IC/88/407

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

A MODIFIED SKYRME MODEL WITHOUT SKYRME TERM

Qi-xing Shen

INTERNATIONAL
ATOMIC ENERGY
AGENCY

UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

A MODIFIED SKYRME MODEL WITHOUT SKYRME TERM *

Qi-xing Shen **
International Centre for Theoretical Physics, Trieste, Italy.

ABSTRACT

The static properties of nucleons and the mass of the resonance in π - N scattering are calculated in a modified Skyrme model, which includes a six-order term and another independent quartic-derivative Lagrangian instead of Skyrme term. The predictions for static properties of nucleons are improved and the mass of Roper resonance is more close to the measured value over the original Skyrme model.

MIRAMARE - TRIESTE
December 1988

* Submitted for publication.

** Permanent address: Institute of High Energy Physics, Academia Sinica, Beijing, People's Republic of China.

I. INTRODUCTION

The Skyrme model [1] has gained some successes in describing the nucleon and delta since Witten's realization that in the large N_c limit (N_c =number of colors) baryons should indeed emerge as topologically stable solitons in the field of pions [2]. The static properties of baryons can be calculated [3-5]. Pion-nucleon, pion-pion and nucleon-nucleon scattering can also be described [6,9,10] in this model. In this original Skyrme model, the nonlinear $SU(2) \times SU(2)$ chiral Lagrangian is

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_{SK}$$

$$\mathcal{L}_2 = \frac{F_\pi^2}{16} Tr \left[\partial_\mu U \partial^\mu U^\dagger \right]$$

$$\mathcal{L}_{SK} = \frac{1}{32 e^2} Tr \left[(\partial_\mu U) U^\dagger, (\partial_\nu U) U^\dagger \right]^2 \tag{1}$$

The leading term is the usual 2-flavor nonlinear sigma model. The second term $\mathcal{L}_{SK}$ is called the Skyrme term. It was introduced by Skyrme to stabilize the soliton. In Ref.[3] an explicit symmetry breaking term

$$\mathcal{L}_1 = \frac{1}{8} m_\pi^2 F_\pi^2 (Tr U - 2) \tag{2}$$

was added into the Lagrangian (1) in order to introduce the pion mass. The phenomenological aspects of the Skyrme soliton based on these Lagrangians have been explored. The static properties of nucleons have been computed [3-5]. The results are generally within about 30% of the experimental values, but the axial-vector coupling constant g_A and the mean square radius $\langle r^2\rangle_A^{\frac{1}{2}}$ of the axial vector form factor of nucleon are much smaller than the experimental datum. The pion-soliton scattering phase shifts are calculated [6]. Two problems have arisen in Ref.[6]. First, the phase shift rises to a maximum of only about 91° and then slowly declines. It is not like experimental P-wave phase shift which rise quite rapidly up to about 180°. Second, the calculated mass of the Roper resonance is lower than physical value by about 200 MeV.

It will be an interesting problem to see whether we can introduce other higher-order terms instead of the Skyrme term in the Lagrangian thereby keeping stability of the soliton simultaneously and improving the above results.

In Ref.[7] and [8], a six-order Lagrangian

$$\mathcal{L}_6 = -\frac{1}{96m^2} Tr\{[\partial_\mu U U^\dagger, \partial^\nu U U^\dagger][\partial_\nu U U^\dagger, \partial^\rho U U^\dagger][\partial_\rho U U^\dagger, \partial^\mu U U^\dagger]\} \tag{3}$$

was introduced instead of the Skyrme term $\mathcal{L}_{SK}$ and the static properties of nucleons were computed. The results are much better than those obtained from original Skyrme model.

