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THE COSMOLOGICAL CONSTANT PROBLEM

A.D.Dolgov
Institute for Theoretical and Experimental Physics,
Moscow, USSR

Abstract

A review of the cosmological term problem is presented. Baby universe model and the compensating field model are discussed. The importance of more accurate data on the Hubble constant and the Universe age is stressed.

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Two major problem in particle physics originated from cosmology. The first one is the hidden mass problem which presents a serious challenge to experimentalists. Unfortunately, the chances to observe the hidden mass particles directly are negligible for a good lot of existing hypotheses on their nature. In a sence the problem of cosmological constant can be connected with the hidden mass problem because the corresponding vacuum energy may, at least to some extent, provide the hidden mass. Of course, the implication of cosmological constant is much wider than that. One can expect that the understanding why the cosmological constant is so small in the scale of the particle physics will have tremendous impact both on quantum field theory and cosmology. The discrepancy between theoretical expectations for its value and the observational upper bound are 100-50 orders of magnitude. This is a singular example when theoretical order of magnitude estimate differs by such an enormous amount from the real world. But we know that scientific development is based on contradictions between theory and experiment, so "the worse the better", this huge contradiction may lead to a great discovery as, for example, the observed stability of atoms has lead to quantum mechanics.

The cosmological term was introduced into Gereral Relativity equations by Einstein in 1918:

$$R_{\mu\nu}-1/2g_{\mu\nu}R=8\pi G_N T_{\mu\nu}+\Lambda g_{\mu\nu} \qquad (1)$$

The right-hand side of this equation is a source of the gravitational field tensor $R_{\mu\nu\alpha\beta}$ (Rieman tensor) in (almost) the same sence as electromagnetic current j_μ is a source of the Maxwell field tensor F_μ . The first term in the r.h.s., $T_{\mu\nu}$, is the energy-momentum tensor of matter and the second one, $\Lambda g_{\mu\nu}$, is the cosmological term.

The l.h.s. is kinematically conservered

$$D_\mu(R^\mu_\nu-1/2g^\mu_\nu R)\equiv 0 \qquad (2)$$

This implies the conservation of the r.h.s. It is usually assumed that the energy-momentum tensor of matter is conserved

$$D_\mu T^\mu_\nu \equiv T^\mu{}_{;\nu} = 0 \quad (3)$$

This is true if one starts with a Lagrangian field theory in which $T_{\mu\nu}$ is defined as functional derivative of action with respect to metric:

$$T_{\mu\nu} = \delta S/\delta g_{\mu\nu} = \frac{\delta}{\delta g^{\mu\nu}} \int d^4x \sqrt{-g} L \quad (4)$$

where scalar function L is the Lagrangian density. In this case the second term in the r.h.s. must be also conserved and since $g_{\mu\nu;\alpha} = 0$ then

$$\Lambda = \mathrm{const} \quad (5)$$

The cosmological term can be interpreted as vacuum energy-momentum tensor

$$T_{\mu\nu}^{vac} = \rho^{vac} g_{\mu\nu} = \Lambda g_{\mu\nu}/8\pi G_N \quad (6)$$

This is the only tensor compatible with Lorentz invariance and in fact vacuum energy calculation in quantum field theory gives the expression for $T_{\mu\nu}^{vac}$ of this form. Thus there can exist nonvanishing vacuum source of gravitational field in contrast to, say, electrodynamics where nonvanishing vacuum currents $j_\mu^{vac}=0$ break Lorentz invariance.

Einshtein invented cosmological term in order to get a stationary cosmological model. He noticed that eq. (1) with $\Lambda = 0$ does not have stationary solutions for homogeneous and isotropic distribution of matter. Λ-term was introduced to compensate gravitational attraction of matter on large scales. But later on it became clear that the Universe is nonstationary, it expands and Einstein rejected the idea of Λ-term considering it to be the greatest mistake of his life. This term is permitted by general covariance but no other excuse for its existence was known. The principle "everything that is not forbidden is permitted" was not operative in physics at that time. Nevertheless Le Maitre advocated very much the existence of cosmological constant and, as we understand it now, he was right. Quantum field theory demands nonvanishing vacuum energy which must be fantastically larger than the observed upper bound. The first physicist who understood the problem was to my knowledge G.Gamov. In his letter to A.Ioffe in 1930 he wrote about the gravitational interaction of the Dirac sea. The hystory of the problem starts from the paper by Zel'dovich [1] where it was explicitely stated that zero-mode oscillations must gravitate and the contributions of bosonic and fermionic degrees of freedom might be cancelled out.

