HYBRID PARTICLE-FLUID APPROACH TO LOW-FREQUENCY TURBULENCE IN MAGNETICALLY CONFINED PLASMAS*

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J.N. LEBOEUF AND R.H. FOWLER
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37831-8071, U.S.A.

J.H. HAN AND P.M. LYSTER
Institute for Fusion Studies
University of Texas
Austin, Texas 78712, U.S.A.

ABSTRACT. We have developed a hybrid particle-fluid computer model for quasi-neutral microinstabilities in magnetically confined plasmas. The ions are treated in the fluid approximation. The electrons are taken as guiding-center macroparticles. The model has been extensively tested for frequency and fluctuation spectra. Linear and nonlinear studies of drift waves, η_i modes, interchange modes, and trapped electron modes have also been carried out.

I. INTRODUCTION. The hybrid plasma model that we present here treats the electrons as particles and the ions as a fluid [1]. It is based on the successful theoretical treatments of a number of important modes in magnetically confined plasmas, such as the η_i (ion pressure-gradient-driven) modes and the resistive interchange modes. In these cases, the ions are treated in the fluid approximation, while the electrons are handled either with the Boltzmann response $n_e = n_0 \exp(e\phi/T_e)$, where ϕ is the electrostatic potential, or with Ohm's law. For the relevant modes, quasi-neutrality over scales longer than the Debye length λ_{De} is assumed. The typical frequency ω is taken to be much smaller than the ion gyrofrequency $\omega_{ci} = eB/Mc$, and the wavelength λ is long compared to the ion gyroradius $\rho_i = v_{ti}/\omega_{ci}$. Our model is electrostatic so far and retains the electron particle equation of motion, with electric acceleration and mirror force, in the direction parallel to the magnetic field. Therefore, the effects of parallel electron resonances with electrostatic waves are retained. We assume a vanishingly small electron gyroradius so that perpendicular electron guiding center motion in two-dimensional sheared slab geometry is handled by $\mathbf{E} \times \mathbf{B}$, ∇B, and curvature drifts. Electron-ion collisions are incorporated using the Lorentz gas model. Equilibrium spatial gradients are easily maintained with the fluid ion description, combined with the multiple space scale method for the electrons [2]. The electrostatic model equations and the numerical algorithm are described next. Applications to collisionless drift waves and trapped electron modes are presented in Section III, and a brief discussion is given in Section IV.

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II. MODEL EQUATIONS AND ALGORITHM. The ion fluid equations are obtained from Braginskii's plasma transport equations. The relevant ones for ion parallel velocity $v_{\parallel i}$ and ion temperature T_i are

$$Mn\left[\partial_t + \left(\underset{\sim}{v}_E + v_{\parallel i}\hat{b}\right)\cdot\underset{\sim}{\nabla}\right]v_{\parallel i} = -en\nabla_{\parallel}\phi - \nabla_{\parallel}P_i \quad , \tag{1}$$

$$\frac{3}{2}\left[\partial_t + \left(\underset{\sim}{v}_E + v_{\parallel i}\hat{b}\right)\cdot\underset{\sim}{\nabla}\right]T_i = -T_i\nabla_{\parallel}v_{\parallel i} \quad , \tag{2}$$

with $\underset{\sim}{v}_E = (c/B)\underset{\sim}{b}\times\underset{\sim}{\nabla}\phi$ and $P_i = nT_i$. Quasi-neutrality yields the equation for the electrostatic potential ϕ:

$$\rho_s^2\underset{\sim}{\nabla}_{\perp}\cdot\left[\left(\partial_t + \underset{\sim}{v}_{Di}\cdot\underset{\sim}{\nabla}\right)\underset{\sim}{\nabla}_{\perp}\left(\frac{e\phi}{T_e}\right)\right] = \nabla_{\parallel}\left[n\left(v_{\parallel i} - v_{\parallel e}\right)\right] \quad . \tag{3}$$

with ρ_s the sound gyroradius and $\underset{\sim}{v}_{Di} = \underset{\sim}{v}_E + (c/eBn)\hat{b}\times\underset{\sim}{\nabla}P_i$.