In this paper we show that in order to give the correct behavior of P-wave phase shift of πN scattering it is necessary to add a term

$$\mathcal{L}_4 = \frac{\gamma}{8}[Tr\,\partial_\mu U \partial^\mu U^\dagger]^2 \tag{4}$$

to the Lagrangian. The term $\mathcal{L}_4$ was first introduced by J.F.Donogune et al. to obtain the correct amplitudes of D-wave on $\pi\pi$ scattering [10]. In this modified Skyrme model the mass of the Roper resonance, the energy at which the phase shift passes

through 90°, is closer to the measured value and the corresponding static properties of nucleon are improved over the original Skyrme model.

II. THE MODEL AND FORMULAS

We use the Lagrangian

$$\mathcal{L} = \frac{F_\pi^2}{16} Tr(\partial_\mu U \partial^\mu U^\dagger) + \frac{\gamma}{8}[Tr\,\partial_\mu U \partial^\mu U]^2 - \frac{1}{96m^2} Tr\{[\partial_\mu U U^\dagger, \partial^\nu U U^\dagger][\partial_\nu U U^\dagger, \partial^\rho U U^\dagger][\partial_\rho U U^\dagger, \partial^\mu U U^\dagger]\} + \frac{1}{8} m_\pi^2 F_\pi^2 [Tr\,U - 2] \tag{5}$$

where U is an SU(2) matrix, F_π is the pion decay constant, γ and m are undetermined parameters. As in Ref.[3] the ansatz for a spinning soliton is

$$U(\vec{x}, t) = A(t)\, U_0(\vec{x})\, A^\dagger(t) \tag{6}$$

where A(t) is a time dependent SU(2) matrix, $U_0(\vec{x}) = \exp[iF(r)\vec{\tau}\cdot\hat{x}]$ is a classical static soliton solution and function F(r) is subject to the boundary conditions $F(0)=\pi$ and $F(\infty)=0$. Substituting (6) into (5), we have

$$\mathcal{L} = M + \lambda\, Tr[\partial_0 A \partial_0 A^\dagger] \tag{7}$$

where M is the classical soliton mass

$$M = \frac{\pi}{m^{1/2}} F_\pi^{3/2} (4M_1 + \beta^2 M_2)$$

$$M_1 = \int d\tilde{r} \{ \frac{1}{8} \tilde{r}^2 (F'^2 + \frac{2\sin^2 F}{\tilde{r}^2}) + \frac{1}{\tilde{r}^2} F'^2 \sin^4 F$$

$$-\frac{1}{2}\xi\,\tilde{r}^2\left(F'^2+\frac{2}{\tilde{r}^2}\sin^2F\right)^2\Big\}\,,$$

$$M_2=\int d\tilde{r}\;\tilde{r}^2\,(1-\cos F)\,. \tag{8}$$

and

$$\lambda=\frac{2\pi}{3}\,\frac{F_\pi^{1/2}}{m^{3/2}}\,\Lambda\,.$$

$$\Lambda=\int d\tilde{r}\,\tilde{r}^2\sin^2F\left\{1+8F'^2\frac{\sin^2F}{\tilde{r}^2}-8\xi\left(F'^2+\frac{2}{\tilde{r}^2}\sin^2F\right)\right\} \tag{9}$$

$$\tilde{r}=\sqrt{mF_\pi}\;r\;,\qquad \beta=\frac{m_\pi^2}{mF_\pi}\;,\qquad \xi=\frac{\gamma m}{F_\pi} \tag{10}$$

Primes in Eqs.(8) and (9) refer to $\tilde{r}$ derivative. The variational equation from Eq. (8) is

$$\left\{\frac{\tilde{r}^2}{4}+\frac{2\sin^4F}{\tilde{r}^2}-2\xi\left(3\tilde{r}^2F'^2+2\sin^2F\right)\right\}F''$$

$$+\left\{\frac{2}{\tilde{r}^2}\sin^2F\,\sin 2F-2\xi\left(2\tilde{r}F'+\sin 2F\right)\right\}F'^2$$