It is well known that the ground state energy of quantal oscillator is nonzero

$$E_0 = \omega/2$$

where ω is the oscillatior frequency.

In quantum field theory the field is represented by an infinite set of oscillators in every space point with all possible frequences ω. Hence the energy of the ground state, i.e. the vacuum energy, is equal to

$$\rho_{vac}=E_{vac}/V=\Sigma\omega/2 \dashrightarrow \int d^3p/(2\pi)^3\omega_p/2 = \infty \qquad (7)$$

which is definitely larger than zero. Here $\omega_p = \sqrt{p^2+m^2}$, and m is the mass of the field quanta.

The lucky circumstance is that the fermionic contribution into ρ_{vac} is of the opposite sign. So if there were an equal number of bosons and fermions with the same masses (in pairs) the contribution of quantum fluctuations into total vacuum energy should cancell out. Such a symmetry between bosons and fermions is called supersymmetry. There exists a formal proof that in the limit of exact supersymmetry vacuum energy must vanish. But we know from experiment that supersymmetry is not exact. Masses of bosons and fermions are different and superpartners of the known particles, if exist, must be as heavy as at least several GeV.

If supersymmetry is broken spontaneosly then vacuum energy must be nonvanishing and be of the order of m_{susy} where m_{susy} is the mass scale of the supersymmetry breaking. Presumably $m_{susy} >$ 100 GeV and correspondingly $\rho_{vac} > 10^8$ GeV^4. This conclusion can be avoided if sypersymmetry is local, i.e. the symmetry transformation can be done in different space-time points independently. To compensate the action variation due to difference of the transformation in different points one has to add into the theory vector fields and, what's more, massless tensor field with spin 2. The latter is the graviton field. So this theory automatically includes gravity. In the supergravity frameworks, even if the symmetry is broken, vacuum energy may be zero. There exist models in which vacuum energy rather naturally vanishes on the classical and one-loop level but no explanation is found for the vanishing of the higher order quantum corrections. This issue is discussed in some detail in the recent review paper [2] where one can find the corresponding list of references. This review is much more exhaustive than the present talk and may be recommended for a deeper study of the Λ-term problem. My aim here is to give an elementary introduction to the subject digestable to experimentalists and astronomers.

Thus supersymmetry let one expect that vacuum energy is finite but gives by an order of magnitude estimate a very huge number for it. No specific mechanism leading to vanishing of ρ_{vac} in the frameworks of supersymmetric theories has yet been found.

Advent of gauge theories with spontaneousdy broken symmetry has produced another source of contributions into vacuum energy. The phase transition from symmetric state to the state with broken symmetry is accompanied by a change in the vacuum energy

$$\Delta T_{\mu\nu} = \Delta\rho \, g_{\mu\nu} \qquad (8)$$

where $\Delta\rho_{GUT} = 10^{60}$ GeV^4 for Grand Unification models, $\Delta\rho_{EW} \sim 10^{8}$ GeV^4 for electroweak theory, and $\Delta\rho_{QCD} \approx 0.1$ GeV^4 for quark-hadron phase transition in quantum chromodynamics. These phase transitions took place in the course of the Universe expansion and cooling down [3,4]. So to get $\rho_{vac}=0$ today the Creator must prepare the initial state with $\rho_{vac} \neq 0$ and with such an accuracy that the subsequent phase transitions would cancell it to the degree better than 10^{-100}, which clearly seems to be a difficult job.

I would like to stress that there definitely exists nonzero contribution into vacuum energy of the order of 0.1 Gev^4 from gluon condensate [8]

$$\langle G_{\mu\nu} G^{\mu\nu} \rangle \neq 0$$

The existence of this condensate is, in a sence measured experimentaly. So the situation looks absolutely crazy since this contribution must be exactly cancelled out either by vacuum energy of some other field or by real cosmological term adjusted to ρ_{vac} with accuracy better than 10^{-48}.

