1)= 0.
        C(N)= 0.
        IF (N.EQ.3) GO TO 15
        C(1)=C(3)/(X(4)-X(2)) - C(2)/(X(3)-X(1))
        C(N)=C(N-1)/(X(N)-X(N-2)) - C(N-2)/(X(N-1)-X(N-3))
        C(1)=C(1)*D(1)**2/(X(4)-X(1))
        C(N)=-C(N)*D(N-1)**2/(X(N)-X(N-3))
C
 15     DO 20 I=2,N
           T=D(I-1)/B(I-1)
           B(I)=B(I)-T*D(I-1)
           C(I)=C(I)-T*C(I-1)
20      CONTINUE
C
        C(N)=C(N)/B(N)
        DO 30 IB=1,NM1
           I=N-IB
           C(I)=(C(I)-D(I)*C(I+1))/B(I)
30      CONTINUE
C
        B(N)=(Y(N)-Y(NM1))/D(NM1)+D(NM1)*(C(NM1)+2.*C(N))
        DO 40 I=1,NM1
           B(I)=(Y(I+1)-Y(I))/D(I)-D(I)*(C(I+1)+2.*C(I))
           D(I)=(C(I+1)-C(I))/D(I)
```

```
            C(I)=3.*C(I)
40        CONTINUE
            C(N)=3.*C(N)
            D(N)=D(N-1)
          RETURN
C
50        B(I)=(Y(2)-Y(1))/(X(2)-X(1))
          C(1)=0.
          D(1)=0.
          B(2)=B(1)
          C(2)=0.
          D(2)=0.
          RETURN
          END
C
C ***********************************************************
C
          REAL FUNCTION SLVSP(M,A,X,Y,B,C,D)
          INTEGER N,M
          REAL A,X(M),Y(M),B(M),C(M),D(M)
C
C  THIS SUBROUTINE EVALUATES THE CUBIC SPLINE FUNCTION
C
          INTEGER I,J,K
          REAL DX
          DATA I/1/
           N=M
          IF(I.GE.N)I=1
          IF(A.LT.X(I)) GO TO 10
          IF(A.LE.X(I+1)) GO TO 30
C
10        I=1
          J=N+1
20        K=(I+J)/2
          IF(A.LT.X(K))J=K
          IF(A.GE.X(K))I=K
          IF(J.GT.I+1) GO TO 20
C
30        DX=A-X(I)
          SLVSP=Y(I)+DX*(B(I)+DX*(C(I)+DX*D(I)))
          RETURN
          END
```

```
$NOFLOATCALLS
$DEBUG
      SUBROUTINE SADD(ITER,RN,EMAX,EAV,X,PHI)
c
c NOV. 1985
c Written by SD MILLER to calculate scaled absorbed-dose
c  distributions for beta spectra.
c
c Modified Dec. 1987 by SD MILLER to work with betacomp.f
c  routine for complex beta emitters.
c
c Modified by WD Reece June 1988 to work with any endpoint energy
c less than 8 MeV
c
      dimension rn(1500),temp(1000),X(1000),PHI(1000)
      dimension range(40),energy(40)
      logical pos
c
      data energy /.010,.015,.020,.025,.030,.035,.040,.045,
     1  .050,.055,.060,.065,.070,.075,.080,.085,.090,.095,.10,
     2  .15,.20,.25,.30,.35,.40,.45,.50,.60,.70,.80,.90,1.00,
     3  1.5,2.0,3.0,4.0,5.0,6.0,7.0,8.0/
c
      data range /.00024,.00050,.00083,.00124,.00171,.00225,
     1  .00285,.00351,.00422,.00499,.00580,.00667,.00759,.00855,
     2  .00956,.0106,.0117,.0128,.0140,.0276,.0440,.0625,.0826,
     3  .104,.126,.150,.174,.223,.273,.325,.377,.430,.696,.961,
     4  1.49,2.00,2.50,2.99,3.47,3.95/
c
c Compute maximum electron range based on maximum beta energy
c
      jj=1
c
5     if (emax.lt.energy(1)) then
        xmax=emax*range(1)/energy(1)
        goto 100
      endif
c
10    if (energy(jj).eq.emax) then
          xmax=range(jj)
          jj=jj+1
          goto 100
      endif
      if (energy(jj).gt.emax) then
         xmax=range(jj-1)+(range(jj)-range(jj-1))
     1       *(emax-energy(jj-1))/(energy(jj)-energy(jj-1))
          goto 100
      else
        jj=jj+1
      endif
      goto 10
```

```
100   continue
c
      delx = xmax/float(iter)
      dele = emax/float(iter)
c
c step through each energy in the beta spectra until EMAX.
