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The Covariant Quantum Green-Schwarz Superstring*

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ABSTRACT

First we present the covariantly quantized space-time supersymmetric superstring. The main ingredients are additional auxiliary variables and their corresponding auxiliary gauge symmetries. They allow a Lorentz covariant gauge fixed lagrangian path integral which has the form of a free 2-dimensional conformal field theory with a finite number of 2-dimensional world-sheet fields and ghosts. Next we show that the path integral is anomaly free in 10 space-time dimensions. Then, by a canonical (operator) quantization we obtain in the point-particle limit the covariant equations of motion of the D=10 Super-Yang-Mills (SYM) theory.

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1. Introduction

The manifestly space-time supersymmetric string theory (the Green-Schwarz (GS) superstring [1]), is important both theoretically (for the very consistency of the theory) and phenomenologically (the direct use of the superstring field action might improve the study of the the "low" energy spectrum and dynamics by systematic generation of a superfield effective action for the low-energy states).

Until recently the covariant quantization procedure for the GS superstring could not be carried out without violating the explicit geometrical invariances characterizing the superspace. In particular there existed no super-Poincare covariant quantization of the superspace particle and, consequently, no covariant superfield action for the relevant supersymmetric field theories (N=1 D=10 SYM, N=4 D=4 SYM, N=2 D=10 SUGRA, N=8 D=4 SUGRA etc.).

In a series of papers [2-7] we have performed the super-Poincare covariant quantization of the (extended) superparticles and superstrings and in fact constructed [6,7] (in the point-particle limit) through the BFV-BRST procedure [8] the relevant off-shell unconstrained superfield actions.

The deduction [6,7] of the relevant supersymmetric field theories from the covariantly quantized superstring theory is a crucial test for any correct covariant quantization procedure.

The main tool we introduced in [2-7] was a set of pure gauge auxiliary variables carrying Lorentz-spinor and Lorentz-vector indices which served as "bridges" reducing the D=10 Lorentz group $SO(1,9)$ to an internal $SO(8) \times SO(1,1)$. This allowed us to recast the crucial fermionic "kappa"-gauge symmetry [9,1] of the GS superstring action into a functionally independent (BFV-irreducible) way which directly lead to a manifestly covariant quantization avoiding the need of an infinite tower of ghost-for-ghost fields [10].

In our initial papers on the subject, we employed the canonical operator quantization method. There, relying on the previous experience with off-shell superspace formulations of extended SUSY field theories [11], it was natural to restrict the space of superfield wave functions to be "harmonic superfields" of a form which explicitly displays part of the local symmetries of the auxiliary variables. Moreover, the canonical operator quantization was essential to get a BRST superspace action for D=10 SYM in terms of unconstrained off-shell superfields [6,7].

Recently [12], we extended our formalism to a form appropriate for covariant path-integral quantization of the GS superstring. For the path integral it is necessary to have all the local symmetries expressed dynamically rather than through kinematical conditions on the wave function. Consequently, we had to reexpress the information encoded in the form of the harmonic superfields into the form of additional gauge invariances. For this purpose it proved useful to further reduce the internal $SO(8)$ gauge symmetry for the auxiliary variables to an internal $SO(7)$ one.

We start in section 2 with a review of our [12] covariant BFV hamiltonian (phase-space) form [8] of the path integral (2.30).

In section 3 we deduce from it the covariant gauge-fixed lagrangian path integral (3.25) through functional integration over the canonical momenta. The covariant lagrangian gauge-fixed action (3.26) represents a free 2-dimensional conformal field theory with a finite number of fields and ghosts.

The deduction of the path integral from a well structured hamiltonian formalism is essential in two ways. First, systems with variable structure "constants" in the constraints algebra present certain ambiguities in the measure of the lagrangian path integral [13] which can be resolved systematically by using the hamiltonian techniques. In the absence of such a systematic technical framework, the lagrangian path integral formalism may lead to incorrect results as suggested in [13] and exhibited in detail in [14]*. Second, the well structured algebraic set of constraints (the hamiltonian analog of the gauge invariances of the lagrangian formalism) allows us to have detailed control on the (in)dependency of the gauge invariances and to make sure that they are really eliminating all the auxiliary variables in the covariant path integral**.

* [14] show in a particular case that the path integral obtained through the lagrangian formalism gives a nonunitary S-matrix while the hamiltonian formalism produces the correct one.

** E.g. , it was found by the analysis in [12] that part of the gauge invariances proposed in [15] for the auxiliary variables introduced in [2-7] were functionally dependent.

In addition to the basic result (3.25),(3.26), the present report contains the following new material.

A) Since the auxiliary variables are chiral world-sheet scalars, it is necessary to prove that the auxiliary gauge symmetries are not plagued by quantum anomalies. It is shown in detail in section 3 that actually the potential anomalies cancel ((3.27)-(3.29)).

B) The relation between the path-integral and the canonical operator formalism is explicitly exhibited in section 4 where we derive from the point-particle limit of the covariantly quantized GS superstring the covariant equations of motion of D=10 SYM ((4.25),(4.28))*. As a by-product, we show explicitly that the present formalism [12] is physically equivalent to the initial "harmonic superfield" one [2-7] ((4.15),(4.16)).

As stressed above, we consider the derivation of D=10 SYM a crucial check for the correctness of any GS superstring covariant quantization procedure.

2. Hamiltonian Covariant Path Integral

The usual heterotic GS action in the lagrangian form is [1]:

$$S_{GS}^{heterotic} = \int d\tau d\xi \sqrt{-g}[-\frac{1}{2}g^{mn}\partial_m X^\mu \partial_n X_\mu - 2i(P_-^{nm}\partial_m X^\mu)(\theta\sigma_\mu\partial_n\theta) + \frac{1}{2}g^{mn}(\theta\sigma_\mu\partial_n\theta)(\theta\sigma^\mu\partial_m\theta)] + S' \tag{2.1}$$

where S' is the action for the left-moving fields describing the internal string degrees of freedom. The precise form of S' does not have any effect on the present analysis, therefore it will be supressed.

The variables appearing in (2.1) have the following meaning. g_{mn} $(m,n = 0,1)$ denotes the D=2 dimensional world-sheet metric. The string bosonic coordinates X^μ , $\mu = 0,...,9$, transform as D=10 vectors and D=2 world-sheet scalars. The anticommuting string variables θ_α , $\alpha = 1,...,16$, transform as a left-handed Majorana-Weyl (MW)

* For the analog supersymmetric interacting result in the "harmonic superfield" approach see [6,7].

SO(1,9) spinor and as D=2 world-sheet scalars.

$$P_{\pm}^{mn} \equiv \frac{1}{2}(g^{mn} \pm \frac{\epsilon^{mn}}{\sqrt{-g}}) \tag{2.2}$$

are the D=2 chiral projectors.

