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DYNAMICS OF SYMMETRY BREAKING IN STRONGLY COUPLED QED

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Dedicated to the memory of
Heinz Pagels
Physicist, Philospher, Friend

ABSTRACT

I review the dynamical structure of strong coupled QED in the quenched, planar limit. The symmetry structure of this theory is examined with reference to the nature of both chiral and scale symmetry breaking. The renormalization structure of the strong coupled phase is analysed. The compatibility of spontaneous scale and chiral symmetry breaking is studied using effective lagrangian methods.

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1. MOTIVATIONS.

Gauge theories with slowly running coupling constants appear to have an approximate scale symmetry. Hence, the dynamical breaking of chiral symmetry in such models[1] should be associated with an approximate dynamical breaking of scale symmetry[2]. Quenched, planar QED has no perturbative running and provides an interesting laboratory for the study of dynamical symmetry breaking. The possible existence of a nonperturbative fixed point with large anomalous dimensions in this theory has led to new speculations about the role of technicolor models[3] and properties of fixed point gauge field theories.

2. DYNAMICS OF QUENCHED, PLANAR QED.

Quenched, planar QED contains the basic dynamical structure of gauge field theory and may represent an approximate treatment of QED and/or nonabelian gauge theories with slowly running couplings. Dynamical symmetry breaking is studied through the Schwinger-Dyson equations

$$S_F^{-1}(p) = p - \Sigma(p) = p - m_0 - i(4\pi)^{-4}\int dk\ \{D_{\mu\nu}(k)\cdot e\gamma^{\mu}\cdot S_F(p-k)\cdot e\gamma^{\nu}\} \quad (1)$$

Solutions to these equations have been extensive studied over the years[4,5,6]. There is a unique infrared solution[6,2] to the equations having a dynamical mass scale, $\Sigma(0) = \Sigma_0 \neq 0$,

$$\Sigma(p) = e^t \cdot u(t+t_0), \quad t = \log(p), \quad t_0 = -\log(\Sigma_0) \quad (2)$$

where $u(x)$ satisfies $e^x \cdot u(x) \to 1$ as $x \to -\infty$. The ultraviolet behavior of this solution depends on the gauge coulping constant, α. For strong coupling, $\alpha > \alpha_C = \pi/3$, $u(x)$ is given by

$$\begin{aligned} u(x) &\to A(\alpha)\cdot e^{-2\cdot x}\cdot \sin[\sqrt{\alpha/\alpha_C - 1}\cdot(x+\delta(\alpha))] \\ &\approx \tilde{A}(\alpha)\cdot e^{-2\cdot x}\cdot\{\sin[\sqrt{\alpha/\alpha_C-1}\cdot(x+\delta(\alpha))]/\sqrt{\alpha/\alpha_C-1}\,\} \end{aligned} \quad (3)$$

where $\tilde{A}(\alpha) \approx 1.2$, $\delta(\alpha) \approx .55$ for $\alpha \approx \alpha_C$. For weak coupling, $\alpha < \alpha_C$,

$$u(x) \to \tilde{A}(\alpha) \cdot e^{-2 \cdot x} \cdot \{\sinh[\sqrt{1-\alpha/\alpha_c} \cdot (x + \delta(\alpha))]/\sqrt{1-\alpha/\alpha_c}\} \tag{4}$$

where $u(x,\alpha)$ is analytic in α for fixed x and $\alpha \approx \alpha_c$.

The weak coupling solution has pure power behavior

$$\Sigma(p) = p \cdot u(\log(p/\Sigma_0)) \to m/(p)^{\gamma_m} + \langle\overline{\Psi}\Psi\rangle/(p)^{2-\gamma_m} \tag{5}$$

reflecting the scaling structure[7] of the operator product expansion of the fermion propagator and the finite anomalous dimension of the fermion mass operator, $\overline{\Psi}\Psi$, given by $\gamma_m = 1 - \sqrt{1-\alpha/\alpha_c}$. The presence of both terms in the expansion of Eq.(4) confirms the result[5] that there are no massive solutions in the chiral limit, $m = 0$, for weak coupling.

