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TRAPPED PARTICLE EFFECTS ON ION VELOCITY DISTRIBUTIONS IN THE PRESENCE OF ICRH

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Abstract

An investigation is made of the influence of toroidal effects, in the form of trapped particles, on ion velocity distributions in the presence of ICRH. The analysis is based on a bounced averaged Fokker-Planck equation including RF-driven quasi-linear velocity space diffusion. A pitch angle averaged distribution, suitable for calculating weighted velocity space averages of the distribution, is derived under the assumption that the influence of anisotropy can be neglected. For a given absorbed power, this pitch angle averaged distribution is found to be independent of the aspect ratio.

1. Introduction

Heating of Tokamak plasmas with RF-waves in the ion cyclotron range of frequencies (ICRH) has become one of the major alternatives for supplementary heating. An important consequence of the absorption of the wave energy is a distortion of the velocity distribution function of the heated ion species.

Deviations from thermal Maxwellian form play an important role in determining many "secondary" physical quantities, e.g. absorbed RF-power, collisional power transfer to background plasma particles, fusion reactivity etc. The fundamental paper on the subject of distortion of ion velocity distributions in the presence of ICRH is due to Stix [1], who investigated the effects of ICRH on the distribution function in a minority heating scheme. The analysis was based on the Fokker-Planck equation including a quasi-linear RF-diffusion operator calculated by Kennel and Engelmann [2]. This Fokker-Planck equation has subsequently been studied, both numerically and analytically, in several papers [3-9]. However, toroidal effects, in the form of trapped particles, have been neglected in these papers.

Toroidal effects can be incorporated into the Fokker-Planck model by bounce averaging the Fokker-Planck equation [10, 11]. Numerical studies of the bounce averaged Fokker-Planck equation have been made in Refs [11, 12, 13]. Furthermore, analytical treatments have been carried out in Refs [14, 15]. However, the analysis in Ref. [14] is restricted to minority heating and does not take into account effects due to finite $k_\perp$. The characteristic features of the distribution is studied in Ref. [15], but the analysis does not give any detailed information about the distorted distribution.

The purpose of the present paper is to contribute to the analytical understanding of the bounce averaged Fokker-Planck equation. In particular, a pitch angle averaged distribution, similar to those in [1,7,8], is derived under the assumption that effects due to anisotropy can be neglected. With this approximation it is shown that, for a given absorbed power, the pitch angle averaged distribution becomes independent of the aspect ratio.

2. The Bounce Averaged Fokker-Planck Equation

The bounce averaged Fokker-Planck equation for the distribution function, f, of the RF-heated ions can be written as, cf [10],

$$\frac{\partial f}{\partial t} = \langle\frac{1}{v_\parallel}\rangle^{-1} \langle\frac{1}{v_\parallel} C(f)\rangle + \langle\frac{1}{v_\parallel}\rangle^{-1} \langle\frac{1}{v_\parallel} Q(f)\rangle \tag{1}$$

where the bracket denotes the bounce average

$$\langle\ldots\rangle = \begin{cases} \frac{1}{2\pi}\int_0^{2\pi} (\ldots)d\theta & \text{for passing particles} \\ \frac{1}{2\pi}\int_A^{B} (\ldots)d\theta & \text{for trapped particles (A, B are turning points)} \end{cases}$$

$\langle 1/v_\parallel\rangle^{-1}\langle 1/v_\parallel\ C(f)\rangle$ is the bounce averaged collision operator and $\langle 1/v_\parallel\rangle^{-1}$ $\langle 1/v_\parallel\ Q(f)\rangle$ is the bounce averaged RF-diffusion operator.