$$+\left(\frac{\tilde{r}}{2}-\frac{4}{\tilde{r}^3}\sin^4F\right)F'-\frac{1}{4}\sin 2F-\frac{1}{4}\beta^2\tilde{r}^2\sin F$$

$$+\frac{4}{\tilde{r}^2}\,\xi\,\sin^2F\,\sin 2F=0\,. \tag{11}$$

In the present model the vector and axial-vector currents are respectively

$$V_\mu^a=-i\left(\frac{F_\pi^2}{8}+\frac{\gamma}{2}\,Tr\,\partial_\nu U\partial^\nu U^\dagger\right)Tr\,\tau^a\left(U\partial_\mu U^\dagger+U^\dagger\partial_\mu U\right)$$

$$+\frac{i}{32m^2}\,Tr\Big\{[\tau^a,\partial_\nu UU^\dagger]\big[[\partial^\nu UU^\dagger,\partial_\rho UU^\dagger],[\partial^\rho UU^\dagger,\partial_\mu UU^\dagger]\big]$$

$$+[\tau^a,\partial_\nu U^\dagger U]\big[[\partial^\nu U^\dagger U,\partial_\rho U^\dagger U],[\partial^\rho U^\dagger U,\partial_\mu U^\dagger U]\big]\Big\}$$

$$A_\mu^a=i\left(\frac{F_\pi^2}{8}+\frac{\gamma}{2}\,Tr\,\partial_\nu U\partial^\nu U^\dagger\right)Tr\,\tau^a\left(\partial_\mu U^\dagger U-\partial_\mu UU^\dagger\right)$$

$$+\frac{i}{32m^2}\,Tr\Big\{[\tau^a,\partial_\nu U^\dagger U]\big[[\partial^\nu U^\dagger U,\partial_\rho U^\dagger U],[\partial^\rho U^\dagger U,\partial_\mu U^\dagger U]\big]$$

$$-[\tau^a,\partial_\nu UU^\dagger]\big[[\partial^\nu UU^\dagger,\partial_\rho UU^\dagger],[\partial^\rho UU^\dagger,\partial_\mu UU^\dagger]\big]\Big\} \tag{12}$$

If we continue to use the methods and signs given in Refs.[4] and [5], we can get the expressions for various physical quantities in this modified Skyrme model. The mean square radii of isoscalar and isovector are respectively

$$\langle r^2\rangle_{I=0}=-\frac{2}{\pi}\,\frac{1}{mF_\pi}\int d\tilde{r}\;\tilde{r}^2F'\sin^2F \tag{13}$$

$$\langle r^2\rangle_{I=1}=\frac{1}{mF_\pi}\,\frac{1}{\Lambda}\int d\tilde{r}\;\tilde{r}^4\sin^2F\Big\{1+8F'^2\,\frac{\sin^2F}{\tilde{r}^2}$$

$$-8\xi\left(F'^2+\frac{2}{\tilde{r}^2}\sin^2F\right)\Big\} \tag{14}$$

The expressions for isoscalar and isovector magnetic mean square radii are

$$\langle r^2\rangle_{M,I=0}=\frac{3}{5}\,\frac{1}{mF_\pi}\,\frac{\int d\tilde{r}\;\tilde{r}^4F'\sin^2F}{\int d\tilde{r}\;\tilde{r}^2F'\sin^2F} \tag{15}$$

$$\langle r^2\rangle_{M,I=1}=\frac{3}{5}\,\langle r^2\rangle_{I=1} \tag{16}$$

The mean square radius of strong interaction [11] is

$$\langle r^2\rangle_S=\frac{3}{5}\,\frac{1}{\beta^2mF_\pi}\left\{\beta^2\,\frac{\int d\tilde{r}\;\tilde{r}^5\sin F}{\int d\tilde{r}\;\tilde{r}^3\sin F}-10\right\} \tag{17}$$

