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Abstract - The interaction of monochromatic Alfvén waves with toroidally trapped particles in the intrinsic stochasticity regime is discussed. Both the diffusion in velocities as well as in the radial position of bananas is studied. Using suitable Hamiltonian formalism, the effect of wave parallel components $\tilde{E}_{\parallel}$ and $\tilde{B}_{\parallel}$ is investigated.

By means of the direct numerical integration of the corresponding canonical equations, the stochasticity threshold of both for plasma electrons and for thermonuclear alpha-particles is estimated (neglecting the effect of $\tilde{B}_{\parallel}$). Stochasticity causes the transfer between trapped and untrapped regimes and the induced radial diffusion of bananas. The latter effect can exceed the neoclassical diffusion considerably.

The effect of $\tilde{B}_{\parallel}$ has been estimated only analytically. It consists in the frequency modulation of banana periodic motion, coupled with the possibility of the Mathieu instability.

Nevertheless, for $\tilde{B}_{\parallel}$ corresponding to $\tilde{E}_{\parallel}$, the effect seems to be weaker then the effect of $\tilde{E}_{\parallel}$ when thermonuclear regime is considered.

1. INTRODUCTION

BANANA trapped particles are the source of several unwelcome effects. Their interaction with RF field, either used for auxiliary heating and current drive or spontaneously excited via some kind of plasma instabilities, is therefore of great interest.

Important effects can be expected espacially in resonant regimes (where often intrinsic stochasticity of particles sets on). Resonant interaction of RF waves with the cyclotron motion of banana particles in the stochasticity regime has been investigated e.g. in WHANG and MORALES, 1983. Another possibility is the resonant coupling of the wave with the harmonics of the banana motion. In this paper, the latter effect is investigated. RF field is represented by Alfvén waves.

Alfvén waves (see e.g. review paper by APPERT et al., 1985) have received considerable attention as a means for plasma heating and current drive (ELFIMOV et al., 1983, ELFIMOV, 1983). Moreover, Alfvén waves can be excited by the thermonuclear cone instability (BELIKOV et al., 1978; LISAK et al., 1983).

The purpose of the present paper is to describe the interaction of a monochromatic Alfvén wave with banana trapped particles in the intrinsic stochasticity regime. (Papers of

DOBROWOLNY et al., 1973, CASATI et al., 1979, 1981 and BELIKOV et al., 1985 are closely related). Besides the changes in velocities of banana particles, the changes in the radial position of the bananas are investigated. Hamiltonian formalism is used. A suitable canonical system describing particle motion on the tokamak magnetic field and in RF field is employed. The integration of corresponding canonical equation is performed numerically. Thus, the necessity of further approximations needed for obtaining the standard mapping form (see GÁŠEK et al., 1985, but analogously also in KARNEY, 1979, BELIKOV et al., 1985) is avoided. The results are presented in the form of Poincaré maps which offer the best possibility how estimate the threshold of stochastic instability. The maps also give an excellent insight into the complicated form of the phase space representation.

The stochastic behaviour of both plasma electrons and thermonuclear alpha particles is investigated. In computations, only the parallel component $\widetilde{E}_{\parallel}$ of the monochromatic Alfvén wave is taken into account. From the results, the RF induced diffusion coefficients of banana particles are deduced, and are found to be significant. The effect of the parallel magnetic component $\widetilde{B}_{\parallel}$ of the Alfvén wave on the banana motion is estimated analytically.

Our paper is organized as folows. Chap. 2 describes the canonical formalism used. Chap. 3 given the description of our model and its parameters. Chap. 4 presents the numerical results obtained for the interaction of banana trapped particles with the parallel component $\tilde{E}_{\parallel}$ of the Alfvén wave, both for plasma electrons and alpha-particles. In chap. 5, the effect of the parallel component $\tilde{B}_{\parallel}$ is estimated. Chap. 6 summarizes the results as well as open problems.

2. CANONICAL FORMALISM FOR THE DESCRIPTION OF THE INTERACTION OF TOROIDALLY TRAPPED PARTICLES WITH RF FIELDS

2.1 Particle on the tokamak magnetic field

Let us consider the orthogonal coordinate system r, ϑ, φ, which is embedded in the original orthogonal x, y, z system (see Fig. 1). In this system, the Hamiltonian H of a particle with the mass m and the charge e, moving in the magnetic field with the corresponding vector potential $\vec{A}$ (A_r, A_ϑ, A_φ) is

$$H=\frac{1}{2m}\left[\left(p_r-eA_r\right)^2+\frac{1}{r^2}\left(p_\vartheta-erA_\vartheta\right)^2+\frac{1}{R^2}\left(p_\varphi-eRA_\varphi\right)^2\right], \quad (1)$$

Here, p_r, p_ϑ, p_φ and r, ϑ, φ are canonical conjugated momenta and coordinates.

Let us successively use the following system of the generating functions (for the symbolics, see GOLDSTEIN, 1951)

$$F_2^{(1)}(r, \theta, \varphi, P_{\xi_1}, P_{\xi_2}, P_{\xi_3}) = \\ = P_{\xi_1} \int^{r} \left(\frac{m\omega_c}{2}\right)^{1/2} dr' + P_{\xi_2}\,\theta + P_{\xi_3}\,\varphi, \tag{2}$$

where $\omega_c = \frac{e}{m} B_\varphi \left[r - \psi(r, \theta, \varphi) \right]$.

The function ψ (the shift of the argument r of B_φ) will be defined later on.

Let, further,

$$F_2^{(2)}(\xi_1, \xi_2, \xi_3, P'_{\xi_1}, P'_{\xi_2}, P'_{\xi_3}) = \\ = P'_{\xi_1} \sqrt{e r A_\theta} + P'_{\xi_2}\,\xi_2 + P'_{\xi_3}\,\xi_3 \tag{3}$$

where we choose $\xi_1'^{2} = 2m\omega_c\,(r - \psi)\,r^2$.

Let us further use following cononical transformation (its canonicity can be proved by means of the Poisson brackets)

$$\xi_1'^2 = P_1 + P_2 + 2\sqrt{P_1 P_2}\ \sin(Q_1 + Q_2)$$

$$p'^2_{\xi_1} = P_2 - P_1$$

$$p'_{\xi_3} = P_3$$

$$\xi_2' = \mathrm{arc\,tg}\,\frac{\sqrt{2P_1}\ \cos Q_1 + \sqrt{2P_2}\ \sin Q_2}{\sqrt{2P_1}\ \sin Q_1 + \sqrt{2P_2}\ \cos Q_2} \tag{4}$$

$$\xi_3' = Q_3$$

$$\xi_1' p'_{\xi_1} = 2\sqrt{P_1 P_2}\ \cos(Q_1 + Q_2).$$

Analogous transformation but for Descartes coordinations was used by MORSE and FESCHBACH, 1953. Using transformations (2), (3), (4), the original Hamiltonian (1) can be transformed into the following one

$$H = \omega_c P_1 + \frac{1}{2mR^2}\left(P_3 - eRA_\varphi\right)^2 \tag{5}$$

where, of course, R and A_φ must be expressed in the coordinate system P_i, Q_i . (For cylindrical geometry, see similar procedure in LACINA, 1964 and KRLÍN, 1967. See also close procedure in HINTON, 1981). KAUFMAN, 1972 presents more general formalism.

In (5), the adiabatic approximation has been used. Moreover, using the free parameter in the definition of the vector potencial, we can choose $\psi(r_0)=0$, were r_0 is the radial coordinate of the chosen magnetic surface. The simplicity of the final form (5) is the result of omission of small fast oscillating terms, which is justified for $|r - r_0| \ll R_0$. In this case, it is

$$\omega_c P_1 = \frac{1}{2} m v_\perp^2 \qquad (6)$$

$$\frac{1}{2mR^2}\left(P_3 - eRA_\varphi\right)^2 = \frac{1}{2} m v_\parallel^2 \qquad (7)$$

where $v_\perp$ is the component of the velocity $\vec{v}$ in the poloidal plane ($r-\theta$ section) and $v_\parallel$ the component of $\vec{v}$ perpendicular to this plane (in $\vec{\varphi}$ direction). In tokamaks, $B_\theta / B_\varphi \ll 1$, and difference between the perpendicular energy in the $r-\theta$ section and the perpendicular energy in the plane, formed by the normal and the binormal to the field line, is quite negligible. (For usual reactor parameters a = 1.5 m, R_0 = 6.4 m, B_T = 6 T, q = 2 is the relative difference less then 10^{-2}). To this accuracy, we can indentify (6) and (7) with the perpendicular and parallel energy in the natural orthogonal system (bounded to the field lines).

The canonical coordinates and momenta can be interpreted as follows. The coordinate Q_1 is the angle of the cyclotron rotation, the expressions $\rho_c = \sqrt{\frac{2P_1}{eB_\varphi}}$ and $\rho_D = \sqrt{\frac{2P_2}{eB_\varphi}}$ give the radii of the cyclotron rotation and of the guiding centre, respectively, and Q_2 is the angle of the guiding centre. The cyclicity in Q_1 and Q_3 provides two invariants of the motion,

$$P_1 = const. , \qquad P_3 = const.$$

Since the relation between the momentum P_1 and the magnetic momentum μ is

$$\mu = \frac{e}{m} P_1 ,$$

the invariancy of P_1 results in the invariancy of μ .

The Hamiltonian (5) can be used for the discussion of the motion of a particle in the tokamak magnetic field. The analytical discussion has been presented in GÁŠEK et al., 1986. Using (5) and the conjugated canonical equations, it is possible to obtain the first integral in the form

$$t = \frac{q_0 R_0}{v} \frac{\sqrt{2}\,(R_0 + r_0')}{(r_0' R_K)^{\frac{1}{2}}} \Pi \left\{ arc \sin \left[\frac{R_K (1 - \cos Q_2)}{R\,(1 - \cos Q_{2max})} \right], \right.$$

$$\left. \frac{2 r_0'}{R_K} \sin^2 \frac{Q_{2max}}{2} , \quad \frac{R_0 - r_0'}{R_K} \sin \frac{Q_{2max}}{2} \right\} \tag{8}$$

(see also DNESTROVSKII and KOSTOMAROV, 1982). Here, v is the particle velocity, r_0' is the initial radial position for $Q_2 = 0$, Q_{2max} is the maximum of the poloidal angle Q_2 , q_0 is the averaged safety factor and $R_K = R_0 + r_{refl} \cos Q_{2max}$. $\Pi(\varphi, \alpha^2, k)$ is the elliptic integral of the third kind, $R = R_0 + r \cos Q_2$, r_{refl}, Q_{2max} are coordinates of the reflection point of the banana.

The complexity of (8) requires additional approximations (especially for obtaining the inverse form of $Q_2(t)$. In its simpliest form

$$Q_2 = Q_{2\,max} \sin \omega_B t \; ; \qquad \omega_B = \frac{v}{q R_0} \sqrt{\frac{r_0'}{2 R_0}} \; .$$

Further approximations are required, if the RF field is included. This is, why we preferred the direct numerical integration of the canonical equations, derived from the Hamiltonian (5), supplemented with the rf field term.

2.2 Particles in the tokamak magnetic field and in RF field

RF field is generally described by its vector potential $\vec{A}_{rf}$ and by its scalar potential φ_{rf}. The electrostatic component of the wave field, represented by the potential φ_{rf}, has no coupling to our coordinate system P_i, Q_i, and its influence is exactly described by adding the term $e\,\varphi_{rf}$ to the foregoing Hamiltonian. However, the canonical system P_i, Q_i is intrinsically coupled with the magnetic field $\vec{B} = \vec{B}_\varphi + \vec{B}_\theta$ (via the vector potential $\vec{A}$). The introduction of the new vector potential

$$\vec{\hat{A}} = \vec{A} + \vec{A}_{rf}$$

requires therefore that a new canonical system P_i', Q_i' be introduced. Provided we retain the original system P_i, Q_i, considering $\vec{A}_{rf}$ as a small perturbation

$$\left|\vec{A}\right| \gg \left|\vec{A}_{rf}\right|$$

but include $\vec{A}_{rf}$ into the Hamiltonian, the solution will only be approximate. The Hamiltonian for a particle in the tokamak magnetic field and in RF field with $\vec{A}_{rf} \neq 0$, $\varphi_{rf} \neq 0$ is

$$H \doteq \hat{\omega}_c P_1 + \frac{1}{2mR^2}\left[P_3 - eR\left(A_\varphi + A_{\varphi_{rf}}\right)\right]^2 + \tag{9}$$

$$+ e\varphi_{rf}.$$

Here

$$\hat{\omega}_c = \frac{e\hat{B}_\varphi}{m}$$

$$\hat{B}_\varphi = rot_\varphi\,(\vec{A} + \vec{A}_{rf}) = B_\varphi + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r A_{\varphi_{rf}}\right) - \right.$$

$$\left. - \frac{\partial A_{r_{rf}}}{\partial Q_2}\right]$$

In computations, only $\vec{A}_{rf} = 0$ is employed. In this case, the exact Hamiltonian can be used

$$H = \omega_c P_1 + \frac{1}{2mR^2}\left(P_3 - eRA_\varphi\right)^2 + e\varphi_{rf}. \tag{10}$$

The effect of $\vec{A}_{rf} \neq 0$ is discussed qualitatively in chap. 5.

