



## Small Scale Structure on Cosmic Strings \*

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### Abstract

I discuss our current understanding of cosmic string evolution, and focus on the question of small scale structure on strings, where most of the disagreements lie. I present a physical picture designed to put the role of the small scale structure into more intuitive terms. In this picture one can see how the small scale structure can feed back in a major way on the overall scaling solution. I also argue that it is easy for small scale numerical errors to feed back in just such a way. The intuitive discussion presented here may form the basis for an analytic treatment of the small scale structure, which I argue in any case would be extremely valuable in filling the gaps in our present understanding of cosmic string evolution.

## 1 Introduction

The last year has seen further advances in our intuitive and analytical understanding of cosmic string networks [1,2,3,4,5]. Some of these advances were presented in other papers at this workshop. Unfortunately, we still find ourselves relying on computers to understand many important aspects of cosmic strings. Still worse, different computer programs give different answers to

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the same physical questions. Still worse yet, the differences can be traced to differences in the algorithms *at the scale of resolution*.

Any numerical approximation to continuum physics has a “scale of resolution” which by definition is where numerical errors are made. When one embarks on a numerical project it is with the hope that these errors will not have a large effect on the physics one wishes to extract from the calculation. In the best of all possible worlds we would be in the happy position today of having two [1,2] (going on three[6,7]) different numerical schemes all telling us the same things about cosmic string physics. Instead, we are busy trying to understand which numerical errors are better at approximating physics.

In the early days of cosmic strings we hoped that one scale,  $R_H$  (the Hubble radius) was the only important scale. The cosmic strings would simply enter a scaling solution for which all important physics occurred on scales close to  $R_H$ . Since then we have come to appreciate string statistical mechanics [8,9] which tells us that there is another scale, namely “0”, in the problem. A box of cosmic string will try to equilibrate and convert all the long string into the smallest possible loops. The expanding universe and the existence of non self-intersecting loops stop the equilibration process. However, to what small scale the equilibration process is allowed to proceed becomes a more delicate question. For example, any chance to interact which is artificially put in would drive the scale of smallest loops down even further. If physical interactions are neglected, artificially large loops would be allowed to survive. In this talk I try to develop an intuitive picture of how the small scale structure evolves and how it can feed back on the whole string network. I advocate this intuitive picture as the starting point for a more thorough analytical description of small scale physics on cosmic strings.

I will also discuss a somewhat different issue. As I have just mentioned, the physics which determines the smallest physical scale in the problem is very delicate. In principle, however, such problems can still be handled numerically if the resolution scale can be made small enough. It is possible, however, that different numerical algorithms could give different physical pictures *no matter how small* the resolution scale is made. I will suggest that it is easy, in the case of cosmic strings, to introduce numerical errors on the scale of resolution which profoundly affect the large scale physics. These effects can remain finite even for arbitrarily small resolution scales.

I should say at the outset that I do not consider myself the first to become

concerned with small scale structure on string networks. Most notably, Bennett and Bouchet [2,4,5,10,11] have been articulating some of these concerns, and have gone to great lengths to develop numerical simulations which can keep track of small scale structure. The new simulations by Allen and Sheldard [6] also pay particular attention to small scale structure on the strings. In the process of thinking through the material presented here I have become more sympathetic to the point of view that small scale structure may be present on the strings, and may cause at least some deviations from the standard scaling behavior. I hope to pass along these sympathies in this talk. However, I feel the point has yet to be proven, particularly because of the possibility that we are being misled by numerical artifacts. Since the possibility of substantial numerical artifacts due to *enhanced* small scale structure has received relatively little attention, I will be emphasizing this possibility here.

## 1.1 Differing pictures of cosmic string physics

Before plunging into the details, let us assess what is at stake. After all, there are many aspects of string evolution on which there is general agreement. The one scale model works very well at describing the evolution of long strings, which do indeed scale with the Hubble radius. The long strings are described as random walks with a “step size”  $\xi$  ( $\propto R_H$ ) which is also roughly their mean separation. This behavior is maintained as  $R_H$  grows via the breaking off of pieces of long string in the form of loops.

It is also becoming clear that almost all the gravitational impact of the strings will be due to these long strings [12]. Thus, the relationship between cosmic strings and the formation of large scale structure in the universe can be studied (up to factors  $O(1)$ ) without understanding any of the unresolved issues in cosmic string evolution. The overall density of long strings is uncertain by no more than a factor of four, which is not serious given the current understanding of structure formation. (Actually, some of the issues I will raise in this talk suggest that the uncertainties in the long string density could be underestimated.)

The real confusion has to do with the sizes of loops coming off the network of long strings. Although they are almost certainly too small to compete gravitationally with the long strings, the small loops hold the key to whether

cosmic strings are ruled out altogether. The issue here is the gravity wave bound as determined from millisecond pulsar timing [13]. In most models with cosmic strings the dominant decay channel for cosmic string loops is into gravity waves. The absence of gravity wave effects in millisecond pulsar timing experiments puts bounds on the density in gravity waves today.

