IC/88/366

INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS

OFF-DIAGONAL LONG-RANGE ORDER AND SUPERCONDUCTING PARTICLE DENSITY

Gen-Di Pang

Fu-Cho Pu

and

Bao-Heng Zhao

INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

IC/88/366

International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

OFF-DIAGONAL LONG-RANGE ORDER AND SUPERCONDUCTING PARTICLE DENSITY*

Gen-Di Pang ** , Fu-Cho Pu **
International Centre for Theoretical Physics, Trieste, Italy

and

Bao-Heng Zhao
Graduate School, Academia Sinica, Beijing, People's Republic of China.

MIRAMARE - TRIESTE
December 1988

* Submitted for publication.

** Permanent address: Institute of Physics, Academia Sinica, Beijing, People's Republic of China.

ABSTRACT

Relations between the existence of ODLRO in the reduced density matrix ρ_n (for all $n \geq 1$) and that of superconducting currents in a hollow cylinder is discussed. Some concrete behaviours of ρ_n, including the power law decaying behaviour, in the coordinate space representation are considered and the corresponding effective superconducting particle densities are given. Some restraint conditions caused by the existence of the ODLRO (including quasi-ODLRO) have been found and discussions on the minimum unit of flux quantization are also included.

1. INTRODUCTION

In 1962, C.N. Yang introduced the important concept of off–diagonal long–range order (ODLRO) and put forward the following propositions in his paper [1]: a) the phase He II of liquid He is characterized by the existence of ODLRO in ρ_1 for the equilibrium density matrix of the interacting He atoms and b) the superconducting state is characterized by the existence of ODLRO in $< e_1'e_2'|\rho_2|e_1e_2 >$, where e_1, e_2, e_1' and e_2' represent electron states, for the ensemble in thermal equilibrium. After that, some efforts are made to confirm these two propositions or to show a general relation between the existence of ODLRO and that of superconductivity, among them are that of Bloch's [2] and that of Kohn and Sherrington [3], the latter showed qualitatively the bearing of the existence of ODLRO on that of the superfluidity. In this paper we further consider some concrete behaviours of the density matrix ρ_n, including the power law decaying behaviour, to discuss the relations between the existence of ODLRO in ρ_n for any $n \geq 1$ and the appearance of the superconducting currents in a hollow cylinder and to give quantitatively the effective superconducting particle densities. In the following, we first describe the model in Sec.2, then give the relationship between ODLRO (including quasi–ODLRO) and superconductivity in Sec.3, where some restraint conditions caused by the existence of ODLRO (or quasi–ODLRO) are found. Discussions on the minimum unit of flux quantization and the conclusions are included in Secs.4 and 5 respectively.

2. MODEL

Similar to Bloch's model [2], we consider the axially symmetrical equilibrium state of N identical particles obeying either Bose of Fermi statistics with mass m and charge e in a circular cylinder of length L, large compared to the outer radius R and the wall thickness d. We assume d is small not only in comparison with R but also the penetration depth $\lambda << R$. Denote r and z the distance from the cylinder axis and a distance in the direction parallel to the axis, and we use the length $x = r\theta$, instead of the angle θ around the axis, as the third position variable and denote the corresponding tangential direction as the x direction. Since $d << R$, we may write $x = R\theta$. In the system considered, there also exists a static magnetic field, described by a vector potential A with only the x component and being independent of x and z. Due to the axial symmetry, we only need to deal with currents in the x direction, which is given by [2]

$$I' = \frac{e}{2\pi R} \sum_{\ell} \frac{\hbar}{mR} (\ell - \alpha) < \ell|\rho_\ell|\ell > \tag{2.1}$$

where $< \ell|\rho_\ell|\ell > \equiv < a_\ell^+ a_\ell >$ is the equilibrium density matrix with the orbital angular momentum ℓ (in units $\hbar$) around axis and $\alpha = \Phi/\Phi^* = 2\pi RA/[2\pi\hbar c/e]$ is the magnetic flux, contained within this radius R of the cylinder, measured in units of $\Phi^* = 2\pi\hbar c/e$.

If the current I' in the equilibrium state depends on the vector potential A, for instance, they are related by the London equation, then the system is superconducting, since in that case the electric field E is not needed to maintain I' undecaying. Therefore whether the system is superconducting or not is determined by the dependence or independence of I' on A, which in turn is determined by the behaviours of the reduced density matrix ρ_n in the coordinate space representation, as seen below.