Up to the present day no convincing resolution of the Λ-term problem is known. There are several possibilities discussed in the literature, but no one is absolutely satisfactory. A list of possible ways of the solution which is by no means complete looks as follows:

1. A compensating field (like axion?).
2. Baby universes.
3. Anthropic principle.
4. Modification of gravity.
5. ? ...

The order reflects my own preferences which may not coincide with those possessed by some other physicists. In what follows the first two cases are mostly discussed and only a few comments are made on the last ones. Those who are interested in them are addressed to review [2]. One thing which is definite is that the cancellation of the cosmological term is a low energy phenomenon which is operative at long time and distance scales.

Modification of gravity does not seem very appealing to me because General Relativaty is a too nice theory to be spoiled. But who knows... It may be premature to judge about the beauty of not yet existing modification.

Anthropic principle states that the Universe must have appropriate conditions for life otherwise there would be no observer who could put a question about e.g. cosmological term. Using this idea S.Weinberg [4] has found the upper bound on the (positive) vacuum energy density $\rho_{vac} < 100\rho_m$, where ρ_m is the contemporary energy density of matter in the Universe. With a larger ρ_{vac} the Universe expansion rate would be too high to permit the galaxy formation. A comparable (by the absolute value) bound is valid for negative ρ_{vac} since very large $(-\rho_{vac})$ would result in too small time duration from the Big Bang till recollapse.

As was claimed by Linde [7] anthropic constraint on the vacuum energy density can be made as strong as $\rho_{vac} < 10^{-10000}$ g/cm^3 if one assumes the validity of chaotic inflation scenario with eternally existing Universe and eternally and continuously existing Life.

Arguments using anthropic principle would be more close to religion than to science if there were no chances for existence of universes (or parts of the Universe) with quite different conditions, physical laws, and so on. In the approach based on the baby universes different values of the so called fundamental constants like particle masses, coupling constants, and at last Λ-term can be realized. Thus it can give a justification of anthropic principle. In fact one expects even more. Baby universe model may permit to calculate the probability distribution for all the constants and so makes them caculable. In particular the probability of vanishing Λ-term is claimed to be infinitely large in comparison with all the other values of Λ. Here lies an essential difference between the baby universe model and the compensating field model. The latter predicts that vacuum energy is not exactly cancelled out but only up to terms of the order of m_{Pl}^2/t^2 i.e. of the order of the critical energy density. So there is a way to distinguish between these two models.

Before going into further theoretical details I shall briefly comment on the present observational status of Λ-term. If $\Lambda=0$ the Universe age can be expressed as follows

$$t_u \approx H_o^{-1}/(1+1/2\sqrt{\Omega})=9.8 \text{ Gyr } h_{100}^{-1}/(1+1/2\sqrt{\Omega}) \quad (9)$$

where $H_o = (100 \text{ km/s/mpc})h_{100}$ is the Hubble constant and $\Omega=\rho/\rho_c$ is the ratio of the average energy density in the Universe to the critical energy density $\rho_c=3H^2/8\pi G$. Inflationary universe model predicts $\Omega=1\pm10^{-4}$. Observations give a smaller value $\Omega=0.1\div0.3$ but they are sensitive to the clustered matter only and not to that uniformly distributed. For the latter the result presented here by Rowan-Robinson is valid which is $\Omega=0.7^{+.3}_{-.16}$. The chances that inflationaty model is true are very high so it seems safe to assume that $\Omega=1$. It was stated by Peebles in his summary talk at this conference that $h_{100}=0.65\pm0.15$ so if $\Lambda=0$ the Universe age must be smaller than 13.5 Gyr. As Rood and Schramm have told us the age of globular clusters and nuclear chronology are compatible with 12 Gyr $< t_u <$ 18 Gyr. So we are still uncertain. If however the bound claimed by Rocca-Volmerange, $t_u > 17$ Gyr, is valid we either have to admit that $\Lambda\neq0$ or $\Lambda=0$, $h_{100}=0.5$ and $\Omega=0.1$ i.e. the inflationary model is wrong. Constraints on decceleration parameter, q_0, presented here by Guiderdoni and Triay seem to be in favour of nonvanishing Λ but systematic errors could be large.