It turns out that the smaller the loops are initially, the more easily the gravity wave bound is avoided. The results of present simulations differ greatly as to the sizes of typical loops coming off the network, and thus on the nature of the gravity wave bound. One group, Albrecht and Turok (AT) [1] finds that the bound is on the verge of completely ruling out the most interesting cosmic string models, while another, Bennett and Bouchet (BB) [2], finds the bound to be much looser.

So it is the question of small loops sizes where the most substantial uncertainties and disagreements lie. In fact, the real differences between the two conflicting simulations lie not so much in the sizes of loops that are produced, but in the trends that are observed. The AT simulations are consistent with the loop production scaling with  $R_H$ . The BB simulations suggest the loop production scale decreases compared with the Hubble radius. It is when these different trends are extrapolated over cosmological time scales that the physical consequences differ significantly.

## 2 Investigating the differences

When I first heard of the trends observed in the BB simulation I was quite surprised, and I tried to find an intuitive picture that would support their results. What I will do now is describe what I find surprising about the BB results, and what I have learned in trying to get a physical feeling for what is going on. Much of the material presented here has emerged from discussions Neil Turok and I have had on this subject [14].

I take as my starting point the one scale model, which everyone agrees describes the long strings quite well. This model was first proposed by Kibble [15], and it has been further developed in [10] and [1]. In this model the long strings, or at least their large scale features, are random walks of step size  $\xi$ . The mean separation of the strings is also  $O(\xi)$  so the string length density is  $\propto \xi^{-2}$ . It is generally agreed that the long string can start from

just about any sufficiently “random” initial conditions and reach the “fixed point” scaling solution. When the scaling solution is reached,  $\xi$  evolves in proportion to  $R_H$ . We are still dependent on the numerical simulations to determine the constant of proportionality. The one scale model has also been shown to apply to the network as it approaches scaling [1].

The main idea behind the one scale model is that loop chopping process stabilizes when all length scales are the same, up to geometrical factors which turn out to be  $O(1)$ . If the mean separation of the strings is much larger than the scale of wiggles on individual strings, these wiggles will cause loops to break off and the long strings will straighten out. The straightening will continue until the individual long strings are straight on the scale of the separation of different strings. At that point the interactions among different strings will randomize any individual long string, and give it the random walk behavior. The fact that an individual wiggly string is straightened out by the chopping off of loops is supported by statistical mechanics [8,9].

If the mean separation is initially smaller than the scale of wiggles, the inter-string interactions will immediately act to introduce smaller scale wiggles. Thus, no matter what the initial conditions, it seems natural for the one scale behavior to settle in. It is important here that the motion of the strings is relativistic, so the relevant time scales are also  $O(\xi)$ . When the effects of the expanding universe are taken into account, the one scale behavior leads directly to the fixed point scaling evolution discussed above.

Although this one scale picture describes the general properties of the long string quite well, Bennett and Bouchet argue that wiggles on long strings do *not* get thoroughly straightened out by the chopping process, and there is a residue of small wiggles which builds up over time [2]. The idea seems to be that while statistically the straightening process is favored, the actual string evolution does not explore phase space sufficiently well to realize this behavior. None the less, they find that the scale of typical loops coming off the network is given by the scale of the small scale wiggles.

The notion that phase space is not explored thoroughly by the strings does not seem impossible to me, since the network is never truly in equilibrium. However, I originally found the proposal by Bennett and Bouchet puzzling because the scale of small scale wiggles (which I call  $\Delta$  here) is the scale on which *most* of their loops are produced. Clearly there are a great deal of interactions occurring on the scale  $\Delta$ . Why should these interactions be

insufficient to equilibrate the string on that scale? Furthermore, given that  $\Delta$  is such an important scale, why is the overall rate at which energy comes off the long string independent of  $\Delta$ ? Bennett and Bouchet report that the rate of energy loss only depends on  $\xi$ , which is the standard one scale result.

To further explore these issues I will work with the flat spacetime string evolution equations. We will not be making significant errors in neglecting the expansion of the universe because we are concerned with scales much smaller than  $R_H$ .