It is convenient to express the bounce averaged operators in the midplane coordinates (v,ξ) (the midplane is defined as the plane at the poloidal angle $\theta = 0$), where v is the velocity and $\xi = v_{\parallel 0}/v$ (subscript 0 denotes the midplane) is the cosine of the pitch angle at the midplane. The velocity, v, is invariant along a particle orbit and the cosine of the pitch angle, $\mu = v_\parallel/v$, can be expressed in ξ as

$$\mu^2 = 1 - \frac{B(\theta=0)}{B(\theta)}(1-\xi^2) = 1 - \frac{1}{b(\theta)}(1-\xi^2) \tag{2}$$

where $b(\theta) \equiv B(\theta)/B(\theta=0)$. The magnetic field is given by

$$B(\theta) = B(\theta=0)\frac{R(\theta=0)}{R(\theta)} = B(\theta=0)\frac{1+\varepsilon}{1+\varepsilon\cos\theta} \tag{3}$$

where $\varepsilon = r/R$ is the inverse aspect ratio.

In the variables (v,ξ), the bounce averaged collision operator takes the form [10]

$$\langle\frac{1}{v_\parallel}\rangle^{-1}\langle\frac{1}{v_\parallel}C(f)\rangle = -\frac{1}{v^2}\frac{\partial}{\partial v}\left[v^2\,\alpha(v)f\right] +$$

$$\frac{1}{2v^2}\frac{\partial^2}{\partial v^2}\left[v^2\beta(v)f\right] + \frac{\gamma(v)}{4v^2}\frac{1}{\xi\langle\frac{v}{v_\parallel}\rangle}\frac{\partial}{\partial\xi}\left[\frac{1-\xi^2}{\xi}\langle\frac{1}{b}\frac{v_\parallel}{v}\rangle\frac{\partial f}{\partial\xi}\right] \qquad (4)$$

where α, β and γ are the collision coefficients describing dynamical friction, energy diffusion and pitch angle scattering respectively, [1].

If the number of non-Maxwellian tail particles is small compared to the number of particles in the low energy bulk, we may neglect self collisions between the tail particles. Furthermore, the distribution function, f, is almost Maxwellian in the low energy range. It is therefore reasonable to approximate the distribution function, f, of the heated ions with a Maxwellian when the collision coefficients are calculated. The collision operator will then be linear in f.

The bounce averaged RF-diffusion operator can be written as [10]

$$\langle\frac{1}{v_\parallel}\rangle^{-1}\langle\frac{1}{v_\parallel}Q(f)\rangle =$$

$$\frac{K_n}{(1+\varepsilon)\langle\frac{v}{v_\parallel}\rangle}\sigma(|\xi| - \xi_R)\sqrt{\frac{b_p}{\xi^2-\xi_R^2}}\left\{\frac{1}{v^2}\frac{\partial}{\partial v}\left[v(1-\xi^2)H_n(\bar{v}_\perp)\frac{\partial}{\partial v}(vf)\right]\right.$$

$$-\frac{1}{v^2}\frac{\sqrt{\xi^2-\xi_R^2}}{\xi}\frac{\partial}{\partial\xi}\left[\sqrt{\xi^2-\xi_R^2}\,(1-\xi^2)H_n(\bar{v}_\perp)\frac{\partial}{\partial v}(vf)\right]$$

$$+\frac{1}{v^2}\frac{\sqrt{\xi^2-\xi_R^2}}{\xi}\frac{\partial}{\partial\xi}\left[(\xi^2-\xi_R^2)(1-\xi^2)H_n(\bar{v}_\perp)\frac{1}{\xi}\frac{\partial}{\partial\xi}(\sqrt{\xi^2-\xi_R^2}f)\right.$$

$$\left.-\frac{1}{v^2}\frac{\sqrt{\xi^2-\xi_R^2}}{\xi}\frac{\partial}{\partial v}\left[v(1-\xi^2)H_n(\bar{v}_\perp)\frac{\partial}{\partial\xi}(\sqrt{\xi^2-\xi_R^2}f)\right]\right\} \qquad (5)$$

where

$$H_n(\bar{v}_\perp) = J_{n-1}^2(\bar{v}_\perp) + \Lambda J_{n+1}^2(\bar{v}_\perp) \tag{6}$$