For strong coupling, we must use an ultraviolet cutoff, Λ, and examine the boundary condition for the bare mass[4,5]

$$m_0 = (\Lambda/2) \cdot [u' + 3 \cdot u](t_\Lambda + t_0), \quad t_\Lambda = \log(\Lambda) \tag{6}$$

In the chiral limit, $m_0 = 0$, there exists a massive solution with scale

$$\Sigma_0 = e^{\delta} \cdot \Lambda \cdot e^{-(\theta/\sqrt{\alpha/\alpha_c - 1})}, \quad 0 < \theta < \pi \tag{7}$$

This solution will correspond to dynamical chiral symmetry breaking with a finite fermion mass scale, Σ_0, only in the Miransky limit[8] with

$$\alpha = \alpha(\Lambda) \to \alpha_c + \alpha_c \cdot \theta^2/\log^2(\Lambda/\kappa), \quad \theta \to \pi, \quad \text{as } \Lambda \to \infty \tag{8}$$

However this solution is incomplete as it neglects four fermion operators which are generated from the gauge interactions. Some four fermion operators have large anomalous dimensions in the planar limit, $d(\overline{\Psi}\Psi)^2 = 6 - 2 \cdot \gamma_m \to 4$, $\alpha \to \alpha_c$, and are relevant (or marginal) operators in the continuum limit. Hence we must consider instead a modified "Nambu-Jona-Lasinio" model[9] where the gauge interactions are included in the lagrangian

$$L_{MNJL} = \overline{\Psi}\{iD - \mu_0\}\Psi + (G_0/2) \cdot [(\overline{\Psi}\Psi)^2 + (\overline{\Psi} i\gamma_5 \Psi)^2] \tag{9}$$

In planar approximation, the Schwinger-Dyson equations are simply modified by the inclusion of a fermion tadpole contribution to the bare mass term which changes the mass boundary condition of Eq.(6)

$$m_0 = \mu_0 - G_0 \cdot \langle\overline{\Psi}\Psi\rangle = (\Lambda/2) \cdot [u' + 3\cdot u](t_\Lambda + t_0) \qquad (10)$$

and

$$\langle\overline{\Psi}\Psi\rangle = (1/2\pi^2)\cdot(\alpha_C/\alpha)\cdot e^{3\cdot t_\Lambda} \cdot [u'+u](t_\Lambda + t_0) \qquad (11)$$

The gap equation is modified to read, $G \equiv (G_0 \cdot \Lambda^2/\pi^2)\cdot(\alpha_C/\alpha)$,

$$\mu_0 \cdot \Lambda = (\Lambda^2/2) \cdot [(1+G)\cdot u' + (3+G)\cdot u](t_\Lambda + t_0)$$

$$= (\tilde{A}/2) \cdot \Sigma_0^2 \cdot [(1-G)\cdot \sin(\theta)/\sqrt{\alpha/\alpha_C - 1} + \cos(\theta)], \quad \alpha > \alpha_C \qquad (12)$$

$$\theta = \sqrt{\alpha/\alpha_C - 1} \cdot (\log(\Lambda/\Sigma_0) + \delta)$$

$$= (\tilde{A}/2) \cdot \Sigma_0^2 \cdot [(1-G)\cdot \sinh(\tilde{\theta})/\sqrt{1-\alpha/\alpha_C} + \cosh(\tilde{\theta})], \quad \alpha < \alpha_C \qquad (13)$$

$$\tilde{\theta} = \sqrt{1-\alpha/\alpha_C} \cdot (\log(\Lambda/\Sigma_0) + \delta)$$

For $\alpha > \alpha_C$, the vacuum solution requires $0 < \theta < \pi$ as before.

In our planar approximation, the full scattering amplitudes are modified by the four fermion interactions which generate the bubble diagrams of the NJL model dressed by the radiative corrections of the gauge interactions. For our calculation we need to know the full, dressed vertex functions as well as the bubble functions. Fortunately we are able to compute the exact solutions in terms of the asymptotic behavior of the self-energy function, $u(x)$ [2]. The results for the scalar and pseudoscalar vertex and bubble functions are

$$\Gamma_S{}^0(p,p) = \partial_{m_0}\Sigma(p) = e^{t_\Lambda} \cdot u'(t_\Lambda + t_0)/(\partial m_0/\partial t_0)$$

$$= \Gamma_S{}^R(p,p)/Z_S, \quad Z_S = -\, e^{t_0} \cdot (\partial m_0/\partial t_0) \qquad (14)$$

$$\Gamma_p{}^0(p,p) = \Sigma_0(p)/m_0 = e^{t_\Lambda} \cdot u(t_\Lambda + t_0)/m_0$$
$$= \Gamma_p{}^R(p,p)/Z_p, \quad Z_p = e^{t_0} \cdot m_0 \tag{15}$$

$$B_S(0) = \partial_{m_0} \langle \overline{\Psi}\Psi \rangle = \partial_{t_0} \langle \overline{\Psi}\Psi \rangle / (\partial m_0/\partial t_0)$$
$$= -(1/2\pi^2) \cdot (\alpha_C/\alpha) \cdot e^{(3 \cdot t_\Lambda + t_0)} \cdot [u'' + u'](t_\Lambda + t_0)/Z_S \tag{16}$$

$$B_p{}^0(0) = \langle \overline{\Psi}\Psi \rangle / m_0$$
$$= (1/2\pi^2) \cdot (\alpha_C/\alpha) \cdot e^{(3 \cdot t_\Lambda + t_0)} \cdot [u' + u](t_\Lambda + t_0)/Z_p \tag{17}$$