We choose φ_{rf}, $\tilde{B}_\parallel$ in the form of waves, propagating around the toroidal

$$\varphi_{rf} = \varphi_0 \sin\left(k_\parallel R_0 Q_3 - \omega t\right)$$

$$\tilde{B}_{\parallel} = \tilde{B}_{\parallel}^{(0)} \sin \left(k_{\parallel} R_0 Q_3 - \omega t \right).$$

Since our paper deals mostly with the effect of $\tilde{E}_{\parallel}$, let us shortly discuss the dynamics of a particle whose motion is described by the Hamiltonian

$$H = \omega_c P_1 + \frac{1}{2mR^2} \left(P_3 - eRA_\varphi \right)^2 + e\,\varphi_0 \sin \left(k_{\parallel} R_0 Q_3 - \omega t \right). \quad (11)$$

The banana motion itself (for $\varphi_0 = 0$) represents a rather complicated dynamical system of coupled oscillations in toroidal, poloidal and radial directions (with, further, the cyclotron motion). It can be therefore expected that the effect of RF field with $\varphi_0 \neq 0$ can strongly change the character of the banana motion, bringing it into the regime of the intrinsic stochasticity (for this phenomenon, see e.g. LICHTENBERG and LIEBERMAN, 1983). RF field can strongly interact with banana particles, if it is resonantly coupled with banana bounce frequency ω_B

$$\omega - n\,\omega_B = 0 \qquad (n \text{ is integer})$$

or with cyclotron frequency ω_c

$$\omega - n\,\omega_c = 0 .$$

In our paper we discuss the first case. Supposing $\omega \ll \omega_{ce,i}$, and, consequently, the invariancy of P_1 and μ , the primary effect is given by the canonical equation for P_3 ,

$$\frac{dP_3}{dt} = -e\,\tilde{E}_{\parallel}^{(0)} R_0 \cos \left(k_{\parallel} R_0 Q_3 - \omega t \right); \quad \tilde{E}_{\parallel}^{(0)} = k_{\parallel}\,\varphi_0 . \quad (12)$$

To obtain analytical solution of this equation, perturbation method is usually used. Q_3 is substituted by a zero solution, e.g. in its simpliest form

$$Q_3^{(0)} = Q_{3\,max} \sin \omega_B t + \alpha t$$

and the RHS of (12) is Fourier expanded. Then, an approximate solution of (12) can be obtained (see e.g. BELIKOV et al., 1985, GÁŠEK et al., 1985, both starting from the solution of the analgous problem of KARNEY, 1979). We do not follow this procedure; however, we shall use it for a preliminary choice of parameters so as to reach the desired stochastic regime. Usually, the resonance $\omega - n\,\omega_B = 0$ requires $n \gg 1$ $(\omega/\omega_B \sim 10^1 \div 10^2)$ In this case, the resonant Fourier component $J_n\left(k_\parallel R_0 Q_{3\,max}\right)$ is non-negligible only for $k_\parallel R_0 Q_{3\,max} \gtrsim n$. In this region, the stochastic regime is to be expected.

3. THE DESCRIPTION OF THE MODEL AND ITS PARAMETERS

According to usual assumptions, we consider the toroidal magnetic field B_φ is the form

$$B_\varphi = B_{\varphi_0} \frac{R_0}{R} \qquad (13)$$

where B_{φ_0} is the magnetic field on the axis. The poloidal magnetic field B_θ , produced by the toroidal current of density $\vec{i}(r)$ is determined as

$$\mathrm{rot}\, \vec{B}_\theta = \mu_0 \vec{i}(r) .$$

In principle, the current density profile $\vec{i}(r)$ can be choser in an arbitrary form. For simplicity, we take

$$i_\varphi(r) = i_0 = const. \tag{14}$$

Since the currents of different origin (either inductive of non-inductive) may have very different radial profiles, the homogeneous current density (14) seems to be a reasonable choice to start with.

We consider simple circular cross-section and parameters, close to the hybrid reactor project (GLUKHICH, 1978). The minor radius $a = 1.5$ m, the major radius $R_0 = 6.4$ m and the toroidal magnetic field $B_\varphi = 6$ T. To estimate the effect of the poloidal magnetic field on particle dynamics, we choose two current densities, namely $i_0^{(1)} = 0.566$ MA m^{-2} (which gives total current $I_{tot} = 4$ MA and the safety factor $q(r = a) = 2.65$) and the density $i_0^{(2)} = 1$ MA m^{-2} with $I_{tot} = 7$ MA and with $q(r = a) = 1.49$. The averaged plasma density n_0 lies in the regime $n_0 \sim 10^{20}$ m^{-3} and the averaged plasma temperature $T_e \sim T_i \sim 10$ keV. We consider D-T plasma.

The kinetic mode of a monochromatic Alfvén wave is consider. This mode has both the parallel magnetic component $\tilde{B}_\parallel$ and parallel electric component $\tilde{E}_\parallel$ (see FISCH and KARNEY, 1981, STIX, 1975, ELFIMOV et al., 1983 and ELFIMOV, 1983). The corresponding relation between $\tilde{E}_\parallel$ and $\tilde{B}_\parallel$ is (ELFIMOV, 1983 and FISCH and KARNEY, 1981)

$$\tilde{E}_{\parallel} = -i \, \frac{|k^2| \, v_{Te}^2 \, c_A^2 \, k_{\parallel}}{\omega_{ce} \, \omega^2} \, \tilde{B}_{\parallel} \tag{15}$$

where $|k^2| = k_{\perp}^2 + k_{\parallel}^2$, c_A is the Alfvén velocity and ω is the frequency

$$\omega \doteq k_{\parallel} c_A \, .$$

Bound to the given magnetic field B_{φ} and the density range, we consider three cases,

$$\omega = 3\text{x}10^7 \text{ s}^{-1} \quad , \quad k_{\parallel} = 4 \text{ m}^{-1}$$
$$\omega = 1.5\text{x}10^7 \text{ s}^{-1} \quad , \quad k_{\parallel} = 2 \text{ m}^{-1}$$
$$\omega = 1.34\text{x}10^7 \text{ s}^{-1} \quad , \quad k_{\parallel} = 3 \text{ m}^{-1}.$$

(For kinetic modes itself, see HASEGAWA A. and CHEN L. (1976), Physics Fluids 19, 1924).

4. NUMERICAL RESULTS - REGIMES OF THE INTRINSIC STOCHASTICITY FOR PLASMA ELECTRONS AND FOR THERMONUCLEAR ALPHA-PARTICLES

Let us consider the Hamiltonian of a particle in the magnetic tokamak field and in the field of the electrostatic wave in the form (11).

The canonical equations

$$\frac{dP_2}{dt} = -\frac{\partial H}{\partial Q_2} \, , \, \frac{dP_3}{dt} = -\frac{\partial H}{\partial Q_3} \, , \, \frac{dQ_2}{dt} = \frac{\partial H}{\partial P_2} \, , \, \frac{dQ_3}{dt} = \frac{\partial H}{\partial P_3}$$

can be - after some algebra - rewritten into a form more suitable for numerical integration

$$\frac{d}{dt}(\Delta r) = \frac{a_1}{R(R+R_0)}\left[-a_4 R - (\Delta r)^2 (r_0 + r)^2\right] \sin Q_2 -$$

$$- \frac{a_2}{r_0} \cos\left(k_\parallel R_0 Q_3 - \omega t\right)$$

$$\frac{dQ_2}{dt} = \frac{a_1}{r R\left(R+R_0\right)}\left\{-a_4 R + \Delta r \left\{4 r_0^2\, \bar{R}_0 + \right.\right.$$

$$\left.\left. + \Delta r\left[6 r_0 \bar{R}_0 + \Delta r\left(4\bar{R}_0 - 2R_0 + \Delta r \cos Q_2\right)\right]\right\}\right\} \qquad (16)$$

$$\frac{dQ_3}{dt} = -\frac{a_3}{R^2}\Delta r\left(r + r_0\right)$$

$$\frac{dr_0}{dt} = \frac{a_2}{r_0}\cos\left(k_\parallel R_0 Q_3 - \omega t\right).$$

Here, R_0 is the major radius of the tokamak, r_0 is the instantaneous centre of the banana and Δr is the instantaneous devitaion of the particle from this centre.[+)] In the absence of RF field, the banana centre is the radius at which $\frac{dQ_2}{dt} = 0$. For $Q_2 = 0$,

$$v_\parallel\left(R_0 + r\right) + a_3\,\Delta r\left(r + r_0\right) = 0.$$

+) see Fig. 14

Further $r = r_0 + \Delta r$; $R = R_0 + r \cos Q_2$

$\bar{R}_0 = R_0 + r_0 \cos Q_2$; $a_1 = \frac{1}{8} \frac{e \mu_0^2 i_0^2 R_0}{m B_{Y_0}}$

$a_2 = - \frac{2 E_{\parallel}}{\mu_0 i_0}$; $a_3 = \frac{1}{4} \frac{e}{m} \mu_0 i_0 R_0$

$a_4 = 16 \frac{\omega_{c0} P_1 m}{e^2 \mu_0^2 i_0^2 R_0}$; $\omega_{c0} = \frac{e B_{Y_0}}{m}$.

4.2 The interaction of the monochromatic Alfvén wave with plasma electrons

Figures 2 - 9 present the results of numerical integration of the equation (16) in the form of the Poincaré surface of section. For all cases, $Q_2 = 0$, $r = r_{min}$ (the minimal radius). On each picture, the parallel axis given u where u is the phase defined as

$$u = mod_{2\pi} \left[\omega t + \pi \right] - \pi . \qquad (17)$$

For all cases, the initial value $u = 0$. The vertical axis gives the values of the parallel velocity $v_{\parallel}$ at the moment when banana particle passes $Q_2 = 0$, $r = r_0'$. The motion is surveyed for thirty or sixty banana periods. In Figs. 2 - 8, the wave parameters are ω = 3x10^7 s^{-1}, $k_{\parallel}$ = 4 m^{-1}. For comparison, ω = 1.5x10^7 s^{-1}, and $k_{\parallel}$ = 2 m^{-1} were chosen in Fig. 9.

The latter parameters are close to those used in the reactor study by KIROV et al., 1985, 1986. Figs. 2 - 5 present the effect of increasing $\tilde{E}_{\parallel}^{(0)}$ (10, 20, 50, 100 V m^{-1}) on the banana particles behaviour for thirty banana periods and for $\dot{\iota}_0$ = 1 MA m^{-2}. The transition from the regular to stochastic regimes with the increase of $\tilde{E}_{\parallel}^{(0)}$ is rather fast. While for $\tilde{E}_{\parallel}^{(0)}$ = 10 V m^{-1} in Fig. 2 almost the whole space is covered by smooth regular curves (with the exception of the small area near $v_{\parallel} \sim 1 \times 10^7$ m s^{-1}), in the case of $\tilde{E}_{\parallel}^{(0)}$ = 100 V m^{-1} (Fig. 5), the whole space is covered by erratically appearing dots. Obviously, the onset of the stochasticity is apparent first at lower velocities. Therefore, the lower velocity region was investigated in more detail, to facilitate the comparison of the stochasticity threshold for different wave and plasma parameters (Figs. 6 - 9). Fig. 6 shows the detail of the case in Fig. 3. The remarkably complicated form of this plot is worth noting. Figs. 7 and 8 present the same part of the surface of section for $\tilde{E}_{\parallel}^{(0)}$ = 10 and 20 V m^{-1}, respectively, but for $\dot{\iota}_0$ = 0.566 MA m^{-2}. When compared to Figs. 2 and 6, the pictures give the evidence of the decrease in the stochasticity threshold with the decrease in the toroidal current. This is in agreement with the simple overlapping criterion for the stochasticity threshold (BELIKOV et al., 1985)

$$K = \frac{\pi^2}{2} \frac{s^2 \omega_r^2}{\lambda^2 \omega_B^2} J_s(\bar{\lambda}) > 1 \qquad (18)$$

(here, $s\,\omega_B$ is the resonant harmonics,

$$\omega_v = \frac{e k_\parallel E_\parallel}{m_e} \; ; \quad \omega_B^2 = \frac{r}{R} \frac{\mu B_\varphi}{q^2 R^2} \; ; \quad \mu = \frac{v_\perp^2}{2 B_\varphi}$$

$$\lambda = k_\parallel q R \; ; \quad \bar{\kappa} = 2 x \lambda \; ; \quad \chi^2 = \frac{\varepsilon - \mu B_\varphi (1-\epsilon)}{2 \epsilon \mu B_\varphi}$$

$$\epsilon = \frac{r}{R} \; ; \quad \varepsilon = v^2 \,).$$

For $\tilde{E}_\parallel^{(1)}$ given, ω_B decreases and K increases with decreasing B_θ (and i_0).