3. RELATIONSHIP BETWEEN ODLRO AND SUPERCONDUCTIVITY

Since we only deal with the currents in x direction in the present case, so the other variables r, z and the spin variable η may be traced over, for example

$$< x_1'|\rho_1|x_1 >= \sum_\eta \int\int < x_1' r z \eta|\rho_1|x_1 r z \eta > drdz \tag{3.1}$$

In the ℓ representation, we have

$$< \ell_1|\rho_1|\ell_1 >= \frac{(N-n)!}{(N-1)!} \sum_{\{\ell_i\}}^{2\leq i\leq n} < \ell_1, \ldots, \ell_n|\rho_n|\ell_1, \ldots \ell_n > \tag{3.2}$$

while

$$< \ell_1, \ldots, \ell_n|\rho_n|\ell_1, \ldots \ell_n >= (2\pi R)^{-n} \int \ldots \int dx_1 \ldots dx_n dx_1' \ldots dx_n'$$
$$< x_1, \ldots, x_n|\rho_n|x_1', \ldots x_n' > \exp\{i \sum_i \ell_i (x_i - x_i')/R\}, \tag{3.3}$$

where ℓ_i takes integral value $0, \pm 1 \ldots$ We mention here a useful property of ρ_n, which states

$$< \ldots, \ell_\alpha, \ldots \ell_\beta, \ldots |\rho_n| \ldots, \ell_\alpha, \ldots, \ell_\beta, \ldots >$$
$$=< \ldots, \ell_\beta, \ldots, \ell_\alpha, \ldots |\rho_n| \ldots, \ell_\beta, \ldots, \ell_\alpha, \ldots > \tag{3.4}$$

From (2.1) and (3.2–3) we see that I' is related to $< x_1, \ldots, x_n|\rho_n|x_1' \ldots x_n' >$, the explicit form of which can only be derived from microscopic theory for given system. Here, will not give the derivation of the explicit form but investigate the problem of the existence of the superconducting currents in the system, provided the behaviours of $< x_1, \ldots, x_n|\rho_n|x_1', \ldots, x_n' >$ in the coordinate space representation are known. We first discuss a perfect case.

1. Perfect case

Assume

$$< x_1,\ldots,x_n|\rho_n|x_1',\ldots,x_n' > \equiv g_n(X,\{\xi_i\},\{\xi_i'\}) \\ = g_n(0,\{\xi_i\},\{\xi_i'\})\exp(-ip_nX/R), \qquad (3.5)$$

where $X \equiv x_1 - x_1'$, $\xi_i \equiv x_i - x_1$ and $\xi_i' \equiv x_i' - x_1'$ for $2 \le i \le n$. Clearly, ρ_n has ODLRO as defined by Yang [1]. From (3.3) and (3.5) we now have

$$< \ell_1,\ldots,\ell_n|\rho_n|\ell_1,\ldots\ell_n >= \\ \Delta(M_n - p_n) < \ell_1,\ldots\ell_n|\rho_n|\ell_1,\ldots\ell_n > \qquad (3.6)$$

where $M_n = \sum_{i=1}^n \ell_i$ and $\Delta(y)$ is the Kronecker delta. Eq.(3.6) means the distribution function in the n-dimensional space spanned by axes $\ell_1,\ldots\ell_n$, $< \ell_1,\ldots,\ell_n|\rho_n|\ell_1,\ldots,\ell_n >$ may take non zero value only on the $(n-1)$-dimensional curve $\sum_i \ell_i = p_n$. Now we have the following theorems:

Theorem 1 If $< \ell_1,\ldots,\ell_{2n}|\rho_{2n}|\ell_1,\ldots,\ell_{2n} >$

$$= \Delta(M_{2n} - p_{2n}) < \ell_1,\ldots,\ell_{2n}|\rho_{2n}|\ell_1,\ldots,\ell_{2n} >$$

then

$$\rho_n(p_n) = \sum_{\{\ell_i\}}^{1\le i\le n} < \ell_1,\ldots,\ell_n|\rho_n|\ell_1,\ldots,\ell_n > \\ \Delta(M_n - p_n) = \rho_n(-p_n + p_{2n}) \qquad (3.7)$$