The gap equations, Eqs.(12,13) now have nontrivial solutions for all values of the gauge coupling constant, α: $G = G(\alpha, \Lambda/\Sigma_0)$. We may now search for scale invariant fixed points. We might expect to find a continuum limit for general values of the coupling, G_0, but only particular values on the induced four fermion interactions may preserve the scale invariance as was the case for scale invariant $\eta\varphi^6$ theory[10]. Actually, no scale invariant fixed point was found[2] and the apparent scale symmetry of quenched, planar QED is broken even when the induced four fermion interactions are incorporated.

However, the continuum limit is modified by the presence of the four fermion interactions. There is now a nontrivial continuum limit along a critical line, $G = G(\alpha) > 1$ and $\alpha < \alpha_C$, as emphasized by a number of authors[11]. The existence of this critical line at weak coupling is somewhat surprizing as the effective anomalous dimensions of the four fermion operators should make them irrelevant at weak coupling, $d(\overline{\Psi}\Psi)^2 = 6 - 2 \cdot \sqrt{1 - \alpha/\alpha_C} > 4$. This result for weak coupling may be an artifact of the factorization properties of the planar approximation, although it may also indicate an interesting renormalizable phase of the gauged, Nambu - Jona-Lasinio model.

3. DILATONS: FACT OR FANCY.

The dynamical generation of the fermion mass scale, Σ_0, breaks both chiral and scale symmetry. If these symmetries are not explicitly broken, then we expect corresponding Goldstone pole in the appropriate S-matrix elements. With the inclusion of the four fermion interactions, we expect the Goldstone poles to come from the bubble sums and not the ladder diagrams of the pure gauge theory. Therefore, we must examine the renormalized bubble denominators for zeros reflecting the existence of Goldstone poles in the full amplitudes. The renormalized scalar and pseudoscalar denominator functions are given by

$$D_P{}^R(0) = Z_P{}^2 \cdot (1/G_0 + B_P{}^0(0)) = (1/4\pi^2) \cdot \tilde{A}^2 \cdot \Sigma_0{}^2 \cdot (\alpha_C/\alpha) \cdot (1/G)$$
$$\cdot\{\sin(\theta)/\sqrt{\alpha/\alpha_C - 1} + \cos(\theta)\}\cdot\{(1-G)\cdot\sin(\theta)/\sqrt{\alpha/\alpha_C - 1} + (1+G)\cdot\cos(\theta)\}$$
$$= 0 \quad \text{(chiral limit)} \tag{18}$$

$$D_S{}^R(0) = Z_P{}^2 \cdot (1/G_0 + B_P{}^0(0)) = (1/4\pi^2) \cdot \tilde{A}^2 \cdot \Sigma_0{}^2 \cdot (\alpha_C/\alpha) \cdot (1/G)$$
$$\cdot\{(1+\alpha/\alpha_C)\cdot\sin(\theta)/\sqrt{\alpha/\alpha_C - 1} + \cos(\theta)\} \cdot\{[(2-G)+(1+G)\cdot(\alpha/\alpha_C - 1)$$
$$\cdot\sin(\theta)/\sqrt{\alpha/\alpha_C - 1} + (1+3\cdot G)\cdot\cos(\theta)\}$$
$$= (1/4\pi^2) \cdot \tilde{A}^2 \cdot \Sigma_0{}^2 \cdot \{2\cdot\alpha_C/\alpha + 1 + 1/G\} \quad \text{(chiral limit)} \tag{19}$$

The above formula for the scalar denominator in the chiral limit is valid in both weak and strong coupling. Clearly, the scalar denominator does not vanish at strong or weak coupling confirming previous results[2,8,12], but condradicting recent claims of a fixed point along the critical line[13]. There is a spurious vanishing for repulsive coupling, $G < 0$, which is due to the vanishing of the scalar vertex renormalization factor, Z_S, but this pole in the bubble sum is exactly cancelled by a related pole in the ladder diagrams with no resulting singularity in the S-matrix elements.