In Fig. 9, similar case as in Fig. 8 is displayed ($\tilde{E}_\parallel^{(0)} = 20$ V m^{-1}, 0,566 MA m^{-2}), but the wave parameters ω and $k_\parallel$ are two times lower. According to the criterion (18), the threshold should be higher ($\sqrt{2}$ times). The comparison of Fig. 9 to Figs. 7 - 8 shows good agreement with this expectation.

Simultaneously with the change of velocity, the change of the banana radial position takes place. For selected initial velocities $v_\parallel(0)$, this change is presented in Fig. 10. Here, $\Delta r'$ is the deviation of the banana point $Q_\perp = 0$, $r = r_0'$ from the initial banana centre r_0 at the moment of completion of each subsequent banana orbit. In Fig. 10, i_0 = 1 MA m^{-2} and three representative cases are depicted (curve (1) for $v_\parallel(0) = 7.5 \times 10^6$ m s^{-1}, $\tilde{E}_\parallel^{(0)} = 20$ V m^{-1}, curve (2) for $v_\parallel(0) = 7.5 \times 10^6$ m s^{-1}, $\tilde{E}_\parallel^{(0)} = 100$ V m^{-1} and curve (3) for $v_\parallel = 7.5 \times 10^6$ m s^{-1}, $\tilde{E}_\parallel^{(0)} = 200$ V m^{-1} . Curve (1) is almost periodical. This reflects the regular situation in the corresponding plot in Fig. 3. In the sequence (2) is still some regularity, but randomness starts to be predominant. Curve (3) shows strong enhancement of displacements.

Supposing that the correlation ceases after each banana period, the induced diffusion coeficient $\mathcal{D}_{ind}$ can be simply estimated by the formula

$$\mathcal{D}_{ind} = \frac{1}{\overline{\tau}_B} \langle \Delta_i^2 \rangle . \tag{19}$$

Here, Δ is the change of $\lambda_{min}(Q_2 = 0)$ after one period, $\langle \Delta_i^2 \rangle$ is its mean square value, and $\overline{\tau}_B$ is the avaraged bounce period. Further, it is possible to estimate the diffusion lenght l_{diff} , defined as

$$l_{diff} = \sqrt{4 \mathcal{D}_{ind} \Delta t} \; ; \quad \Delta t = 1s \; . \tag{20}$$

Let us compare the induced diffusion coefficient $\overline{\mathcal{D}_{ind}}$, defined as

$$\overline{\mathcal{D}_{ind}} = \sqrt{\frac{r}{R}} \, \mathcal{D}_{ind} \tag{21}$$

(thus taking into account the number of trapped particles) with the neoclassical diffusion coefficient, $\mathcal{D}_{neo}$ which is in its simpliest form (RAWLS, 1979)

$$\mathcal{D}_{neo} = \sqrt{\frac{r}{R}} \, \nu \, \rho_p^2 \; ; \quad \rho_p = \frac{m v}{e B_p}$$

where ν is the 90° scattering frequency.

Let us consider the case $\dot{i}_{\phi} = 1$ MA m^{-2}, $\lambda = a$ and the averaged temperature $T_e \sim 10$ keV. Then for the case $\tilde{E}_{\parallel}^{(0)} = 100$ V m^{-1} and $v_{\parallel} = 7.5\text{x}10^{6}$ m s^{-1} we obtain $\mathcal{D}_{neo} \sim 0.89\ \overline{\mathcal{D}_{ind}}$, $l_{diff} \sim$ $\sim 6\text{x}10^{-3}$ m and for $\tilde{E}_{\parallel}^{(0)} = 200$ V m^{-1} and $v_{\parallel} = 7.5\text{x}10^{6}$ m s^{-1} we obtain $\mathcal{D}_{neo} \sim 0.156\ \overline{\mathcal{D}_{ind}}$, $l_{diff} \sim 2\text{x}10^{-2}$ m (neglecting the rest of the singularity). These results show that the induced diffusion can play an important role. To avoid this rather strong effect, the amplitude $\tilde{E}_{\parallel}^{(0)}$ (used either for heating or for current drive) needs to be lower then $\tilde{E}_{\parallel}^{(0)} \sim 10^{2}$ V m^{-1}.

It is of interest to compare the stochasticity threshold parameter K, given by the analytical estimate (18) with our numerical results. Let us approximate $\bar{\lambda}$ in (18) by $\bar{\lambda} \doteq$ $\doteq k_{\parallel} R_0 Q_{3m}$, where Q_{3m} is the amplitude of banana oscillations in the azimuthal direction. Let us further choose, for the discussion, the case $\dot{i}_{\phi} = 1$ MA m^{-2}, $v_{\perp} = 7\text{x}10^{7}$ m s^{-1}, $v_{\parallel} =$ $= 1.4\text{x}10^{7}$ m s^{-1}, $\lambda = a$. In this case, $Q_{3m} \doteq 1.2$ rad. Using $E_{\parallel} = 20$ V m^{-1} (i.e. the situation in Fig. 3), we obtain $K \sim$ ~ 0.6. The analytical criterion indicates that the regime is close to the stochasticity threshold. This agrees with the onset of the stochasticity in only a small area of Fig. 3.

To obtain a global view on this kind of the interaction, a broader and more complete spectrum of parameters needs to be surveyed. Nevertheless, some features seems to be evident.

The stochasticity threshold is, for usual tokamak parameters and for bananas close to the plasma boundary, in the region $\tilde{E}_{\parallel}^{(0)} \sim 10^{1} \div 10^{2}$ V m^{-1}, in good agreement with the overlapping criterion.

The induced radial diffusion of bananas can significantly exceed the neoclasical diffusion and it should be estimated in all cases, when large monochromatic waves are used.

The threshold amplitude decreases with the decrease of toroidal current density (and, correspondingly, with the decrease of the poloidal magnetic field).

4.3 The interaction of a monochromatic Alfvén wave with thermonuclear alpha-particles

Generally, the bounce period of alpha-particles $\tau_{B\alpha}$ with the born fusion energy W_α = 3.52 MeV is longer than typical bounce period of plasma electrons τ_{Be} ($\tau_{B\alpha} / \tau_{Be} \sim$ $\sim 10^1 \div 10^2$). To obtain the resonant interaction on not too high harmonics $n\omega_{B\alpha}$, we choose therefore a lower frequency $\omega = 1.34\times10^7\ s^{-1}$, lowering $k_\parallel$ only to $k_\parallel = 3\ m^{-1}$. The aim of the calculation was to estimate whether the stochasticity threshlod can be reached for realistic wave amplitudes. The simple overlap criterion (18) gives $\tilde{E}_\parallel^{(a)} = 10^2 \div 10^3\ V\ m^{-1}$. We present only the result for the current density $j_0 = 1\ MA\ m^{-2}$. Since the thickness of alpha-particle bananas is already considerable, we choose the initial radius $r_0 = 1$ m. Unlike to the previous, the velocity component $v_\perp$ is given by the condition $\frac{1}{2} m \left(v_\perp^2 + v_\parallel^2 \right) = W_\alpha$. We have considered two cases, $\tilde{E}_\parallel^{(a)} =$ $= 200\ V\ m^{-1}$ (Fig. 11) and $\tilde{E}_\parallel^{(a)} = 1000\ V\ m^{-1}$ (Fig. 12). A set of initial values $v_\parallel$ $\left(\theta_\ell = 0,\ t = 0 \right)$, close to the separatrix $v_\parallel \sim 5\times10^6\ m\ s^{-1}$ was taken. For $\tilde{E}_\parallel^{(a)} = 200\ V\ m^{-1}$ the stochasticity is only in an embryonic state. A larger - scale stochasticity appears for $\tilde{E}_{\parallel}^{(o)}$ = 1000 V m^{-1}. Fig. 13 presents the sequence of the radial displacement $\Delta r'$ of the particles for three cases - - (1) for $\tilde{E}_{\parallel}^{(o)}$ = 200 V m^{-1}, $v_{\parallel}(t=0)$ = 5.5x10^6 m s^{-1}, (2) for $E_{\parallel}$ = 1000 V m^{-1}, $v_{\parallel}(t=0)$ = 5.7x10^6m s^{-1} (both for i_0 = = 1 MA m^{-2}) and (3) for $\tilde{E}_{\parallel}^{(o)}$ = 1000 V m^{-1}, $v_{\parallel}(t=0)$ = = 5x10^6 m s^{-1} and i_0 = 0.566 MA m^{-2}. Whereas curve (1) indicates almost a regular motion, curve (2) and curve (3) (close to the separatrix) start to be chaotic. For the case (2), we have estimated the diffusion lenght l_{diff} (20). For Δt = 1 s, l_{diff} $\sim$ 1.3 m. Since the typical reactor energy confinement time τ_E $\sim$ 1 s, this diffusion may prove to be significant.

Since the realistic amplitudes of $\tilde{E}_{\parallel}^{(o)}$ are expected in the region $\tilde{E}_{\parallel}^{(o)} \sim 10^1 \div 10^2$ V m^{-1} it seems that alpha-particles will not enter a large-scale stochasticity regime (with the exception of regions close to the separatrix). Nevertheless, more expressive results can be expected for cases with hollow-profile toroidal current, such as may occur e.g. at lower hybrid drive. Here, due to the low poloidal field B_{θ} in larger region of plasma volume, stronger stochasticity may be expected. On the other hand, it may prove interesting to study the RF induced diffusion as a potential means for removing of the alpha-particle ash from the plasma (forthe case of ICR interaction, this application has been mentioned in RIYOPOULOS et al., 1986).

5. EFFECT OF THE PARALLEL COMPONENT $\tilde{B}_{\parallel}$ OF THE RF FIELD ON THE BANANA MOTION

Till now, we have investigated only the influence of the electric component $\tilde{E}_{\parallel}$ of the Alfvén wave. The parallel component $\tilde{B}_{\parallel}$ appearing primarily, will also interact with particles.

Neglecting the banana precession, we can consider the banana motion in the Q_3 coordinate in its simpliest harmonic form

$$\frac{d^2 Q_3}{dt^2} + \omega_B^2 Q_3 = 0 \;; \quad \omega_B = v\,(2\,\kappa\,R_0)^{-\frac{1}{2}} B_\theta / B_\varphi \;. \qquad (22)$$

Let us express Q_3 as $Q_3 = Q_3^{(0)} + Q_3^{(1)}$ where $Q_3^{(0)} = Q_{3m} \sin \omega_B t$ and $Q_3^{(1)}$ is the perturbation, and let $B_\varphi = B_{\varphi_0} + \tilde{B}_{\parallel}$, where $\tilde{B}_{\parallel} = \tilde{B}_{\parallel}^{(0)} \sin(k_{\parallel} R_0 Q_3 - \omega t)$. Applying usual perturbation procedure, expanding $\tilde{B}_{\parallel}$ into the Fourier components, considering the main resonance $(2n+1)\omega_B - \omega = 0$ and neglecting the effect of all other terms. we obtain the equation for $Q_3^{(1)}$ in the Mathieu form

$$\frac{d^2 Q_3^{(1)}}{d\tau^2} + \left[\, 1 + 2\alpha\, J_{2n+1}(x) \cos 2\tau \,\right] Q_3^{(1)} = 0 \qquad (23)$$

The solution of (23) in the first unstable region (HAYASHI, 1953) may be expressed as $Q_3^{(1)} \sim e^{\mu\tau}$

Here, $\tau = \omega_B t \;, \quad x = k_{\parallel} R_0 Q_{3m} \;, \quad \alpha = \dfrac{\tilde{B}_{\parallel}}{B_{\varphi_0}}$

$$\mu \le \frac{1}{2} \, \frac{\tilde{B}_{\parallel}}{B_{\varphi_0}} \, J_{2n+1}(x) \,.$$

The effect of the magnetic component $\tilde{B}_{\parallel}$ can be neglected for

$$\xi = \frac{1}{4\pi}\left(\frac{k_\perp^2 + k_\parallel^2}{k_\parallel}\right)\frac{v_{Te}^2}{R\,\omega_{Be}^2} \gg 1 \,.$$

Here we have compared the effect of the electric and magnetic wave components (considered separately). The effect of $\tilde{E}_\parallel$ has been estimated from the perturbation solution of the canonical equations, related to the Hamiltonian (10). To estimate the effect of $\tilde{B}_\parallel$, we take the largest contribution, given by the original equation (22). Further we use the relation (15). For electrons at thermonuclear temperature T_e = 10 keV, for $k_\parallel$ = 4 m^{-1}, $k_\perp$ = 1.6 m^{-1}, R_0 = 6.4 m and for typical calculated value τ_B = 3×10^{-6} s (for $v_\parallel$ = 0.75×10^7 ms^{-1} and for $v_\perp$ = 7×10^7 ms^{-1}) we obtain $\xi \sim 37$. In this regime, the effect of $\tilde{B}_\parallel$ seems to be substantially weaker than of $\tilde{E}_\parallel$. (For $T_{e,i} \sim 1$ keV, the two effects are comparable).