Proof

$$\rho_n(p_n) = \sum_{\{\ell_i\}}^{1\le i\le 2n} < \ell_1,\ldots\ell_{2n}|\rho_{2n}|\ell_i,\ldots,\ell_{2n} > \\ \{(N-2n)!/(N-n)!\}\Delta(M_n - p_n)\Delta(M_{2n} - p_{2n}) \\ = \sum_{\{\ell_i\}}^{1\le i\le 2n} < \ell_1,\ldots,\ell_{2n}|\rho_{2n}|\ell_1,\ldots,\ell_{2n} > \\ \frac{(N-2n)1}{(N-n)!}\Delta(\sum_{i=1}^n \ell_i - p_n)\Delta(\sum_{i=n+1}^{2n} \ell_i + p_n - p_{2n}) \ . \\ = \sum_{\{\ell_i\}}^{\ell\le i\le 2n} < \ell_1,\ldots,\ell_{2n}|\rho_{2n}|\ell_1,\ldots,\ell_{2n} > \\ \frac{(N-2n)!}{(N-n)!}\Delta(\sum_{i=n+1}^{2n} \ell_i - p_{2n} + p_n)\Delta(\sum_{i=1}^n \ell_i - p_{2n}) \\ = \rho_n(-p_n + p_{2n})$$

In particular, when $n = 1$, we have

$$< \ell_1|\rho_1|\ell_1 >=< -\ell_1 + p_2|\rho_1| - \ell_1 + p_2 > \qquad (3.8)$$

that means the density of particles at state ℓ_1 must equal to that at $-\ell_1 + p_2$, which indicates the forming of a pair between these two states.

Theorem 2 If

$$< \ell_1,\ldots,\ell_n|\rho_n|\ell_1,\ldots,\ell_n > \\ = \Delta(M_n - p_n) < \ell_n,\ldots,\ell_n|\rho_n|\ell_1,\ldots,\ell_n >$$

then

$$\sum_\ell \ell < \ell|\rho_1|\ell >= Np_n/n \ . \qquad (3.9)$$

Proof

$$\sum_\ell \ell < \ell|\rho_1|\ell > = \frac{(N-n)!}{(N-1)!}\sum_{\{\ell_i\}}^{1\le i\le n} \ell_1 < \ell_1,\ldots,\ell_n|\rho_n|\ell_1,\ldots,\ell_n > \\ = \frac{(N-n)!}{(N-1)!}\sum_{\{\ell_i\}}^{1\le i\le n} < \ell_1,\ldots,\ell_n|\rho_n|\ell_1,\ldots,\ell_n > \\ (\ell_1 + \ldots + \ell_n)/n \\ = Np_n/n$$

Here we have used Eq.(3.4). From Theorem 2 and Eq. (2.1) we have

$$I' = -e\hbar(2\pi R^2 m)^{-1}(\alpha - p_n/n)N \qquad (3.10)$$

Since $p_n = \sum_{i=1}^n \ell_i$ takes integral values, I' is generally not zero in value (for normal systems I' is always equal to zero in equilibrium states), unless α equals $p_n/n = 0$, $\pm 1/n$, $\pm 2/n, \ldots$, which indicates the flux quantization as shown by Yang [1]. The minimum unit of flux quantization is 1/n, where n is the smallest n for which ODLRO occurs and the reason will be given in Sec. 4.

Eq.(3.10) is actually the London equation with the effective number of superconducting particles N_s equal to the total number of particles N. However, there are non-perfect cases with $N_s < N$, which will be discussed below.

2. Non–perfect cases

Now assume *) $g_n(X,\{\xi_i\},\{\xi_i'\}) = \beta_n Q_n \exp\{-ip_n X/R\}$

$$g_n(0,\{\xi_i\},\{\xi_i'\})/X^{r_n} \text{ when X is large} \tag{3.11}$$

where $Q_n = (1-r_n)(2\pi R/N)^{r_n}$, $r_n \neq 1$ and $0 < \beta_n \leq 1$.

In the case when $r_n = 0$, ρ_n also has ODLRO as defined by Yang [1]. For $r_n \neq 0$, $g_n(X,\{\xi_i\},\{\xi_i'\}) \to 0$ when $X \to \infty$. We will show below that when $0 < r_n < 1$, there still exists a weak macroscopic superconducting current. ρ_n is said to have quasi–ODLRO in this case.