There is, however, an interesting partial cancelation in the scalar denominator function. Dimensional analysis, with the known anomalous dimensions, would predict that the scalar denominator should diverge with the cutoff

$$D_S{}^R(0) \to \Lambda^{2\cdot\sqrt{1-\alpha/\alpha_C}} \to \infty \qquad (20)$$

as the four fermion operators should be irrelevant at weak coupling. Instead, the scalar denominator remains finite along the critical line[11] as prevously discussed. This behavior may be an artifact of the factorization treatment of the four fermion interactions or may imply an interesting weak coupling, renormalizable phase of the full theory.

A final possibility would be that the observed behavior at zero momentum represents a decoupled dilaton, much like the pseudoscalar Goldstone boson in the normal NJL model. In this situation we could have $F^2{}_{\mathrm{dilaton}} \to \infty$ and $m^2{}_{\mathrm{dilaton}} \to 0$ as $\Lambda \to \infty$. To rule out this possibility, we must compute the momentum derivative of the scalar denominator, $\partial_{p^2} D_S{}^R(p^2)_{p^2=0} \approx F^2{}_{\mathrm{dilaton}}$, to determine whether it remains finite or becomes divergent. It is not possible to get an exact expression for the result, but an estimate of the diagrams indicates that it remains finite and the decoupled dilaton scenario is not viable.

By our analysis of the symmetry structure, the gauged NJL model the chiral symmetry is preserved and can be dynamically broken even at weak gauge coupling, $\alpha < \alpha_C$. The scale symmetry is explicitly broken, as in the pure gauge case, and there is no dilaton, or decoupled dilaton, in either theory. The four fermion operators seem to remain as relevant operators even though their physical dimension seems too large, as $d(\overline{\Psi}\Psi)^2 > 4$, although this feature may also be an artifact of the approximations. Finally, it is still possible that a scale invariant theory may exist beyond the quenched, planar theory with the presence of additonal relevant interactions.

4. RENORMALIZATION.

In this section we discuss the renormalization properties of the quenched, planar theory. The continuum limit seems to require a particular cutoff dependence of the bare coupling constants. This cutoff dependence would seem to imply nonperturbative β-functions describing the strong coupling phase of the the theory[8,2]. Normally these β-functions would be related to the physical behavior of the amplitudes of the theory through the application of the renormalization group equations. However, the quenched, planar theory does not seem to allow for a dynamical running of the gauge coupling constant and the normal renormalization properties of the theory are brought into question.

We could study the dynamical running of the coupling constants if we could explicitly integrate the high momentum behavior of the theory and study directly the renormalization flow of the coupling constants. Despite the great simplifications of the quenched, planar theory, it would seem to be quite difficult to integrate the high momentum parts of the various rungs of the ladder and self-energy diagrams as in Figure 1. From this diagram, it is clear that the four fermion interactons must be generated from the high momentum structure of the pure gauge theory and must be included in the renormalization flow if these interactions become relevant.

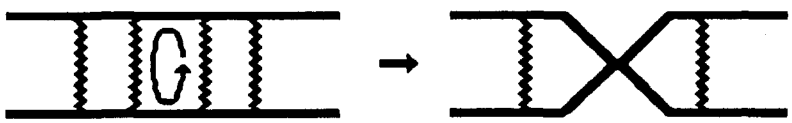

Figure 1: Renormalizaton Diagrams

Although we can not integrate the specific diagrams directly, we can compute their effect indirectly by using the fact that our theory is defined using a momentum cutoff on the fermion self-energy. By varying the cutoff, Λ, while holding the low energy physics constant, we can effectly study the renormalization flow of the coupling constants.

For the gauge coupling constant near the critical coupling, the fermion self-energy function has a nearly universal behavior. For moderate momentum, $\Sigma_0 < p << \Lambda$, we use Eqs.(2,3,4) to obtain,

$$\Sigma(p) \to \tilde{A}(\alpha) \cdot(\Sigma_0{}^2/p) \cdot(\log(p/\Sigma_0) + \delta(\alpha)) \qquad (21)$$

To hold low energy physics fixed we shall use the generated fermion mass scale, Σ_0, as an invariant quantity. An alternative would be the renormalized fermion condensate,

$$\langle\overline{\Psi}\Psi\rangle_R = Z_S \cdot\langle\overline{\Psi}\Psi\rangle = -(1/2\pi^2) \cdot\tilde{A}^2 \cdot(\alpha_C/\alpha) \cdot\Sigma_0{}^3 \qquad (22)$$

where we have used Eqs.(11,14,6) for the pure gauge theory result although exactly the same answer is obtained in the gauged, NJL model. We should be able to use Σ_0, $\langle\overline{\Psi}\Psi\rangle$, and $\Sigma(p)$ as the low energy parameters. This would seem to imply that α must be held fixed during the renormalization flow in agreement with the diagram structure.

