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LOCAL BOSONIC SYMMETRIES IN THE STRING'S ACTION

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ABSTRACT

The string action enjoys Sierra's bosonic local symmetry on which Siegel's κ-symmetry closes. This bosonic symmetry is analyzed in Dirac's Hamiltonian formalism and its generators are identified.

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In an interesting paper, Sierra observed[1] that the actions for a massless relativistic particle and for the superparticle[2] enjoy invariance under local transformations, which include the usual reparametrization invariance. The local Siegel symmetry closes on these bosonic symmetries[3]. In this letter, following arguments similar to Sierra's[1], and using Dirac's formalism for constrained Hamiltonian systems[4, 5, 6], we show that the bosonic string enjoys a similar local symmetry and find its generators.

The bosonic string is described by the constraints

$$\begin{aligned} \mathcal{H}_\perp &= \frac{1}{2}\left(P^2 + (\partial_\sigma X)^2\right) \approx 0 \\ \mathcal{H}_1 &= P \cdot \partial_\sigma X \approx 0 \end{aligned} \tag{1}$$

where $X^\mu(\tau,\sigma)$ is the position of the string in space–time, and $P^\mu(\tau,\sigma)$ is its conjugate momentum, with equal time Poisson bracket

$$[P^\mu(\sigma), X^\nu(\sigma')] = \eta^{\mu\nu}\delta(\sigma - \sigma'). \tag{2}$$

Instead of (1), we find more convenient the linear combinations

$$\mathcal{H}_\pm = \frac{1}{2}(\mathcal{H}_\perp \pm \mathcal{H}_1) = \frac{1}{4}Q_\pm^2, \tag{3}$$

with

$$Q_\pm^\mu = P^\mu \pm \partial_\sigma X^\mu. \tag{4}$$

The string action

$$S = \int d\tau d\sigma \left[P^\mu \dot{X}^\mu - \lambda_+\mathcal{H}_- - \lambda_-\mathcal{H}_-\right] \tag{5}$$

enjoys the following local gauge invariance:

$$\begin{aligned} \delta X^\mu &= \frac{1}{2}\left(\epsilon_+ Q_+^\mu + \epsilon_- Q_-^\mu\right) \\ \delta P^\mu &= \frac{1}{2}\partial_\sigma\left(\epsilon_+ Q_+^\mu - \epsilon_- Q_-^\mu\right) \\ \delta\lambda_+ &= \dot{\epsilon}_+ + (\partial_\sigma\lambda_+)\epsilon_+ - \lambda_+\partial_\sigma\epsilon_+ \\ \delta\lambda_- &= \dot{\epsilon}_- - (\partial_\sigma\lambda_-)\epsilon_- - \lambda_-\partial_\sigma\epsilon_- \end{aligned} \tag{6}$$

This is the natural generalization of Sierra's gauge invariance for the particle. The variation of the action (5) under (6) is

$$\begin{aligned} \delta S &= \frac{1}{4}\int_{\sigma_1}^{\sigma_2} d\sigma(\epsilon_+ + \epsilon_-)(P^2 - \partial_\sigma X^2)\Big|_{\tau_1}^{\tau_2} \\ &+ \frac{1}{2}\int_{\tau_1}^{\tau_2} d\tau \int_{\sigma_1}^{\sigma_2} d\sigma \partial_\sigma \left\{\epsilon_+(Q_+^\mu \dot{X}_\mu - 2\lambda_+ \mathcal{H}_+) - \epsilon_-(Q_-^\mu \dot{X}_\mu - 2\lambda_- \mathcal{H}_-)\right\}, \end{aligned} \tag{7}$$

which vanishes only if $\epsilon_\pm = 0$ at the end points. Hence, the symmetry (6) does not admit the interpretation of $P^2 - (\partial_\sigma X)^2$ as a symmetry generator (as in Ref. 1 for the particle). This was to be expected, because the string is a general-covariant system and the gauge symmetry is not internal[7,8,9]. The interpretation of Ref. 1 is possible, nevertheless, if we require the action to be invariant under the transformations

$$\begin{aligned} \delta X^\mu &= \frac{1}{2}\left(\beta_+^\mu Q_+^2 + \beta_-^\mu Q_-^2\right) \\ \delta P^\mu &= \frac{1}{2}\,\partial_\sigma\left(\beta_+^\mu Q_+^2 - \beta_-^\mu Q_-^2\right), \end{aligned} \tag{8}$$

with $\delta\lambda_\pm$ to be determined. One finds that δS vanishes if and only if

$$\begin{aligned} \delta\lambda_+ &= 2\beta_+^\mu \partial_\sigma(\lambda_+ Q_{+\mu}) - \beta_+^\mu \dot{Q}_{+\mu} \\ \delta\lambda_- &= -2\beta_-^\mu \partial_\sigma(\lambda_- Q_{-\mu}) - \beta_-^\mu \dot{Q}_{-\mu} \end{aligned} \tag{9}$$

with periodic boundary conditions for $\lambda_\pm$. Identifying the total derivative term as

$$\delta S = \int d\tau d\sigma \partial_\tau \partial_\sigma \left\{\beta \cdot (\text{generator or constraint})\right\},$$

the generators of the β-symmetry for the string are

$$F_\pm^\mu = P^\mu Q_\pm^2 \approx 0. \tag{10}$$

Could these considerations be generalized to the case of higher-dimensional extended objects, such as membranes? The answer is not clear, since in general one does not know how to write the constraints as a perfect square Q^2. Work along this lines is in progress[10].

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