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"Toroidal coupling and frequency spectrum of tearing modes"

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TOROIDAL COUPLING AND FREQUENCY SPECTRUM OF TEARING MODES

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Abstract : The frequency spectrum of tearing modes is analyzed with the help of a mode coupling model including toroidal effects in the MHD regions and various non linear effects in the resonant layers. In particular it is shown that the sudden damping of the mode rotation and the simultaneous enhancement of the growth rate observed in tokomaks, could be explained as a bifurcating solution of the dispersion equation.

I. INTRODUCTION

The structure of the tearing modes in tokomaks is the result of two processes (Furth et al, 1963 ; Ruthertord 1973 ; Basu and Coppi, 1981) first, the redistribution of the equilibrum current along the perturbed magnetic surfaces in the MHD regions outside the resonant layers wich liberates energy from the poloidal magnetic energy reservoir ; second, the resonant plasma response in these layers, and the wall resistive response, allowing the dissipation of the liberated poloidal energy. The purpose of this article is to incorporate those processes in a compact variational principle expressing the self consistency of the modes. The formalism takes into account the linearized MHD equations, including the toroidal effects in the plasma bulk between the resonances (Edery and al, 1981). The resonant layers are supposed to be in a non linear regime controlled by magnetic island geometry, in which case the plasma response is determined by diamagnetism combined with transverse transport (particle diffusion or viscosity) (Samain, 1984). For a given toroidal wave number N, the variational principle leads to a system of coupled equations for the real frequency ω, for the growth rate γ, and for the amplitudes $\tilde{\psi}_m$ (r_m) of each poloidal harmonic m of the perturbation, taken on the corresponding resonant surface r_m where the safety factor q (r_m) = m/N.

As an application we will consider the bifurcation of a mode from an oscillating state to a quasistationary state (Edery et al 1989) which appears at a critical ratio of the coupling coefficients with the resonant layers and with the wall. That bifurcation is introduced by the non linear effects within the islands, leading to critical island widths, contrary to the model given in (Nave and Wesson, 1987) where the island sizes are only governed by the Rutherford regime.

II. VARIATIONAL PRINCIPLE

The equilibrum magnetic field

$$\underset{\sim}{B} = \nabla F \times \nabla (\varphi + q(F)\Theta)$$

introduces intrinsic coordinates, namely, the poloidal flux F embrassed by the magnetic surfaces, the toroidal polar angle φ around the major axis and the poloidal angle variable Θ such that $\nabla_{//}\Theta/\nabla_{//}\varphi = 1/q(F)$. It is convenient to label the magnetic surfaces by the radial scale r(F) such that $B_0 r dr = -q\, dF$ [8], B_o being the field on the magnetic axis. The plasma equilibrium is characterised by density and temperature profiles n(r), T(r) and by a rotation angular velocity $\Omega(r)$ of the plasma around the major axis.

Up to second order in the inverse aspect ratio a/R, a tearing perturbation may be approximated by

$$\delta B = \mathrm{rot}\ (\delta\Psi\nabla\varphi) = \nabla\delta\Psi \times \nabla\varphi \tag{1}$$

with

$$\delta\Psi = \tilde{\Psi}(r, \Theta) \exp (iN\varphi) \exp (\gamma - i\omega t) + C.C \tag{2}$$

The Fourier analysis

$$\tilde{\Psi}\ (r,\Theta) = \sum_m \Psi_m(r) \exp i\, m\Theta \tag{3}$$

introduces the resonances of the mode on the surfaces $r = r_m$ where $q = m/N$.

A chain of magnetic islands is produced near the surface r_m if $\Psi_m(r_m)$ is a non null constant, with an half width

$$\delta_m = \left| 8\ \Psi_m/B_o S \right|^{1/2}_{r=r_m} \tag{4}$$

$$S = r \frac{d}{dr}\ (1/q)$$

The self consistency of the structure $\left\{\tilde{\Psi}(r,\Theta),\ \gamma,\ \omega\right\}$ can be expressed by extremalizing in Ψ^* the functional

$$F\ (\tilde{\Psi}^*,\tilde{\Psi},\gamma,\omega) = -\frac{1}{\mu_o}\iiint \nabla\tilde{\Psi}^*.\nabla\tilde{\Psi}\ \frac{d^3x}{R^2} + \iiint \tilde{\Psi}^*\ \tilde{j}_\varphi\ \frac{d^3x}{R} \qquad (5)$$

where the plasma current response $\tilde{j}_\varphi(r,\Theta)$ must be specified as a functional of $\tilde{\Psi}(r,\Theta)$, within the MHD bulk as well as within the resonant layers. Accordingly we will write

$$F = F_{MHD} + F_{reson}$$

From (5) we deduce the quantities 2ω Im (F) and 2N Im (F) which are the power and the Ø momentum rate coupled to the plasma respectively.