Since in (22) a set of possible resonances appears, the stochastic random walk among these resonances can be expected. This effect has been already mentioned for others models of interaction by RAM et al., 1984, WHITE et al., 1984 and KRLÍN, 1981.

6. CONCLUSION

The interaction of Alfvén waves with banana-trapped particles results - for sufficiently large wave amplitudes - in the stochastic motion in the whole space. The diffusion in velocities leads to the possibility of the detrapping of particles and it can, e.g., enlarge the efficiency of the Alfvén wave current drive (BELIKOV et al., 1985).

The radial diffusion - induced neoclassical diffusion - can strongly influence the energy balance. The stochasticity threshold for plasma electrons is in the region $\tilde{E}_{\parallel}^{(o)} \sim 10^1 \div 10^2$ V m^{-1}. The stochasticity of alpha-particles requires larger amplitudes (of the order $\tilde{E}_{\parallel}^{(o)} > 10^2$ V m^{-1}). If this stochasticity sets up, rapid diffusion of alpha-particles appears. This might have a negative influence on the energy balance for a stationary current driven reactor or for so called driven reactor. On the other hand, the removing of low energy alpha-particles (ash) from the plasma volume via this effect might prove feasible and is worth further study. (The diffusion of particles in three actions, describing the particle motion in an axisymetric system - - as the consequence of the breaking of adiabatic invariants by the perturbing RF fields - has been considered by KAUFMAN (1972). Here the actions are the magnetic moment, the canonical angular momentum and the toroidal flux enclosed by the drift surface; the quasilinear diffusion is studied).

We have investigated the interaction that leaves the magnetic moment μ constant. For other types of waves (IC, EC), the change of μ can also cause the stochastization of banana motion. Moreover, similar effect can appear by the interaction of lower hybrid waves with alpha-particles (concerning this interaction, see WONG and ONO, 1984 and KRLÍN et al., 1985). This effect is now under study. (The relation between the diffusion in velocity and configuration space - especially for the case of the inhomogeneous magnetic field - requires further investigation, also with regard to the very recent paper of ZASLAVSKII et al., 1986).

A more exact approach requires, of course, to complete the present model of stochastization with the effect of classical

Coulomb scattering (in a way similar to that of FROESCHLÉ, 1975 and KARNEY et al., 1982). Moreover, the discussion of the effect of non-monochromaticity of the applied RF field is necessary.

Last but not least, a broader region of parameters, having the influence on the interaction, has to be taken into account.

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Figure captions

Fig. 1. The coordinate system in toroidal geometry.

Fig. 2. The surface of section, representing the parallel velocity $v_\parallel$ and the phase μ of banana electrons after each banana period for $\theta_L = 0$, $r = r_{min} \cdot \mu$ is modulo of the phase ωt transposed into the interval $\langle -\pi, \pi \rangle$. For all cases, $v_\perp(0) = 7\mathrm{x}10^7$ ms^{-1}. Applied RF field parameters: $\omega = 3\mathrm{x}10^7$ s^{-1}, $k_\parallel = 4$ m^{-1}, $\tilde{E}_\parallel^{(m)} = 10$ V m^{-1}. The current density $i_0 = 1$ MA m^{-2}. $r_0 = 1.5$ m. The detrapping takes place for $v_\parallel(0) \gtrsim 5.5\mathrm{x}10^7$ ms^{-1}. Thirty bananas.

Fig. 3. The same as in Fig. 2, but for $\tilde{E}_\parallel^{(m)} = 20$ V m^{-1}. The resonant islands appear in the region $v_\parallel = 1.38\mathrm{x}10^7$ ms^{-1} ($\omega/\omega_B = 12$), $v_\parallel = 3.75\mathrm{x}10^7$($\omega/\omega_B = 13$) and $v_\parallel =$ $= 4.5\mathrm{x}10^7$ms^{-1}($\omega/\omega_B = 14$).

Fig. 4. The same as in Fig. 2, but for $\tilde{E}_\parallel^{(m)} = 50$ V m^{-1}.

Fig. 5. The same as in Fig. 2, but for $\tilde{E}_\parallel^{(m)} = 100$ V m^{-1}.

Fig. 6. The detail of Fig. 3. Sixty bananas.

Fig. 7. The surface of section for the same parameters as in Fig. 2, but for $i_0 = 0.566$ MA m^{-2}. Sixty bananas.

Fig. 8. The same as in Fig. 7, but for $\tilde{E}_\parallel^{(m)} = 20$ V m^{-1}. The resonant island is evident especially in the region $v_\parallel =$ $= 1.3\mathrm{x}10^7$ ms^{-1}($\omega/\omega_B = 22$).

Fig. 9. The surface of section for the case $\omega = 1.5\mathrm{x}10^7$ s^{-1}, $k_\parallel = 2$ m^{-1}, $\tilde{E}_\parallel^{(m)} = 20$ V m^{-1}, $i_0 = 0.566$ MA m^{-2}, $v_\perp(0) = 7\mathrm{x}10^7$ ms^{-1}. Sixty bananas. Large resonant island appears in the region $v_\parallel = 1\mathrm{x}10^7$ms^{-1}($\omega/\omega_B = 10$).

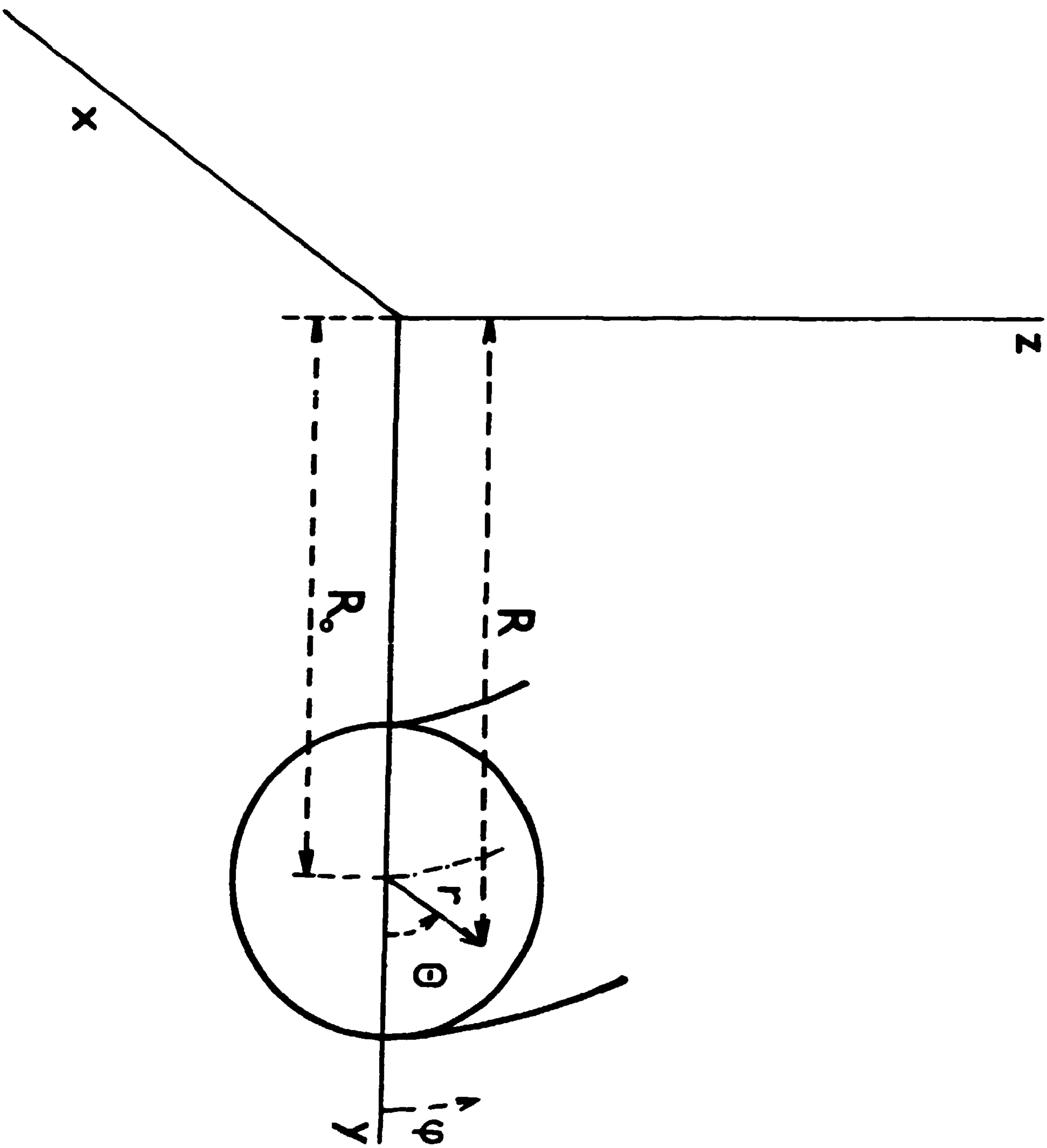
Fig. 10. The radial devitation $\Delta r'$ of banana electons from the initial centre of the banana $r = r_0$ after i-th banana period. The lines between successive points help only to follow the individual cases. Curves (1), (2), (3) are plotted for $\tilde{E}_{\parallel}^{(w)} = 20, 100, 200$ V m^{-1}, respectively. $v_{\parallel}(0) = 7.5\text{x}10^6$ ms^{-1}. The remaining parameters as in Fig. 2.

Fig. 11. The surface of section for alpha-particle bananas. Applied RF field parameters: $\omega = 1.34\text{x}10^7$ s^{-1}, $k_{\parallel} = 3$ m^{-1}, $\tilde{E}_{\parallel}^{(w)} = 200$ V m^{-1}. The current density $j_0 = 1$ MA m^{-2}, $r_0 = 1$ m. The detrapping takes place for $v_{\parallel}(0) \gtrsim 5.8\text{x}10^6$ ms^{-1}. Thirty bananas. The resonant islands appear in the region $v_{\parallel} = 4.6\text{x}10^6$ms^{-1} ($\omega/\omega_B = 47$) and $v_{\parallel} = 5.2\text{x}10^6$ ms^{-1} ($\omega/\omega_B = 53$).

Fig. 12. The same as in Fig. 11, but for $\tilde{E}_{\parallel}^{(w)} = 1000$ V m^{-1}.

Fig. 13. The sequence of radial devitations $\Delta r'$ from the initial centre of bananas for alpha-particles vs the number of banana periods. Curve (1) is plotted for $E_{\parallel} =$ $= 200$ V m^{-1}, $v_{\parallel}(0) = 5.5\text{x}10^6$ ms^{-1}, curve (2) for $\tilde{E}_{\parallel}^{(w)} = 100$ V m^{-1}, $v_{\parallel}(0) = 5.7\text{x}10^6$ m s^{-1} (both for $i_0 = 1$ MA m^{-2}), curve (3) for $\tilde{E}_{\parallel}^{(w)} = 1000$ V m^{-1}, $v_{\parallel}(0) =$ $= 5\text{x}10^6$ m s^{-1} and for $i_0 = 0.566$ MA m^{-2}.