c
        do 199 i=1,iter
199        temp(i) = 0.
c
c pick an energy
c
      write(*,*)
c
      do 299 k=1,iter
c
      e = dele*(float(k)-0.5)
      prob = 0.
           do 298 m = 1,10
           prob = prob + rn( (k-1)*10 + m)
  298      continue
      prob = 0.1*prob
c
c normalize the phi function before use
c
      write(*,398)k,iter
  398 format('+ on iteration',i4,' of',i4)
        sum = 0.
c
      do 399 i=1,iter
      xs= float(i)*delx
c
      imax = i
      adder = spens(e,xs)
      if(adder.eq.0.0)then
      goto400
      else
c
      sum = sum + adder*delx*3.14159*4.*xs*xs
      endif
c
399   continue
c
  400 if (sum.ne.0.) then
      do 499 i=1,imax
      xs = float(i)*delx
        temp(i) = temp(i) + dele*spens(e,xs)*prob*e/sum
499   continue
      endif
c
299   continue
c
```

```
        do 599 i=1,iter
          x(i) = float(i)*delx
          phi(i) = temp(i)/eav
599     continue
c
c
      return
      stop
      end
c
c ***********************************************************
c
      REAL FUNCTION SPENS(ENERGY,DISTANCE)
C
C THIS FUNCTION CALCULATES PHI OF A MONOENERGETIC ELECTRON USING DATA
C FROM THE SPENSER TABLES. PHI IS THE FRACTION OF THE TOTAL ENERGY
C DEPOSITED AT "DISTANCE" PER UNIT MASS.  PHI IS CALCULATED KNOWING THE
C INITIAL ELECTRON ENERGY AND THE DISTANCE FROM THE SOURCE TO THE DOSE
C POINT.
C WRITTEN BY WD REECE DEC 1987
C
      DIMENSION X90(9),E(9),FX90(13),F(13,9)
      DIMENSION ELOW(5),X90LOW(5),EUP(15),X90T(15)
      DIMENSION B(13,9),C(13,9),D(13,9),B1(15),C1(15),D1(15)
C
      COMMON /COMSPLI/ B,C,D,B1,C1,D1
C
      DATA E/4.,2.,1.,0.7,0.4,0.2,0.1,0.05,0.025/
      DATA X90/1.57,.74,.328,.207,.096,.0334,.0106,.00318,.00097/
      DATA EUP/0.,0.004,0.006,0.008,0.010,0.02,0.025,0.05,.1,
     &.2,.4,.7,1.,2.,4./
      DATA X90T/0.,0.000042,0.00008,0.000130,0.000194,0.00064,
     &0.00097,0.00318,0.0106,0.0334,0.096,.207,.328,.74,1.57/
C
      DATA FX90/0.,.1,.2,.3,.4,.5,.6,.7,.8,.9,1.,1.1,1.15/
      DATA F/0.75,.75,.76,.78,.80,.83,.89,.99,1.09,1.13,.97,.45,.12,
     &0.68,.69,.71,.74,.78,.84,0.92,1.04,1.16,1.20,1.00,.47,.18,
     &0.61,.62,.65,.70,.76,.85,0.95,1.08,1.22,1.29,1.03,.49,.20,
     &0.57,.59,.62,.68,.75,.85,0.96,1.10,1.25,1.31,1.04,.50,.21,
     &0.52,.54,.58,.65,.73,.84,0.98,1.13,1.30,1.37,1.06,.51,.22,
     &0.48,.50,.55,.62,.72,.84,1.00,1.20,1.35,1.39,1.09,.51,.22,
     &0.45,.47,.53,.61,.71,.85,1.01,1.19,1.37,1.42,1.11,.50,.22,
     &0.43,.45,.51,.59,.71,.86,1.03,1.21,1.37,1.41,1.13,.49,.20,
     &0.43,.46,.52,.61,.73,.87,1.04,1.22,1.35,1.40,1.14,.46,.17/
C
      DATA ELOW/0.020,0.010,0.008,0.006,0.004/
      DATA X90LOW/0.00064,0.000194,0.000130,0.00008,0.000042/
      DATA IFIRST/0/
C
      IF(IFIRST.EQ.0)THEN
      DO 100 J = 1,9
100   CALL SPLIN2D(13,J,FX90,F,B,C,D)
```

```
      CALL SPLINE(15,EUP,X90T,B1,C1,D1)
      IFIRST = 1
      ENDIF
C
C FIND FRACTION OF X90 DISTANCE
C
      IF (ENERGY.LT.0.004) THEN
      SPENS=0.0
      RETURN
      ENDIF
C
      DO 10 I=2,9
      IF(ENERGY.GE.E(I))GOTO15
10    CONTINUE
C
C  ENERGY IS LESS THAN 25 KEV
C
      DS = SLVSP(15,ENERGY,EUP,X90T,B1,C1,D1)
      FRACTION=DISTANCE/DS
      DX90=1.0/DS
      FACT = DX90/FRACTION
      IF(FRACTION.GE.1.17)THEN
      SPENS = 0.