The guiding center orbit equations for the electrons are

$$\frac{dv_{\parallel q}}{dt} = \frac{e}{m}\nabla_{\parallel}\phi - \frac{\mu}{m}\hat{b}\cdot\underset{\sim}{\nabla}B \quad , \tag{4}$$

$$\underset{\sim}{v}_{\perp q} = \underset{\sim}{v}_E - \frac{c\hat{b}}{eB}\times\left(\mu\underset{\sim}{\nabla}B + mv_{\parallel q}^2\frac{\hat{b}}{B}\cdot\underset{\sim}{\nabla}\underset{\sim}{B}\right) \quad , \tag{5}$$

$$\frac{d\underset{\sim}{x}_q}{dt} = v_{\parallel q}\hat{b} + \underset{\sim}{v}_{\perp q} \quad . \tag{6}$$

The density and average parallel electron velocity are calculated on a spatial grid as

$$n(\underset{\sim}{x}_g) = \sum_{q\in g} S(\underset{\sim}{x}_g - \underset{\sim}{x}_q) \quad , \tag{7}$$

$$v_{\parallel e}(\underset{\sim}{x}_g) = \sum_{q\in g} v_{\parallel q}S(\underset{\sim}{x}_g - \underset{\sim}{x}_q) \quad . \tag{8}$$

The Eulerian fluid equations are advanced explicitly in time using the two-step scheme of the reduced MHD code RSF [3] on the two-dimensional spatial grid. Spatial derivatives are handled by finite differences and the Laplace operator is inverted by using fast Fourier transforms. The Lagrangian electron orbit equations are advanced in time using a two-step predictor-corrector algorithm [4]. Operations between particle space and grid space are handled using the dipole scheme. The most stringent constraint on the time step is imposed by the so-called electron trapping condition $k_{\parallel}v_{te}\Delta t < 1$, followed by the need to resolve the fastest frequency in the system $[(\omega_{ce}\omega_{ci})^{1/2}k_{\parallel}/k]\Delta t < 1$ (for a homogeneous plasma) and the usual CLF condition on the fastest (fluid) velocity on the mesh.

The theoretical fluctuation spectrum, $W(k)$, supported by our model in the case of a thermal, homogeneous plasma, is $W(k)/(T_e/2) = (v_A/c)^2/(1 + k_{\perp}^2\rho_s^2)$. It predicts a significant reduction in the background fluctuation (noise) level with smaller ratios of Alfvén to light speed, v_A/c, which is verified in the computer calculations. We have also derived the linear dispersion relation that our model obeys and have obtained excellent agreement between it and the calculations in the case of sound waves in a thermal. homogeneous plasma, as well as density-gradient-driven drift waves and η_i modes in an inhomogeneous plasma [1].

III. APPLICATION TO COLLISIONLESS DRIFT AND TRAPPED ELECTRON INSTABILITIES. Computer calculations of the collisionless drift wave instability in a $2\frac{1}{2}$-D shearless slab using the hybrid code (periodic, frozen density profile) have been compared with those from a conventional particle code (bounded, evolving profile). Figure 1 shows that the growth rate of the electrostatic energy is higher by 50% and its saturation level higher by a factor of 2 for the hybrid code. The density fluctuation levels in both codes are within the mixing length estimate of $\tilde{n}/n_0 = 1/k_\perp L_n \approx 0.35$.

Computer calculations of trapped electron modes have also been performed with a bounded version of the hybrid code. The magnetic field is chosen such that $B_x = -k_y\psi_0 \sin(k_x x)\sin(k_y y)$, $B_y = k_x\psi_0 \cos(k_x x)\cos(k_y y)$, $B_z = \text{const}$, with $k_x = 2\pi/L_x$, $k_y = 2\pi/L_y$, and $L_x = 2L_y$. This is a simplified model for a straight stellarator. We vary the depth of the magnetic well through ψ_0 to control the fraction of trapped electrons. Typical orbits for such electrons (in the absence of electric field) are shown in Fig. 2. Exploratory results with self-consistent electric fields included are shown in Fig. 3. The time evolution of the electrostatic energy is displayed without and with trapped electrons. Note the significant increase of the field energy, with 72% of the electrons executing at least one bounce.