The mean square radius of axial form factor can be written as

$$\langle r^2\rangle_A = \frac{3}{5m^2 D}\int d\tilde{r}\,\tilde{r}^4\Big\{F' + \frac{2\sin 2F}{\tilde{r}} + \frac{16}{\tilde{r}^3}F'^2\sin^2F\sin 2F + \frac{8}{\tilde{r}^4}F'\sin^4F - 8\xi\Big(F'^2+\frac{2}{\tilde{r}^2}\sin^2F\Big)\Big(F'+\frac{2}{\tilde{r}}\sin 2F\Big)\Big\} \tag{18}$$

where

$$D = \frac{F_\pi}{m}\int d\tilde{r}\,\tilde{r}^2\Big\{F' + \frac{\sin 2F}{\tilde{r}} + \frac{8}{\tilde{r}^3}F'^2\sin^2F\sin 2F + \frac{8}{\tilde{r}^4}F'\sin^4F - 8\xi\Big(F'^2+\frac{2}{\tilde{r}^2}\sin^2F\Big)\Big(F'+\frac{\sin 2F}{\tilde{r}}\Big)\Big\} \tag{19}$$

In this model we obtain the axial coupling constant

$$g_A = \begin{cases} -\frac{\pi}{3}D & \text{for} \quad m_\pi = 0 \\ -\frac{2\pi}{9}D & \text{for} \quad m_\pi \neq 0 \end{cases} \tag{20}$$

where D is given in Eq.(19). The Goldberger-Treiman relation

$$g_{\pi NN} = \frac{2m_\pi}{F_\pi}g_A \tag{21}$$

are still correct both for $m_\pi \simeq 0$ and $m_\pi \neq 0$. The magnetic moments for the proton and neutron are respectively

$$\mu_p = \frac{1}{4}\left(g_{I=0} + g_{I=1}\right)$$
$$\mu_n = \frac{1}{4}\left(g_{I=0} - g_{I=1}\right) \tag{22}$$

where the isoscalar g factor $g_{I=0}$ and isovector g factor $g_{I=1}$ can be written as follows

$$g_{I=0} = -\frac{2M_N}{\pi^2}\frac{m^{1/2}}{F_\pi^{3/2}}\frac{1}{\Lambda}\int d\tilde{r}\,\tilde{r}^2 F'\sin^2F$$
$$g_{I=1} = \frac{8\pi M_N}{9}\frac{F_\pi^{1/2}}{m^{3/2}}\Lambda \tag{23}$$

The pion-nucleon sigma term given in Ref. [3] is now

$$\sigma = \pi\beta^2\frac{F_\pi^{3/2}}{m^{1/2}}\int d\tilde{r}\,\tilde{r}^2(1-\cos F) \tag{24}$$

Finally, the masses of nucleon and baryon delta are respectively

$$M_N = M + \frac{3}{8\lambda}$$
$$M_\Delta = M + \frac{15}{8\lambda} \tag{25}$$

where M and λ are given in Eqs.(8) and (9). Hence we have the relations of the between parameters F_π, m and M_N, M_Δ, pion-mass m_π as follows

$$m^4 = \frac{4\pi^2}{81}\frac{\Lambda^2}{M_1}(M_\Delta - M_N)^2\cdot\Big\{\frac{1}{9}\Lambda(M_\Delta - M_N)(5M_N - M_\Delta) - m_\pi^2 M_2\Big\}$$
$$F_\pi = \frac{81\,m^3}{16\pi^2\Lambda^2(M_\Delta - M_N)^2} \tag{26}$$

where M_1, M_2 and Λ are given in Eqs.(8) and (9). Therefore parameters F_π, m and β are fixed by inputting physical mass M_N, M_Δ and m_π.