      RETURN
      ENDIF
      IF(FRACTION.LT.1.15)THEN
      SPENS = SLVSP2D(13,9,FRACTION,FX90,F,B,C,D)
      SPENS = 0.25*SPENS*DX90*FACT*FACT/3.14159
      RETURN
      ELSE
      SPENS = (1.17-FRACTION)*F(13,9)*50.
      SPENS = 0.25*SPENS*DX90*FACT*FACT/3.14159
      RETURN
      ENDIF
C
15    DELE=(ENERGY-E(I-1))/(E(I)-E(I-1))
      DS = SLVSP(15,ENERGY,EUP,X90T,B1,C1,D1)
      FRACTION=DISTANCE/DS
      DX90=1.0/DS
      FACT = DX90/FRACTION
C
16    IF(FRACTION.GE.1.17)THEN
      SPENS=0.
      RETURN
      ELSE
      IF(FRACTION.GE. 1.15  )THEN
C
C FRACTION IS BETWEEN 1.15 AND 1.17
C
      SPENS=(F(13,I-1)+DELE*(F(13,I)-F(13,I-1)))*(1.17-FRACTION)*50.
      SPENS=SPENS/(4.0*3.14159)
      SPENS=SPENS*DX90*FACT*FACT
```

```
      RETURN
      ENDIF
C
21    SPENS=(1.-DELE)*SLVSP2D(13,I-1,FRACTION,FX90,F,B,C,D)
     & +           DELE*SLVSP2D(13,I,FRACTION,FX90,F,B,C,D)
      SPENS=SPENS*DX90*FACT*FACT/(4.0*3.14159)
C
      RETURN
      ENDIF
      END
C*************************************************************************
      SUBROUTINE SPLIN2D(M,IE,X,Y,B,C,D)
      INTEGER N,M
      REAL X(13),Y(13,9),B(13,9),C(13,9),D(13,9)
      INTEGER NM1,IB,I
      REAL T
C
      N=M
      NM1=N-1
      IF(N.LT.2) RETURN
      IF(N.LT.3) GO TO 50
C
      D(1,IE)=X(2)-X(1)
      C(2,IE)=(Y(2,IE)-Y(1,IE))/D(1,IE)
      DO 10 I=2,NM1
         D(I,IE)=X(I+1)-X(I)
         B(I,IE)=2.*(D(I-1,IE)+D(I,IE))
         C(I+1,IE)=(Y(I+1,IE)-Y(I,IE))/D(I,IE)
         C(I,IE)=C(I+1,IE)-C(I,IE)
10    CONTINUE
C
      B(1,IE)=-1.*D(1,IE)
      B(N,IE)=-1.*D(N-1,IE)
      C(1,IE)= 0.
      C(N,IE)= 0.