Define $\bar{g}_n(X,\{\xi_i\},\{\xi_i'\}) = \beta_n f(X) g_n(0,\{\xi_i\},\{\xi_i'\})$

$$\exp(-ip_n X/R), \tag{3.12}$$

where

$$f(X) \equiv \begin{cases} 1, & \text{when } r_n \neq 0 \text{ and } |X| < Q_n^{1/r_n} \equiv c \\ Q_n/|X|^{r_n}, & \text{when } r_n \neq 0 \text{ and } |X| \geq c \\ 1, & \text{when } r_n = 0 \end{cases}$$

then

$$g_n(X,\{\xi_i\},\{\xi_i'\}) = \bar{g}_n(X,\{\xi_i\},\{\xi_i'\}) + \Delta g_n(X,\{\xi_i\},\{\xi_i'\}) \tag{3.13}$$

where

$$\begin{aligned} \Delta g_n(X,\{\xi_i\},\{\xi_i'\}) &\equiv g_n(X,\{\xi_i\},\{\xi_i'\}) \\ &\quad - \bar{g}_n(X,\{\xi_i\},\{\xi_i'\}) \\ &\to 0, \text{ when X is large } (X \geq a \text{ finite number } \bar{A}) \end{aligned}$$

Since

$$\begin{aligned} \frac{N!}{(N-n)!} &= \sum_k \sum_{\{\ell_i\}}^{1\leq i\leq n} < \ell_1,\ldots,\ell_n|\rho_n|\ell_1,\ldots,\ell_n > \Delta(M_n - k) \\ &= \sum_k [2\pi R]^{-1} \int \ldots \int dx_1' dX d\xi_2 \ldots d\xi_n \\ &\quad [g_n(X,\{\xi_i\},\{\xi_i\}) + \Delta g_n(X,\{\xi_i\},\{\xi_i\})] \exp(ikX/R) \\ &= \{\sum_k \beta_n N^{(1-r_n)} \Delta(k-p_n) + \sum_k F_n(k)\} \frac{(N-n)!}{(N-n)!} \end{aligned} \tag{3.14}$$

*) Conditions for the existence of persistent currents may be weaker and in fact we only need

$$\int \ldots \int d\xi_2 \ldots d\xi_n g_n(X,\{\xi_i\},\{\xi_i\})$$

$$= \beta_n Q_n \exp\{-ip_n X/R\}/X^{r_n} \frac{N!}{2\pi R(N-n)!}$$

in the following discussions, where $0 \leq r_n < 1$ and $0 < \beta_n \leq 1$.

when $R \to \infty$, where

$$F_n(k) = \frac{1}{2\pi R} \int \ldots \int dx_1' dX d\xi_2 \ldots d\xi_n \frac{(N-n)!}{(N-1)!}$$

$$\Delta g_n(X,\{\xi_i\},\{\xi_i\}) \exp(ikX/R).$$

If we assume

$$\Delta g_n(X,\{\xi_i\},\{\xi_i\}) \leq B g_n(0,\{\xi_i\},\{\xi_i\}) \tag{3.15}$$

where B is a finite number, then we have

$$\begin{aligned} F_n(k) &= \int_0^{\bar{A}} dX \{2\pi R\}^{-1} \int \ldots \int dx_1' d\xi_2 \ldots d\xi_n \frac{(N-n)!}{(N-1)!} \\ &\quad \Delta g_n(X,\{\xi_i\},\{\xi_i\}) \exp(ikX/R) \\ &< BN \int_0^{\bar{A}} dX \exp(ikX/R)/(2\pi R) << \beta_n N^{1-r_n} \end{aligned} \tag{3.16}$$

when $0 \leq r_n < 1$ and $N \to \infty$. Eq.(3.14 – 16) means that the distribution function

$$\sum_{\{\ell_i\}}^{1\leq i\leq n} < \ell_1 \ldots, \ell_n|\rho_n|\ell_1, \ldots \ell_n > \Delta(M_n - k)$$

has a very high peak when $k = p_n$, and we call this phenomenon the condensation of p_n in the n–dimensional angular momentum space and denote the height of the peak as the condensation quantity, which equals to $\beta_n N^{1-r_n}$ in the present case. Similar to Theorems 1 and 2, we can easily prove the following two theorems:

Theorem 3 If $\sum_{\{\ell_i\}}^{1\leq i\leq n} < \ell_1, \ldots \ell_{2n}|\rho_{2n}|\ell_1, \ldots, \ell_{2n} >$

$$\Delta(M_{2n} - p_{2n}) = \frac{(N-1)!}{(N-2n)!} \beta_{2n} N^{1-r_{2n}}$$

then

a)

$$\begin{aligned} \tilde{\rho}_n(p_n) &\equiv \frac{(N-2n)!}{(N-n)!} \sum_{\{\ell_i\}}^{1\leq i\leq 2n} < \ell_1, \ldots, \ell_{2n}| \\ &\quad \tilde{\rho}_{2n}|\ell_1, \ldots, \ell_{2n} > \Delta(M_n - p_n)\Delta(M_{2n} - p_{2n}) \\ &= \tilde{\rho}_n(-p_n + p_{2n}) \end{aligned}$$

b)

$$\sum_{p_n} \tilde{\rho}_n(p_n) = \frac{(N-1)!}{(N-n)!}\beta_{2n} N^{(1-r_{2n})} \tag{3.17}$$

Theorem 4 If

$$\sum_{\{\ell\}}^{1\le i\le n} < \ell_1, \dots \ell_n|\rho_n|\ell_1, \dots, 1_n >$$

$$\Delta(M_n - p_n) = \frac{(N-1)!}{(N-n)!}\beta_n N^{(1-r_n)}$$

then

a)

$$\sum_\ell \ell < \ell|\tilde{\rho}_1|\ell > = \beta_n N^{(1-r_n)} p_n/n$$

b)

$$\sum_\ell < \ell|\tilde{\rho}_1|\ell > = \beta_n N^{(1-r_n)} \tag{3.18}$$

where

$$< \ell|\tilde{\rho}_1|\ell > \equiv \frac{(N-n)!}{(N-1)} \sum_{\{\ell_n\}}^{1\le i\le n} < \ell_1, \dots, \ell_n|\rho_n|\ell_1, \dots, \ell_n >$$

$$\Delta(M_n - p_n)$$

Theorem 4 means that the condensation of ρ_n (for $0 \le r_n < 1$) in the n–dimensional momentum space leads to the restraint condition, on the movement of the particles, that is, effectively, angular momentum a fraction $\beta_n N^{(1-r_n)}$ of particles' only takes discrete values $\beta_n N^{(1-r_n)} p_n$ (since p_n only takes integral values). We say, the movement of this fraction of particles is now restrained. This is vitally important for the existence of the macroscopic superconducting currents, as shown by the following relations:

$$< \ell|\rho_1|\ell > = < \ell|\tilde{\rho}_1|\ell > + < \ell|(\rho_1 - \tilde{\rho}_1)|\ell >$$

and

$$I' = e\hbar(2\pi R^2 m)^{-1} \sum_\ell (\ell - \alpha) < \ell|\rho_1|\ell >$$

$$= -e\hbar(2\pi R^2 m)^{-1}(\alpha - p_n/n)\beta_n N^{(1-r_n)}$$

$$+ e\hbar(2\pi R^2 m)^{-1} \sum_\ell (\ell - \alpha) < \ell|\rho_1 - \tilde{\rho}_1|\ell >$$

$$\equiv H(n) + J(n) \tag{3.19}$$

Because of the restraint condition, the first term $H(n)$ in (3.19) is, like Eq.(3.10), generally not zero and have a macroscopic contribution to the superconducting current I' when

$0 \le r_n < 1$, since $\beta_n N^{(1-r_n)}$ is still a macroscopic quantity. As for the second term $J(n)$, if there is no restraint conditions like Eq.(3.18) for the second part of particles $\sum_\ell < \ell|(\rho_1 - \tilde{\rho}_1)|\ell >$, then the second term has no contributions to the current I', $J(n)$ is zero as that for the normal particles (a normal system has no ODLRO or quasi–ODLRO for all ρ_n, and, hence, has no restraint conditions and $\sum_\ell \ell < \ell|\rho_1|\ell > = 0$). In that case we have

$$I' = -e\hbar(2\pi R^2 m)^{-1}(\alpha - p_n/n)\beta_n N^{(1-r_n)}, \tag{3.20}$$

which is also London's equation with $N_s = \beta_n N^{(1-r_n)} < N$. However, there may exist ODLRO or quasi–ODLRO for $\rho_{n'}$, when $n' > n$, $\rho_{n'}$ may be condensed in the n'–dimensional angular momentum space with a larger value of the condensation quantity. So it is possible that the movement of more particles are restrained by a similar restraint condition (it is reasonable to assume $p_{n'}/n'$ takes the same discrete values as p_n/n does, some discussions are given in Sec.4), then N_s should be the largest effective number of particles whose movement is restrained. So the effective superconducting particles can be generally written as

$$N_s \ge \beta_n N^{(1-r_n)} \quad \text{for} \quad 0 \le r_n < 1 \tag{3.21}$$

From the above discussions, we see that superconductivity is caused by the macroscopic condensation of ρ_n in the n–dimensional angular momentum space spanned by $\ell_1, \dots, \ell_n$. For the present cases discussed, we need $0 \le r_n < 1$ and $0 < \beta_n \le 1$ to observe the macroscopic superconducting currents in the system.