## 2.1 The Kibble-Turok Sphere and small scale structure

The strings are described by  $\vec{x}(\sigma, t)$  where  $\vec{x}$  is the position and  $\sigma$  runs along the string. The flat spacetime string equations (the Nambu equations) are [16,17]

$$\ddot{\vec{x}}(\sigma, t) = \vec{x}''(\sigma, t) \quad (1)$$

subject to the gauge choice

$$\dot{\vec{x}} \cdot \vec{x}' = 0 \quad (2)$$

and

$$(\dot{\vec{x}})^2 + (\vec{x}')^2 = 1 \quad (3)$$

where  $\dot{\vec{x}} \equiv \partial\vec{x}/\partial t$  and  $\vec{x}' \equiv \partial\vec{x}/\partial\sigma$ . Equation (2) means there are no longitudinal modes, while Eq. (3) chooses a particular parameterization with constant energy per unit  $\sigma$ . The general loop solution can be written in terms of “right-movers” ( $\vec{a}$ ) and “left-movers” ( $\vec{b}$ ) and is given by:

$$\vec{x}(\sigma, t) = \frac{1}{2} [\vec{a}(\sigma - t) + \vec{b}(\sigma + t)] \quad (4)$$

Equations (2) and (3) translate into

$$(\vec{a}')^2 = (\vec{b}')^2 = 1 \quad (5)$$

so  $\vec{a}'$  and  $\vec{b}'$  lie on the surface of a sphere with radius 1 (the “Kibble-Turok sphere”). Any solution to (1) represents a pair of curves on the Kibble-Turok sphere, one for  $\vec{a}'(\sigma - t)$  and one for  $\vec{b}'(\sigma + t)$ . String self intersections can result in sharp bends in the strings which, in the thin string limit used here

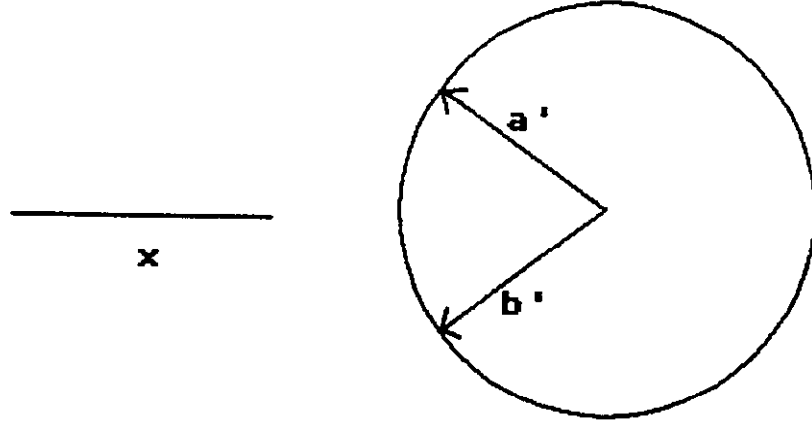


Figure 1: A straight segment of string with its representation on the Kibble-Turok sphere

look like discontinuities in  $\vec{a}'$  and  $\vec{b}'$  (or kinks). These kinks propagate along the string but do not become less discontinuous.

Let us, for the moment, idealize the random walks of the one scale model as perfectly straight string segments connected by kinks. The curves  $\vec{a}(\sigma)$  and  $\vec{b}(\sigma)$  are then also generally composed of straight segments connected by kinks. A given segment of the string is made up of a left moving segment and a right moving segment, so

$$\vec{x}' = \frac{1}{2} [\vec{a}' + \vec{b}'] \quad (6)$$

and

$$\dot{\vec{x}} = \frac{1}{2} [\vec{a}' - \vec{b}'] . \quad (7)$$

As time evolves the left and right-movers go their separate ways and different  $\vec{a}$  and  $\vec{b}$  segments get matched up to make new string segments. A given string segment appears on the Kibble-Turok sphere as two points, one for  $\vec{a}'$  and one for  $\vec{b}'$  (see Fig. 1).

Now let us introduce some smaller scale structure on the string, and let us approximate it too by straight links connected by kinks. From now on I will use the convention that “segments” refer to the large scale pieces (of length  $\xi$ ) which make up the one scale random walks. I will use the term

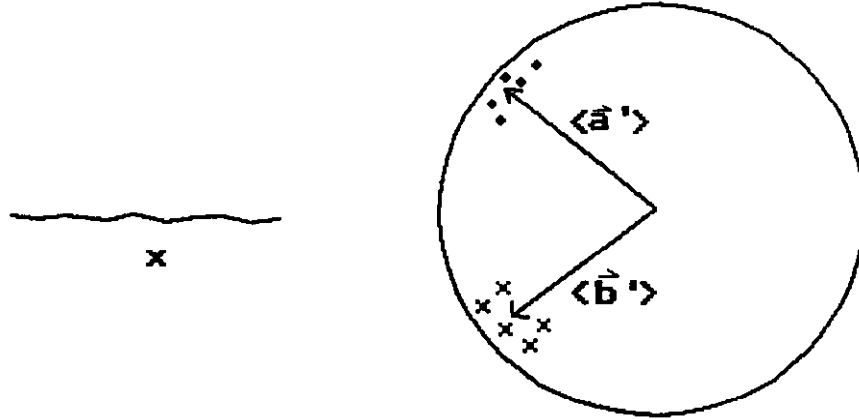


Figure 2: Small wiggles correspond to the scattering of points around their average values on the Kibble-Turok sphere. Note that  $\langle \vec{a}' \rangle$  and  $\langle \vec{b}' \rangle$  lie *inside* the sphere. Here the diamonds are the  $\vec{a}'$ 's and the x's are the  $\vec{b}'$ 's

“links” for the straight bits that make up whatever small scale structure I may be discussing. The segments are no longer represented by pairs of points on the Kibble-Turok sphere, but by many points.