      IF (N.EQ.3) GO TO 15
      C(1,IE)=C(3,IE)/(X(4)-X(2)) - C(2,IE)/(X(3)-X(1))
      C(N,IE)=C(N-1,IE)/(X(N)-X(N-2)) - C(N-2,IE)/(X(N-1)-X(N-3))
      C(1,IE)=C(1,IE)*D(1,IE)**2/(X(4)-X(1))
      C(N,IE)=-C(N,IE)*D(N-1,IE)**2/(X(N)-X(N-3))
C
15    DO 20 I=2,N
         T=D(I-1,IE)/B(I-1,IE)
         B(I,IE)=B(I,IE)-T*D(I-1,IE)
         C(I,IE)=C(I,IE)-T*C(I-1,IE)
20    CONTINUE
C
      C(N,IE)=C(N,IE)/B(N,IE)
      DO 30 IB=1,NM1
         I=N-IB
         C(I,IE)=(C(I,IE)-D(I,IE)*C(I+1,IE))/B(I,IE)
30    CONTINUE
```

```
C
      B(N,IE)=(Y(N,IE)-Y(NM1,IE))/D(NM1,IE)+D(NM1,IE)*(C(NM1,IE)+
     &          2.*C(N,IE))
      DO 40 I=1,NM1
         B(I,IE)=(Y(I+1,IE)-Y(I,IE))/D(I,IE)-D(I,IE)*(C(I+1,IE)+
     &             2.*C(I,IE))
         D(I,IE)=(C(I+1,IE)-C(I,IE))/D(I,IE)
         C(I,IE)=3.*C(I,IE)
40    CONTINUE
         C(N,IE)=3.*C(N,IE)
         D(N,IE)=D(N-1,IE)
      RETURN
C
50    B(I,IE)=(Y(2,IE)-Y(1,IE))/(X(2)-X(1))
      C(1,IE)=0.
      D(1,IE)=0.
      B(2,IE)=B(1,IE)
      C(2,IE)=0.
      D(2,IE)=0.
      RETURN
      END
C
C ***********************************************************
C
      REAL FUNCTION SLVSP2D(M,IE,U,X,Y,B,C,D)
      INTEGER N,M
      REAL U,X(13),Y(13,9),B(13,9),C(13,9),D(13,9)
C
C  THIS SUBROUTINE EVALUATES THE CUBIC SPLINE FUNCTION
C
      INTEGER I,J,K
      REAL DX
      DATA I/1/
      N=M
      IF(I.GE.N)I=1
      IF(U.LT.X(I)) GO TO 10
      IF(U.LE.X(I+1)) GO TO 30
C
10      I=1
        J=N+1
20      K=(I+J)/2
        IF(U.LT.X(K))J=K
        IF(U.GE.X(K))I=K
        IF(J.GT.I+1) GO TO 20
C
C  EVALUATE SPLINE
C
30      DX=U-X(I)
        SLVSP2D=Y(I,IE)+DX*(B(I,IE)+DX*(C(I,IE)+DX*D(I,IE)))
        RETURN
        END
```

APPENDIX B

VERIFICATION CALCULATIONS

TABLES B.1-B.3

Verification Calculations Comparing SADDE and Berger (1971) for ^{49}Ca

TABLE B.1. Values Used by VARSKIN for ^{49}Ca

Values Generated by SADDE

0.8750 0.4688 1.0003

1.2484 1.2370 1.2255 1.2141 1.2026 1.1995 1.2010 1.1967 1.1847 1.1653

1.1326 1.0970 1.0531 1.0063 0.9580 0.9065 0.8522 0.7966 0.7399 0.6832

0.6264 0.5698 0.5135 0.4583 0.4046 0.3042 0.2164 0.0881 0.0234 0.0030

Values Published By Berger (1971)

0.8741 0.4690 0.9999

1.2500 1.2300 1.2200 1.2100 1.2100 1.2100 1.2000 1.1900 1.1800 1.1500

1.1200 1.0900 1.0500 1.0000 0.9500 0.9000 0.8400 0.7800 0.7300 0.6700

0.6100 0.5500 0.5000 0.4400 0.3900 0.3000 0.2100 0.0800 0.0200 0.0000

TABLE B.2. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Generated by SADDE for ^{49}Ca

a) Source Characteristics

Average Beta Energy: 0.875 MeV
X-90 Distance: 0.4688 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 0.844 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	2.60
0.8438	2.2369	1.17

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	2210.
0.0014	2130.
0.0054	1380.
0.0122	543.
0.0216	205.