Furthermore, $\bar{v}_\perp = (k_\perp v/\omega_{ci})\sqrt{b_R(1-\xi^2)}$, $b_R = b(\theta_R)$ is b at the resonance point, $k_\perp$ is the perpendicular wave number, $\Lambda = |E_-|^2/|E_+|^2$ where E_+ and E_- are the amplitudes of the left and right hand components of the RF-wave, K_n is a constant proportional to $|E_+|^2$, J_{n-1} and J_{n+1} are Bessel functions of order n-1 and n+1 where n denotes the heating mode (RF-wave frequency $\omega = n\omega_{ci}$) and the pitch angle ξ_R is determined by

$$\xi_R = \left[1 - \frac{1}{b_R}\right]^{1/2} \tag{7}$$

The step function, $\sigma(|\xi| - \xi_R)$, accounts for the fact that trapped particles which have turning points with poloidal angles less than the poloidal angle at the resonance point will not sample the resonance region. Effects due to finite $E_\parallel$ and $k_\parallel$ have been neglected in the RF-operator. However, this is justified since typically $|E_\parallel| \ll |E_+|, |E_-|$, and effects due to finite $k_\parallel$ can be neglected if $\omega/k_\parallel \gg v_\parallel$, cf [1], this is the case, in the velocity range of interest, for most ICRH scenarios [6].

3. The Pitch Angle Averaged Distribution

Many physically important quantities, like absorbed RF-power, collisional power transfer to background plasma particles, fusion reactivity etc, are weighted velocity space averages of the distribution function. The averages over a magnetic flux surface of these quantities are often of interest. Thus, averages of the following form are important to calculate.

$$\bar{S} = \frac{\int 2\pi R\, r d\theta \int S(v,\mu) f(v,\mu) d^3v}{\int 2\pi R\, r d\theta} \tag{8}$$

The volume element in velocity space can be written, in the midplane coordinates, as

$$d^3v = b\,\frac{\xi}{\mu}\,d^3v_o = \frac{R(\theta=0)}{R(\theta)}\,\frac{\xi}{\mu}\,d^3v_o \tag{9}$$

where d^3v_o is the volume element in the midplane coordinates.

In many cases of interest, S is independent of μ (e.g. when S represents collisional RF-power transfer to background particles or fusion reactivity). In such cases, eq. (8) can be rewritten as

$$\bar{S} = \frac{R(\theta=0)\int 2\pi v^2 dv\ S(v) \int_{-1}^{1} f\langle\frac{v}{v_\parallel}\rangle \xi d\xi}{\frac{1}{2\pi}\int R d\theta} =$$

$$= \int S(v)\ F(v)\ 4\pi v^2 dv \tag{10}$$

where we have defined the pitch angle averaged distribution, F(v), as, cf [15]

$$F(v) = \frac{1+\varepsilon}{2}\int_{-1}^{1} f\langle\frac{v}{v_\parallel}\rangle\ \xi d\xi \tag{11}$$

Furthermore, note that

$$\frac{1}{2}\int_{-1}^{1} \langle\frac{v}{v_\parallel}\rangle\ \xi d\xi = \frac{1}{2\pi}\int_{o}^{2\pi} d\theta \int_{\sqrt{1-1/b}}^{1} \frac{1}{\sqrt{1-b(1-\xi^2)}}\ \xi d\xi = \frac{1}{1+\varepsilon} \tag{12}$$

4. Approximate model for F(v)

We write the distribution function $f(v,\xi,t)$ as

$$f(v,\xi,t) = F(v,t)\left[1+h(v,\xi,t)\right] \tag{13}$$

where the function $h(v,\xi,t)$ accounts for deviations from an isotropic distribution, and satisfies the condition

$$\int_{-1}^{1} h(v,\xi,t) \left\langle \frac{v}{v_\parallel} \right\rangle \xi d\xi = 0 \tag{14}$$

If f is isotropic, then $f=F(v)$.

