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An Empirical Model for the Luminosity of the Fermilab Tevatron Collider *

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AN EMPIRICAL MODEL FOR THE LUMINOSITY OF THE FERMILAB TEVATRON COLLIDER

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Abstract Using the accelerator data collected during the 1988-89 collider run, correlations have been established between the proton-antiproton collider luminosity, the antiproton stack intensity and the proton bunch parameters. These correlations are used to determine empirically the value of the proton transverse emittance which maximizes the collider luminosity. Also, the correlations are used to predict the effects of planned improvements in collider subsystem performance on the luminosity.

INTRODUCTION

This paper will discuss a model which has been developed based on data collected during the 1988-89 run of the Tevatron proton-antiproton collider[1,2]. The goal of the model is to parameterize the collider luminosity in terms of three independent variables:

1. The number of stored antiprotons in the Accumulator at 8 GeV ($\bar{N}_S$);
2. The number of protons injected into the Tevatron at 150 GeV (N_{150});
3. The transverse emittance of the proton beam in the Tevatron at 150 GeV (ϵ_{150}).

From these variables, for a given store the model predicts the initial luminosity of the collider at 900 GeV, and the integrated luminosity during the first 12 hours of the store.

The dependent variables in the model are derived from the independent variables using quadratic empirical fits to the data collected during the collider run. The luminosity is then expressed directly in terms of the dependent variables. In all cases, invariant transverse emittances will correspond to 95% values, and will be in units of π mm-mrad. Emittances in the x and y planes will be averaged to form a single transverse emittance. Intensities will be given in units of 10^{10}.

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SPECIFICS OF THE MODEL

Dependent variables related to antiproton transfer from the Accumulator to the Tevatron

To establish antiproton beams in the Tevatron, a fraction of f_u of the total intensity $\bar{N}_s$ in the Accumulator is unstacked (with an rf bucket of area $\epsilon_L = 1$ ev-sec) and injected into the Main Ring at 8 GeV. The injected beam intensity is

$$\bar{N}_8 = \bar{N}_s\, f_u,$$

where $f_u = 0.8 \text{ erf } (\sqrt{\pi}\ 6\epsilon_L\ \rho_0(\bar{N}_s)/2\bar{N}_s)$.

The constant 0.8 is an empirically determined number. In this expression the peak core longitudinal density (in 10^{10}/ev-sec) is $\rho_0(\bar{N}_s)$, a function of the stack size $\bar{N}_s$. The invariant transverse emittance of the antiproton beam in the Accumulator is $\bar{\epsilon}_8(\bar{N}_s)$ and is also function of $\bar{N}_s$.

After injection into the Main Ring, antiprotons are accelerated to 150 GeV and injected into the Tevatron. The number of antiprotons at 150 GeV in the Tevatron is

$$\bar{N}_{150} = 0.9\ \bar{N}_8 \xi_i(\bar{\epsilon}_8),$$

where $\xi_i(\bar{\epsilon}_8)$ is the injection efficiency into the Main Ring, which is a function of the antiproton beam transverse emittance (because of the limited Main Ring transverse aperture). The constant 0.9 is an empirically determined number.

During the processes of injection and acceleration in the Main Ring, and during transfer into the Tevatron at 150 GeV, the antiproton beam has been observed to experience transverse emittance growth of roughly 4π mm-mrad. Hence the antiproton transverse emittance at 150 GeV in the Tevatron, $\bar{\epsilon}_{150}$, is equal to $\bar{\epsilon}_8$ plus 4π.

Dependent variables related to protons and antiprotons in the Tevatron

Under normal circumstances, protons are injected into the Tevatron at 150 GeV, followed by antiprotons at 150 GeV.

Both beams are then accelerated simultaneously to 900 GeV. Subsequently, the low-β quadrupole system is energized (this is called the "low-β squeeze") to reduce β^* to about 0.5 m at the interaction point.