0.0338	88.8
0.0486	43.8
0.0662	23.8
0.0864	13.9
0.1094	8.59
0.1350	5.49
0.1634	3.61
0.1944	2.42
0.2282	1.64
0.2646	1.12
0.3038	0.761
0.3456	0.513
0.3902	0.341
0.4374	0.219
0.4874	0.135
0.5400	0.077
0.5954	0.041
0.6534	0.019
0.7142	0.007
0.7777	0.002
0.8438	0.000

TABLE B.3. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Published by Berger (1971) for ^{49}Ca

a) Source Characteristics

Average Beta Energy: 0.874 MeV
X-90 Distance: 0.4690 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 0.844 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	2.58
0.8442	2.2388	1.16

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	2190.
0.0014	2120.
0.0054	1370.
0.0122	540.
0.0216	206.
0.0338	89.0
0.0486	43.6
0.0662	23.6
0.0864	13.8
0.1094	8.48
0.1351	5.41
0.1634	3.58
0.1945	2.40
0.2283	1.62
0.2647	1.11
0.3039	0.743
0.3458	0.503
0.3903	0.328
0.4376	0.211
0.4876	0.131
0.5403	0.076
0.5956	0.038
0.6537	0.017
0.7145	0.006
0.7780	0.002
0.8442	0.000

TABLES B.4-B.6

Verification Calculation Comparing SADDE and Berger (1971) for ^{60}Co

TABLE B.4. Values Used by VARSKIN for ^{60}Co

Values Generated by SADDE

0.0980 0.0307 1.0000

2.4601 2.3435 2.2270 2.1104 1.9976 1.9288 1.6524 1.4645 1.3261 1.2095

1.1094 1.0187 0.9368 0.8609 0.7909 0.7236 0.6637 0.6057 0.5525 0.5029

0.4561 0.4123 0.3716 0.3339 0.2989 0.2368 0.1841 0.1039 0.0520 0.0219

Values Published by Berger (1971)

0.0965 0.0312 1.0000

3.0600 2.8200 2.5900 2.3900 2.2100 2.0500 1.5200 1.4300 1.3000 1.1800

1.0800 1.0000 0.9100 0.8400 0.7700 0.7000 0.6400 0.5900 0.5300 0.4800

0.4400 0.4000 0.3600 0.3200 0.2800 0.2200 0.1700 0.1000 0.0500 0.0200

TABLE B.5. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Generated by SADDE for ^{60}Co

a) Source Characteristics

Average Beta Energy: 0.098 MeV
X-90 Distance: 0.0307 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 0.055 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.0548	0.0094	127.
0.5642	1.0000	1.20

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	3860.
0.0001	3860.
0.0004	3850.
0.0008	3800.
0.0014	3680.
0.0022	3450.
0.0032	3080.
0.0043	2610.
0.0056	2100.
0.0071	1610.
0.0088	1190.
0.0106	851.
0.0126	600.
0.0148	417.
0.0172	287.
0.0197	196.
0.0225	133.
0.0253	88.5
0.0284	58.2
0.0317	37.6
0.0351	23.7
0.0387	14.4
0.0424	8.41
0.0464	4.63
0.0505	2.36
0.0548	0.000

TABLE B.6. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Published by Berger (1971) for ^{60}Co

a) Source Characteristics

Average Beta Energy: 0.097 MeV
X-90 Distance: 0.0312 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 0.056 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.0557	0.0098	118.
0.5642	1.0000	1.15

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	3680.
0.0001	3680.
0.0004	3670.
0.0008	3620.
0.0014	3500.
0.0022	3270.
0.0032	2920.
0.0044	2460.
0.0057	1960.
0.0072	1500.
0.0089	1110.
0.0108	785.
0.0128	555.
0.0151	382.
0.0175	262.
0.0201	179.
0.0228	119.