4. THE MINIMUM UNIT OF FLUX QUANTIZATION

Yang [1] pointed out that the minimum unit of flux quantization is $1/n$, where n is the smallest n for which ODLRO occurs. Here, we give some discussions on the basis of the following theorem

Theorem 5 If

$$\sum_{\{\ell_i\}}^{1\le i\le n} < \ell_1, \dots, \ell_n|\rho_n|\ell_1, \dots \ell_n >$$

$$\Delta(M_n - p_n) = \lambda_n \quad \text{in order of} \quad N^n(\sim N^n)$$

$$\text{then} \quad \sum_{\{\ell_i\}}^{1\le i\le 2n} < \ell_1, \dots, \ell_{2n}|\rho_{2n}|\ell_1, \dots \ell_{2n} >$$

$$\Delta(M_{2n} - p_{2n}) \sim N^{2n} \quad \text{for} \quad p_{2n} = 2p_n$$

Here we have assumed $N \to \infty$. Theorem 5 can be easily confirmed by using the following relation

$$Tr[\sum_{\{\ell_i\}}^{1\le i\le n} a^+_{\ell_n}\dots a^+_{\ell_1} \quad a_{\ell_1}\dots a_{\ell_n}\Delta(M_n-p_n)-\lambda_n]^2>0$$

where

$$<\ell_1,\dots,\ell_n|\rho_n|\ell_1,\dots,\ell_n>\equiv Tra^+_{\ell_n}\dots a^+_{\ell_1}a_{\ell_1}\dots a_{\ell_n}$$

Since

$$\sum_{\{\ell_i\}}^{1\le i\le n}<\ell_1,\dots,\ell_n|\rho_n|\ell_1,\dots,\ell_n>=\frac{N!}{(N-n)!}\sim N^n\ ,$$

Theorem 5 means that if ρ_n has ODLRO or it is condensed in the n–dimensional momentum space, then ρ_{2n} will also be condensed in the 2n–dimensional momentum space with $\sum_{i=1}^{2n} l_i = 2p_n$, or ρ_{2n} has also ODLRO. The important thing is $p_{2n}=2p_n$ only takes value $0,\pm2,\pm4$, if p_n takes $0,\pm1,\pm2\ \dots$ Therefore, if we use ρ_{2n}, instead of p_n, to discuss the unit of flux quantization, then the unit of flux quantization is still 1/n, not 1/2n, where n is the smallest n for which ODLRO occurs.

5. CONCLUSION

From a simple model, we show the relations between the existence of ODLRO (including quasi-ODLRO) in ρ_n for any $n\ge1$ and that of the superconducting currents. Some concrete behaviours of ρ_n are considered. It is indicated from our discussions that the superconductivity is caused by the macroscopic condensation of ρ_n in the n–dimensional momentum space, which happens when $0<\beta_n\le1$ and $0\le r_n<1$ for the present discussed cases. The effective number of superconducting particles N_s equals the largest number of particles whose movement is restrained by the restraint condition caused by the existence of ODLRO or quasi-ODLRO in ρ_n's. It is also argued why the minimum unit of flux quantization is $1/n$ instead of $1/2n$, where n is the smallest n for which ODLRO occurs.

Acknowledgments

Two of the authors (G.D.P. and F.C.P.) would like to thank to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. G.D. P. would also like to thank Professor Z.Q. Zhang for helpful discussions.

REFERENCES

[1] C.N. Yang, Rev. Mod. Phys. **34**, 694 (1962).

[2] F. Bloch, Phys. Rev. **137**, A787 (1965).

[3] W. Kohn and D. Sherrington, Rev. Mod. Phys. **42**, 1 (1970).

Stampato in proprio nella tipografia
del Centro Internazionale di Fisica Teorica