The flow of the four fermion coupling constant, G, for constant Σ_0 can be computed directly from the gap equations Eqs.(12,13) in the chiral limit.

$$G = (\tan(\theta)/\sqrt{\alpha/\alpha_C - 1} + 1)/(\tan(\theta)/\sqrt{\alpha/\alpha_C - 1} - 1)$$

$$\rightarrow 1 + 2/(\log(\Lambda/\Sigma_0) + \delta - 1), \quad \tan(\theta) \approx \theta \tag{23}$$

where this limit is valid for both the weak and strong coupling regions. The physical running of the four fermion coupling is required to keep the low energy physics fixed. This flow is shown in Fig.(2) where the bare coupling, $G_0 = G \cdot (\alpha/\alpha_C)$, is plotted as a function of the cutoff.

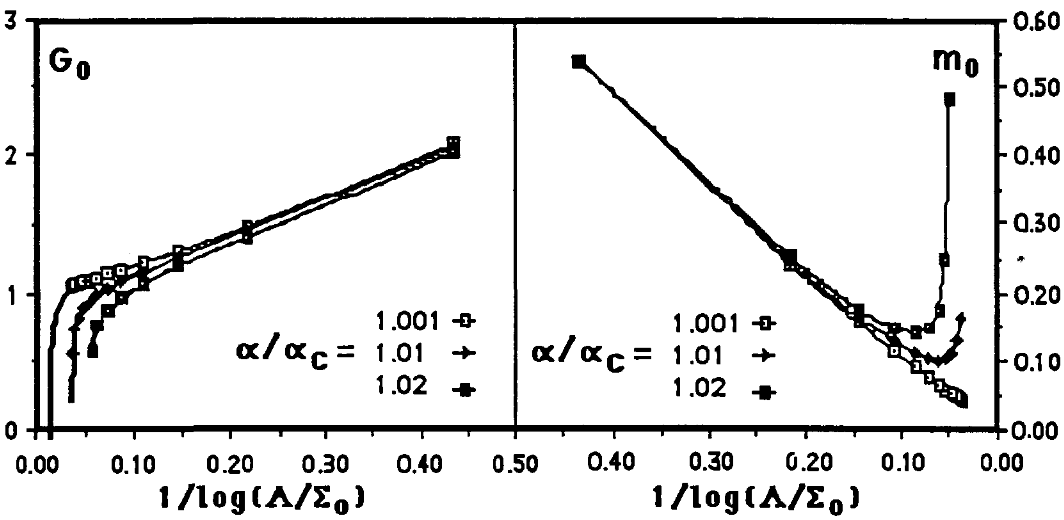

Figure 2: Renormalization Flows G_0 and m_0

The flow is given for various values of the gauge coupling, α, which does not flow with the cutoff in ladder approximation. The flow indicates the presence of an apparent ultraviolet fixed point[2] as $G \rightarrow 1$, $\alpha \rightarrow \alpha_C$. If we require that $G_0(\Lambda)$ be held fixed, for example at the pure gauge theory value: $G_0(\Lambda) = 0$, then a spurious renormalization of the gauge coupling constant would seem to be required to maintain a stable low energy theory. However, all relevant coupling constants must be included if the low energy theory is to be fully renormalized and the four fermion interactions are generated. Away from the chiral limit the bare mass parameter, m_0, will also flow with the cutoff and this dependence is also shown in Figure 2. Actually, the renormalization flow shown for $\alpha > \alpha_C$ can not be maintained to the continuum limit as the vacuum eventually becomes unstable due to short distance effects. However the continuum limit can be achieved for the theory in the range, $0 < \alpha \leq \alpha_C$. We have not investigated the stability of the full low energy effective theory and further running of other relevant coupling constants may be required to achieve a full renormalization of the complete theory.

We have examined the renormalization flow of the quenched, planar theory which requires the dynamical running of the four fermion coupling constants but not the gauge coupling constant. A smooth continuum limit seems to be associated with an ultraviolet fixed point where $G \to 1$ and $\alpha \to \alpha_C$ as well as the apparent critical line at weak coupling[11]. A more complete analysis of the renormalization properties of the full theory is needed to establish the complete continuum limit.