Figures 2 and 3 show how different small scale structure might appear on the Kibble-Turok sphere. The points representing the individual links are scattered around in the general area of the averages  $\langle \vec{a}' \rangle$  and  $\langle \vec{b}' \rangle$ . The greater the degree of scatter, the more sharply the wiggles appear on the string.

## 2.2 Backtracking

As time evolves, different  $\vec{a}$  and  $\vec{b}$  segments will be paired up together. This process guarantees that at any time there will be segments somewhere on the string network where  $\langle \vec{a} \rangle$  and  $\langle \vec{b} \rangle$  point in nearly opposite directions from each other. On these segments the  $\langle \vec{x}' \rangle$  is smaller than average, and  $\langle \dot{\vec{x}} \rangle$  is larger than average (see equations (6) and (7)). If the segments were truly straight these occurrences would correspond to places where the string was moving close to the speed of light. The small value of  $\langle \vec{x}' \rangle$  would then just reflect the fact that energy per unit physical length ( $(\vec{x}')^{-1}$ ) is larger due to the high kinetic energy. (Remember, we have parameterized



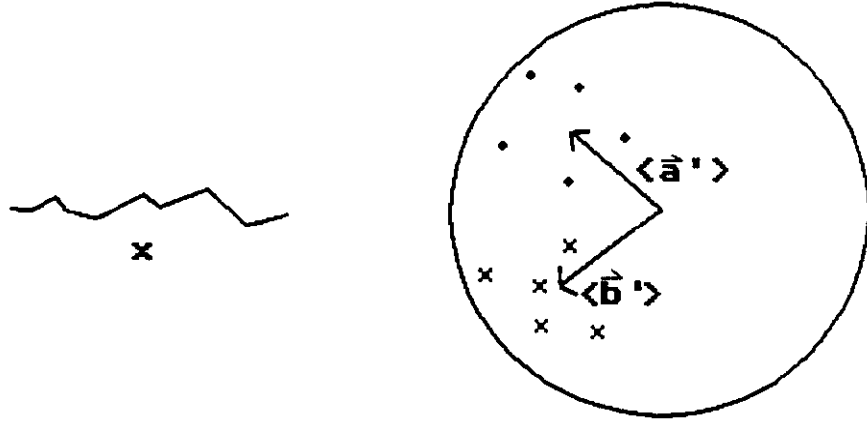


Figure 3: Sharper wiggles correspond to broader scatter on the Kibble-Turok sphere.

the string so there is constant energy per unit  $\sigma$ .)

However, when  $\langle \vec{a} \rangle \approx - \langle \vec{b} \rangle$  on a wiggly segment there is another effect that can occur, which I illustrate in Fig. 4. Due to the scatter on the Kibble-Turok sphere, there can be individual links which point opposite to the direction of the whole segment. In other words, the presence of wiggles on  $\langle \vec{a} \rangle$  and  $\langle \vec{b} \rangle$  can result in back-tracking when the overall length of the segment is short. The increased energy per unit physical length associated with small  $\langle \vec{a}' \rangle$  comes not only from kinetic energy in this case, but from the crinkling up of the string as well.

It seems natural that a string which back-tracks on itself will have a certain number of self intersection, and these intersections will usually break off loops of a size  $O(\Delta)$ , the size of the back-tracking links. However, the overall length which back-tracks depends on the length of the whole segment because the  $\langle \vec{a} \rangle \approx - \langle \vec{b} \rangle$  condition which causes back-tracking persists over a length  $\xi$ .

It is particularly interesting that the tendency to back-track is determined not by the *number* of wiggles per segment ( $\xi/\Delta$ ), but by the degree of scatter, or “sharpness” of the wiggles. Figure 5 shows a segment with more, but smaller wiggles than those depicted in Fig. 3. In some sense the segment in Fig. 5 might seem to be a better approximation to a straight segment, but it has just as much scatter as the segment in Fig. 3. One would expect the