III. MHD INTERVALS

In the plasma bulk between the resonances, the magnetic perturbation preserves the topology of the magnetic surfaces. The current $\tilde{j}_\varphi$ is then determined in terms of $\tilde{\Psi}$, expressing that the plasma recovers an equilibrium compatible with those surfaces. The energy produced is (Bussac et al, 1975) :

$$F_{MHD} = -\delta W = -\frac{1}{\mu o}\iiint/\nabla\tilde{\Psi}/^{2} d^3x/R^2 + \\ + R_e\iiint \tilde{U}^*\ (\nabla\tilde{\Psi}\ x\ \nabla\varphi)\ \nabla\ (Rj_\varphi)d^3x/RB_o \qquad (6)$$

where j_φ is the toroidal component of the equilibrum current density and $\tilde{U}$ (r,Θ) is the time integrated electrostatic potential perturbation, expressed in terms of $\tilde{\Psi}\ (r,\Theta)$ by

$$\tilde{\Psi}(r,\Theta) = R\underline{B}.\underline{\nabla}\ \tilde{U}(r,\Theta)\ /\ B_\varphi \qquad (7)$$

which implies between the m harmonics the relation :

$$U_m(r) = -i\Psi_m(r) / (N - m/q(r)).$$

The integrals in (6) are of course taken ouside the resonant layers extending symmetrically on each side of the resonant surfaces $r = r_m$. The expression (6) does not contain the term proportional to $\partial p/\partial \varphi$ (Hastie, Nice 1988) reflecting the redistribution of the plasma pressure p(r) on the new magnetic surfaces. Indeed, for tearing modes, the pressure term in F_{MHD} consists of integrals localized at the boundary of the resonant layers, and can be introduced in the functional F_{reson}.

The extremalization of F_{MHD} with respect to Ψ^* in each MHD interval, determines $\Psi(r,\theta)$ in those intervals if the boundary values $\Psi(r_m,\theta)$ are specified. The extremal value of F_{MHD} may then be expressed in terms of the slope jumps $[\underline{n}.\nabla\tilde{\Psi}]$ of $\tilde{\Psi}$ (r,θ) across each layer where $\underline{n}$ is the unit vector normal to the layer :

$$\delta W = \frac{1}{\mu o} \sum_m \iint_{r=r_m} \tilde{\Psi}^* \; [\underline{n}.\nabla\tilde{\Psi}] \; d^2x/R^2 \qquad (8)$$

In principle that extremal value depends on the amplitudes Ψ_m (r_m') of all harmonics m on all resonant surfaces r_m', i.e., it is a real hermitian form in $\Psi_m(r_m')$ and $\Psi^*_m(r_m')$. However, as it will be seen below, F_{res} depends on Ψ^* through the restricted set of amplitudes $\Psi^*_m(r_m)$ only. The principle that $F_{MHD} + F_{reson}$ must be an extremum in Ψ^* then implies that F_{MHD} is also extremum in Ψ^*_m (r_m'), $m' \neq m$.

The functions $\Psi_m(r)$ in the MHD intervals are then determined in terms of the amplitudes $\Psi_m(r_m)$ only, leaving F_{MHD} in the hermitian form :

$$F_{MHD} = \sum_{mm'} T_{mm'} \, \Psi_{m'}(r_{m'}) \; \Psi^*_m \; (r_m) \qquad (9)$$

In cylindrical geometry the quantities $T_{mm'}, m'=m$ are the classical logarithmic slope jump on each resonance $r = r_m$. At second order in r/R the quantities $T_{mm'}$, $m' = m \pm 1$ have the following analytical expression in terms of the cylindrical tearing profiles $f_m(r) = \Psi_m(r)/\Psi_m(r_m)$ and of the Shaffranov equilibrum parameter Λ (r) (Edery et al, 1981) :

$$T_{m,\ m+1} = \frac{r_m}{R_o} \int_o^a dr \left\{ r^2(\Lambda+1)\ f'_m\ f'_{m+1} + \frac{r}{2}\ (r\Lambda' + 2\Lambda + 3) \right.$$
$$+ ((m+1)\ f_{m+1}\ f'_m - mf_m\ f'_{m+1})$$
$$\left. - m(m+1)\ (\Lambda + 2)\ f_m f_{m+1} \right\} \quad (10)$$

$$f' \equiv \frac{df}{dr}$$

IV. RESONANT LAYERS

The resonant current $\bar{j}_\varphi$ which determines F_{res} is calculated assuming a non linear regime where the electrons reach equilibrium in the islands created by the perturbation $\delta\Psi$. In each resonant layer m, such a regime imposes islands with a significant half width δ_m (eq.(4)), so that they are ergodically explored by the electrons before they are extracted by the effect of the phase velocity ω/m of the mode along Θ, or by the effect of the diffusion due to the microturbulence present in the plasma ; that condition is expressed by the inequality