Nonvanishing neutrino mass can help to resolve the problem because the Gerstein-Zeldovich bound

$$m_\nu < 400 \text{ eV } (9.8 \text{ Gyr}/t_u - h_{100})^2, \qquad (10)$$

if we are lucky, may be violated. That would mean that Λ-term is not zero. The positive result on neutrino mass obtained by the ITEP group is still neither confirmed nor rejected. Hopefully it will be done (in one or other way) in the nearest future.

To conclude the modern data seem to trend to a nonvanishing Λ-term but of course they are not decisive.

The idea that the cosmological constant is most probably zero originated from the Hawking's paper [8]. Shortly his arguments are the following. The probability of a field configuration is assumed to be defined by action S in Euclidean space-time which is achieved by analytic continuation to imaginary time, $t\rightarrow it$. S is taken to be the Einstein action with Λ-term and without matter:

$$S = (m_{Pl}^2/16\pi)\int d^4x\sqrt{g}(-R+2\Lambda) \qquad (11)$$

It is also assumed that quasiclassical approximation is valid so S is calculated on the solution of the classical equations of motion. In the case under consideration the latter are the Enstein equations and their solution is the four-dimensional Euclidean sphere with radius $\sqrt{3/\Lambda}$ (if $\Lambda > 0$). Hence, the action is ($S_{cl}=-3\pi m_{Pl}^2/\Lambda$ and one could expect that the probability of the cosmological term being equal to Λ in a universe is

$$W \sim exp\{3\pi m^2_{Pl}/\Lambda\} \qquad (12)$$

So universes with vanishingly small Λ are infinitely more probable than any other.

This result heavily rests on the Euclildean approach and the sign nondefiniteness of the gravitational action. In a sence these two statements are contratictory. The continuation to Euclidean space is made in order to achieve the convergence of the integral over fields by transforming the oscillating function $exp(iS)$ into the decreasing one, $exp(-S)$. But this is not the case if S is not positive definite. One hopes however that the method is nevertheless correct and will be justified in the future.

Another objection against this result is that Λ is not a dynamical variable in this approach but a constant. To overcome this the third quintized theory of baby universe was proposed (for the list of references and the detailed review see paper [9]). This is an absolutely new theory which does not follow from quantum field theory (second quantized theory) as quantum field theory does not follow from quantum mechanics (first quantized theory) and quantum mechanics does not follow from classical mechanics. Still some analogy between lower and higher quantized theories is kept. Quantum field theory is constructed as quantum mechanics of a system with infinitely many degrees of freedom when the value of a field in any space point is considered as quantum variable. Third quantized theory deals with whole universes in any three-geometry. Fundamental objects in quantum field theory are elementary particles and those in third quantized theory are universes. In analogy with particle creation-annihilation operators a^+_p and a_p one introduces universe creation-annihilation operators A^+_j and A_j where index j refers to different kinds of universes.

It is assumed that only small size baby universes ($l \sim m^{-1}_{Pl}$) are essential for our Universe. The formation of large scale universes is suppressed as $exp(-m^2_{Pl}l^2)$. Whether this is the case or not is still an open question. Another assumption is that the interaction between the baby universes is negligible and so the action is the bilinear function of A_j and A^+_j:

$$S = \Sigma f_j A_j A_j$$

Here f_j can be represented as the ingegrals over whole space-time (to be more exact, over Euclidean four dimensional space) of scalar functions L_j:

$$f_j = \int d^4x \sqrt{g} L_j$$

This follows from the condition that baby universes do not posses nonvanishing energy and momentum because they are closed. By the same reason they do not posses any conserved charge. Functions L_j depend upon the properties of the baby universes and are of the form

$$L_0 = 2\Lambda - R,$$
$$L_1 = m\bar{\psi}\psi,$$
$$L_2 = gV_\alpha \bar{\psi}\gamma^\alpha\psi \text{ , etc}$$

Averaging over small size baby universes gives an effective Lagrangian in our large Universe. One sees that all the "fundamental constants" like m, g, etc. can have arbitrary values depending upon the average $< A_j^+ A_j >$. The latter generally are different for different large universes.

Nonrenormalizable couplings like, e.g. $(\bar{\psi}\psi)^2$ must also be present. Their absence at the available energies is probably explained by their power law rescaling, $\sim (E/m_{Pl})^n$, whereas renormalizable couplings rescale as logarithms of energy.