In the canonical hamiltonian formalism, the local invariances of the lagrangian-form action (2.1) are expressed as constraints (T_L, T_R, D^α) which appear in the hamiltonian-form action multiplied by Lagrange multipliers $(\Lambda_L, \Lambda_R, \Lambda_\alpha)$:

$$S_{GS}^{heterotic} = \int d\tau d\xi [P_\mu \partial_\tau X^\mu + p_\theta^\alpha \partial_\tau \theta_\alpha - \Lambda_L T_L - \Lambda_R T_R - \Lambda_\alpha D^\alpha] \tag{2.3}$$

In Eq.(2.3) $T_{L,R}$ are the left (right-) reparametrization (Virasoro) constraints (primes indicate differentiation with respect to the string parameter ξ):

$$T_L \equiv (P_\mu - X'_\mu)^2 \tag{2.4}$$

$$T_R \equiv \Pi^2 - 4i\theta'_\alpha D^\alpha \tag{2.5}$$

where

$$\Pi^\mu \equiv P^\mu + X'^\mu + 2i\theta\sigma^\mu\theta' \tag{2.6}$$

The spinorial fermionic constraints D^α appearing in (2.3) are:

$$D^\alpha \equiv -ip_\theta^\alpha - (P^\mu + X'^\mu + i\theta\sigma^\mu\theta')(\sigma_\mu\theta)^\alpha \tag{2.7}$$

The Poisson brackets (PB) of these constraints form a singular 16×16 matrix whose rank on the constraints shell is 8, i.e. (2.7) contains a mixture of first-class and second-class constraints. The quantization procedure of a system is straightforward only if the constraints are functionally independent and are separated into sets whose PB matrix is either zero (first class) or invertible (second class) on the constraint surface.

To separate covariantly the first and second class parts of D^α , one introduces the auxiliary variables [2-7]:

i) seven $SO(1,9)$ vectors:

$$u^p_\mu \ , \ p = 1, ..., 7 \ , \ \mu = 0, ..., 9 \tag{2.8}$$

ii) two $SO(1,9)$ MW spinors

$$v^{\pm\frac{1}{2}}_\alpha \ , \ \alpha = 1, ..., 16 \tag{2.9}$$

In the sequel the following objects and notations will be useful :

$$u^8_\mu \equiv \sqrt{2}(v^{+\frac{1}{2}}\sigma_\mu v^{-\frac{1}{2}}) \tag{2.10}$$

$$u^+_\mu \equiv v^{+\frac{1}{2}}\sigma_\mu v^{+\frac{1}{2}} \tag{2.11}$$

$$u^-_\mu \equiv v^{-\frac{1}{2}}\sigma_\mu v^{-\frac{1}{2}} \tag{2.12}$$

$$u^a_\mu \equiv (u^p_\mu, u^8_\mu) \ , \quad a = (p, 8) \tag{2.13}$$

$$A^\pm \equiv A^\mu u^\pm_\mu, \quad A^a \equiv A^\mu u^a_\mu \tag{2.14}$$

$$\sigma^{\mu_1 \cdots \mu_n} \equiv \sigma^{[\mu_1}\sigma^{\mu_2} ... \sigma^{\mu_n]}$$

In (2.14), A^μ is an arbitrary $SO(1,9)$ Lorentz vector. The indices $p, \pm\frac{1}{2}$ in (2.8)(2.9) transform as a $SO(7)$-vector and $SO(1,1)$ charge $\pm\frac{1}{2}$ respectively, where $SO(7) \times SO(1,1)$ is an **internal** symmetry group. The index a in (2.13),(2.14) is short-hand for the pair of indices $(p, 8)$ which transform as a $SO(8)$ vector.

With the help of the auxiliary variables (2.8),(2.9), the fermionic constraints (2.7) can be separated covariantly into independent first-class part (kappa-gauge symmetries):

$$D^{+\frac{1}{2}a} \equiv v^{+\frac{1}{2}}\sigma^{a} \not{\Pi} D \ , \ a=1,..,8 \tag{2.15}$$

where $\not{\Pi} \equiv \Pi_\mu \sigma^\mu$ (with Π_μ being defined in (2.6)), and second-class part :

$$G^{+\frac{1}{2}a} \equiv \frac{1}{2} v^{-\frac{1}{2}} \sigma^{a+} D \ , \ a=1,..,8 \tag{2.16}$$

The auxiliary variables (2.8)(2.9) are introduced on the string world sheet as reparametrization scalars. Their dynamics is governed by a hamiltonian which contains only a linear combination of independent first-class constraints equal in number with the number (=102) of the auxiliary variables, thus insuring that the latter carry no dynamical degrees of freedom at least classically. However, after quantization anomalies might appear in the local symmetries generated by these constraints. That this is not the case is proved in the next section. This insures that the introduction of the auxiliary variables does not affect the physical content of the theory.

The resulting action, physically equivalent to the original one, but easier to quantize covariantly, reads :

$$\tilde{S} \equiv \tilde{S}_{GS}^{heterotic} + S_{auxiliary} \tag{2.17}$$

where

$$\begin{aligned} \tilde{S}_{GS}^{heterotic} \equiv \int d\tau d\xi [P_\mu \partial_\tau X^\mu + p_\theta^\alpha \partial_\tau \theta_\alpha \\ -\Lambda_L \tilde{T}_L - \Lambda_R T_R - \Lambda_a^{-\frac{1}{2}} D^{+\frac{1}{2}a} - M_a^{-\frac{1}{2}} G^{+\frac{1}{2}a}] \end{aligned} \tag{2.18}$$

where now the left Virasoro constraint T_L acquires contribution from the auxiliary variables:

$$\tilde{T}_L \equiv (P_\mu - X'_\mu)^2 - 4(\pi_u^p u'_p + \pi_v^{\mp\frac{1}{2}} (v^{\pm\frac{1}{2}})') \tag{2.19}$$

and $\Lambda_a^{-\frac{1}{2}}, M_a^{-\frac{1}{2}}$ denote the Lagrange multipliers for the covariantly disentangled fermionic constraints (2.15),(2.16) . Note the second term in $\tilde{T}_L$ (2.19) which says that the auxiliary variables (u, v) transform under reparametrizations as left-moving world-sheet scalars.

The auxiliary part of the action (2.17) is:

$$S_{auxiliary} = \int d\tau d\xi [\pi_v^{\pm\frac{1}{2}} \partial_\tau v^{\mp\frac{1}{2}} + \pi_u^p \partial_\tau u_p - \Lambda_{MN} D^{MN} - \mathcal{M}_{AB} \Psi^{AB}] \quad (2.20)$$

In (2.19),(2.20) $(\pi_v^{\pm\frac{1}{2}})^\alpha, (\pi_u^p)^\mu$ denote the canonical momenta conjugate to $v_\alpha^{\mp\frac{1}{2}}, u_\mu^p$:

$$\{(\pi_v^{\mp\frac{1}{2}})^\alpha(\xi), v_\beta^{\pm\frac{1}{2}}(\eta)\}_{PB} = -\delta_\beta^\alpha \delta(\xi - \eta),$$

$$\{(\pi_u^p)_\mu(\xi), u_\nu^q(\eta)\}_{PB} = -\delta^{pq} \eta_{\mu\nu} \delta(\xi - \eta),$$

(ξ, η denote the string world-sheet parameter at fixed world-sheet time τ; in most cases they will be supressed for brevity). $\Lambda_{MN}, \mathcal{M}_{AB}$ are Lagrange multipliers corresponding to the constraints on the auxiliary variables D^{MN}, Ψ^{AB}. The latter have the following geometrical meaning . The 50 orthonormality constraints Ψ^{AB}:

$$\begin{aligned} \Psi^{pq} &\equiv u_\mu^p u^{q\mu} - \delta^{pq} = 0 \\ \Psi^{p8} &\equiv u_\mu^p u^{8\mu} = 0 \\ \Psi^{p\pm} &\equiv u_\mu^p u^{\pm\mu} = 0 \\ \Psi^{+-} &\equiv u^{+\mu} u_\mu^- + 1 = 0 \end{aligned} \quad (2.21)$$

(with $u_\mu^8, u_\mu^\pm$ as in (2.10)-(2.12)) imply that the auxiliary variables form on-shell an orthonormal frame of ten $SO(1,9)$ vectors (the "missing" orthonormality conditions are authomatically fulfilled *off-shell* by construction, due to the D=10 Fierz identities [12]).