$$\frac{1}{\tau_{erg}} \geq \frac{1}{\tau_{ext}} \quad (11)$$

with

$$1/\tau_{erg} = \text{Min}\ (\dot{K}\ \delta_m V_e,\ \dot{K}^2\ \ \delta_m^2\ V_e^2\ /\ \nu_{ei}),$$

$$1/\tau_{ext} = \text{Max}\ (\omega,\ D_e/\delta_m^2)\ ;\ \dot{K} = \frac{m}{r}\ \frac{S}{R}$$

where $V_e = (2T/m_e)^{1/2}$, ν_{ei} = collision rate of electrons (e) with ions (i), D_e = electron diffusion coefficient. It is shown in (Samain, 1984) that such a regime imposes the density and temperature profiles for electrons in terms of the imperturbed diamagnetic frequency of electrons (outside the layer) :

$$\omega_e^* = \frac{m}{r}\ T\ \frac{1}{eB_o}\ \frac{\partial}{\partial r}\ \log\ (n,T)$$

However the electrostatic potential U must be given. Taking into account the ion response and expressing the neutrality, the potential U may be determined in terms of the unperturbed plasma frequency $\Omega(r)$. We will consider situations where

$$\max(\omega, D_i/\delta_m^2) > \dot{K}_{//}\ \delta_m V_i$$

or $$\omega > Di/\delta_m^2. \qquad (12)$$

The quantities n, T, U are then approximately constant over the perturbed magnetic surfaces within the layer. The conditions (11) and (12) are typically satisfied in the case of tearing modes observed by Mirnov probes.

The resonant current response $\tilde{j}_\varphi$ is identical to $j_{//}$ up to second order in r/R_o ; $j_{//}$ is determined from the charge continuity equation

$$\mathrm{div}\ (j_{//}) = - \mathrm{div}\ (j\ ) \qquad (13)$$

$$j_\perp = e\ \emptyset_e - e\emptyset_i + j_{\perp 1}$$

where $\emptyset_e$, $\emptyset_i$ are the radial fluxes of electrons and ions due to the microturbulence and $j_{\perp 1}$ represents the transverse ion current due to inertia, F.L.R. drift curvature and viscosity effects. The $e(\emptyset_e - \emptyset_i)$ term in $j_\perp$ is found to be largely dominant if the microturbulent modes exchange Θ momentum with the plasma over a radial range larger than the island width δ_m, a situation which removes the local ambipolarity constraint $\emptyset_e = \emptyset_i$; the calculations are given in (Samain, 1984) and lead to :

$$F_{reson} = \sum_m H_m\ \Psi_m(r_m)\ \Psi^*_{m'}(r_m') \qquad (14)$$

where for each layer m of area $\mathscr{A}$ along Θ and φ :

$$H_m = - R_m\gamma + iK_m\ (\omega - \omega_e^*(r_m) - N\Omega(r_m))$$

$$R_m = 0{,}8\,\mathcal{S}\,\frac{\delta_m}{\eta\, R^2}$$

$$K_m = 44\,\mathcal{S}\,\frac{ne^2}{T}\;\frac{D_e}{\dot{K}_{/\!/}^{2}\delta_m^3}\;\frac{1}{R^2} \qquad (15)$$

η = plasma resistivity

The term $-\gamma R_m$ represents the Rutherford effect. Indeed (13) determines $j_{/\!/}$ except for an additive constant over each perturbed magnetic surface. That constant results from the balance between the resistive effect and the inductive effect in the island frame, proportional to the growth rate γ. The above situation where the radial range of Θ momentum transfer by the microturbulent modes exceeds the island width δ is the only case in the considered non linear regime where the mode is influenced by the electron diamagnetic frequencies ω^*_e. For larger island widths, the microturbulent modes adjust themselves to the perturbed magnetic surfaces in the resonant layers and local ambipolarity $\underline{\phi}_e - \tilde{\phi}_i = 0$ applies. In this case the value of F_{reson} is determined by the viscosity forces acting on ions. The functional F_{res} is still given by (14) (15) putting $\omega^*_e = 0$ and reducing K_2 by a factor of order ρ_i^2/δ_m^2 where ρ_i is the Larmor radius of ions.

V. APPLICATION : MODE LOCKING

For a given equilibrium, the extremalization of the functional $F_{MHD} + F_{Res}$ with respect to the $\Psi^*_m(r_m)$, leads to the set of equations :

$$\sum_{m'} \left\{ T_{mm'} + H_m\,(\gamma,\omega,/\Psi_m/)\;\delta_{mm'} \right\} \Psi_{m'}(r_{m'}) = 0 \qquad (16)$$

which produces the values of γ, ω and the ratios $\Psi_{m'}(r_m')/\Psi_m(r_m)$.