Now cosmological constant has become variable so one can talk about its probability distribution. Of course, in one universe Λ=const but it can be different in different universes.

Summing over all noninteracting baby universes ane gets an extra exponential in comparison with eq.(12) and obtains the following probability distribution

$$W \sim \exp\{\exp(3\pi m_{Pl}^2/\Lambda)\} \tag{13}$$

This result and to the large extent the method belong to Coleman [10].

The theory seems to be very promising. All the fundamental constants might be calculable quantities if their their probability distributions prove to be peaked at some specific values as it was demonstrated for Λ-term. At the moment however the way seems to be long and hard. First of all the validity of Euclidean approach and quasiclassical approximation can be questioned. Universe behaviour in real (not imaginary) time is obscure. If the Universe was created with the characteristic Planck scales how was it sensitive to a quantity which is more than one hundred orders of magnitude smaller? And at last but not the least how the objects which do not interact with our Universe, since their energy and momentum are zero from the point of view of observers in our world, can influence the Universe evolution? All these questions reflect our poor understanding of the relation between real and imaginary time.

Now let us turn to the compensating field models. The idea is extremely simple and is the following. The curvature of space-time is assumed to induce the formation of a classical field condensate which by its back reaction cancells out its own source. As a result the exponential expansion of the universe turns into power law one. As a toy model let us consider the theory with the Lagrangian [11].

$$L = - M^2/16\pi(R-2\Lambda) + 1/2(f_{,\alpha}f^{,\alpha} - \xi R f^2) \qquad (14)$$

It leads to the following equation of motion of scalar field f:

$$f_{;\alpha}^{;\alpha} + \xi R f = 0 \qquad (15)$$

If $R<0$, long-wave fluctuations of f are unstable and there exist rising solutions of this equation. This rise is exponential while the total energy-momentum tensor is dominared by the vacuum term $T_{\mu\nu} \approx (\Lambda M^2/8\pi) g_{\mu\nu}$:

$$exp\{(-3/2+\sqrt{9/4-12\xi})\sqrt{\Lambda/3}\ t\} \qquad (16)$$

The solution may be taken spatially homogeneous because the Universe expansion $a \sim \exp(\sqrt{\Lambda/3}\ t)$ quickly smooths down the inhomogeneities of f. When f becomes large its contribution into $T_{\mu\nu}$ must be taken into account. Using the Einstein equations one obtains

$$R=\frac{4\Lambda M^2+8\pi(6\xi-1)\dot{f}^2}{M^2+8\pi\xi(6\xi-1)f^2} \quad (17$$

With encreasing f, R becomes smaller and the exponential rise of f slows down. At large t $f\approx\sqrt{\Lambda}\,Mt$, $R\sim t^{-2}$, $H=\dot{a}/a\sim t^{-1}$. Thus we have got the desired result that the sc factor behaves as a power of t, $a\sim t^{k}$. Unfortunately this is hieved by the price of vanishing of the gravitational coupl constant:

$$G(t)=(M^2-8\pi\xi f^2)^{-1}\sim\Lambda^{-1}(Mt)^{-2} \quad (18)$$

To get the presently measured value $G_N=m_{Pl}^{-2}=10^{-38}$ GeV^{-2} one has to assume that

$$M\approx m_{Pl}/(\sqrt{\Lambda}\,t_u) \quad (19)$$

where $t_u\approx 5\cdot 10^{17}$ sec is the Universe age. Of course, in such a naive version the model can not stand cosmological tests. In particular there exist a very strong upper bound on possible time variation of G_N. The bound might be avaded if there would be a conspiracy between time variation of different coupling constants and masses. Probably the model of this type can be worked out. An idea of consistent variation of masses together with G(t) was considered by Fujii [12].

The problem of vanishing of $G_N(t)$ persisted in the models with more general coupling of f with gravity as it has been shown by Ford [13]. Attempts to resolve it with the help of the conformal anomaly did not prove to be very succesful [11,14]. It was claimed in ref.[15] that in theories with torsion one is able to get rid of the cosmological constant along the lines considered without destroying gravity.