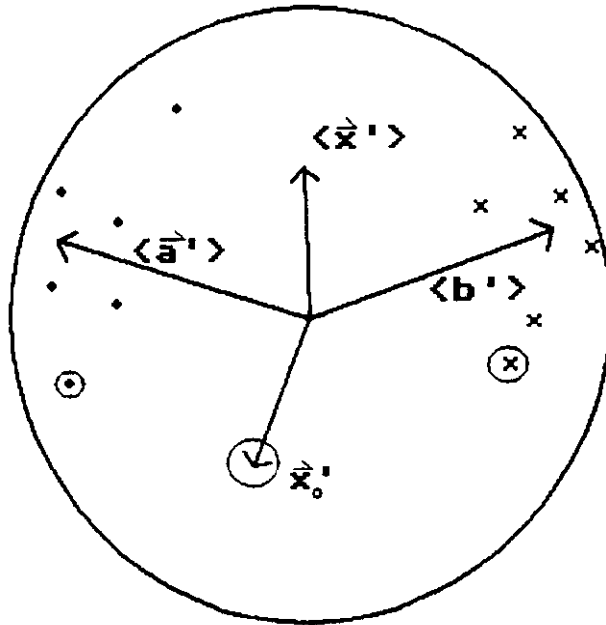


Figure 4: When  $\langle \vec{a} \rangle \approx - \langle \vec{b} \rangle$  some links (for example the circled points) point in a direction opposite to that of the segment and cause “back-tracking”.

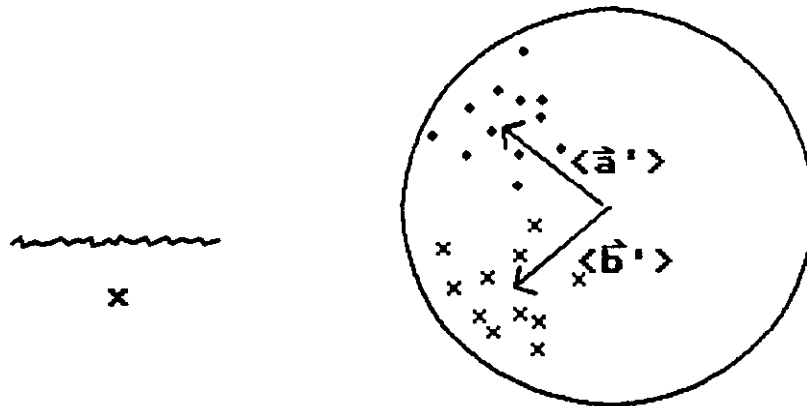


Figure 5: This segment may look smoother than the one in Fig. 3, but it has the same degree of scatter on the Kibble-Turok sphere

tendency to back-track to be just as great in either case. When  $\langle \vec{a} \rangle \approx - \langle \vec{b} \rangle$  it would seem that the same proportion of energy would be tied up in crinkling rather than in kinetic energy, so the smoother looking segment in Fig. 5 is really no better an approximation to a truly smooth segment (for which small  $\vec{x}'$  would be exclusively due to higher kinetic energy).

This discussion has lead to a simple picture in which the scale of loops breaking off the long string is set by the scale of wiggles,  $\Delta$ , while the rate at which energy comes off the long string seems to be set by the length scale of segments,  $\xi$ , over which  $\vec{a}$  and  $\vec{b}$  are each correlated. I will return to a more careful treatment of this point in section 3.

Let me emphasize that the back-tracking occurs specifically when  $\langle \vec{a} \rangle \approx - \langle \vec{b} \rangle$ . This means that over most of the network (where  $\vec{a}$  and  $\vec{b}$  are *not* anti-parallel) the wiggles may show no particular tendency to back-track. Non the less, these very wiggles will, from time to time, find themselves in situations (when  $\langle \vec{a} \rangle \approx - \langle \vec{b} \rangle$ ) where they cause back tracking to occur.

### 2.3 Time evolution of small scale structure

Let us now try to understand how the small scale structure evolves in time. In flat spacetime the left and right-moving links available to the network never really change, and what is important is how they are rearranged with time. Here we will still assume the rough validity of the one scale model, which describes the equilibration of long strings into small loops via the growth of the single scale,  $\xi$ . We will focus on how the links get re-distributed to form segments which have correlations on the (ever increasing) scale  $\xi$  plus a growing bath of infinitesimal loops. It will be useful to define the angle  $\theta$  which represents the characteristic angle between  $\vec{a}$  and  $\langle \vec{a} \rangle$  on a typical segment of long string. (The angle should be the same for both left and right-movers.) If all the vectors  $\vec{a}'$  for a given segment were plotted on the Kibble-Turok sphere, they would occupy a roughly cone-shaped region (see Fig. 6 ) with an angle proportional to  $\theta$ .

The chopping off of loops from long string is the key process in the evolution of  $\theta$ , but it produces two competing effects. The chopping off of a loop removes some links from the long string, and leaves two links that were once separate partially intact, and connected by a kink. All this would appear visually on the Kibble-Turok sphere as the removal of some of the points, or

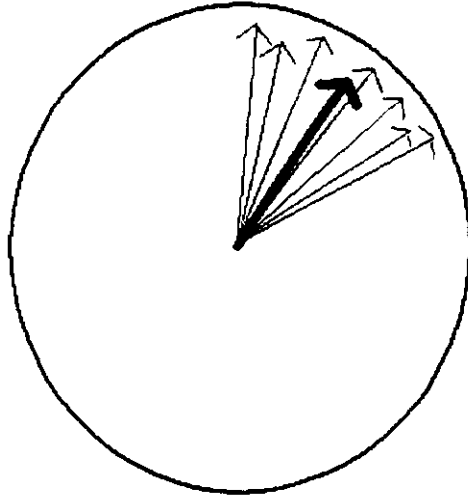


Figure 6: A right-moving segment represented on the Kibble-Turok sphere with vectors for all the links drawn in (forming a “cone” with angle  $\propto \theta$ ). The heavy vector represents  $\langle \vec{a} \rangle$ .