The 52 constraints D^{MN} imply that the physics is invariant under local $SO(1,9)$ rotations of the orthonormal frame (u^p, u^8, u^+, u^-) and under transformations of the $v's$ which leave this frame invariant (the last gauge invariance is expressed by the constraints $\tilde{D}^{8p}$):

$$D^{pq} \equiv -u^p \pi_u^q + u^q \pi_u^p - \frac{1}{2} \sum_{+,-} v^{\pm\frac{1}{2}} \sigma^{pq} \pi_v^{\mp\frac{1}{2}} \quad (2.22)$$

$$D^{8p} \equiv -u^8 \pi_u^p - \frac{1}{2} \sum_{+,-} v^{\pm\frac{1}{2}} \sigma^8 \sigma^p \pi_v^{\mp\frac{1}{2}} \tag{2.23}$$

$$D^{-+} \equiv -\frac{1}{2} \sum_{+,-} v^{\pm\frac{1}{2}} \sigma^{-+} \pi_v^{\mp\frac{1}{2}} \tag{2.24}$$

$$D^{+p} \equiv -u^+ \pi_u^p - \frac{1}{2} \sum_{+,-} v^{\pm\frac{1}{2}} \sigma^{+p} \pi_v^{\mp\frac{1}{2}} \tag{2.25}$$

$$D^{-p} \equiv -u^- \pi_u^p - \frac{1}{2} \sum_{+,-} v^{\pm\frac{1}{2}} \sigma^{-p} \pi_v^{\mp\frac{1}{2}} \tag{2.26}$$

$$D^{+8} \equiv -\frac{1}{2} \sum_{+,-} v^{\pm\frac{1}{2}} \sigma^{+8} \pi_v^{\mp\frac{1}{2}} \tag{2.27}$$

$$D^{-8} \equiv -\frac{1}{2} \sum_{+,-} v^{\pm\frac{1}{2}} \sigma^{-8} \pi_v^{\mp\frac{1}{2}} \tag{2.28}$$

$$\tilde{D}^{8p} \equiv -\frac{1}{2} \sum_{+,-} (\pm) v^{\pm\frac{1}{2}} \sigma^{8p} \pi_v^{\mp\frac{1}{2}} \tag{2.29}$$

Let us stress that all constraints in (2.18),(2.20) except $G^{+\frac{1}{2}a}$ (2.16) are first-class.

Consequently, the gauge-fixed path integral will contain the following elements:

a) the functional integration over the canonical variables $X^\mu, \theta_\alpha, u, v$ and their conjugated momenta $P_\mu, p_\theta^\alpha, \pi_u, \pi_v$;

b) the functional integration over the Lagrange multipliers $\Lambda_L, \Lambda_R, \Lambda^{-\frac{1}{2}a}, M^{-\frac{1}{2}a}, \Lambda_{MN}, \mathcal{M}_{AB}$;

c) delta functions imposing the gauge fixings χ for the first-class constraints $\tilde{T}_L, T_R$ (rep) , $D^{+\frac{1}{2}a}$ (kappa), D^{MN} (rot) , Ψ^{AB} (norm);

d) the determinants Δ_{FP} of the matrices formed by the PB of first-class constraints with their gauge-fixing conditions;

e) the inverse square root of the determinant of the PB of the fermionic second-class constraints $G^{+\frac{1}{2}a}$ $(= det^{-4}[\Pi^+])$ *).

The gauge fixing for the (kappa) invariance is represented in the path integral by a factor:

$$\delta(\chi^{(\kappa)}) \equiv \delta(v^{+\frac{1}{2}}\sigma^a\theta)$$

which implies a (local) determinant:

$$\Delta_{FP}^{(\kappa)} \equiv det^{-8}[\Pi^+].$$

The hamiltonian path integral result is therefore of the form [12]:

$$\begin{aligned} Z = \int & DX^\mu D\theta_\alpha Du Dv \\ & DP^\mu Dp_\theta^\alpha D\pi_u D\pi_v \\ & D\Lambda_L D\Lambda_R D\Lambda^{-\frac{1}{2}a} DM^{-\frac{1}{2}a} D\Lambda_{MN} D\mathcal{M}_{AB} \\ & exp\{i\tilde{S}\} \\ & \delta(\chi^{(rep)})\delta(v^{+\frac{1}{2}}\sigma^a\theta)\delta(\chi^{(rot)})\delta(\chi^{(norm)}) \\ & \Delta_{FP}^{(rep)} det^{-8}[\Pi^+]\Delta_{FP}^{(rot)}\Delta_{FP}^{(norm)} \\ & det^{-4}[\Pi^+] \end{aligned} \tag{2.30}$$

The correctness of the gauge fixings $\delta(\chi^{(rot)})\delta(\chi^{(norm)})$ and the determinants $\Delta_{FP}^{(rot)}, \Delta_{FP}^{(norm)}$ for the auxiliary constraints depend on the lack of anomalies. We will study them below explicitly in the lagrangian framework.

*) $\Pi^+ \equiv u_\mu^+ \Pi^\mu = v^{+\frac{1}{2}} \Pi v^{+\frac{1}{2}}$, cf. notations (2.14).

3. Lagrangian Covariant Path Integral

Integration over the momenta P^μ, p^α_θ and the fermionic Lagrange multipliers $\Lambda^{-\frac{1}{2}a}, M^{-\frac{1}{2}a}$ in (2.30) leads to the lagrangian gauge-fixed covariant path integral:

$$
\begin{aligned}
Z = \int & DX^\mu D\theta_\alpha Du Dv D(\pi_u)_m D(\pi_v)_m Dg_{mn} D\Lambda^{MN}_n D\mathcal{M}^{AB} \\
& exp\{iS^{lagrangian}\} \\
& \delta(\chi^{(rep)})\delta(\chi^{(rot)})\delta(\chi^{(norm)})\delta(v^{+\frac{1}{2}}\sigma^a\theta) \\
& \Delta^{(rep)}_{FP}\Delta^{(rot)}_{FP}\Delta^{(norm)}_{FP}\Delta_{local}
\end{aligned}
\tag{3.1}
$$

where

$$S^{lagrangian} = S^{heterotic}_{GS} + S^{lagrangian}_{auxiliary} \tag{3.2}$$

$S^{heterotic}_{GS}$ is the same as (2.1) while:

$$
\begin{aligned}
S^{lagrangian}_{auxiliary} \equiv \int d\tau d\xi \sqrt{-g}[& (\pi_v^{\mp\frac{1}{2}})_n P^{nm}_- \partial_m v^{\pm\frac{1}{2}} + (\pi^q_u)_n P^{nm}_- \partial_m u_q \\
& - \Lambda^{MN}_n P^{mn}_+ D^{MN}_m - \mathcal{M}^{AB}\Psi^{AB}]
\end{aligned}
\tag{3.3}
$$