5. EFFECTIVE DYNAMICS OF SPONTANEOUSLY BROKEN SCALE AND CHIRAL SYMMETRIES.

Recently a NO-GO theorem has been proposed[14] which suggests a basic conflict between the low energy theorems for scale and chiral symmetry. We will show, by the explicit construction of an effective lagrangian, that both scale and chiral symmetry can be realized in the Goldstone model in the presence of explicit symmetry breaking of "fermion" mass term. The low energy theorems of both scale and chiral symmetry are shown to be satisfied by the the amplitudes generated by this effective lagrangian.

In a scale invariant gauge theory, only the fermion mass terms should explicitly break the scale symmetry. The divergence of the scale and axial currents should be given by

$$\partial^\mu D_\mu = \Theta_{\mu\mu} = (1+\gamma_m) \cdot \bar{\Psi}\, m\, \Psi$$

$$\partial^\mu A^a{}_\mu = \overline{\Psi}\,\{\lambda^a/2, m\}\cdot i\cdot\gamma_5\,\Psi \tag{24}$$

where m is the fermion mass matrix and γ_m is the anomalous dimension for the fermion mass operator, $\overline{\Psi}\Psi$. Dynamical breaking of both scale and chiral symmetry would imply that Goldstone bosons carry the scale dimension and chirality. We can introduce the Goldstone fields, $\pi(x)$ and $D(x)$, by

$$U(\pi) = \exp\{\, i\boldsymbol{\lambda}\cdot\pi(x)/f_\pi \,\}$$
$$S(D) = \exp\{\, D(x)/F_D \,\} \tag{25}$$

where $U(\pi)$ is a dimensionless matrix with $SU(N)\otimes SU(N)$ flavor symmetry and $S(D)$ is the dimension one, scale field. If the Goldstone bosons saturate the low energy theorems then all operators must have a Goldstone realization in terms of $\pi(x)$ and $D(x)$. The fermion bilinear operator will have the representation

$$\overline{\Psi}_{Rj}\Psi_{Li} = -r_0\cdot F^2{}_\pi\cdot(S(D))^{3-\gamma_m}\cdot\{U(\pi)\}_{ij} \tag{26}$$

where the S factor generates the correct dimension and $U(\pi)$ the correct chirality.

The effective action is given in terms of the nonlinear lagrangian

$$L = (1/2)\cdot F^2{}_D\cdot(\partial_\mu S)^2 + (1/4)\cdot F^2{}_\pi\cdot S^2\cdot \mathrm{tr}\{\, \partial^\mu U^+\partial_\mu U \,\}$$
$$+\, r_0\cdot F^2{}_\pi\cdot(S)^{3-\gamma_m}\cdot \mathrm{tr}\{\, U^+\cdot m + m\cdot U \,\} \tag{27}$$
$$-\,(1/2)\cdot r_0\cdot F^2{}_\pi\cdot(3-\gamma_m)\cdot \mathrm{tr}\{\, m \,\}\cdot S^4$$

where the coupling have been defined so that the classical vacuum state will have $\langle S\rangle_0 = 1$, $\langle\{U\}_{ij}\rangle_0 = \delta_{ij}$ or $\langle D\rangle_0 = \langle\pi\rangle_0 = 0$. The full energy momentum tensor is determined from the above lagrangian to be

$$\Theta_{\mu\nu} = F^2{}_D\cdot\{\, \partial_\mu S\partial_\nu S - (1/2)\cdot g_{\mu\nu}\cdot(\partial_\alpha S)^2 \,\}$$

$$+ (1/4) \cdot F^2_\pi \cdot S^2 \cdot tr\{ \partial_\mu U^+ \partial_\nu U + \partial_\nu U^+ \partial_\mu U - g_{\mu\nu} \cdot \partial^\alpha U^+ \partial_\alpha U \}$$

$$- g_{\mu\nu} \cdot r_0 \cdot F^2_\pi \cdot S^{3-\gamma_m} \cdot tr\{ U^+ \cdot m + m \cdot U \} \tag{28}$$

$$+ g_{\mu\nu} \cdot (1/2) \cdot r_0 \cdot F^2_\pi \cdot (3-\gamma_m) \cdot tr\{ m \} \cdot S^4$$

$$- F^2_D \, (1/6) \cdot \{ \partial_\mu \partial_\nu - g_{\mu\nu} \cdot \partial^2 \} \, S^2$$

where the final term is a necessary "improvement" term. Using the classical field equations derived from Eq.(26), the trace of the energy momentum tensor in Eq.(27) is given by

$$\begin{aligned} \Theta_{\mu\mu} &= -(1+\gamma_m) \cdot r_0 \cdot F^2_\pi \cdot S^{3-\gamma_m} \cdot tr\{ U^+ \cdot m + m \cdot U \} \\ &= (1+\gamma_m) \, \overline{\Psi} \, m \, \Psi \end{aligned} \tag{29}$$

in agreement with Eq.(24). Of course the axial current divergence is also correctly given by using the classical field equations.