0.0258	80.8
0.0289	52.4
0.0322	32.8
0.0357	20.6
0.0393	12.6
0.0432	7.60
0.0472	4.23
0.0514	2.09
0.0557	0.924

TABLES B.7-B.9

Verification Calculations Comparing SADDE and Berger (1971) for ^{90}Y

TABLE B.7. Values Used by VARSKIN for ^{90}Y

Values Generated by SADDE

0.9535 0.5260 0.9999

1.2342 1.2279 1.2217 1.2155 1.2096 1.2058 1.1924 1.1823 1.1654 1.1399

1.1108 1.0751 1.0384 0.9976 0.9542 0.9092 0.8618 0.8116 0.7590 0.7045

0.6484 0.5912 0.5335 0.4759 0.4189 0.3103 0.2138 0.0747 0.0127 0.0003

Values Published by Berger (1971)

0.9367 0.5170 1.0000

1.3300 1.2900 1.2500 1.2200 1.2000 1.1800 1.1900 1.1800 1.1600 1.1300

1.1000 1.0700 1.0300 0.9900 0.9400 0.8900 0.8400 0.7900 0.7400 0.6900

0.6300 0.5800 0.5200 0.4700 0.4100 0.3100 0.2200 0.0800 0.0200 0.0000

TABLE B.8. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Generated by SADDE for ^{90}Y

a) Source Characteristics

Average Beta Energy: 0.953 MeV
X-90 Distance: 0.5260 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 0.947 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	2.57
0.9468	2.8161	0.924

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	2140.
0.0015	2040.
0.0061	1220.
0.0136	442.
0.0242	162.
0.0379	69.0
0.0545	33.7
0.0742	18.2
0.0969	10.6
0.1227	6.49
0.1515	4.15
0.1833	2.73
0.2181	1.84
0.2560	1.26
0.2969	0.867
0.3408	0.598
0.3878	0.408
0.4378	0.272
0.4908	0.176
0.5469	0.107
0.6059	0.060
0.6680	0.030
0.7332	0.012
0.8013	0.004
0.8725	0.001
0.9468	0.000

TABLE B.9. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Published by Berger (1971) for ^{90}Y

a) Source Characteristics

Average Beta Energy: 0.937 MeV
X-90 Distance: 0.5170 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 0.931 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	2.55
0.9306	2.7205	0.948

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	2220.
0.0015	2120.
0.0060	1270.
0.0134	456.
0.0238	164.
0.0372	69.6
0.0536	34.8
0.0730	18.8
0.0953	10.9
0.1206	6.67
0.1489	4.25
0.1802	2.81
0.2144	1.89
0.2516	1.28
0.2918	0.877
0.3350	0.602
0.3812	0.413
0.4303	0.276
0.4824	0.179
0.5375	0.108
0.5956	0.063
0.6566	0.032
0.7206	0.014
0.7876	0.005
0.8576	0.001
0.9306	0.000

TABLES B.10-B.12

Verification Calculations Comparing SADDE and Berger (1971) for ^{106}Rh

TABLE B.10. Values Used by VARSKIN for ^{106}Rh

Values Generated by SADDE

1.4200 0.8005 0.9963

1.1481 1.1488 1.1495 1.1502 1.1520 1.1527 1.1670 1.1736 1.1714 1.1576

1.1380 1.1099 1.0751 1.0348 0.9885 0.9365 0.8803 0.8208 0.7592 0.6964

0.6336 0.5715 0.5109 0.4524 0.3963 0.2937 0.2059 0.0808 0.0195 0.0016

Values Published by Berger (1971)

1.4283 0.7920 1.0002

1.1700 1.1600 1.1500 1.1400 1.1400 1.1400 1.1600 1.1600 1.1600 1.1400

1.1200 1.0900 1.0600 1.0200 0.9700 0.9200 0.8700 0.8100 0.7500 0.6900

0.6300 0.5700 0.5100 0.4500 0.4000 0.3000 0.2100 0.0800 0.0200 0.0000

TABLE B.11. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Generated by SADDE for ^{106}Rh

a) Source Characteristics

Average Beta Energy: 1.420 MeV
X-90 Distance: 0.8005 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 1.441 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	2.59
1.4409	6.5224	0.420

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	1950.
0.0023	1760.
0.0092	714.
0.0207	200.
0.0369	68.1
0.0576	28.6
0.0830	14.0
0.1130	7.63
0.1475	4.48
0.1867	2.77
0.2305	1.79
0.2790	1.19
0.3320	0.803
0.3896	0.549
0.4519	0.375
0.5187	0.255
0.5902	0.170
0.6663	0.111
0.7470	0.070
0.8323	0.043
0.9222	0.024
1.0167	0.012
1.1158	0.006
1.2196	0.002
1.3279	0.001
1.4409	0.000

TABLE B.12. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Published by Berger (1971) for ^{106}Rh

a) Source Characteristics

Average Beta Energy: 1.428 MeV
X-90 Distance: 0.7920 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 1.426 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	2.62
1.4256	6.3846	0.434

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	2010.