Another attempt to save gravity was made in a model with vector field V_α instead of scalar one [16]. The Lagrangian has the form

$$L = 1/4 F_{\alpha\beta}F^{\alpha\beta} + 1/2(D_\alpha V^\alpha)^2 + \xi R(m_1^2/16\pi)\ln(1+V^2/m_2^2) \qquad (20)$$

where $F_{\alpha\beta} = \partial_\alpha V_\beta - \partial_\beta V_\alpha$ is the field strength. Potential of this type may arise due to radiative correction. In this model field V_μ is unstable and asymptotically increases as

$$V_\mu \sim \delta_{\mu t}(At + Bt^{-1}) \qquad (21)$$

Its back reaction cancelis out vacuum energy so that $R \sim t^{-2}$ and $H \sim t^{-1}$. In contrast to the case of scalar field, energy-momentum tensor of V_μ is asymptotically proportional to $g_{\mu\nu}$ so that all the components of the total energy-momentum tensor go down as the Univers expands. Effective gravitational coupling constant decays only logarithmically,

$$G^{eff}(t) = [m_0^2 + m_1^2 \ln(1+A^2t^2)]^{-1} \qquad (22)$$

which may be consistent with observations.

Energy density in this theory is not positive definite but field quantization over classical background (21) seems safe.

It is noteworthy that the cancellation of cosmological term in such models proceeds only on large scales comparable with the horizon. In fact gravitational field drives the formation of condensate of f or V_μ if it is homogeneous at the distance

$$l > M(32\pi\rho/\xi l)^{-1/2} \qquad (23)$$

where ρ is the energy density. So gravitational field of usual astronomical bodies is not influenced by this mechanism. It could only be essential in black holes.

All the models considered have the common feature that the vacuum energy is not completely cancelled out but only up to terms which give rise to the power expansion law. It seem to be the generic feature of compensating field models. This could change the standard scenario of the Universe evolution, influence large scale structure formation and so on. The cosmological models with unstable scalar field have been considered in refs.[17,18] and by Sato at this meeting. In the frameworks of such models large values of the Hubble constant and the Universe age can be compatible.

In this short talk I am unable to cover many other interesting possibilities. Their large number shows the importance of the problem as well as it is still far from the resolution. I am not so optimistic as to believe that it will be solved to the next conference but I hope that it will be done in this millenium and in one of the future Rencontres de Moriond a single and correct model of Λ-term cancellation will be presented.

References

1. Zel'dovich, Ya.B (1968), Uspekhi Fiz. Nauk, 95, 209.
2. Weinberg, S. (1989), Rev. Mod. Phys., 61, 1.
3. Kirzhnits, D.A. (1972) Pis'ma ZhETF, 15, 745.
4. Kirzhnits, D.A. and Linde, A.D. (1972), Phys.Lett., 42B, 385.
5. Shifman, M.A., Vainshtein, A.I., and Zakharov, V.I. (1979) Nucl.Phys. B147, 385.
6. Weinberg, S. (1987) Phys.Rev.Lett. 59, 2607.
7. Linde, A.D. (1988) DESY 88-147. Plenary talk at the XXIV International Conference on High Energy Physics.
8. Hawking, S.W. (1984) Phys.Lett., 134B, 403.
9. Strominger, A. (1988) Baby Universes. Lectures presented at the TASI summer school. Brown University.
10. Coleman, S. (1988) Nucl.Phys., B310, 643.
11. Dolgov, A.D. (1982) Nuffield Workshop on the Very Early Universe. Cambridge, 1982. Ed. G.Gibbons, S.W.Hawking, and S.T.C.Siklos (Cambrdge University Press, Cambridge, 1983).
12. Fujii, Y. (1988) University of Tokio preprint UT-Komaba 88-14.
13. Ford, L.H. (1987) Phys.Rev. D35, 2339.
14. Peccei, R.D., Sola, J., and Wetterich, C. (1987) Phys.Lett. 195B, 183.
15. R. de Ritis, P.Scudellaro, C.Stornaillo (1988) Universita di Napoli preprint.
16. Dolgov, A.D. (1985) Pis'ma ZhETF, 41, 280.
17. Freese, K., Adams, F.C., Frieman, J.A., and Mottola (1987) Nucl.Phys. B287, 797.
18. Peebles, P.J.E. and B.Ratra (1987) Astrophys. J., 325, L17-L20.