During the processes of acceleration and low-β squeeze, both proton and antiproton beams suffer some loss and some transverse emittance growth. The principal mechanism responsible for these detrimental effects is the beam-beam interaction between the protons and the antiprotons. The strength of this interaction is proportional to the transverse density of the beam. The transverse densities are

$$x_{150} = \frac{N_{150}}{\epsilon_{150}} \text{ (protons)}; \qquad \bar{x}_{150} = \frac{\bar{N}_{150}}{\bar{\epsilon}_{150}} \text{ (antiprotons)} .$$

Because of the role of the beam-beam interaction in determining beam loss and transverse emittance growth in the Tevatron, the empirical functions describing these quantities for each particle species are parameterized in terms of the transverse density of the other species:

$$f_1\ (\bar{x}_{150}) = \frac{N_{1b}}{N_{150}}\ ; \qquad f_2\ (\bar{x}_{150}) = \frac{\epsilon_{1b}}{\epsilon_{150}}$$

$$\bar{f}_1\ (x_{150}) = \frac{\bar{N}_{1b}}{\bar{N}_{150}}\ ; \qquad \bar{f}_2\ (x_{150}) = \frac{\bar{\epsilon}_{1b}}{\bar{\epsilon}_{150}}$$

In these equations, N_{1b} and ϵ_{1b} are the proton intensity and invariant transverse emittances at low-β; $\bar{N}_{1b}$ and $\bar{\epsilon}_{1b}$ are the corresponding quantities for antiprotons. The functions f_1 and f_2 parameterize proton loss and transverse emittance growth from 150 GeV to the end of the low-β squeeze; $\bar{f}_1$ and $\bar{f}_2$ are the corresponding quantities for antiprotons.

Luminosity

The collider initial luminosity L can then be written in terms of dependent variables as

$$L = k\frac{N_{1b}\ \bar{N}_{1b}}{\epsilon_{1b} + \bar{\epsilon}_{1b}}$$

where k is a constant which is calculated from machine parameters. The calculated value of k is $9.6\times10^8\ \pi$ mm-mrad/cm^2/sec.

The integrated luminosity, L_I, depends both on L and on the luminosity lifetime, τ. It has been observed that τ is strongly correlated with the proton transverse density at low-β, i.e.,

$$\tau = \tau(x_{1b}),\ \text{where}\ x_{1b} = \frac{N_{1b}}{\epsilon_{1b}}\ \text{and}\ \tau\ \text{is in hours.}$$

The integrated luminosity in the model is computed assuming that τ is constant over a period T_0, so that

$$L_I = L\tau(1\text{-}\exp(-\tau/T_0)) ,$$

in which T_0 is taken as 12 hours. Although the luminosity lifetime actually varies with time during a store, we assume constancy with time in this model for purposes of simplicity.

RESULTS AND CHECK OF THE MODEL

The values of the dependent variable function parameters are given in Table I. The model calculates directly the luminosity (initial and integrated) from the independent variables and the model parametric functions specified above. For each store in the collider run, there is an independent determination of the initial luminosity, based on observed collision rates at the CDF detector. Hence a direct check of the model is a store-by-store comparison of the model's prediction for initial luminosity with the measurements at CDF. This direct comparison is presented in Fig. 1. The rms fractional deviation between the measurements and the model prediction is about 14%.

USES OF THE MODEL

Once verified, the model can be used in two ways. First, the model independent variables can be varied to maximize the luminosity. Secondly, the parameters of the model can be artifically adjusted to reflect upgrades to the accelerator systems. In this way the model allows a prediction to be made for the impact of the upgrades on the collider initial and integrated luminosity.

AN EMPIRICAL MODEL FOR THE LUMINOSITY

TABLE I: Dependent Variable Function Parameters

$$F(x) = a+b\ x+c\ x^2$$

f(x)	x	a	b	c
ρ_0	$\bar{N}_s$	0.772	0.096	-0.0002
$\bar{\epsilon}_8$	$\bar{N}_s$	4.9	0.139	-0.0007
ξ_i	$\bar{\epsilon}_8$	1.01	-0.0064	-0.0009
f_1	$\bar{x}_{150}$	0.939	0.0123	-0.119
f_2	$\bar{x}_{150}$	1.05	-0.0737	0.0395
$\bar{f}_1$	x_{150}	1.02	-0.0246	-0.0053
$\bar{f}_2$	x_{150}	1.02	-0.0387	0.0665
τ	x_{1b}	23.9	-3.06	-1.05