Fig. 14. The banana geometry and symbols, introduced in the eq. (16). The initial banana centre is denoted by A .



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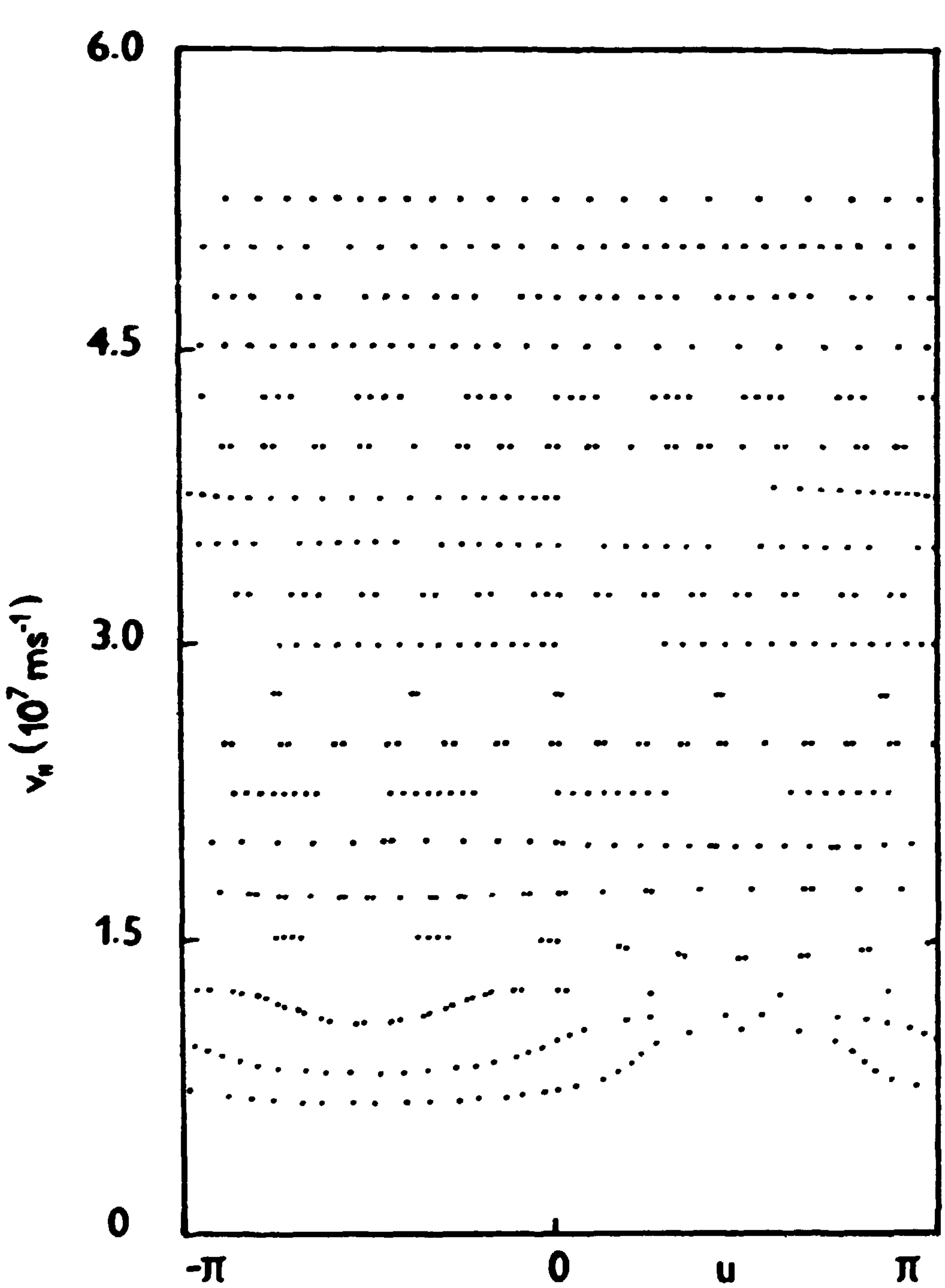

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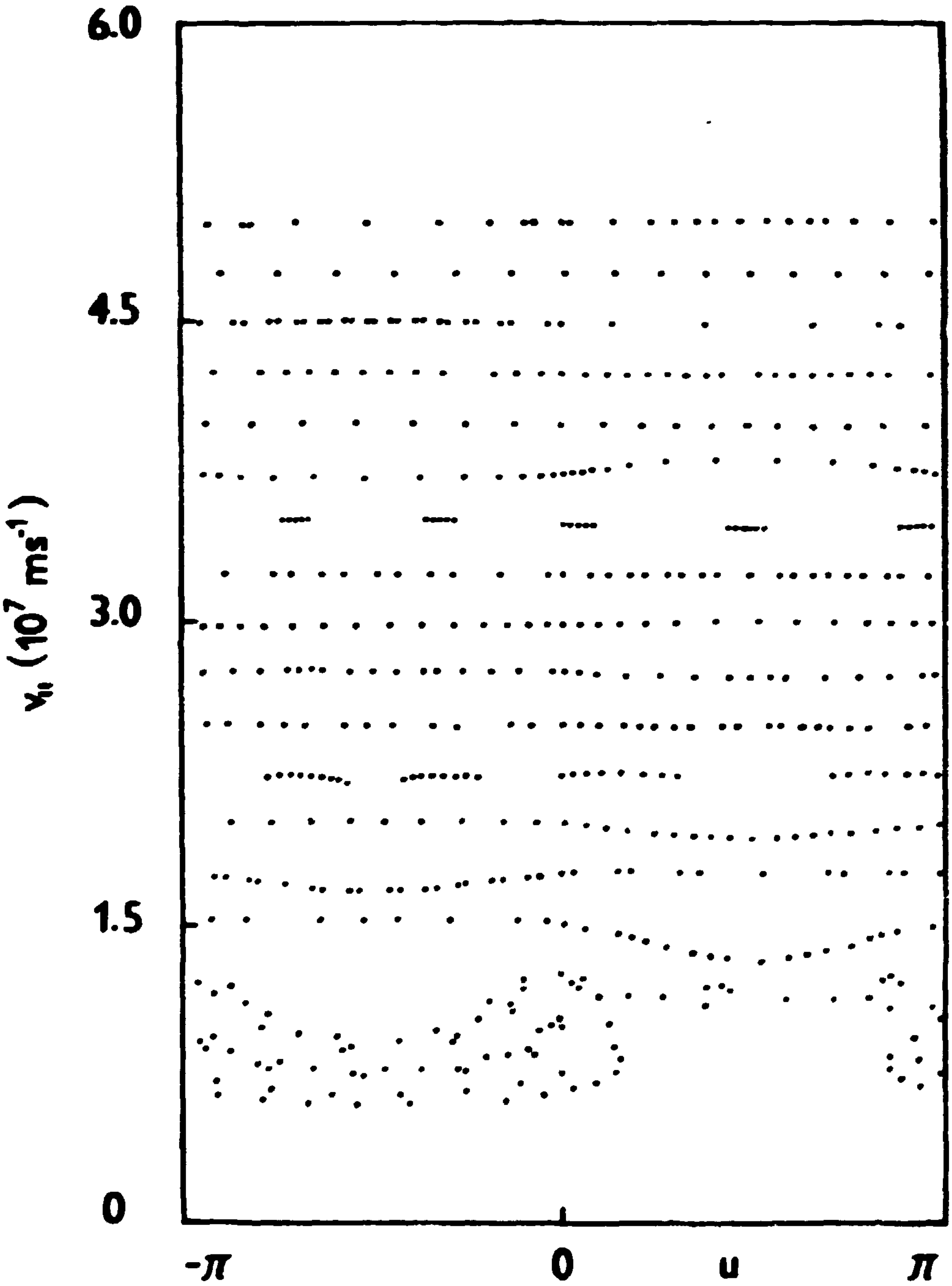

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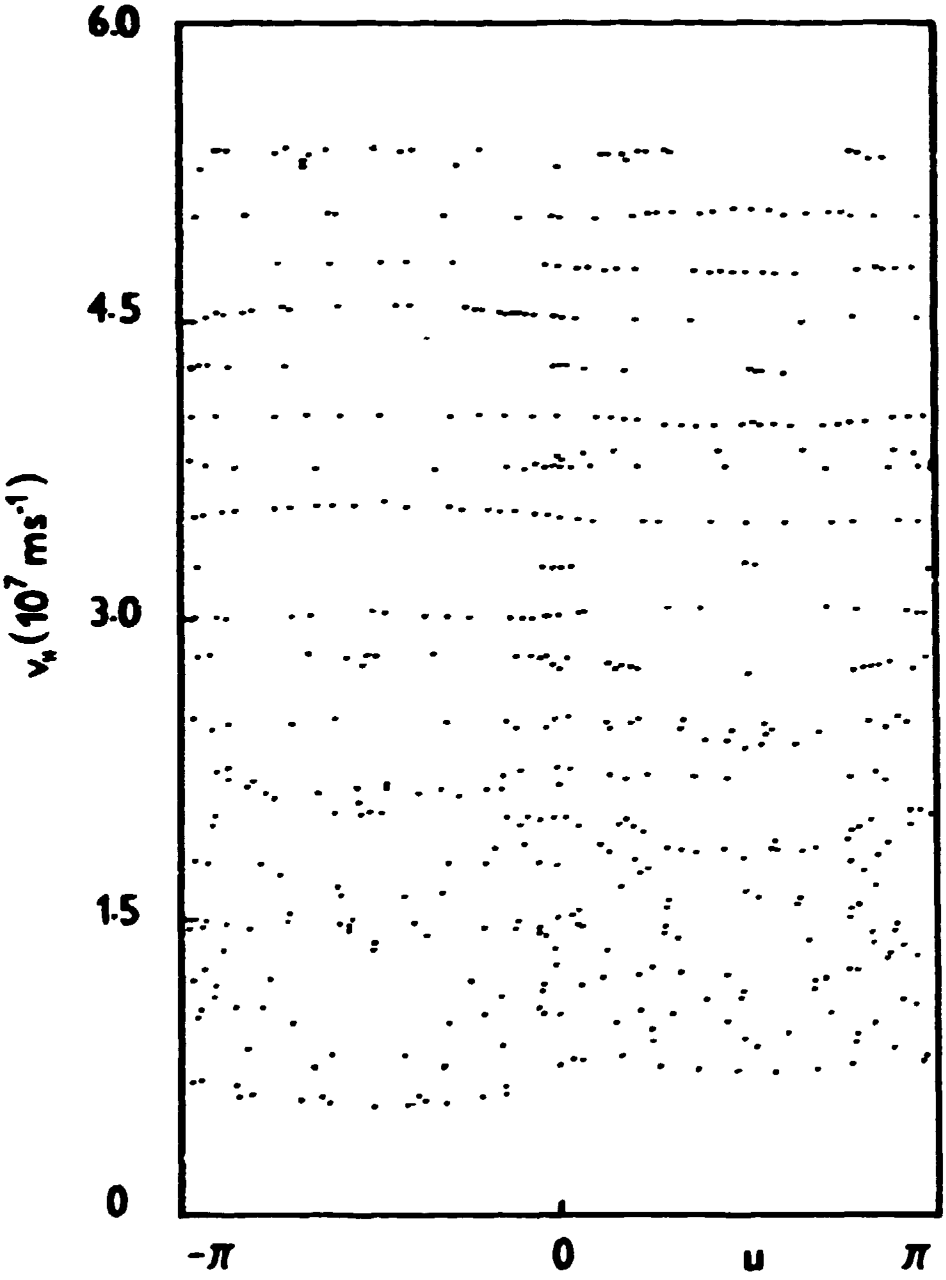