vectors corresponding to the links. But the loop chopping acts to straighten the long string, so it has a tendency to merge the “cones” corresponding to nearby segments together, into one longer segment. This is not done by moving any of the individual vectors, but rather by removing ones on the outer edges until a new merged cone takes on an identity of its own. This effect clearly tends to broaden  $\theta$ , since as a given segment grows it incorporates new links which probably are not initially well aligned.

On the other hand, viewed individually, the chopping off of loops tends to preferentially remove segments which backtrack, since any closed loop *must* backtrack in some place, as depicted in Fig. 7. This effect reduces  $\theta$ . The competition between this effect and the merging of cones, which broadens  $\theta$ , must be well understood if one is to determine the evolution of  $\theta$ . (I should add here that it is *not* enough to count the kinks per unit  $\xi$  as a measure of wiggleness of the segments. The effect of a small loop breaking off increases the number of kinks per  $\xi$ , but also smooths out the segment. A segment with many kinks can be very smooth if  $\theta$  is small!)

The statistical mechanics of strings tells us that the chopping process acts to straighten out the long string, but we have seen how this can have two opposing effects on the evolution of  $\theta$ . On the scale of the mean long string

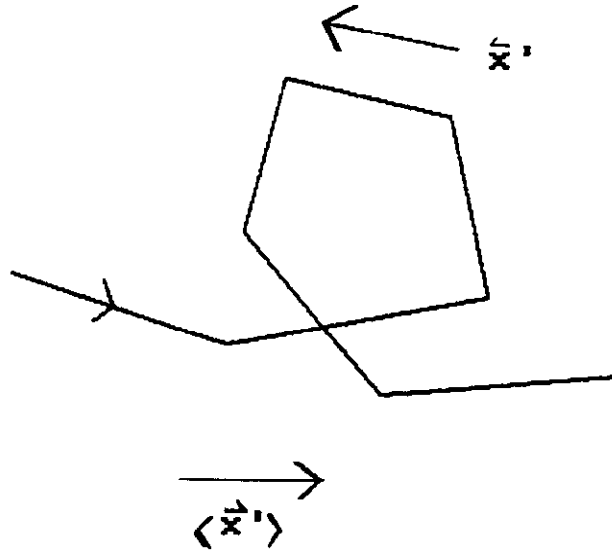


Figure 7: Loop chopping preferentially breaks off back-tracking (or larger  $\theta$ ) links

separation ( $\approx \xi$ ) the straightening involves building up long segments out of pieces of shorter ones that point in different directions. It is not surprising that this process sharpens the wiggles and broadens  $\theta$ . On scales smaller than  $\xi$  where there already is some straightness (or correlation among links), the effect of chopping is to enhance this straightness and reduce  $\theta$ .

The interactions between different long strings tends to control the straightening process by randomizing the long string on large scales. On the Kibble-Turok sphere this has the effect of dividing up cones which become “over crowded” (segments that have become longer than average) by assigning parts of them to different strings. The main effect of this process is to insure the strings are randomized on a scale  $\xi$  which is roughly uniform throughout the network, as we have already assumed. The interactions of the long strings with small loops, however, provides another way that  $\theta$  can grow, since this effect adds randomly oriented links to the long string network.

## 2.4 An example: standard flat spacetime simulations

I now apply the above discussion to the standard flat spacetime cosmic string simulations [18,19], first performed by Smith and Vilenkin. More recently there has been a renewed interest in such simulations [20,3,21] because of the inconclusive nature of the expanding universe computations. In con-

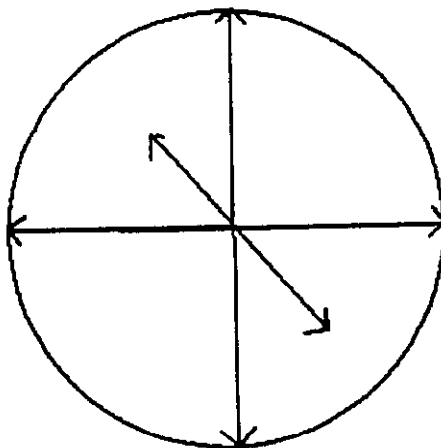


Figure 8: All strings in the standard “cubic” flat spacetime simulations look like this on the Kibble-Turok sphere.

trast to the expanding universe case the flat spacetime simulations find *exact* solutions to the Nambu equations, correct on all length scales. This is accomplished, however, by choosing very special initial conditions.