In going to the lagrangian path integral the following notations were used:

$$
\begin{aligned}
\sqrt{-g}g^{00} &= -[2(\Lambda_L + \Lambda_R)]^{-1}, \\
\sqrt{-g}g^{01} &= (\Lambda_R - \Lambda_L)(\Lambda_L + \Lambda_R)^{-1}, \\
\sqrt{-g}g^{11} &= 8\Lambda_L\Lambda_R(\Lambda_L + \Lambda_R)^{-1}
\end{aligned}
\tag{3.4}
$$

$$\pi_v^{\pm\frac{1}{2}} = \sqrt{-g}P^{0m}_+(\pi_v^{\pm\frac{1}{2}})_m \quad , \quad \pi^q_u = \sqrt{-g}P^{0m}_+(\pi^q_u)_m \tag{3.5}$$

Similarly, D^{MN}_m have exactly the same form as the hamiltonian constraints D^{MN} (2.22)-(2.29) with all canonical momenta $\pi_v^{\pm\frac{1}{2}}$, π^q_u substituted with $(\pi_v^{\pm\frac{1}{2}})_m$, $(\pi^q_u)_m$.

As usual, we take $\chi^{(rep)}$ to be the conformal gauge for the world-sheet metric g_{mn} (3.4), D= 2 light-cone vector components are denoted as usual $\partial_z, \partial_{\bar{z}} \equiv \frac{1}{2}(\partial_1 \mp \partial_0)(= \frac{1}{2}(\partial_1 \mp i\partial_2)$ in euclidean space).

Although in (3.3) the Lagrange multiplier Λ_n^{MN} and the momenta $(\pi_v^{\pm\frac{1}{2}})_n$, $(\pi_u^q)_n$ become D=2 world-sheet vectors, only one independent component $\Lambda_{\bar{z}}^{MN}$, $(\pi_v^{\pm\frac{1}{2}})_z$, $(\pi_u^q)_z$ actually enters due to the D=2 self-duality projectors $P_{\pm}^{mn}$ (2.2).

Δ_{local} in (3.1) is a local determinant factor of the form :

$$\Delta_{local} \equiv \int D\chi^{-\frac{1}{2}a} exp\{i \int d\tau d\xi \sqrt{-g} [\bar{\chi}^{-\frac{1}{2}a} \rho_n \frac{1}{2}(1-\rho_5)\chi_a^{-\frac{1}{2}}] P_{-}^{nm} \partial_m X^\mu u_\mu^+\} \tag{3.6}$$

where $\chi^{-\frac{1}{2}a}$ are world-sheet spinor bosonic ghosts and ρ_n, ρ_5 denote the world-sheet Dirac matrices. In the conformal gauge:

$$\Delta_{local} = det^4[(\partial_{\bar{z}} X^\mu) u_\mu^+] \tag{3.7}$$

The orthonormality constraints Ψ^{AB} in the auxiliary action contain only u, v and no momenta, therefore, they cannot be anomalous. Thus it is correct to introduce in the path integral the delta functions appropriate for their fixing:

$$\delta(\chi^{(norm)}) \equiv \delta(\Omega^{AB})$$

where

$$\Omega^{AB} \equiv \{\frac{1}{2}\pi_u^{(p} u^{q)},$$
$$(\pi_u^p)_\mu (v^{\pm\frac{1}{2}} \sigma^\mu v^{\pm\frac{1}{2}}), (\pi_u^p)_\mu \sqrt{2}(v^{+\frac{1}{2}} \sigma^\mu v^{-\frac{1}{2}}), \frac{1}{4}(v^{+\frac{1}{2}} \pi_v^{-\frac{1}{2}} + v^{-\frac{1}{2}} \pi_v^{+\frac{1}{2}})\} \tag{3.8}$$

The FP determinant corresponding to Ω^{AB} is constant on the surface $\Psi^{AB} = 0$ (2.21).

The rotation part of auxiliary action (3.3) (in the conformal gauge for g_{mn}), upon inserting the explicit form of the constraints (2.22)-(2.29), has the form:

$$\int d\tau d\xi \{\pi^p [\partial_{\bar{z}} u_p + 2\Lambda_{pq} u^q + O(u,v)] + \pi_v^{\mp\frac{1}{2}} [\partial_{\bar{z}} v^{\pm\frac{1}{2}} + O(u,v)]\} \tag{3.9}$$

The terms collectively denoted in (3.9) and below as $O(u,v)$ do not affect the path integral due to the following argument.

The path integral has the following general form:

$$Z = \int DuD(\pi_u)Dyexp\{i\int d\tau d\xi\ \pi_u(L(y)u + V(u,y))\ + W[u,y]\} \quad (3.10)$$

where u stands for (u,v);
y denotes the rest of the path integral variables;
$L(y)$ is a linear differential operator;
$V(u,y)$ is a polynomial in u containing only terms higher than linear;
$W[u,y]$ is an arbitray functional of u. In the present specific case, $W[u,y]$ is the sum of the logarithms of the ghost determinants plus the fermionic part of the string action (see (3.12)-(3.19),(3.22),(3.23) below).

Now, it is straightforward to show that:

$$\begin{aligned} Z &= \int DuDy\delta[L(y)u + V(u,y)]exp\{iW[u,y]\} \\ &= det^{-1}[L(y)]exp\{iW[0,y]\} \\ &= \int DuD(\pi_u)Dyexp\{i\int d\tau d\xi \pi_u L(y)u\}exp\{W[0,y]\} \end{aligned} \quad (3.11)$$

i.e. the functional integral does not depend on $V(u,y)$ and $W(u,y) - W(0,y)$ which can be disregarded for any practical purpose. We will keep indicating their presence in the intermediate results (by the collective notation $O(u,v)$) only to make easier for the reader to follow the algebra. In the final results (3.25), (3.26), we will appropriatelly invoque (3.11) and supress the $O(u,v)$ terms.

Given the fact that only the derivatives sandwiched between π^p and u^q become covariantized by the appearence of the Lagrange multipliers Λ_{pq} as "gauge fields" in (3.9), the only auxiliary gauge invariances liable with anomalies are the ones corresponding to $SO(7)$ rotations D^{pq} (2.22). The invariances corresponding to the internal orthonormal frame rotations $D^{8p}, D^{-+}, D^{\pm p}, D^{\pm 8}, \tilde{D}^{8p}$ (2.23)-(2.29) can be fixed with no further worries. They contribute the factors:

$$\delta(\Lambda^{8p})det[\partial_{\bar{z}}\delta^{pq} + 2\Lambda^{pq}] \quad (3.12)$$

$$\delta(\Lambda^{\pm p}) det^2[\partial_{\bar{z}}\delta^{pq} + O(u,v)] \tag{3.13}$$

$$\delta(\Lambda^{+-})\delta(\Lambda^{\pm 8})\delta(\tilde{\Lambda}^{8p}) det^{10}[\partial_{\bar{z}}] \tag{3.14}$$