In order to obtain an equation for $F(v)$, we multiply the bounce averaged Fokker-Planck equation with $(1+\varepsilon)\langle v/v_\parallel\rangle\xi$ and integrate it over ξ, the resulting equation can be written as

$$\frac{\partial F}{\partial t} = \frac{1}{v^2} \frac{\partial}{\partial v} \left[AF + B \frac{\partial F}{\partial v}\right] \tag{15}$$

where

$$A = -\alpha v^2 + \frac{1}{2} \frac{\partial}{\partial v} (\beta v^2) + K_n v\, R_2(v,t) \tag{16}$$

$$B = \frac{1}{2} \beta v^2 + K_n v^2\left[G(v) + R_1(v,t)\right] \tag{17}$$

$$G(v) = \int_{\xi_R}^{1} \sqrt{b_R}\, \frac{1-\xi^2}{\sqrt{\xi^2-\xi_R^2}}\, H_n(\bar{v}_\perp)\, \xi d\xi \tag{18}$$

$$R_1(v,t) = \int_{\xi_R}^{1} \sqrt{b_R}\, \frac{1-\xi^2}{\sqrt{\xi^2-\varepsilon_R^2}}\, H_n(\bar{v}_\perp) h(v,\xi,t)\xi d\xi \tag{19}$$

$$R_2(v,t) = \int_{\xi_R}^{1} \sqrt{b_R}\, (1-\xi^2)\, H_n(\bar{v}_\perp)\left[v \frac{\xi}{\sqrt{\xi^2-\xi_R^2}} \frac{\partial h(v,\xi,t)}{\partial v} - \sqrt{\xi^2-\xi_R^2}\, \frac{\partial h(v,\xi,t)}{\partial \xi}\right] d\xi \tag{20}$$

Note that the time derivative of the averaged density $\bar{n}$ ($\bar{n} = \int F(v) 4\pi v^2 dv$) is zero, i.e., the collision and RF-operators conserve the number of particles on a flux surface.

In steady state we obtain the following formal solution

$$F(v) = F(0)\exp\left[-\int_0^v \frac{A}{B}\,dv\right] \tag{21}$$

This solution requires the knowledge of the function $h(v,\xi)$. However, in order to calculate $h(v,\xi)$ the full 2D problem must be solved. It has proved very difficult to find an analytic solution that can be used to evaluate $R_1(v)$ and $R_2(v)$. We are therefore forced to consider approximate forms.

4.1 Isotropic approach

A widely used approach is to neglect the influence of anisotropy, i.e., using $h(v,\xi) = 0$. This approximation is valid in the limit of weak anisotropy and should therefore be reasonable for velocities well below the characteristic velocity v_γ, where v_γ is the characteristic velocity associated with the pitch angle scattering [1]. Furthermore, in Ref. [7] this isotropic approximation has been shown to provide useful and accurate results for many physically meaningful velocity space averages, including such high energy characteristics as cololisional power transfer to electrons and fusion reactivity. One might conclude that the isotropic approximation is reasonable when F(v) is used to calculate velocity space averages, although it does not describe the detailed form of the anisotropic high energy tail correctly.

If the isotropic approximation is used then $R_1(v) = 0$, $R_2(v) = 0$ and there only remains to calculate the function G(v).

We write G(v) as

$$G(v) = g_{n-1}(v) + \Lambda\, g_{n+1}(v) \tag{22}$$

where

$$g_n(v) = \int_{\xi_R}^{1} \sqrt{b_R}\,\frac{1-\xi^2}{\sqrt{\xi^2-\xi_R^2}}\,J_n^2(\bar{v}_\perp)\,\xi d\xi \tag{23}$$

It is possible to calculate $g_n(v)$ analytically by expanding the Bessel function in a power serie. According to Ref. [16] we have

$$J_n^2(\bar{v}\sqrt{1-\xi^2}) =$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k\,(2n+2k)!}{4^{n+k}\,[(n+k)!]^2\,(2n+k)!k!}\,\bar{v}^{2(n+k)}\,(1-\xi^2)^{n+k} \tag{24}$$

Thus in order to calculate $g_n(v)$ we must evaluate integrals of the form

$$I_m = \int_{\xi_R}^{1} \frac{(1-\xi^2)^m}{\sqrt{\xi^2-\xi_R^2}}\,\xi d\xi \tag{25}$$