We may now directly check the structure of the low energy theorems for scale and chiral symmetry. We can first determine the masses for the Goldstone particles

$$m^2_\pi = 2 \cdot r_0 \cdot (m_i + m_j) = -(1/F^2_\pi) \cdot (m_i + m_j) \cdot \langle \overline{\Psi} \Psi \rangle_0$$

$$\begin{aligned} m^2_D &= 2 \cdot r_0 \cdot (F_\pi / F_D)^2 \cdot tr\{m\} \cdot (3-\gamma_m) \cdot (1+\gamma_m) \\ &= -(1/F^2_D) \cdot (3-\gamma_m) \cdot (1+\gamma_m) \cdot tr\{m\} \cdot \langle \overline{\Psi} \Psi \rangle_0 \end{aligned} \tag{30}$$

If we use the divergence of the axial current for the interpolating field for the pseudoscalar Goldstone bosons as in Ref.(14), then we may directly evaluate the matrix elements for the trace of the energy momentum tensor. The pseudsoscalar field is given by

$$\Phi^a \equiv \overline{\Psi} \{ \lambda^a/2, m \} 1 \gamma_5 \Psi / F_\pi m^2_\pi \rightarrow \pi^a \cdot (1 + (3-\gamma_m) \cdot D/F_D + \cdots) \tag{31}$$

The appropriate diagrams are shown in Fig.(3) and yield

$$\Gamma = \langle \Phi(p)\ \Phi(p')\ \Theta_{\mu\mu}(p'-p) \rangle_0$$

$$= (p'^2 - m^2_\pi)^{-1} \cdot (p^2 - m^2_\pi)^{-1} \cdot \{ [-2p\cdot p' + 4m^2_\pi]$$

$$- F_D \cdot q^2 (q^2 - m^2_D)^{-1} \cdot (1/F_D) \cdot [-2p\cdot p' + m^2_\pi \cdot (3-\gamma_m)] \} \qquad (32)$$

$$- (p'^2 - m^2_\pi)^{-1} \cdot F_D \cdot q^2 (q^2 - m^2_D)^{-1} \cdot (1/F_D) \cdot (3-\gamma_m)$$

$$- (p^2 - m^2_\pi)^{-1} \cdot F_D \cdot q^2 (q^2 - m^2_D)^{-1} \cdot (1/F_D) \cdot (3-\gamma_m)$$

where it is essential to keep the contributions of the dilaton poles.

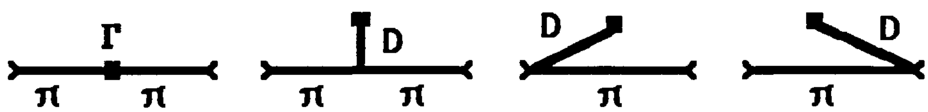

Figure 3: Diagrams for trace matrix elements

The matrix elements for the divergence of the scale current are given by

$$\Gamma_m = \langle \Phi(p)\ \Phi(p')\ (1+\gamma_m)\overline{\Psi} m \Psi(p'-p) \rangle_0$$

$$= (p'^2 - m^2_\pi)^{-1} \cdot (p^2 - m^2_\pi)^{-1} \cdot \{ [(1+\gamma_m) \cdot m^2_\pi]$$

$$- F_D \cdot m^2_D (q^2 - m^2_D)^{-1} \cdot (1/F_D) \cdot [-2p\cdot p' + m^2_\pi \cdot (3-\gamma_m)] \} \qquad (33)$$

$$- (p'^2 - m^2_\pi)^{-1} \cdot F_D \cdot m^2_D (q^2 - m^2_D)^{-1} \cdot (1/F_D) \cdot (3-\gamma_m)$$

$$- (p^2 - m^2_\pi)^{-1} \cdot F_D \cdot m^2_D (q^2 - m^2_D)^{-1} \cdot (1/F_D) \cdot (3-\gamma_m)$$

The scale identity is determined from Eqs.(31,32) and given as

$$\Gamma = \Gamma_m - (3-\gamma_m) \cdot (p'^2 - m^2_\pi)^{-1} - (3-\gamma_m) \cdot (p^2 - m^2_\pi)^{-1} \qquad (34)$$

where $(3-\gamma_m)$ is the scale dimension of pseudoscalar field, Φ, used to compute the matrix element. The relation found above is precisely as expected, contrary to the result found in Ref.(14).