The (conformal gauge) lagrangian path integral can now be rewritten as:

$$\begin{aligned} Z = \int & DX^{\mu} D\psi^{a} Du Dv D(\pi_u)_z D(\pi_v)_z D\Lambda_{\bar{z}}^{pq} D\mathcal{M}^{AB} \\ & exp\{iS\}\Delta_{FP}^{(rep)} \\ & \delta(\Omega^{MN})\delta(\chi^{SO(7)})\Delta_{FP}^{SO(7)} \\ & det[(\nabla_{\bar{z}})_{fund}^{SO(7)} + O(u,v)] det^2[\partial_{\bar{z}} + O(u,v)] det^{10}[\partial_{\bar{z}}] \end{aligned} \tag{3.15}$$

where

$$\begin{aligned} S = -2\int d\tau d\xi [& \partial_z X^{\mu}\partial_{\bar{z}} X_{\mu} + i(\psi_a \nabla_{\bar{z}}^{ab}\psi_b) + \frac{1}{2}\mathcal{M}_{AB}\Psi^{AB} \\ & + (\pi_v^{\mp\frac{1}{2}})_z(\partial_{\bar{z}} v^{\pm\frac{1}{2}} + O(u,v)) + (\pi_u^p)_z(\partial_{\bar{z}} u^p + 2\Lambda^{pq}u_q + O(u,v))] \end{aligned} \tag{3.16}$$

The covariant derivatives entering (3.15),(3.16) are defined as follows:

$$(\nabla_{\bar{z}})_{fund}^{SO(7)} \equiv \nabla_{\bar{z}}^{pq} = \delta^{pq}\partial_{\bar{z}} + 2\Lambda_{\bar{z}}^{pq}, \tag{3.17}$$

where p,q transform as $SO(7)$-vector indices, and:

$$\nabla_{\bar{z}}^{ab} = \delta^{ab}\partial_{\bar{z}} + \Gamma_{\bar{z}}^{ab} - (\partial_{\bar{z}} X^{\mu} u_{\mu}^{c})(\partial_{\bar{z}} X^{\nu} u_{\nu}^{+})^{-1}\Gamma_{\bar{z}}^{+d}(\tilde{S}^{ab})_{cd} \tag{3.18}$$

$$\begin{aligned} & \Gamma_{\bar{z}}^{ab} \equiv u_{\mu}^{a}\partial_{\bar{z}} u^{b\mu} + v^{-\frac{1}{2}}\sigma^{ab}\sigma^{+}\partial_{\bar{z}} v^{-\frac{1}{2}}, \\ & \Gamma_{\bar{z}}^{+a} \equiv u_{\mu}^{+}\partial_{\bar{z}} u^{a\mu}, \\ & (\tilde{S}^{ab})_{cd} \equiv \frac{1}{2}v^{-\frac{1}{2}}\sigma_c\sigma^{ab}\sigma^{+}\sigma_d v^{-\frac{1}{2}} \end{aligned} \tag{3.19}$$

(the latter are precisely the matrices of the $SO(8)$ generators in the harmonic (c)-spinor representation [5,7]). All indices a,b,c,d in (3.18),(3.19) and elsewhere transform as internal $SO(8)$ ones (recall (2.13)). In (3.15) we used the change of variables:

$$\theta_\alpha \rightarrow \theta^{\pm\frac{1}{2}a} = (v^{\pm\frac{1}{2}}\sigma^a\theta) \tag{3.20}$$

with a subsequent rescaling:

$$\theta^{-\frac{1}{2}a} \rightarrow \psi^a = -2(\partial_{\bar{z}}X^\mu u^+_\mu)^{\frac{1}{2}}\theta^{-\frac{1}{2}a} \tag{3.21}$$

Note that the Jacobian of the rescaling (3.21) precisely cancels Δ_{local} (3.7).

Now, it is useful to pass from the gauge $\Omega^{AB} = 0$ (3.8) for the orthonormality constraints Ψ^{AB} (2.21) to another gauge $\mathcal{M}^{AB} = 0$ which significantly simplify subsequent calculations. This passage is effectuated in the standard way by introducing the usual FP "unit". (Recall that Ψ^{AB} (2.21) form an abelian gauge algebra which acts only on π_u, π_v but leaves u, v invariant. The Lagrange multipliers $\mathcal{M}^{AB}$ are, of course, the corresponding gauge fields).

Thus, one obtains in the path integral (3.15) the term

$$\delta(\mathcal{M}^{MN})det[(\nabla_{\bar{z}})^{SO(7)}_{symm,traceless} + O(u,v)]det^{23}[\partial_{\bar{z}} + O(u,v)] \tag{3.22}$$

in place of $\delta(\Omega^{MN})$. Here:

$$\begin{aligned}(\nabla_{\bar{z}})^{SO(7)}_{symm,traceless} \equiv \nabla^{pqrs}_{\bar{z}} =& \delta^{p(r}\delta^{s)q}\partial_{\bar{z}} + 2(\Lambda^{p(r}_{\bar{z}}\delta^{s)q} + \Lambda^{q(r}_{\bar{z}}\delta^{s)p}) \\ &- (trace - parts),\end{aligned} \tag{3.23}$$

is the $SO(7)$-covariant derivative in the symmetric traceless tensor representation with p, q, r, s transforming as $SO(7)$-vector indices.

Ignoring for a moment the possibility of $SO(7)$ anomalies, let us choose the gauge fixing for the internal $SO(7)$ rotations to be $\Lambda^{SO(7)}_{\bar{z}} = 0$ which implies the path integral contribution

$$\delta(\chi^{SO(7)})\Delta^{SO(7)}_{FP} = \delta(\Lambda^{pq}_{\bar{z}})det^{21}[\partial_{\bar{z}}] \tag{3.24}.$$

Now, accounting for (3.12)-(3.14) and (3.24) we can straightforwardly integrate over the lagrange multipliers. Then, (disposing of the $O(u,v)$ terms cf. (3.11)) we obtain the quadratic covariant gauge fixed path integral in terms of a finite number of conformal fields and ghosts:

$$
\begin{aligned}
Z = \int & DX^{\mu} D\psi^{a} DuDvD(\pi_u)_z D(\pi_v)_z \\
& Db^{z} Dc_{zz} D\bar{b}^{\bar{z}} D\bar{c}_{\bar{z}\bar{z}} \\
& D\bar{\eta}_z D\zeta D\bar{\eta}_z D\zeta \\
& exp\{iS_{GS}^{bilinear} + iS'\}
\end{aligned}
\tag{3.25}
$$

$$
\begin{aligned}
S_{GS}^{bilinear} \equiv -2 \int d\tau d\xi [& \partial_z X^{\mu} \partial_{\bar{z}} X_{\mu} + i\psi^{a} \partial_z \psi_a \\
& + (\pi_v^{\mp\frac{1}{2}})_z \partial_{\bar{z}} v^{\pm\frac{1}{2}} + (\pi_u^{p})_z \partial_{\bar{z}} u_p \\
& + \bar{\eta}_z^{MN} \partial_{\bar{z}} \eta_{MN} + \bar{\zeta}_z^{AB} \partial_{\bar{z}} \zeta_{AB} \\
& + b^{z} \partial_{\bar{z}} c_{zz} + \bar{b}^{\bar{z}} \partial_z \bar{c}_{\bar{z}\bar{z}}]
\end{aligned}
\tag{3.26}
$$

where $\bar{\eta}_z$, η, $\bar{\zeta}_z$, ζ are the ghosts for the rotation- (D^{MN}) and orthonormality (Ψ^{AB}) gauge symmetries and b, c are the reparametrization ghosts.