This integral is solved in appendix, and the result is

$$I_m = \frac{2m!!}{(2m+1)!!}\,(1-\xi_R^2)^{m+\frac{1}{2}} = \frac{2m!!}{(2m+1)!!}\,t_R^{-(m+\frac{1}{2})} \tag{26}$$

Thus, we obtain

$$g_n(v) = \sum_{k=0}^{\infty} \frac{(-1)^k\,(2n+2k)!\,(2n+2k+2)!!}{4^{n+k}\,[(n+k)!]^2\,(2n+k)!k!(2n+2k+3)!!}\,\left(\frac{k_\perp v}{\omega_{ci}}\right)^{2(n+k)} \tag{27}$$

An interesting feature of the "isotropic" solution, eq. (21) with $R_1(v) = 0$, $R_2(v) = 0$ and $G(v)$ given by eq. (22) and (27), is that for a given K_n, i.e. for a given absorbed power, the solution is independent of the inverse aspect ratio ε. In fact it is identically the same as in Ref. [7] where toroidal effects have been neglected! One might therefore expect toroidal effects to have only a weak influence on velocity space averaged quantities, $\bar{S}$, if S is independent of the pitch angle. This conclusion is supported by the numerical results of Refs [12,13], where the influence of toroidal effects on the fusion reactivity has been shown to be weak.

5. Effects of anisotropy

Although it is very difficult to treat the problem with anisotropy, some qualitative remarks can be made. In the high energy tail, where the pitch angle scattering is weak, the distribution is peaked around $\xi=\pm\xi_R$, since the RF-induced diffusion transfers the resonant ions towards higher $v_\perp$ until $\xi=\pm\xi_R$. The function $h(v,\xi)$ should therefore be of importance in the high energy range.

The anisotropy will influence $F(v)$, as compared to the isotropic model in two different ways: (i) the function $K_n v^2 R_1(v)$, which acts like a diffusive term, will tend to decrease the slope of the tail, (ii) the function $K_n v R_2(v)$, which acts like a friction term, will tend to increase the slope of the tail. In analogy with the results in Ref. [8], one might expect the first effect to dominate in the tail, whereas in the transition region between bulk and tail distributions the second effect can be important. The success of the "isotropic" model, for calculating velocity space averages can, at least partly, be understood from the fact that these two effects work against each other.

Conclusions

An investigation has been made of the influence of trapped particles on the velocity distribution of RF-heated ions. The analysis was based on a bounce averaged Fokker-Planck equation. A pitch angle averaged distribution function, defined in such a way that it is useful for calculating quantities averaged over velocity space and over a magnetic flux surface, was derived under the assumption that effects due to anisotropy could be neglected. For a given absorbed power, this pitch angle averaged distribution turned out to be independent of the inverse aspect ratio ε. One might therefore conclude that velocity space averaged quantities, which are averaged over a magnetic flux surface, is only weakly influenced by trapped particle effects, unless they are strongly weighted in favour of the anisotropic high energy part of the distribution function.

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Appendix

We want to calculate the following integral:

$$I_m = \int_{\xi_R}^{1} \frac{\xi(1-\xi^2)^m}{\sqrt{\xi^2-\xi_R^2}}\, d\xi \tag{A1}$$

we change variable to $x = \sqrt{\xi^2-\xi_R^2}$, and obtain

$$I_m = \int_0^{\sqrt{1-\xi_R^2}} \left[(1-\xi_R^2) - x^2\right]^m dx \tag{A2}$$

Integrating by parts yields

$$I_m = - \frac{1}{2m+1} \int_0^{\sqrt{1-\xi_R^2}} x^{2m+1} \frac{d}{dx} \left\{ \frac{\left[(1-\xi_R^2)-x^2\right]^m}{x^{2m}} \right\} dx$$

$$= (1-\xi_R^2) \frac{2m}{2m+1} I_{m-1} \tag{A3}$$

since

$$I_o = \sqrt{1-\xi_R^2} \tag{A4}$$

we obtain from (A3)-(A4)

$$I_m = (1-\xi_R^2)^{m+\frac{1}{2}} \frac{2m!!}{(2m+1)!!} \tag{A5}$$

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