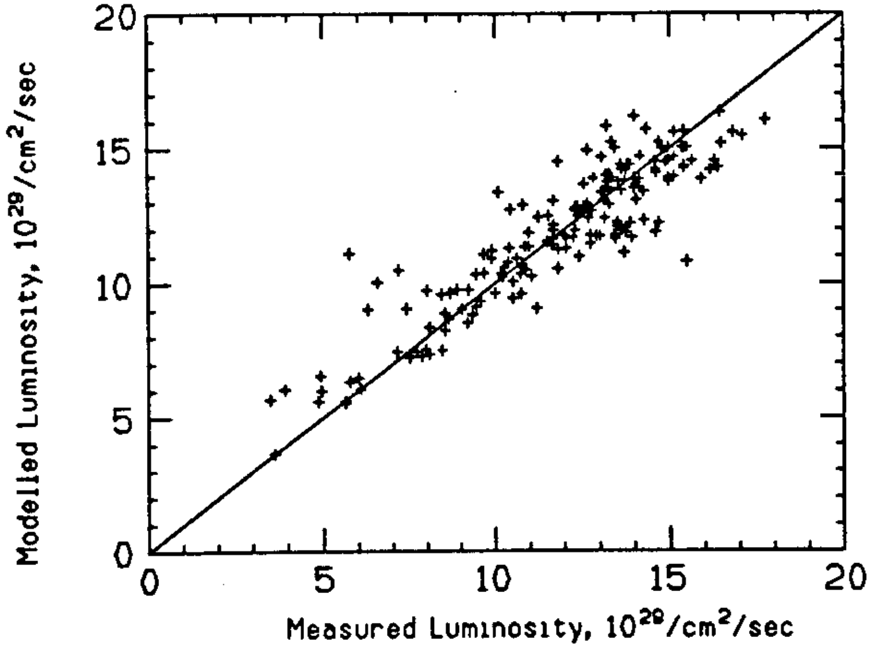

FIGURE 1: Modelled luminosity vs. measured luminosity

Maximization of collider luminosity

In this model, there is no optimal value of antiproton stack size: the initial luminosity simply increases monotonically (but not linearly) with this variable, at least within the range of observations covered here. The situation is different with regard to the proton variables because of the beam-beam interaction. The model shows that there are optimum values for the proton transverse emittance which maximize the initial and integrated luminosity.

The optimization was performed by varying the proton transverse emittance for a fixed stack size $\overline{N}_s$ and proton intensity N_{150} to maximize the initial luminosity. The optimization was then repeated for a range of stack sizes and proton intensities. A similar optimization was carried out to maximize the integrated luminosity. The results of these optimizations were very useful during the collider run in establishing the proton transverse emittance values required operationally to maximize the luminosity delivered to the experiments.

Predictions for performance with upgrades

As an example of the model's predictions for performance enhancements resulting from accelerator upgrades[1], the cases of a bandwidth upgrade of the Accumulator core cooling system, and the installation of a system of beam separators in the Tevatron, have been considered. The bandwidth upgrade in the Accumulator is expected to result in about a factor of 2 increase in both the transverse and longitudinal densities in the core. To incorporate this in the model, the functions $\rho_0(\overline{N}_s)$ and $\overline{\epsilon}_8(\overline{N}_s)$ are simply multiplied by 2 and divided by 2 respectively. The Tevatron separator system will reduce the number of head-on beam-beam interactions per turn from 12 to 2. This is represented in the model by a reduction in the transverse beam densities x_{150}, $\overline{x}_{150}$, and x_{1b}, which determine the strength of the beam-beam interaction, by a factor of 6.

The model predicts that, with these upgrades in place, the luminosity increase over the 1988-89 collider run will be a factor of about 3.9 at a stack size of 75×10^{10} antiprotons and a proton intensity of 9×10^{10} per bunch in six bunches.

CONCLUSION

The empirical model described in this paper was formulated during the 1988-89 collider run to guide collider operation and to understand the correlations between various accelerator parameters and the collider luminosity. It proved to be a useful tool for specifying the optimum proton transverse emittance required to maximize the luminosity in a situation dominated by the beam-beam interaction. It has also been used to estimate quantitatively the performance enhancements to be expected from upgrades anticipated in the near future.

References

1. G. Dugan, "Tevatron Collider: Status and Prospects", contribution to this conference.
2. V. Bharadwaj et al, "The 1988-89 Tevatron Collider Run Summary", contribution to this conference.