Eqs. (3.25),(3.26) are the basic starting point for computations of GS superstring amplitudes upon inserting in (3.25) the appropriate covariant vertex operators (which, in general, will depend also on the auxiliary variables (u, v), cf . [5]).

To prove the legitimacy of the $SO(7)$ gauge fixing (3.24) and of the subsequent integration over the Lagrange multiplier $\Lambda_{\bar{z}}^{pq}$ which which lead us to (3.26), we will show now that the corresponding $SO(7)$ gauge invariance in (3.15),(3.16) is not anomalous.

The integration in (3.15)(3.16) over the auxiliary variables u, v and the momenta π_u, π_v (cf . (3.10),(3.11)) provides the factor:

$$
det^{-10}[(\nabla_{\bar{z}})_{fund}^{SO(7)}] det^{-32}[\partial_{\bar{z}}]
$$

in the functional integral which consequently becomes:

$$
\begin{aligned}
Z = \int & DX^{\mu} D\psi^{a} D\Lambda_{\bar{z}}^{pq} \\
& exp\{-2i \int d\tau d\xi [\partial_z X^{\mu} \partial_{\bar{z}} X_{\mu} + i\psi^{a} \partial_z \psi_a]\} \\
& \Delta_{FP}^{(rep)} \delta(\chi^{SO(7)}) \Delta_{FP}^{SO(7)} \\
& det[(\nabla_{\bar{z}})^{SO(7)}_{symm,traceless}] det^{-9}[(\nabla_{\bar{z}})^{SO(7)}_{fund}] det^{15}[\partial_{\bar{z}}]
\end{aligned}
\tag{3.27}
$$

The terms in (3.27) containing $\nabla_{\bar{z}}$ are estimated by observing that [16]:

$$
ln\ det[(\nabla_{\bar{z}})^{SO(7)}_{repr}] = c_2(repr) W[\Lambda_{\bar{z}}^{SO(7)}]
$$

with $W[\Lambda]$ denoting the standard Wess-Zumino action and $c_2(repr)$ indicating the value of the second Casimir of the corresponding gauge group ($SO(7)$) representation.

Since [17]:

$$
\frac{c_2(\underline{27})}{c_2(\underline{7})} = \frac{54}{6} = 9
$$

where $\underline{27}, \underline{7}$ denote the symmetric traceless tensor and the fundamental representation of $SO(7)$, respectively, one obtains precise cancellation of the dependence on $\Lambda_{\bar{z}}^{SO(7)} \equiv \Lambda_{\bar{z}}^{pq}$ in the quantum effective action:

$$
det[(\nabla_{\bar{z}})^{SO(7)}_{symm,traceless}] det^{-9}[(\nabla_{\bar{z}})^{SO(7)}_{fund}] det^{15}[\partial_{\bar{z}}] = det^{-27-9+15}[\partial_{\bar{z}}]. \tag{3.28}
$$

and, therefore, no anomalies associated with the auxiliary gauge invariances.

Substituting (3.24) and (3.28) into (3.27) one obtains:

$$
\begin{aligned}
Z = \int & DX^{\mu} D\psi^{a} \Delta_{FP}^{(rep)} \\
& exp\{-2i \int d\tau d\xi [\partial_z X^{\mu} \partial_{\bar{z}} X_{\mu} + i\psi^{a} \partial_z \psi_a]\}.
\end{aligned}
\tag{3.29}
$$

The covariant functional integral (3.29) coincides formally with the noncovariant expression which was shown in [18] to be free of conformal anomalies. In particular, (3.29) shows that the introduction of the auxiliary variables u, v does not contribute to the conformal anomalies at all.

4. Massless Field Theory Limit

In [6] we have shown that given a functionally independent covariant first-class system of constraints, one can construct by a BFV-BRST procedure a field theory action whose equations of motion are the partial differential equations obtained by Dirac quantizing these constraints.

Let us show here that the Dirac constraints corresponding to our system describe on-shell, in the point-particle limit, the D=10 Super-Yang-Mills (SYM) theory. For simplicity we will prove here the equivalence with the component-field equations. A proof of the equivalence with the superfield equations of motion was given in [7]. Of course, both proofs are equivalent because of the well known equivalence between the *component-* and superfield formulations of D=10 N=1 SYM [19] (and D=4 N=3,4 SYM [20]). However, it is instructive to see both.

The general procedure of canonical (operator) quantization [21] instructs us to perform the following steps:

i) Go from Poisson brackets (PB) to Dirac brackets (DB) for the canonical variables accounting for the presence of the second class constraints $G^{+\frac{1}{2}a} = 0$ (2.16) and of the pairs formed by the first-class constraints $D^{+\frac{1}{2}a} \equiv v^{+\frac{1}{2}}\sigma^a \not{p} D = 0$*) (2.15), $\Psi^{AB} = 0$ (2.21) together with their associate gauge-fixing conditions $\chi^{(\kappa)\,+\frac{1}{2}a} \equiv v^{+\frac{1}{2}}\sigma^a\theta = 0$,**) $\Omega^{AB} = 0$ (3.8).

ii) All remaining first-class constraints:

$$p^2, \hat{D}^{ab}, \hat{D}^{-+}, \hat{D}^{\pm a}, \hat{\tilde{D}}^{8p} \tag{4.1}$$

are imposed as operators annihilating the wave function of the superparticle. (Recall that a, b are $SO(8)$ indices unifying $SO(7)$ vector- and $SO(7)$ singlet- indices , cf. (2.13)).

*) In the point-particle limit: $\tilde{T}_L, T_R \to p^2$, $D^\alpha \to -ip^\alpha_\theta - \not{p}^{\alpha\beta}\theta_\beta$.

**) Actually, we may choose not to impose classically the covariant κ-gauge fixing conditions $\chi^{(\kappa)\,+\frac{1}{2}a} \equiv v^{+\frac{1}{2}}\sigma^a\theta = 0$ but to add the quantized first-class constraint $\hat{D}^{+\frac{1}{2}a}$ to the set (4.1). The corresponding Dirac constraint equation, of the form $\hat{D}^{+\frac{1}{2}a}\Phi \equiv (p^+\frac{\partial}{\partial\theta_a^{+\frac{1}{2}}} + p^2\theta^{+\frac{1}{2}a})\Phi = 0$, would simply imply that on-shell Φ does not depend on $\theta^{+\frac{1}{2}a}$ (3.18).