0.0023	1820.
0.0091	743.
0.0205	206.
0.0365	70.2
0.0570	29.5
0.0821	14.5
0.1118	7.86
0.1460	4.62
0.1848	2.85
0.2281	1.84
0.2760	1.22
0.3285	0.826
0.3855	0.562
0.4471	0.385
0.5132	0.262
0.5839	0.176
0.6592	0.116
0.7390	0.073
0.8234	0.045
0.9124	0.026
1.0059	0.013
1.1040	0.006
1.2066	0.002
1.3138	0.001
1.4256	0.000

TABLES B.13-B.15

Verification Calculations Comparing SADDE and Berger (1971) for $^{90}Sr/^{90}Y$

TABLE B.13. Values Used by VARSKIN for $^{90}Sr/^{90}Y$

Values Generated by SADDE

0.5800	0.4963	1.9999							
2.7214	2.6153	2.5092	2.4032	2.3012	2.2064	1.7746	1.3965	1.1080	0.9392
0.8740	0.8447	0.8186	0.7901	0.7593	0.7270	0.6936	0.6582	0.6210	0.5823
0.5422	0.5011	0.4592	0.4170	0.3747	0.2921	0.2152	0.0931	0.0249	0.0024

TABLE B.14. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Generated by SADDE for $^{90}Sr/^{90}Y$

a) Source Characteristics

Average Beta Energy: 0.580 MeV
X-90 Distance: 0.4963 cm
Source Strength: 1.00E+03 μCi/sec
All cell damage occurs in an area with radius = 0.893 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	4.66
0.8933	2.5070	1.87

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	5780.
0.0014	5540.
0.0057	3410.
0.0129	1240.
0.0229	428.
0.0357	166.
0.0515	70.9
0.0700	32.3
0.0915	15.5
0.1158	8.03
0.1429	4.75
0.1729	3.11
0.2058	2.10
0.2416	1.45
0.2801	1.01
0.3216	0.702
0.3659	0.487
0.4131	0.333
0.4631	0.221
0.5160	0.141
0.5717	0.085
0.6303	0.046
0.6918	0.022
0.7561	0.009
0.8233	0.002
0.8933	0.000

TABLE B.15. Dose Profiles from a Point Source on the Skin at 0.007 cm Using Values Published by Berger (1971) for $^{90}Sr/^{90}Y$

a) Source Characteristics

	^{90}Sr	^{90}Y
Average Beta Energy:	0.196 MeV	0.937 MeV
X-90 Distance:	0.0787 cm	0.5170 cm
Source Strength:	1.00E+03 μCi/sec	1.00E+03 μCi/sec

All cell damage occurs in an area with radius = 0.931 cm

b) Beta Dose Averaged over an Area of Skin at the Basal Layer

Radius (cm)	Area (cm^2)	Beta Dose (rem)
0.5642	1.0000	4.43
0.9306	2.7205	1.64

c) Calculated Beta Doses at Points on the Skin Basal Layer

Horizontal Distance (cm)	Beta Dose (rem)
0.0000	5600.
0.0015	5350.
0.0060	3160.
0.0134	1080.
0.0238	366.
0.0372	141.
0.0536	61.1
0.0730	27.8
0.0953	13.4
0.1206	7.06
0.1489	4.25
0.1802	2.81
0.2144	1.89
0.2516	1.28
0.2918	0.877
0.3350	0.602
0.3812	0.413
0.4303	0.276
0.4824	0.179
0.5375	0.108
0.5956	0.063
0.6566	0.032
0.7206	0.014
0.7876	0.005
0.8576	0.001
0.9306	0.000

APPENDIX B REFERENCE

Berger, M. J. 1971. "Distribution of Absorbed Dose Around Point Sources of Electrons and Beta Particles in Water and Other Media." Medical Internal Radiation Dose Committee, Pamphlet No. 7, J. Nucl. Med. 12:5.