In order to determining the only parameter ξ in Eq.(11) we now calculate the mass of Roper resonance in πN scattering. The methods used to find the phase shifts are essentially the same as those used in Ref.[6]. For this reason we consider a small fluctuation around the classical soliton and let

$$U(\vec{x},t) = \exp\Big\{iF(r)\vec{\tau}\cdot\vec{x} + i\,\delta F(r)\,e^{-i\omega t}\,\vec{\tau}\cdot\vec{x}\Big\} \tag{27}$$

Substituting (27) into (5) and using Lagragian equation satisfied by this Lagrangian we get the equation of motion for $\delta F(\tilde{r})$

$$
\begin{aligned}
&(\delta F)''\left\{\frac{\tilde{r}^2}{4}+\frac{2}{\tilde{r}^2}\sin^4 F-2\xi\left(3\tilde{r}^2F'^2+2\sin^2 F\right)\right\} \\
&+(\delta F)'\left\{\frac{\tilde{r}}{2}+\frac{4}{\tilde{r}^2}\left(F'\sin^2 F\sin 2F-\frac{1}{\tilde{r}}\sin^4 F\right)\right. \\
&\qquad\left.-4\xi\left(3\tilde{r}^2F'F''+3\tilde{r}F'^2+F'\sin 2F\right)\right\} \\
&+\delta F\left\{\frac{1}{4}\tilde{\omega}^2\tilde{r}^2-\frac{1}{2}\cos 2F-\frac{1}{4}\beta^2\tilde{r}^2\cos F\right. \\
&\qquad+\frac{2}{\tilde{r}^2}\left[2F''\sin^2 F\sin 2F+F'^2\left(2\sin^2 F\cos 2F+\sin^2 2F\right)\right. \\
&\qquad\qquad\left.-\frac{4}{\tilde{r}}F'\sin^2 F\sin 2F+\tilde{\omega}^2\sin^4 F\right] \\
&\qquad-2\xi\left[2F''\sin 2F+2F'^2\cos 2F-\frac{2}{\tilde{r}^2}\left(2\sin^2 F\cos 2F+\sin^2 2F\right)\right. \\
&\qquad\qquad\left.\left.+\tilde{\omega}^2\left(\tilde{r}^2F'^2+2\sin^2 F\right)\right]\right\}=0 \quad .
\end{aligned}
\tag{28}
$$

where $\tilde{\omega}^2=\omega^2/mF_\pi$. δF subjects to the boundary conditions $\delta F(0)=0$ and $\delta F(\infty)=0$. As $\tilde{r}\to\infty\ \delta F(\tilde{r})\to a(\tilde{k})j_1(\tilde{k}\tilde{r})+b(\tilde{k})n_1(\tilde{k}\tilde{r})$, where j_1 and n_1 are the spherical Bessel functions of order 1. $\tilde{k}=\sqrt{\tilde{\omega}^2-\beta^2}$. Therefore P-wave phase shift δ_1 can be got from following expresion

$$
\delta_1(k)=\tan^{-1}\left[-\frac{b(\tilde{k})}{a(\tilde{k})}\right]
\tag{29}
$$

Finally, the energy at which the phase shift passes through 90° is identified with the mass of Roper resonance.

III. THE RESULTS AND DISCUSSION

First of all we obtain, numerically, that as parameter $\xi=0$,

that is, there is no $\mathcal{L}_4$ in Lagrangian $\mathcal{L}$, the phase shift δ_1 does not pass through 90° . As ξ increases slightly, the phase shift δ_1 may pass through 90° but the resonance mass computed is too great. For example, we have resonance mass 2114 MeV for $\xi=0.02$. Experimentally we have the Roper resonance at about 1440 MeV. Secondly we find that the value of resonance mass calculated decreases as ξ increases. But paramenter ξ can not be taking too great in order to guarantee the existence of stable soliton. We find that as $\xi\geqslant 0.0272$ there is no soliton solution. Therefor we chose $\xi=0.0271$ to solve the equation of motion (11). We obtain the resonance mass to be 1522 MeV. Corresponding static properties of nucleon are calculated in this model and listed in Tab.1. We also listed the results for original Skyrme model [3] in Tab.1 in order to taking a comparison. We find that some of the results, in particular F_π , $\langle r^2\rangle_A^{1/2}$ are improved greatly.