One easily gets the following nontrivial DB for $\theta^{-\frac{1}{2}a}$ (3.18):

$$\{\theta^{-\frac{1}{2}a},\theta^{-\frac{1}{2}b}\}_{DB}=-i(4p^{+})^{-1}\delta^{ab} \tag{4.2}$$

which implies, after rescaling (cf. (3.19)):

$$\theta^{-\frac{1}{2}a}\rightarrow\Psi^{a}=-2\sqrt{p^{+}}\theta^{-\frac{1}{2}a} \tag{4.3}$$

the following quantum anticommutation relations:

$$\{\Psi^{a},\Psi^{b}\}=\delta^{ab}. \tag{4.4}$$

As shown in [5,7] , the $SO(8)$ Clifford-algebra (4.4) can be represented in terms of 16×16 matrices $\Gamma^{a}_{(8)}$ constructed entirely out of the auxiliary variables u,v and the $D=10$ σ-matrices:

$$\Psi^{a}=\frac{1}{\sqrt{2}}\Gamma^{a}_{8}=\begin{bmatrix}0 & \frac{1}{\sqrt{2}}(\gamma^{a})_{bc}\\ \frac{1}{\sqrt{2}}(\tilde{\gamma}^{a})_{bc} & 0\end{bmatrix}, \tag{4.5}$$

$$(\gamma^{a})_{bc}=(\tilde{\gamma}^{a})_{cb}\equiv\sqrt{2}v^{+\frac{1}{2}}\sigma_{b}\sigma^{a}\sigma_{c}v^{-\frac{1}{2}} \tag{4.6}$$

As a result of the matrix representation of the quantum variables Ψ^{a} (4.4)-(4.6), the wave function of the system becomes a 16-component spinor:

$$\Phi(p,u,v)=\begin{bmatrix}Y^{+\frac{1}{2}a}(p,u,v)\\ B^{a}(p,u,v)\end{bmatrix}, \tag{4.7}$$

with a fermionic upper half and a bosonic lower half. The assignement of $(+\frac{1}{2})$ $SO(1,1)$ charge to Y^{a} is a matter of convenience.

Taking into account the representation (4.5)-(4.7), and performing a suitable similarity transformation (involving the u's and the v's) on the operators (4.1), we first solve the following constraint equations:

$$\hat{D}^{ab}\Phi \equiv \begin{bmatrix} D^{ab}Y^{+\frac{1}{2}c} + (S^{ab})^c{}_d Y^{+\frac{1}{2}d} \\ D^{ab}B^c + (V^{ab})^c{}_d B^d \end{bmatrix} = 0 \tag{4.8}$$

$$\hat{D}^{-+}\Phi \equiv \begin{bmatrix} (D^{-+} - \frac{1}{2})Y^{+\frac{1}{2}a} \\ D^{-+}B^a \end{bmatrix} = 0 \tag{4.9}$$

$$\hat{\tilde{D}}^{8p}\Phi \equiv \begin{bmatrix} \tilde{D}^{8p}Y^{+\frac{1}{2}a} - (\tilde{S}^{8p})^{ab}Y_b^{+\frac{1}{2}} \\ \tilde{D}^{8p}B^a \end{bmatrix} = 0 \tag{4.10}$$

The notations used in (4.8)-(4.10) are as follows (cf. (2.22)-(2.24),(2.29)):

$$D^{pq} \equiv u_\mu^p \frac{\partial}{\partial u_{\mu q}} - u_\mu^q \frac{\partial}{\partial u_{\mu p}} + \frac{1}{2}(v^{+\frac{1}{2}}\sigma^{pq}\frac{\partial}{\partial v^{+\frac{1}{2}}} + v^{-\frac{1}{2}}\sigma^{pq}\frac{\partial}{\partial v^{-\frac{1}{2}}}) \tag{4.11a}$$

$$D^{8p} \equiv u_\mu^8 \frac{\partial}{\partial u_{\mu p}} + \frac{1}{2}(v^{+\frac{1}{2}}\sigma^{8p}\frac{\partial}{\partial v^{+\frac{1}{2}}} + v^{-\frac{1}{2}}\sigma^{8p}\frac{\partial}{\partial v^{-\frac{1}{2}}}) \tag{4.11b}$$

$$D^{-+} \equiv \frac{1}{2}(v_\alpha^{+\frac{1}{2}}\frac{\partial}{\partial v_\alpha^{+\frac{1}{2}}} - v_\alpha^{-\frac{1}{2}}\frac{\partial}{\partial v_\alpha^{-\frac{1}{2}}}) \tag{4.12}$$

$$\tilde{D}^{8p} \equiv \frac{1}{2}(v^{+\frac{1}{2}}\sigma^{8p}\frac{\partial}{\partial v^{+\frac{1}{2}}} - v^{-\frac{1}{2}}\sigma^{8p}\frac{\partial}{\partial v^{-\frac{1}{2}}}), \tag{4.13}$$

$(\tilde{S}^{ab})_{cd}$ is the same as in (3.17) and $(V^{ab})_{cd}$, $(S^{ab})_{cd}$ denote the generators of the $SO(8)$ (v) and (s) representations [5,7]:

$$(V^{ab})_{cd} = \delta^{ac}\delta^{bd} - \delta^{ad}\delta^{bd}$$

$$(S^{ab})_{cd} \equiv \frac{1}{2}v^{+\frac{1}{2}}\sigma_c\sigma^{ab}\sigma^-\sigma_d v^{+\frac{1}{2}} \tag{4.14}$$

Solving the equations (4.8)-(4.10) one is lead to the general solutions (in momentum space):

$$B^a(p,u,v) = u^a_\mu \sum_{\{\kappa\}\{\lambda\}\{\rho\}\{\tau\}} (\frac{u^+_{\kappa_1}}{p^+})...(\frac{u^-_{\lambda_1}}{p^-})...u^+_{\rho_1}...u^-_{\tau_1}...B^{\mu\{\kappa\}\{\lambda\}\{\rho\}\{\tau\}}(p) \tag{4.15}$$

(recall $p^\pm \equiv v^{\pm\frac{1}{2}} \not{p} v^{\pm\frac{1}{2}}$),

$$Y^{+\frac{1}{2}a}(p,u,v) = (v^{+\frac{1}{2}}\sigma^a)^\alpha \sum_{\{\kappa\}\{\lambda\}\{\rho\}\{\tau\}} (\frac{u^+_{\kappa_1}}{p^+})...(\frac{u^-_{\lambda_1}}{p^-})...u^+_{\rho_1}...u^-_{\tau_1}... Y_\alpha^{\{\kappa\}\{\lambda\}\{\rho\}\{\tau\}}(p) \tag{4.16}$$

The coefficient fields in the expansions (4.15),(4.16) satisfy certain algebraic conditions of symmetry and tracelessness with respect to their Lorentz indices described in [7].

Actually, in our older works [2-7]*, the form (4.15),(4.16) displaying a specific dependence on the auxiliary variables** was input as a definition of the space of wave functions ("harmonic superfields"***). Now, it becomes clear that the space of D=10 harmonic superfields used in [2-7] is nothing but the space of general solutions of the equations (4.8)-(4.10). Therefore our present formalism [12] is physically equivalent to our original harmonic one [2-7].

Solving now the equations

$$\hat{D}^{+a}\Phi \equiv \begin{bmatrix} D^{+a}Y^{+\frac{1}{2}a} \\ D^{+a}B^a \end{bmatrix} = 0 \tag{4.17}$$

---

* As opposed to the new formalism in [12].

** The specific form (4.15)(4.16) is characterized by the complete saturation of the internal $SO(8) \times SO(1,1)$ indices among the u's and the v's. This, in turn means that the coefficient fields in the expansion carry only Lorentz-indices but not internal $SO(8) \times SO(1,1)$ indices.