In these standard simulations one chooses initial conditions which are made up straight segments connected by kinks, similar to the string configurations I have been discussing. The segments on the left and right-movers are restricted to all be the same length (which I call  $\Delta$ ), and to point along one of the  $x$ ,  $y$ , or  $z$  axes, in either direction. For such initial conditions the intersections always occur at kinks, and the restricted form of the left and right-movers continues to hold throughout the evolution. I will call these special string solutions “cubic solutions”.

It is interesting to analyse these cubic solutions in the context of scatter on the Kibble-Turok sphere. The representation of any of the solutions on the Kibble-Turok sphere sits entirely on  $\pm x$ ,  $\pm y$ , and  $\pm z$  axes as depicted in Fig 8. In order for a “straight” segment to be represented in some other direction, it must be composed of these six components in suitable proportions. Clearly, no matter how small  $\Delta$  is, there will be the same degree of scatter on the Kibble-Turok sphere, and  $\theta$  will always be  $O(45^\circ)$ .

For these special solutions, the different forces which evolve  $\theta$  must cancel exactly, since  $\theta$  is independent of time. Although exact, these special solutions will always be very bad approximations to smooth strings. No matter

how small one makes  $\Delta$  there will be back-tracking due to the scatter on the Kibble-Turok sphere, whereas back-tracking will not be present on a smooth string.

This is an example where one might be tempted to consider a “continuum limit” where as  $\Delta \rightarrow 0$  any smooth string configuration could be represented by a cubic solution. Such a limit would not be correct, however, because “numerical errors” (namely back-tracking) on the scale  $\Delta$  would cause loop production on the “scale of resolution”,  $\Delta$ , which would not occur for a truly smooth string. These errors would not get smaller as  $\Delta \rightarrow 0$ , because even though the sizes of individual erroneous loops would decrease, more would be produced.

I should add that the back-tracking effect is not small. For example, if a pair of neighboring right moving links is randomly assigned a pair of left moving links, chances are  $1/36$  that a closed loop is formed. Considering that the overall chopping efficiency is  $O(1/10)$  in a scaling network, back-tracking must play a significant role.

Cubic solutions are inadequate at approximating smooth strings because they are stuck with left and right-movers that only point in a few directions. A real string network, of course, contains links pointing in a greater variety of directions. In principle the loop chopping can systematically remove the most deviant links and result in much smoother segments of string. One wonders, however, how efficient the dynamics would be at making sure the right segments were in the right places to produce smooth segments. (In the language of the previous section, I am asking if the forces which reduce  $\theta$  are all that efficient.) In the limit where this efficiency is poor, the real physics of cosmic strings might closely resemble the physics of the cubic solutions.

### 3 Estimating loop production: Is there scaling?

Let us try to estimate the effect of “back-tracking” on the rate of energy loss from long string. In particular, I will discuss the total length lost into all loops from a segment of length  $\xi$  in a time  $\xi$ . Only in the case where this rate depends only on  $\xi$  can one expect the one scale model to be valid. The arguments in this section are rough, and mainly meant to give a feeling for

the complexity of the problem and for the likelihood of scaling to occur.

Let us start by considering one (length  $\xi$ ) right-moving segment as it runs past a left-moving segment (also length  $\xi$ ), a process which takes a time  $\xi$ . During this time different individual left and right moving links will be matched up with one another in succession. A given series of right-moving links will break off when it meets up with a series of left-moving links which has equal length in  $\sigma$  and equal but opposite net length in physical space. (That is,  $\vec{a}_2 - \vec{a}_1 = -(\vec{b}_2 - \vec{b}_1) \implies \vec{x}_1 = \vec{x}_2$  which is the condition for a self intersection. [22] ) In the course of a time  $\xi$  a series of right-moving links will have more than one chance to meet up with the "perfect" series of left moving links to close off and form a loop.

Naively, one might think that if one halved  $\Delta$  (holding  $\xi$  fixed), twice as much back-tracking would occur on a length  $\xi$ , and that would exactly compensate for the fact that the typical loops produced would be half as large. In that case the total length of string lost from a length  $\xi$  in a time  $\xi$  would be independent of  $\Delta$ , and the one scale model would be valid. One reason why this picture is too naive is that the smaller one makes  $\Delta$ , the more chances a given group of, say, five links has of finding a match in a time  $\xi$ . The number of chances is proportional to  $\xi/\Delta$ , so this is one way the scale  $\Delta$  can enter the problem.

The scale  $\Delta$  can enter in other ways as well. For example, as  $\Delta$  gets smaller there will be more opportunities for longer bits of string (made up of more links) to back-track on themselves. This is just because a length  $\xi$  will be made up of more links. Both the effects I have mentioned indicated the energy loss rate will increase as  $\Delta$  gets small. This suggests the possibility that for small enough  $\Delta$ , loop production is sufficient to really smooth out the long strings on that scale. Furthermore, this critical value of  $\Delta$  could well wind up being proportional to  $\xi$ , in which case the one scale model could survive. A systematic analysis would be needed to determine if this is the case.