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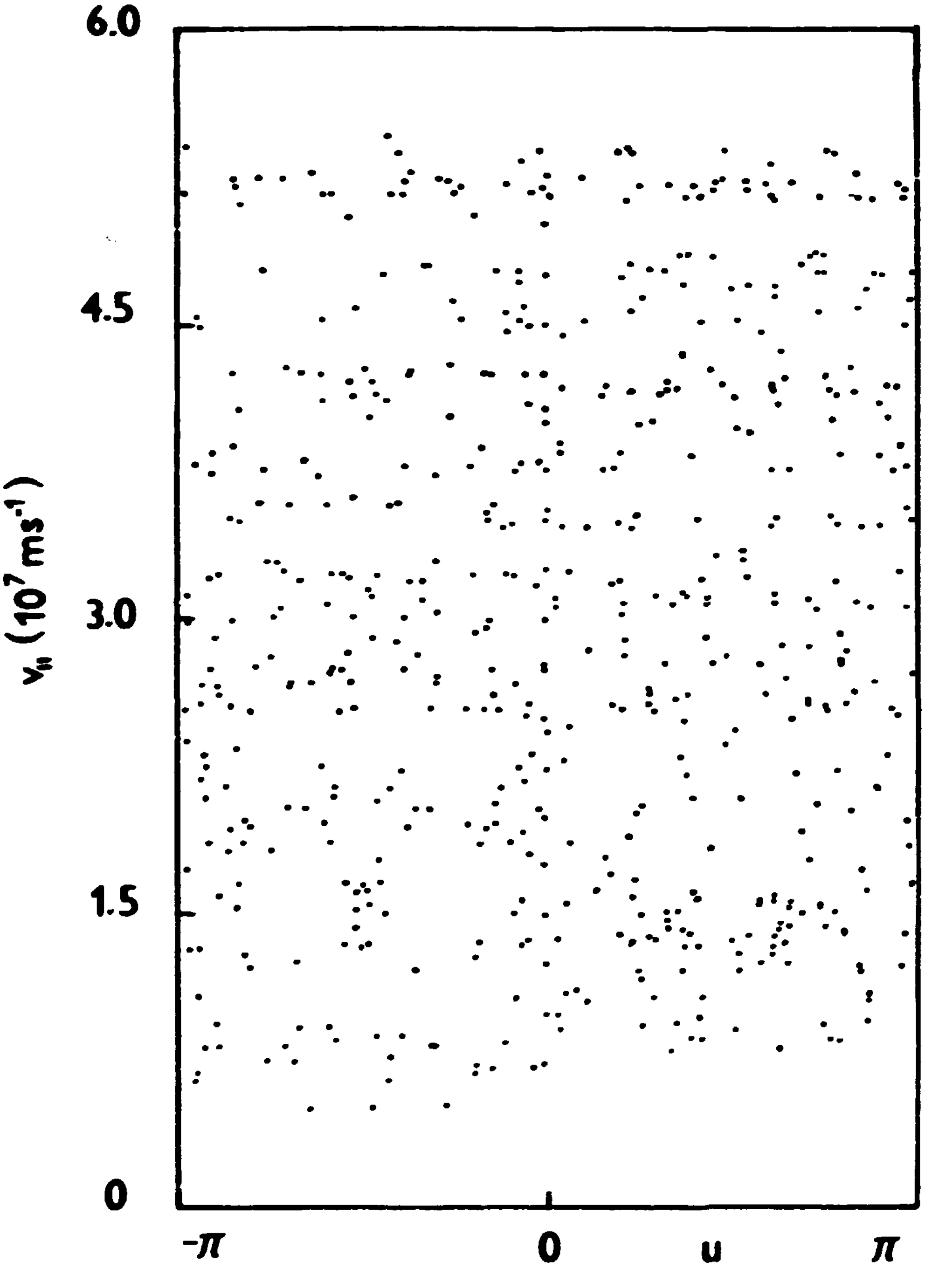

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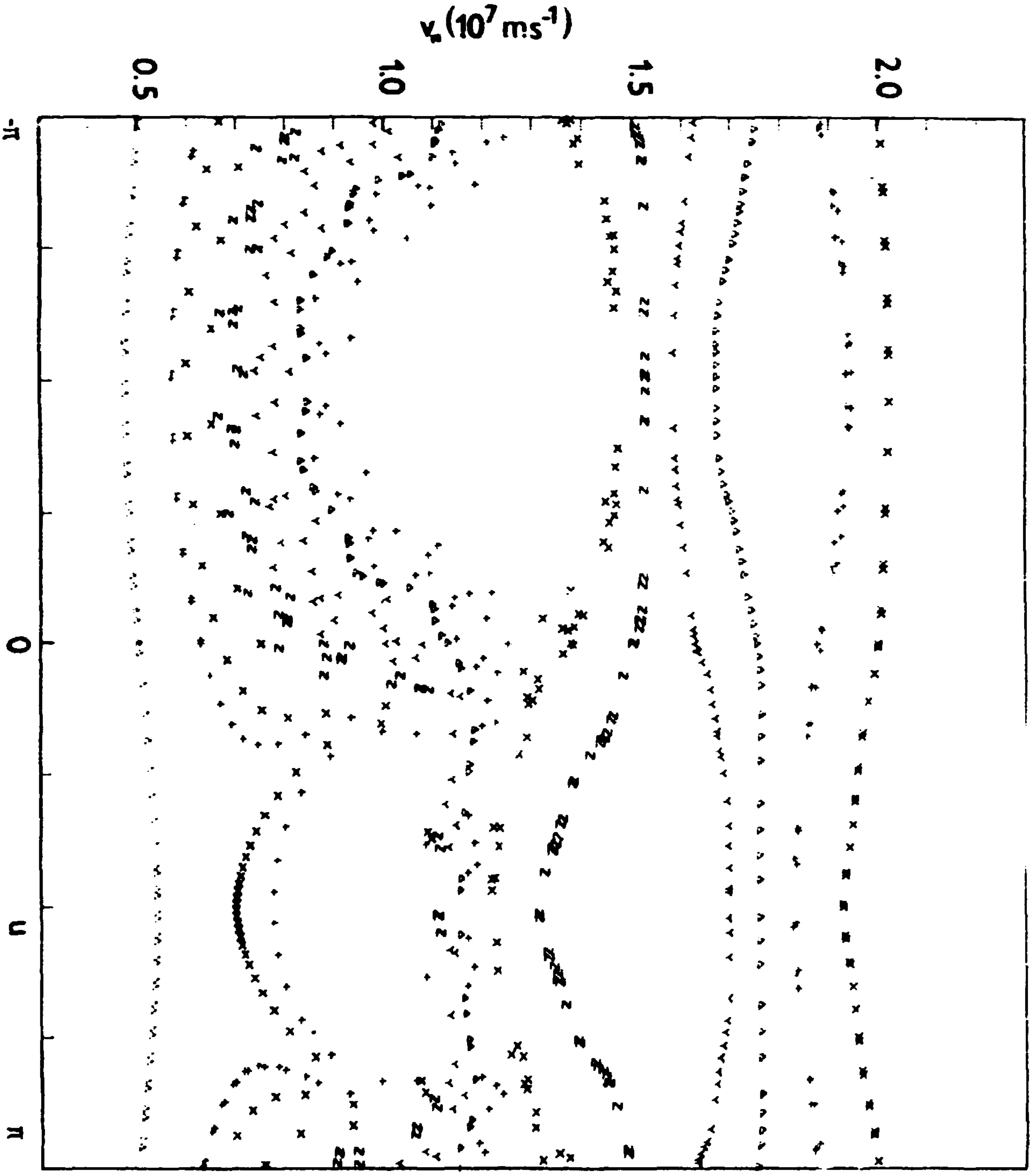

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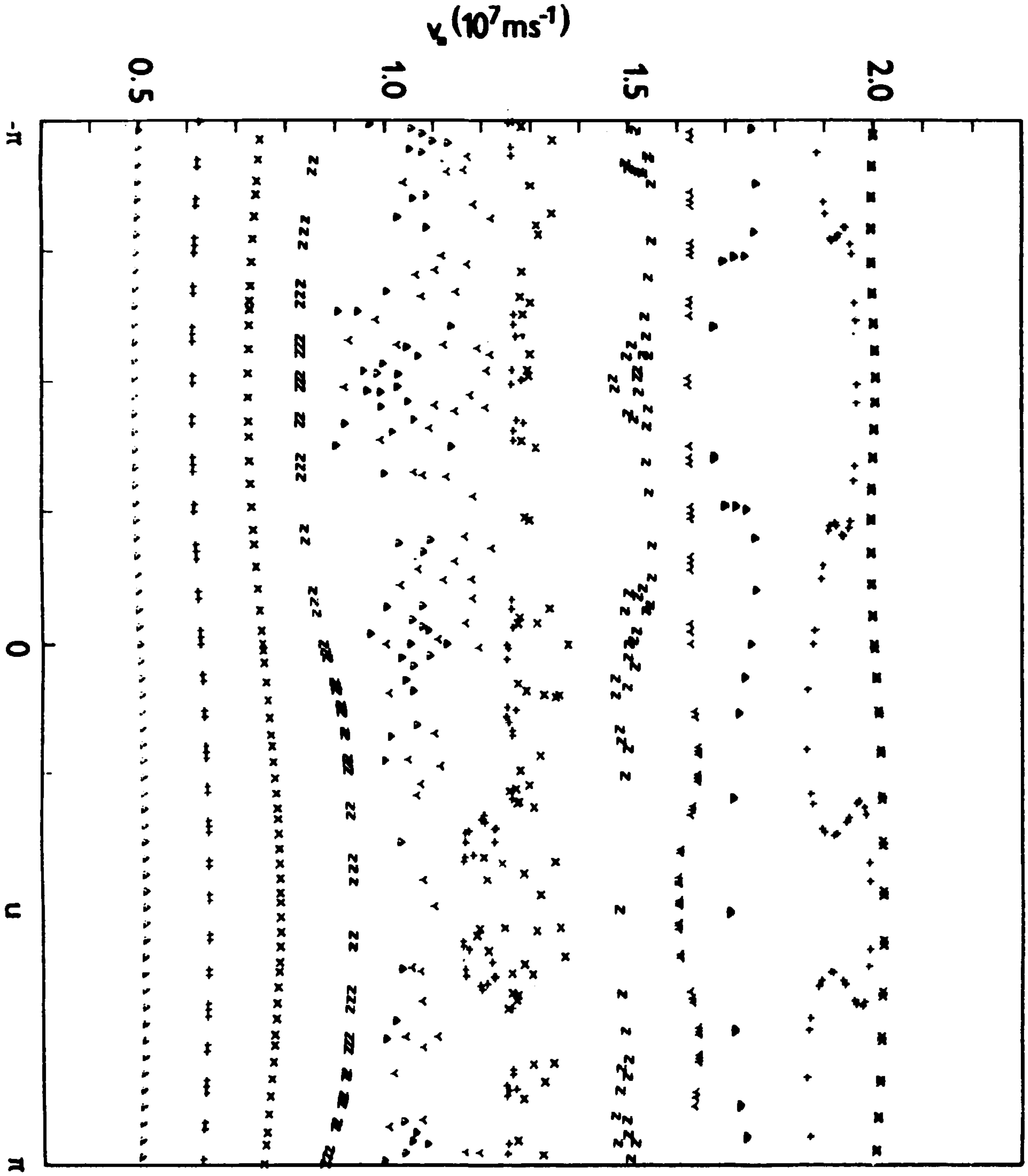

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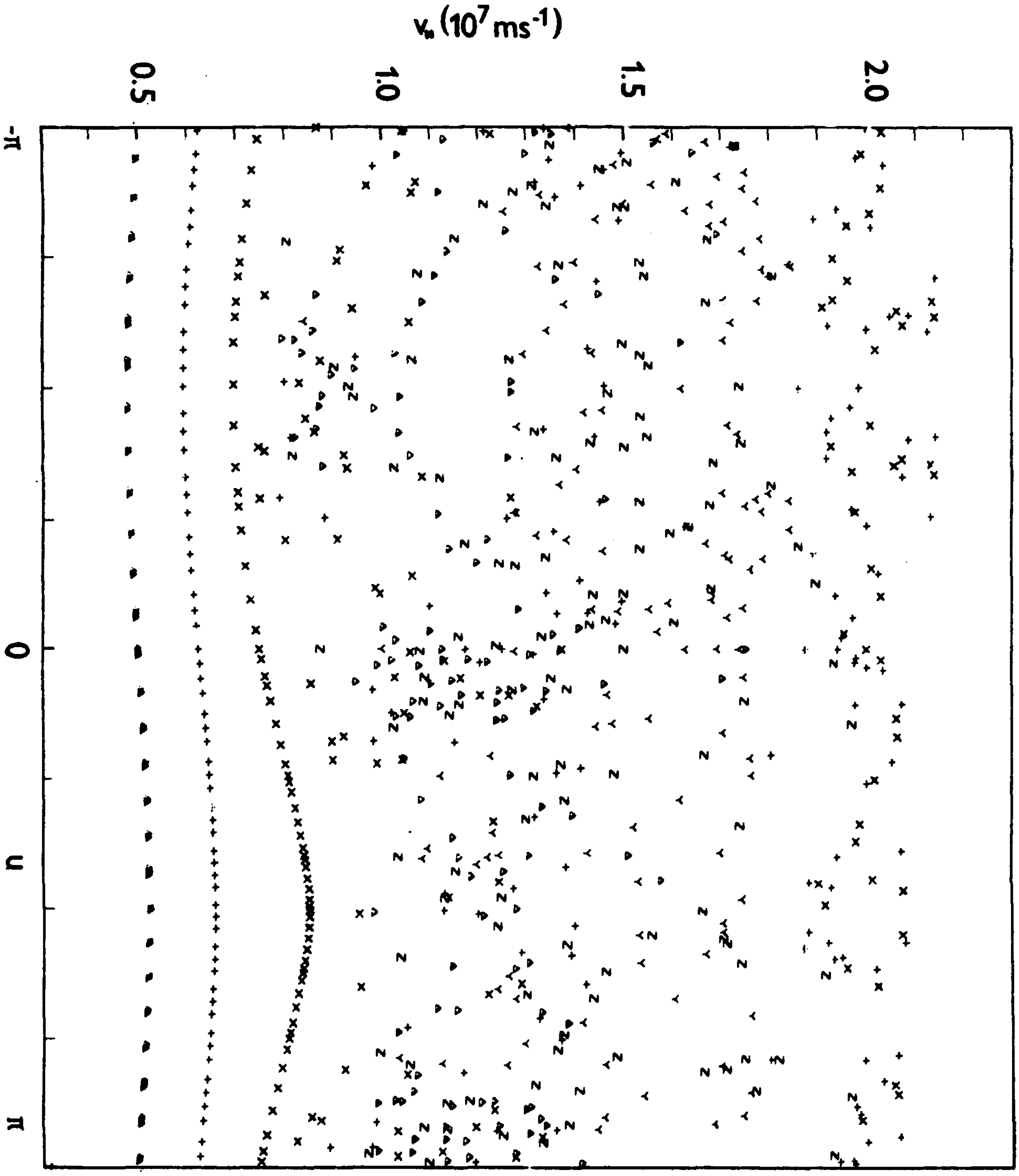