*** The name "harmonic" came from the fact that fields of the form (4.15)(4.16) as functions of (u,v) are actually functions on a (non-compact) homogenous space $\frac{\mathcal{L}}{SO(8)\times SO(1,1)}$ where $\mathcal{L}$ is the subspace spanned by the (u,v) fulfilling the orthonormality constraints Ψ^{AB} (2.21). Similar constructions previously appeared in the D=4 harmonic superspace formalism [11].

with (cf. (2.25)(2.27)):

$$D^{+p} \equiv u^{+}\frac{\partial}{\partial u^{p}} + \frac{1}{2}v^{-\frac{1}{2}}\sigma^{+p}\frac{\partial}{\partial v^{-\frac{1}{2}}}$$

$$D^{+8} \equiv \frac{1}{2}v^{-\frac{1}{2}}\sigma^{+8}\frac{\partial}{\partial v^{-\frac{1}{2}}} \tag{4.18}$$

further limits the form of the general solutions (4.15)-(4.16) by eliminating the terms containing u_μ^-:

$$B^a(p,u,v)|_{D^{+\bullet}\Phi=0} = (u^a_\mu - \frac{p^a}{p^+}u^+_\mu)\sum_{\{\kappa\}}(\frac{u^+_{\kappa_1}}{p^+})...(\frac{u^+_{\kappa_r}}{p^+})B^{\mu\{\kappa\}}(p) \tag{4.19}$$

$$Y^{+\frac{1}{2}a}(p,u,v)|_{D^{+\bullet}\Phi=0} = (v^{+\frac{1}{2}}\sigma^a)^\alpha\sum_{\{\kappa\}}(\frac{u^+_{\kappa_1}}{p^+})...(\frac{u^+_{\kappa_n}}{p^+})Y^{\{\kappa\}}_\alpha(p) \tag{4.20}$$

Finally, one imposes the $\hat{D}^{-a}\Phi = 0$ equations:

$$\hat{D}^{-a}\Phi \equiv \begin{bmatrix} (D^{-a} - \frac{1}{2}\frac{p^a}{p^+})Y^{+\frac{1}{2}b} - \frac{p_c}{p^+}(S^{ac})^b{}_dY^{+\frac{1}{2}d} \\ D^{-a}B^b - \frac{p_c}{p^+}(V^{ac})^b{}_dB^d \end{bmatrix} = 0 \tag{4.21}$$

where (cf (2.26),(2.28)):

$$D^{-p} \equiv u^{-}\frac{\partial}{\partial u^{a}} + \frac{1}{2}v^{+\frac{1}{2}}\sigma^{-a}\frac{\partial}{\partial v^{+\frac{1}{2}}}$$

$$D^{-8} \equiv \frac{1}{2}v^{+\frac{1}{2}}\sigma^{-8}\frac{\partial}{\partial v^{+\frac{1}{2}}} \tag{4.22}$$

and $(V^{ab})_{cd}$, $(S^{ab})_{cd}$ are the same as in (4.14).

Let us consider first the explicit form of the second eq. (4.21) upon inserting (4.22) and (4.19):

$$\delta^{ab}\frac{u^{+\nu}}{(p^+)^2}[\eta_{\mu\nu}p^2 - p_\mu p_\nu][B^\mu(p) + \sum_{\{\kappa\}'}(\frac{u^+_{\kappa_1}}{p^+})...(\frac{u^+_{\kappa_r}}{p^+})B^{\mu\{\kappa\}'}(p)]$$

$$+ (u^b_\mu - \frac{p^b}{p^+}u^+_\mu)\sum_{\{\kappa\}'}\sum_{i=1}^{r}(\frac{u^+_{\kappa_1}}{p^+})...\frac{p^\lambda}{(p^+)^2}(u^a_{\kappa_i}u^+_\lambda - u^+_{\kappa_i}u^a_\lambda)...(\frac{u^+_{\kappa_r}}{p^+})B^{\mu\{\kappa\}'}(p) = 0 \tag{4.23}$$

where now $\{\kappa\}'$ labels the non-empty sets of indices $(\kappa_1, ..., \kappa_r)$.

Comparing the expressions in front of the different structures involving the auxiliary variables we find that:
(a) All higher order coefficient-fields in $B^a(p,u,v)$ vanish:

$$B^{\mu\{\kappa\}'}(p) = 0 \tag{4.24}$$

(b) The lowest-order field $B^\mu(p)$ satisfies the ordinary Maxwell equations (going back to x-space):

$$\partial_\mu F^{\mu\nu} = 0 \ , \ F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu \tag{4.25}$$

In a completely analogous manner, upon substituting (4.22) and (4.20), we for the first eq. (4.21):

$$\frac{1}{2p^+}(v^{+\frac{1}{2}}\sigma^b\sigma^a \not{p})^\alpha [Y_\alpha(p) + \sum_{\{\kappa\}'}(\frac{u^+_{\kappa_1}}{p^+})...(\frac{u^+_{\kappa_r}}{p^+})Y_\alpha^{\{\kappa\}'}(p)]$$
$$+ (v^{+\frac{1}{2}}\sigma^b)^\alpha \sum_{\{\kappa\}'}\sum_{i=1}^{r}(\frac{u^+_{\kappa_1}}{p^+})...\frac{p^\lambda}{(p^+)^2}(u^a_{\kappa_i}u^+_\lambda - u^+_{\kappa_i}u^a_\lambda)...(\frac{u^+_{\kappa_r}}{p^+})Y_\alpha^{\{\kappa\}'}(p) = 0 \tag{4.26}$$

and, therefore:
(a) All higher order coefficient-fields in $Y^{+\frac{1}{2}a}(p,u,v)$ vanish:

$$Y_\alpha^{\{\kappa\}'}(p) = 0 \tag{4.27}$$

(b) The lowest-order field $Y_\alpha(p)$ satisfies the ordinary Dirac equations:

$$\not{p}^{\alpha\beta} Y_\beta(p) = 0 \tag{4.28}$$

This concludes the results for the point-particle limit analysis of the covariant canonically (operator-) quantized GS superstring: The covariant Dirac constraint equations for the wave function completely reduce to the usual Maxwell and Dirac equations for the lowest order ordinary fields $B^\mu(p), Y_\alpha(p)$. The nonlinear supersymmetric extension of this analysis and the construction of the corresponding D=10 SYM superfield action were performed in [7].

5. Conclusions

We have shown that the lagrangian path integral for the covariantly quantized GS heterotic superstring reduces to a two dimensional free conformal field theory with a finite number of fields and ghosts (3.26).

We have found that this path integral is anomaly free in 10 dimensions (3.29).

By performing the corresponding Dirac quantization we have shown the equivalence of the present formulation [12] to the "harmonic superfield" formulation of [2-7] (4.15),(4.16).

We have shown that in the point-particle limit, the Dirac constraint (partial differential) equations obtained this way lead to the covariant component-field D=10 SYM field equations (4.25),(4.28).

We consider the contact with the D=10 SYM theory as a decisive evidence that our superstring covariant quantization procedure is correct and tractable.

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Recent work on the same subject was reported at this conference by C. Hull [22], R. Kallosh, M. Perry, and W. Siegel [23]. It will be interesting to see how the D=10 SYM is reproduced in the point-particle limit by these approaches.

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