I have been tempted to use these arguments to build a specific model of energy loss due to back-tracking. Do do so, however, one needs to flesh out the picture in a number of ways. For example, one needs a model for how  $\theta$  evolves. And one needs a description of how the correlations among the kinks goes from being well correlated on scales  $O(\xi/2)$  to being much less correlated on the scale  $\Delta$ . That is, one needs more details about the starting



point than can be specified by  $\theta$ ,  $\xi$ , and  $\Delta$  alone. One can then model the evolution of these quantities. I have not found a complete set of starting assumptions that I have been happy with, so I have no further progress to report here. However, this seems like an area that could be quite fruitful.

## 4 Conclusions

I have described how small scale wiggles on long string can lead to significant production of loops whose sizes are given by the scale of those wiggles. This effect occurs when right and left-moving segments which are roughly anti-parallel get matched up for a period of time, causing back-tracking (or crinkling) of the string. The length of string and period of time over which this anti-parallel property is approximated can be much larger than the scale set by the wiggles themselves. It is the importance of this larger scale that makes such loop production particularly significant. Bennett and Bouchet [2,4] describe how in their simulations macroscopic segments of long string (“parent regions”) are removed due to the production of “essentially microscopic” loops. What they see seems to fit nicely with the picture I have described here.

Because the small scale structure can have such an important role in the overall evolution of a string network, it must be well understood. For example, as I emphasized in section 2.4, the standard “cubic” solutions used in flat spacetime simulations involve very specific choices of small scale behavior. The fact that these solutions are exact does not mean they answer the questions you want to answer, unless the choice of small scale behavior matches the physical problem at hand. Letting the scale of small structure go to zero does not arbitrarily increase the applicability of the cubic solutions, because macroscopic artifacts of the small scale structure remain.

Likewise, all curved spacetime numerical simulations make some assumptions about the small scale string structure. So far, I believe none of these assumptions have been carefully justified. The AT simulations use an algorithm which introduced numerical diffusion on the scale of resolution. If one accepts the AT results one is assuming that any sharp wiggles lost to the diffusion would not have had much impact if they had been left in. On the other hand, the BB simulations go to great trouble to avoid numerical dif-

fusion, and they keep specific track of kinks. Nonetheless, it is unavoidable that finite numerical schemes introduce numerical artifacts.

An artifact in the BB simulations involves the “merging” of kinks [2] which they say occurs at some string intersections. The details of this merging process have yet to be published, but one is left wondering what the effect of this process is on sharpness of wiggles on the string (that is, on  $\theta$  as described in section 2.3). The production of a loop tends to smooth out the string by preferentially removing the sharpest wiggles (see Fig. 7). In doing so, the number of kinks on the long string is increased. If these kinks are then merged, is the resulting kink sharper? If so, is this artifact sufficient to change the physics results? These are questions which must be answered in order to judge the validity of the algorithm.

It is clear that the assumptions about small scale behavior implicit in the AT and BB algorithms are sufficiently different to yield different results. From the numerical point of view, the best hope for progress lies with the newer algorithms [2,6] which are designed to treat small scale structure more carefully. For example, if the merging in the BB algorithm does appear to be a  $\theta$  increasing process, perhaps the easiest test would be to substitute the merging with a  $\theta$  decreasing version. I believe this would still give less smoothing than AT’s outright numerical diffusion, and if we are lucky, the two merging algorithms might not give substantially different physics.

Still, it would be much more satisfying if an analytic description of the small scale structure on cosmic strings could be developed. Then we could be confident in extrapolating our understanding to cosmic time scales. For example, the discussion in section 3 suggested that the one scale model may not be entirely correct, since the scale of small wiggles may introduce another scale into the problem. We already see indications in flat spacetime simulations that the long strings might be deviating from the simple one scale behavior [3,20,21].

The one scale behavior is observed in all the expanding universe string simulations, but this could to some extent be an illusion. In the expanding universe case there are transients associated with the differences between the initial conditions and the “fixed point” solution. If there are other transients as well, initial conditions can be found where the two transients roughly cancel over the relatively short duration of a numerical simulation, suggesting an incorrect fixed point solution. This problem appeared in the interpreta-

tion of the first string simulations [23], where numerical artifacts resulted in an artificially low quote for the scaling density. Even if the one of the current simulations is essentially free of numerical artifacts, this cancelling of “transients” could be misleading us as to the validity of the one scale model.

In conclusion, the small scale structure on cosmic strings can have a substantial impact on many different length scales. At this point I feel the role of this structure in the evolution of cosmic string networks has yet to be pinned down. Although this issue is being handled with an increasing degree of numerical sophistication[2,4,5,7,6], we would be much better off if we had an analytical understanding of what was going on. The material discussed here was designed to aid the intuition, but it also provides a framework within which to further develop analytical methods.

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