We can use these results to compute the on-shell relations for the meson matrix elements

$$
\begin{aligned}
\langle\pi(p')|\Theta_{\mu\mu}|\pi(p)\rangle &= \lim\,(p'^2-m^2_\pi)\cdot(p^2-m^2_\pi)\cdot\Gamma \\
&= [q^2+2\cdot m^2_\pi] - q^2\cdot(q^2-m^2_D)^{-1}\cdot[q^2+(1-\gamma_m)\cdot m^2_\pi] \qquad (35) \\
&= \langle\pi(p')|(1+\gamma_m)\cdot\overline{\Psi}m\Psi|\pi(p)\rangle
\end{aligned}
$$

It is clear that the presence of the dilaton pole is essential to a consistent evaluation of the low energy theorems for the meson matrix elements of the energy momentum tensor and the divergence of the scale current.

I conclude that there is no NO-GO theorem. There **is** a consistent low energy phenonemology for the Goldstone realization of scale and chiral symmetry. Further, there seems to be no constraint on the value of the anomalous dimension, γ_m. Hence, there is no consistency barrier to the simultaneous realization of scale and chiral symmetry.

Of course, it is still essential that the fundamental theory have the softly broken scale symmetry, as in Eq.(24), to apply the relations of scale current algebra. The results of the previous sections have shown that the nontrivial phase of quenched, planar QED is associated with a hard, explicit breaking of the scale symmetry and, hence, the scale current algebra can not be applied to this system.

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REFERENCES

1] W.J. Marciano, Phys. Rev. **D21**(1980)2425.

2] W.A. Bardeen, C.N. Leung and S.T. Love, Phys. Rev. Lett. **56**(1986)1230; C.N. Leung, S.T. Love and W.A. Bardeen, Nucl. Phys. **B273**(1986)649.

3] T. Akiba and T. Yanigida, Phys. Lett. **169B**(1986)432; T. Appelquist, D. Karabali and L.C.R. Wijewardhana, Phys. Rev. Lett. **57**(1986)957, T. Appelquist and L.C.R. Wijewardhana, Phys. Rev. **D35**(1987)774; ibid, Phys. Rev. **D36**(1987)568; K. Yamawaki, M. Bando and K. Matumoto, Phys. Rev. Lett. **56**(1986)1335; K. Aoki, M. Bando, H. Mino, T. Nonoyama, H. So and K. Yamawaki, Nagoya Preprint, DPNU-88-7; B. Holdom, Phys. Rev. Lett. **60**(1988)1233.

4] K. Johnson, M. Baker, R. Willey, Phys. Rev. **136**(1964)B1111; ibid, Phys. Rev. **163**(1967)1699.

5] T. Maskawa and H. Nakajima, Prog. Theor. Phys **52**(1974)1326; ibid, **54**(1975)860.

6] R. Fukuda and T. Kugo, Nucl. Phys. **B117**(1976)250.

7] S.L. Adler and W.A. Bardeen, Phys. Rev. **D4**(1971)3045; R. Jackiw and K. Johnson, Phys. Rev. **D8**(1973)2386; H. Pagels and S. Stokar, Phys. Rev. **D20**(1979)2947.

8] P.I. Fomin, V.P. Gusyinin, V.A. Miransky and Yu. Sitenko, Riv. Nuovo Cimento **6**(1983)1; P.I. Fomin, V.P. Gusyinin, V.A. Miransky, Phys. Lett. **78B**(1978)136; V.P. Gusynin and V.A. Miransky, Phys Lett. B198(1987)79; ibid, Phys. Lett. **B198**(1987)362.

9] Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122(1961)345.

10] M. Bander, W. Bardeen and M. Moshe, Phys. Rev. Lett. **52**(1984)1188;

11] K. Aoki, Japan Physical Society Meeting (April, 1988); K. Kondo, H. Mino, K. Yamawaki, Nagoya Preprint, DPNU-88-18 T. Appelquist, M. Soldate, T. Takeuchi and L.C.R. Wijewardhana, Yale Preprint, YCTP-P19-88.

12] B. Holdom, Phys. Lett. **200B**(1988)338; ibid., Phys. Lett. **187B**(1987)357.

13] K. Kondo, H. Mino and K. Yamawaki, Nagoya Preprint DPNU-88-18; K. Yamawaki, College Park Superstrings, 1987:514; K. Yamawaki, M. Bando and K. Matumoto, Phys. Rev. Lett. **56**(1986)1335.

14] V.P. Gusynin, V.A. Kushnir and V.A. Miransky, Kiev Preprint ITP-88-84E.