Other successful applications in slab geometry include the study of the nonlinear development of ideal and resistive interchange modes and of η_i modes.

IV. DISCUSSION. The hybrid particle-fluid approach is proving to be a powerful tool in the study of low-frequency turbulence in magnetically confined plasmas. It provides a direct assessment of electron transport in the presence of fluid-like modes.

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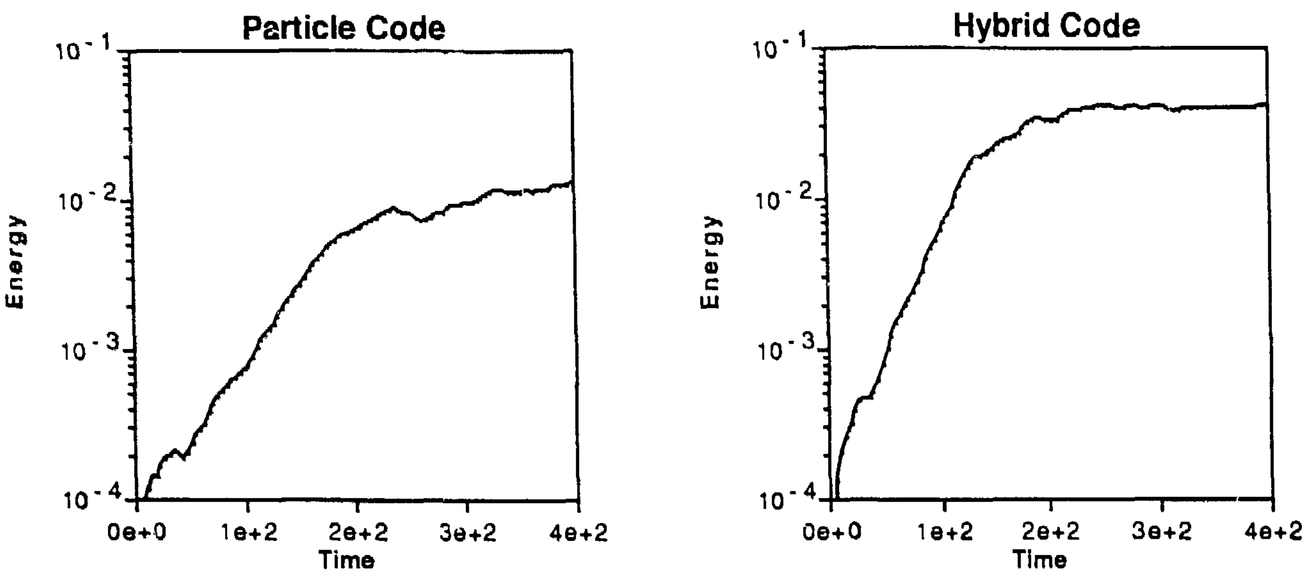

Fig. 1. Collisionless drift wave instability: time evolution of the electrostatic energy with the particle code and with the hybrid code.