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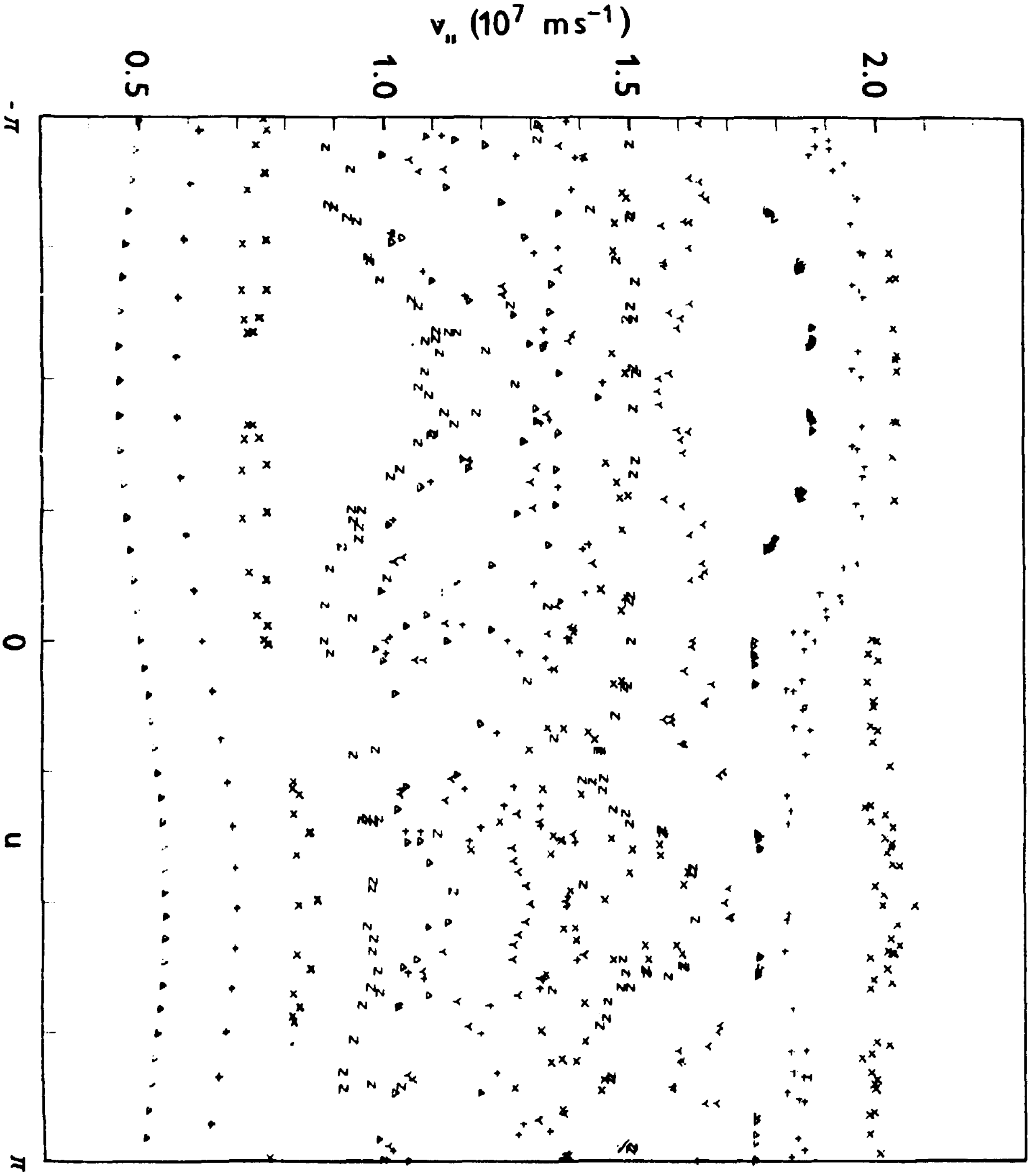

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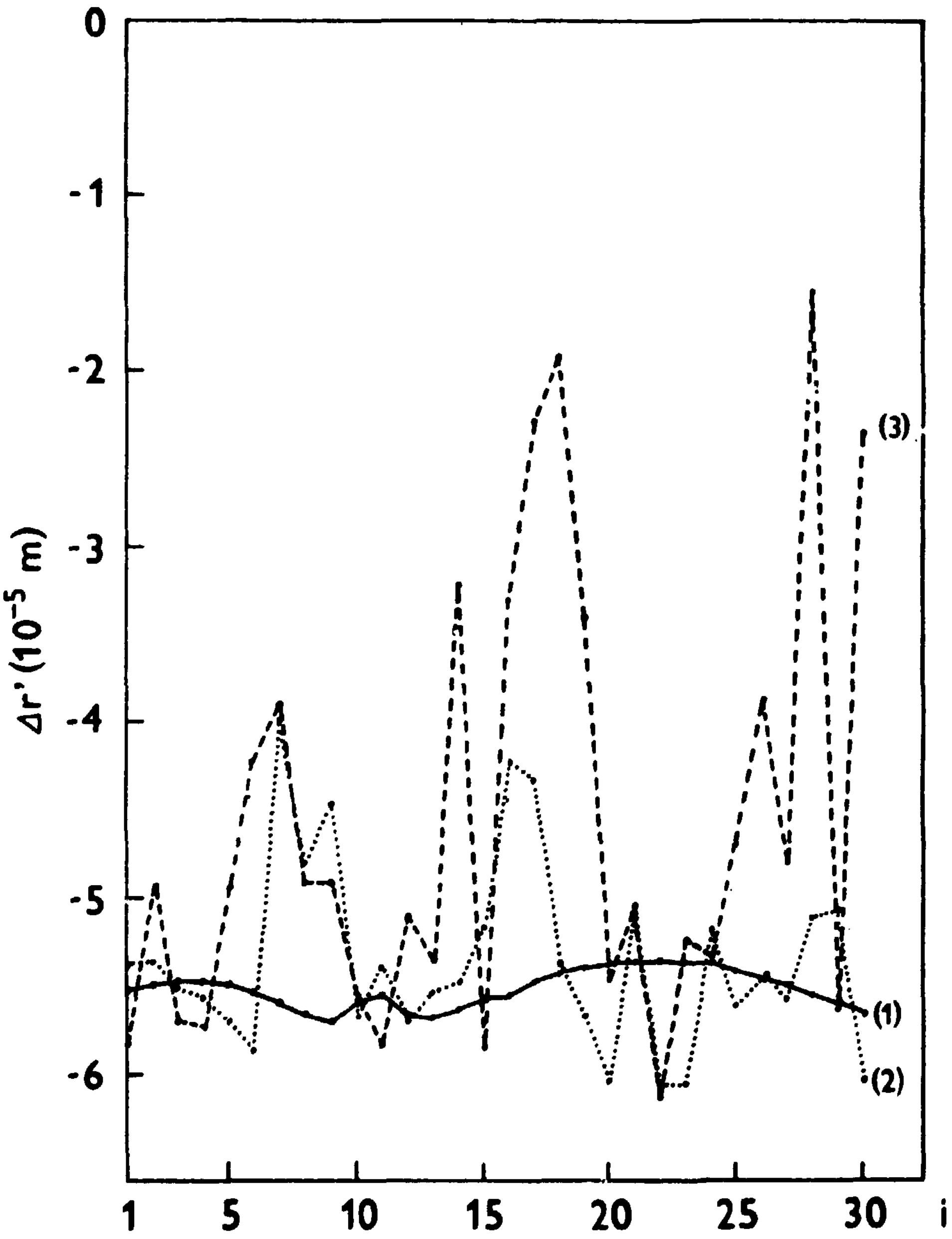
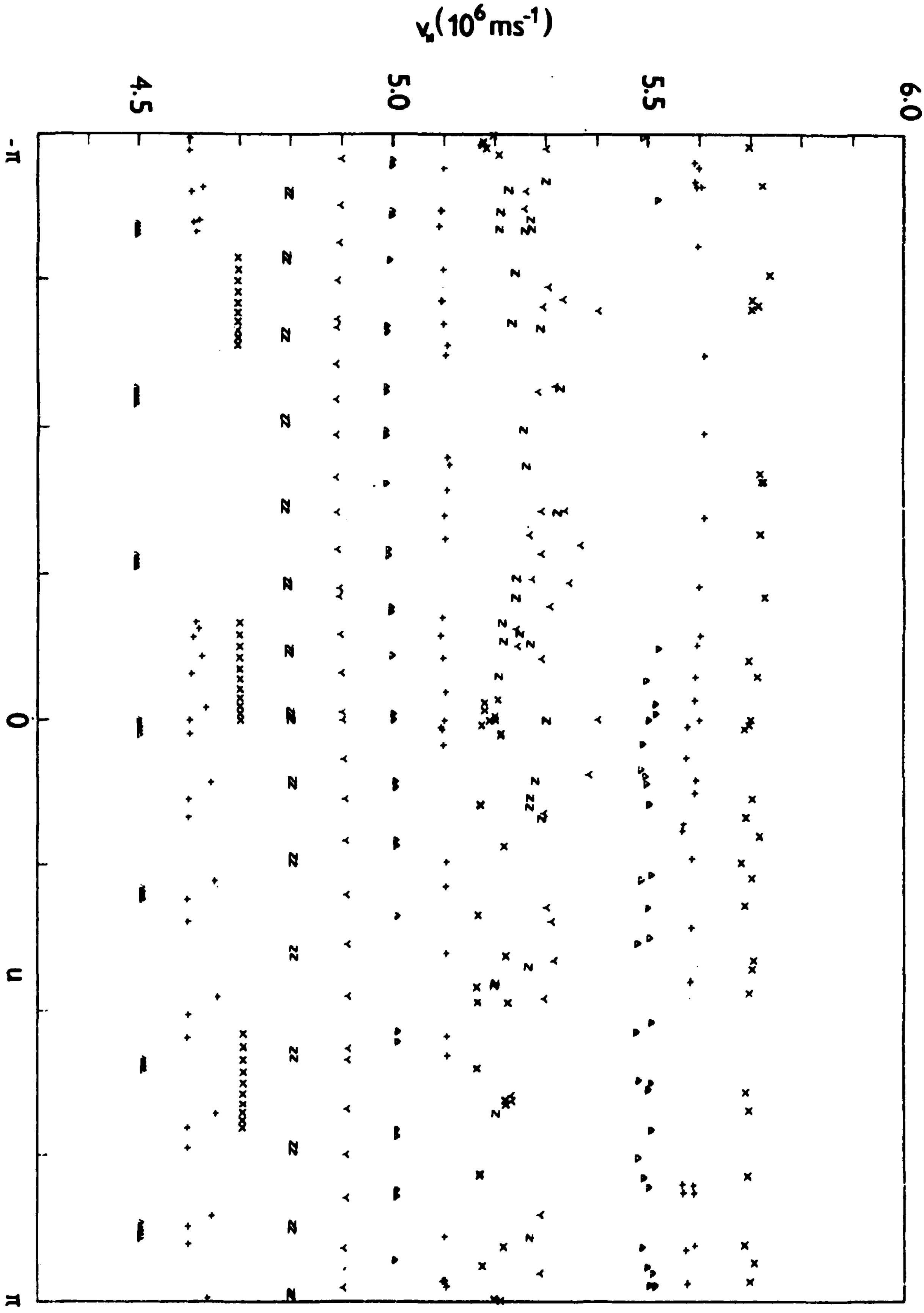


Fig. 10.