To summarize, we calculate the static propeties of nucleons, P-wave phase shift and resonance mass for πN scattering in a modified Skyrme model, which includes a six-order Lagrangian and another independent quartic-derivative Lagrangian instead of Skyrme term in original Skyrme model. The results show that most of the static properties of nucleons are improved and the mass of the resonance is more close to measured value over the original Skyrme model.

ACKNOWLEDGMENTS

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where part of this work was done. He would also like to thank Professor Bing-an Li for helpful discussions.

REFERENCES

1. T.H.R.Skyrme, Proc. R. Soc. London, A260(1961)127; A262(1961)237; Nucl. Phys. 31(1962)550,556.

2. E.Witten, Nucl. Phys. B160(1979)57; B223(1983)422; B223(1983)433.

3. G.S.Adkins and C.R.Nappi, Nucl. Phys. B233(1984)109.

4. G.S.Adkins, C.R.Nappi and E.Witten, Nucl. Phys. B228 (1983)552.

5. Bing An Li and Qi Xing Shen. Commun. in Theor. Phys. 6(1986)65.

6. J.D.Breit and C.R.Nappi, Phys. Rev. Lett. 53(1984)889; U.B.Kaufuss and U.G.Meissner, Phys. Lett. 154B(1985)193; K.F.Liu, J.S.Zhang and G.R.E.Black, Phys. Rev. D30(1984) 2015.

7. A.Jackson et al., Phys. Lett. 154B(1985)101.

8. J.R.Wen and Tao Huang, High Energy Physics and Nuclear Physics, 12(1988)286;
   Bing An Li and Qi Xing Shen, High Energy Physics and Nuclear Physics, 12(1988)616.

9. M.Oka, K.F.Liu and Hong Yu, Phys. Rev. D34(1986)1575;
   M.Lacombe, B.Loiseau, R.Vinh and W.N.Cottingham, Phys. lett. 169B(1986)121; Phys. Lett. 161B(1985)31;
   A.Jackson, A.D.Jackson and V.Pasquier, Nucl. Phys. A432(1985) 567;
   U.-G.Meissner and U.B.Kaulfuss, Phys. Rev. C30(1984)2058.

10. J.F.Donognue, E.Golowich and B.R.Holstein, Phys. Rev. Lett. 53(1984)747.

11. Bing An Li, High Energy Physics and Nuclear Physics, 11(1987)426.

TABLE 1. Results of the present model compared against the results of original Skyrme model [3] and against experiment.

Physical quantity	Present model	Original model	Experimental value
m_N (MeV)	Input	Input	939
m_Δ (MeV)	Input	Input	1232
m_π (MeV)	Input	Input	138
F_π (MeV)	136	108	186
$\langle r^2\rangle^{1/2}_{I=0}$ (fm)	0.70	0.68	0.72
$\langle r^2\rangle^{1/2}_{I=1}$ (fm)	0.95	1.04	0.88
$\langle r^2\rangle^{1/2}_{M,I=0}$ (fm)	0.66	0.74	0.81
$\langle r^2\rangle^{1/2}_{M,I=1}$ (fm)	0.74	0.80	0.80
$\langle r^2\rangle^{1/2}_{A}$ (fm)	0.53	0.28	0.72
$\langle r^2\rangle^{1/2}_{S}$ (fm)	0.65	0.80	
μ_p	1.98	1.97	2.793
μ_n	-1.22	-1.24	-1.91
g_A	0.75	0.65	1.23
$g_{\pi NN}$	10.4	11.9	13.5
$g_{\pi N\Delta}$	15.6	17.8	20.3
$\mu_{N\Delta}$	2.3	2.3	3.3
σ	50	49	36 ± 20

Stampato in proprio nella tipografia
del Centro Internazionale di Fisica Teorica