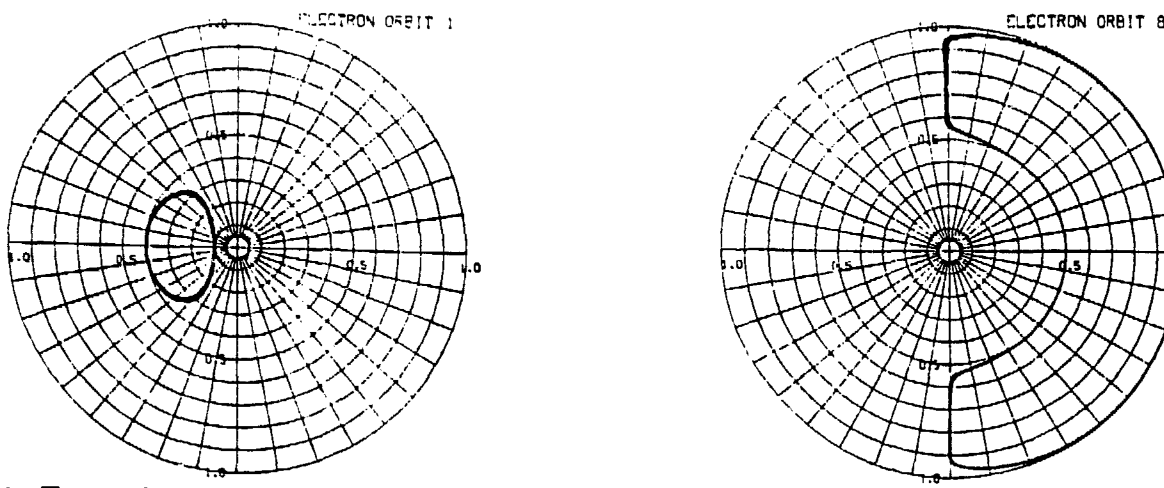

Fig. 2. Trapped electron mode instability: typical trapped electron orbits.

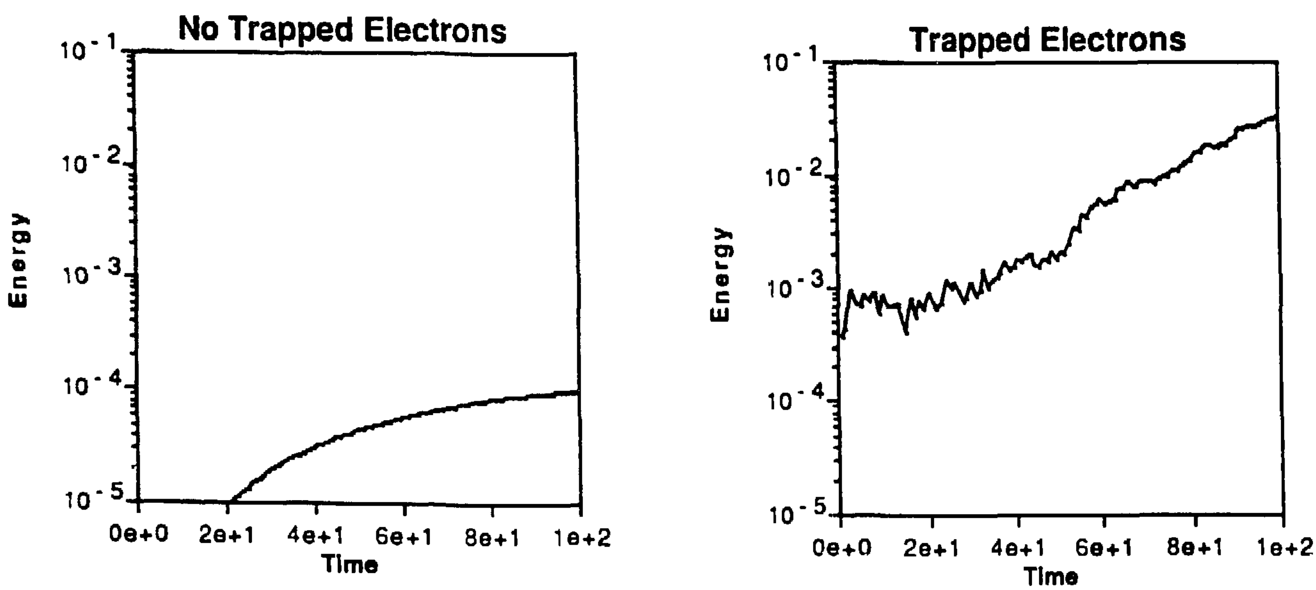

Fig. 3. Trapped electron mode instability: evolution of the electrostatic energy without and with trapped electrons.