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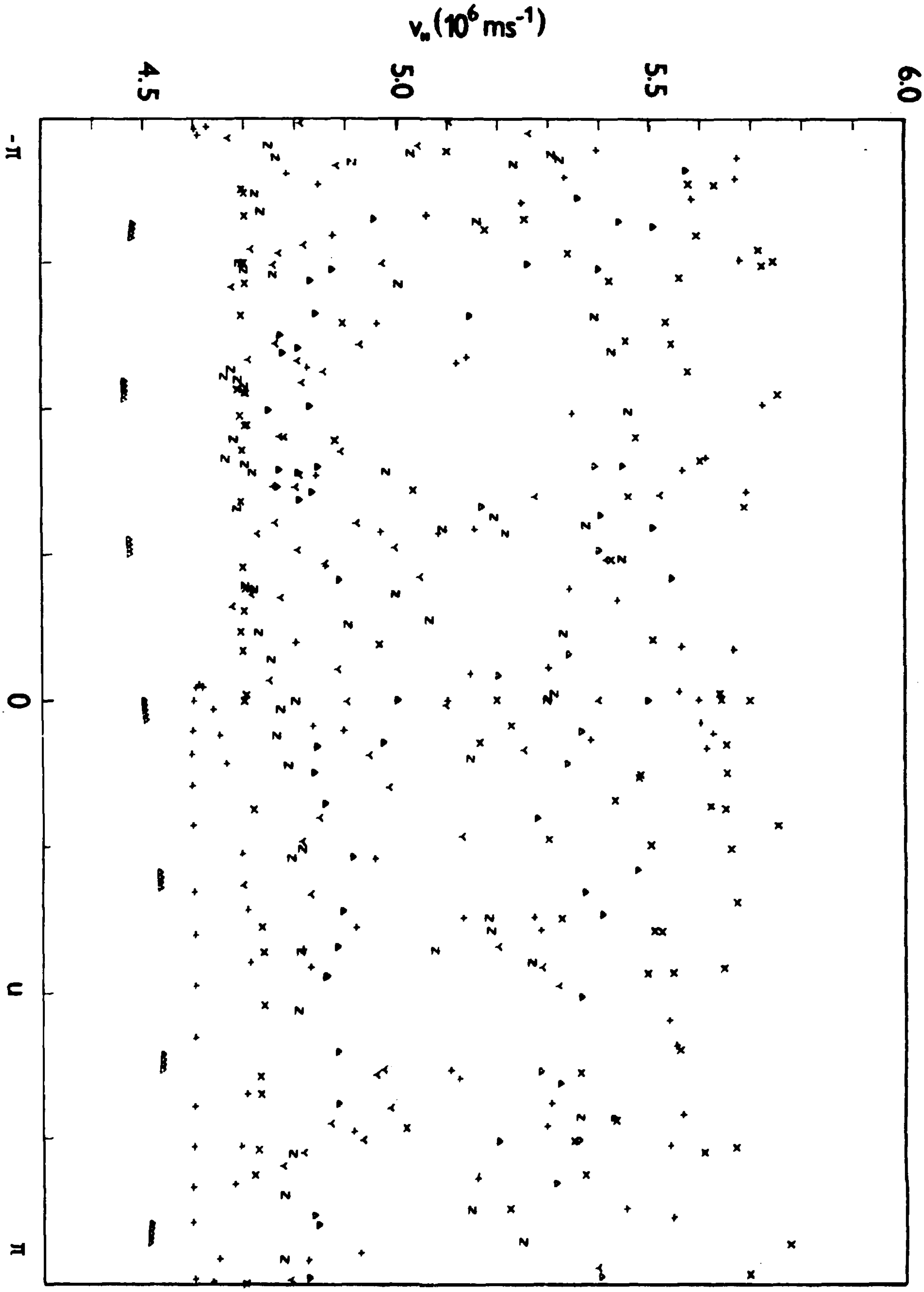
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5.0
5.5
6.0
- π
0
π

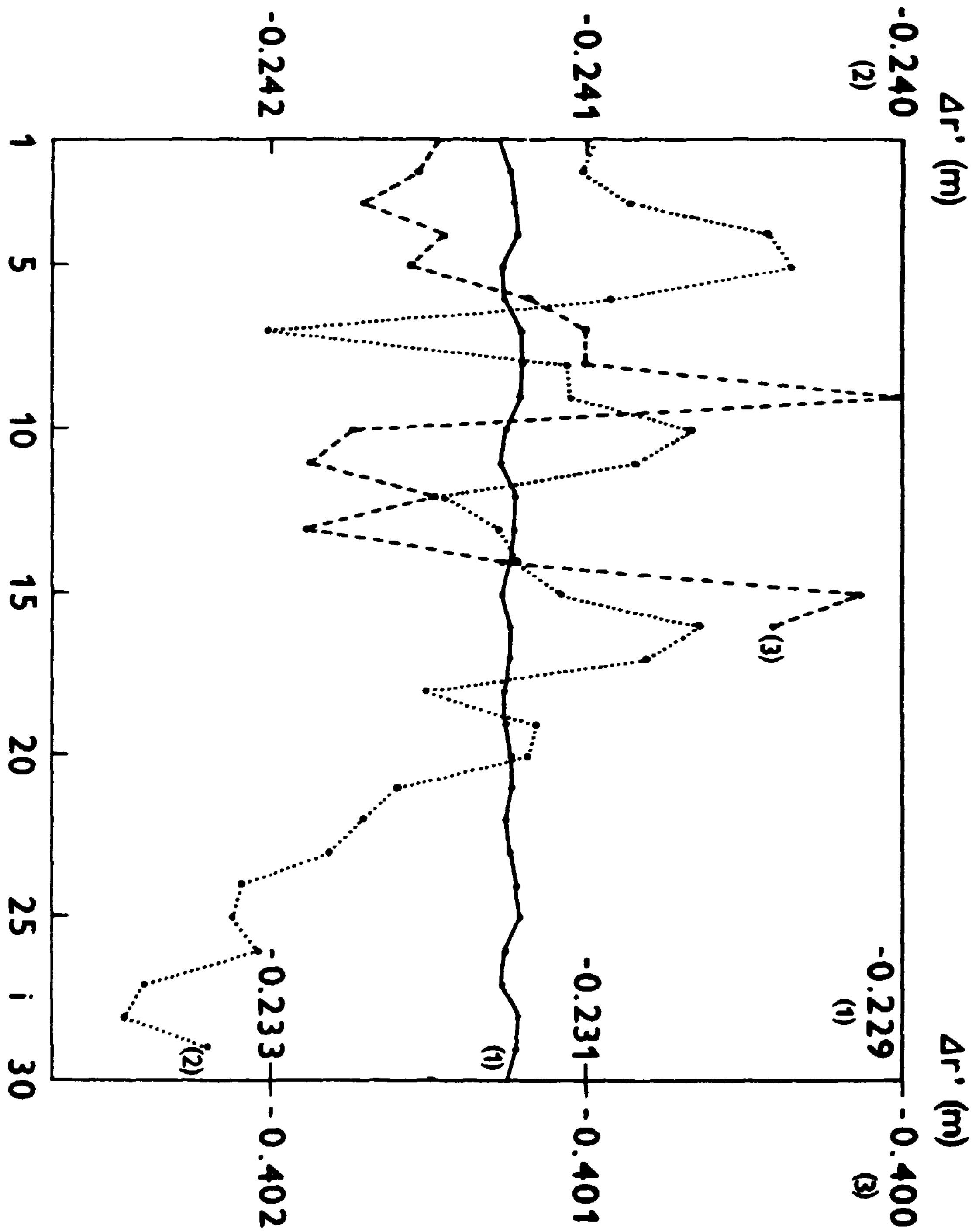
Δr' (m)
-0.240
(2)
-0.241
-0.242
-0.229
(1)
-0.231
-0.233
-0.400
(3)
-0.401
-0.402
1
5
10
15
20
25
i
30
(1)
(2)
(3)

Fig. 13.

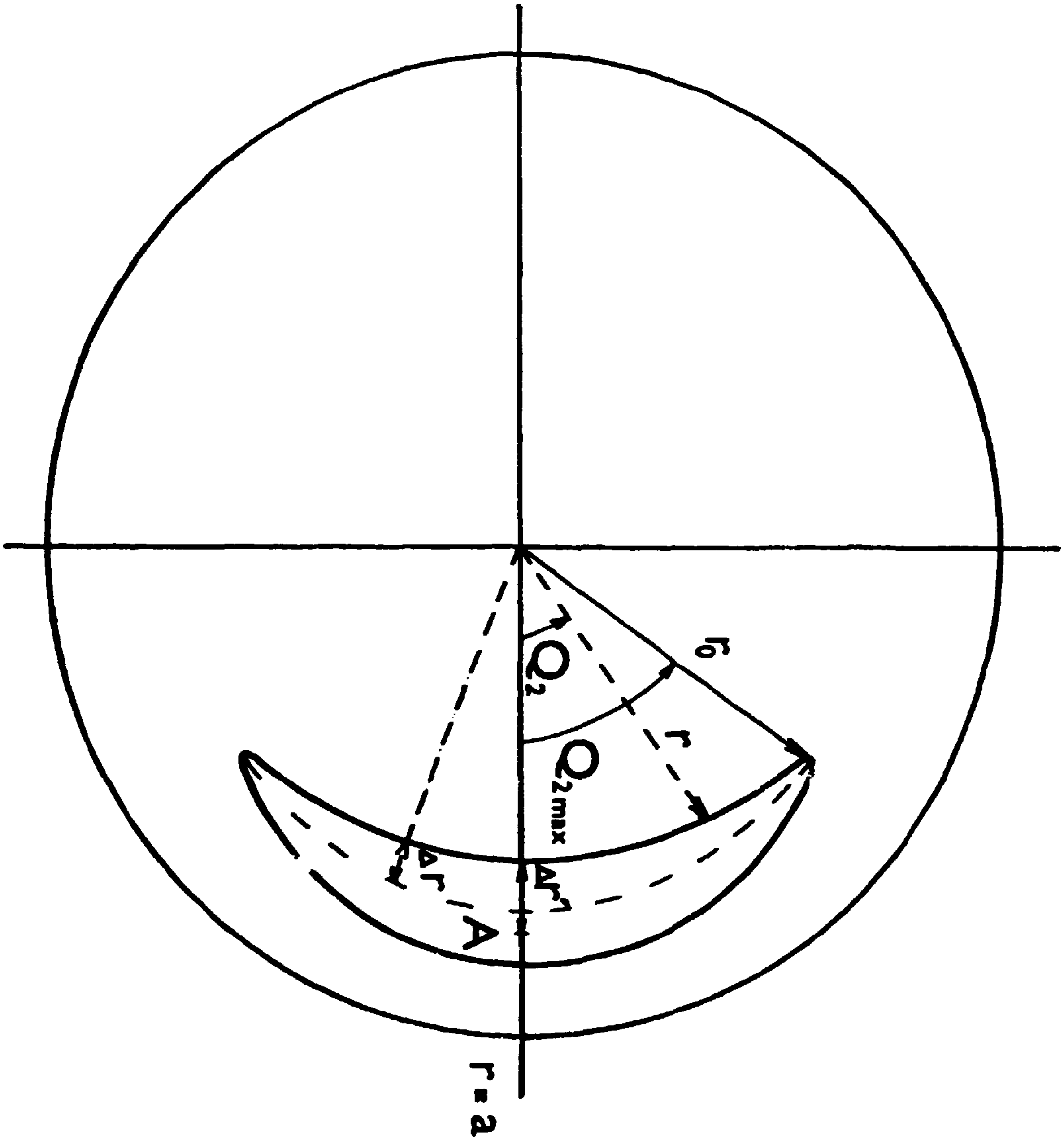

Fig. 14.