The toroidal momentum rate P_φ transferred to each resonant layer m by the mode is given by the relevant contribution in $2N\Im m(F)$, i.e :

$$P_\varphi = 2N \; K_m \, [\omega - \omega^*(r_m) - N\Omega(r_m)] \; |\psi_m(r_m)|^2$$

The created forces on each layer are balanced by the viscosity or inertia forces proportional to the plasma angular velocities $\Omega(r_m)$.

A resistive wall of resistivity η_w and width δ_w at radius r_w is similar to the resonant layers, in the sense that it introduces a dissipative term in the functional F. It may be included in the formalism by adding the amplitudes $\Psi_m(r_w)$ to the set of the $\psi_m\,(r_m)$.
The value F_{MHD} excluding the wall is expressed as a real hermitian form in $\Psi_m(r_m)$, $\psi_m(r_w)$ and, $\Psi_m^*(r_m)$, $\psi_m^*(r_w)$ the wall then contributing by

$$F_w = \iiint_{wall} \tilde{j}_\varphi \, \Psi^* \, d^3x/R^2 = \iiint_{wall} \frac{i\omega-\gamma}{\eta_w} \, |\Psi|^2 d^3x/R^2$$

A simple application of the above formalism is the study of the transition of a single helicity tearing mode N = 1, m = 2 from an oscillating state where it mainly interacts with the resonant layer at $r = r_2$ to a stationary state where it strongly interacts witn the wall at $r = r_w$. The basic functionals are :

$$F_{MHD} = [\Delta_2' |\Psi_2|^2 + \Delta_w' |\Psi_w|^2 + C(\Psi_2^* \, \Psi_w + \Psi_w^* \, \Psi_2)] \, \frac{\mathcal{L}}{R^2\mu_0}$$

$$F_{reson} = (- R_2\gamma + iK_2(\omega-\tilde{\omega}_2))|\Psi_2|^2 \tag{17}$$

$$F_w = (- R_w\gamma + iK_w\omega)|\Psi_w|^2$$

where $\Psi_2 = \Psi_2(r_2)$, $\Psi_w = \Psi_2\ (r_w)$,

$$\tilde{\omega}_2 = \omega_2^*(r_2) + N\ \Omega\ (r_2)$$

$$K_w = R_w = \mathscr{A} \frac{\delta w}{\eta_w}\ \frac{1}{R^2}$$

In the case of a perfectly conducting wall ($\eta_w = 0$) we have $\psi_w = 0$ and $F_{MHD}\ \alpha\ \Delta_2'/\psi_2/^2$. On the contrary if the wall is not active ($\eta_w = \infty$), $F_w = 0$ and $F = F_{MHD} + F_{Res} + F_w$ depends on ψ_w through F_{MHD} only. Extremalization with respect to ψ_w^* then gives $F_{MHD}\ \alpha\ (\Delta_2'-C^2/\Delta'_w)/\psi_2/^2$. The quantities $\Delta'_2 < 0$ and $\Delta'_2 - C^2/\Delta'_w > 0$ are the usual Δ' for the two cases $\eta_w = 0$ and $\eta_w = \infty$, respectively. We will assume in what fallows that $-\Delta'_2 \sim C << \Delta'_w$.

The plasma velocity $\Omega\ (r_2)$ is typicaly determined from the balance between the toroidal force exerted by the mode on the resonant layer and the viscosity forces. This balance is expressed by

$$2N\ K_2(\omega-\tilde{\omega}_2)\ /\psi_2/^2 = n\ mi\ D_v R^2\ \left(\frac{\Omega(r_2) - \Omega(o)}{r_2}\right) + \frac{\Omega(r_2)}{r_w-r_2})\quad (18)$$

where D_v is the viscosity transport coefficient (m^2/s) and we have assumed constant $n\ m_i\ D_v\ \partial\Omega/\partial r$ on each side of the resonant layer, up to the wall $r = r_w$ and the center $r = 0$. The elimination of $\Omega(r_2)$ allows to replace in (17) the old expressions of $\tilde{\omega}_2$ and K_2 given by (15) by the new ones :

$$\tilde{\omega}_2' = \omega_e^*(r_2) + N\Omega(0)\ (r_w - r_2)/r_w$$

$$K_2' = \frac{\mathscr{A} D_v R^2}{2N^2}\ \frac{nm_i}{/\psi_2/^2}\ \frac{r_w}{r_2(r_w-r_2)}/(1+\alpha)$$

$$\alpha = \mathscr{A} D_v\ \frac{R^2}{2N^2}\ \frac{nm_i}{/\psi_2/^2}\ \frac{r_w}{r_2(r_w-r_2)}\ /K_2$$

In most practical cases, one finds $\alpha << 1$. The plasma angular velocity $\Omega(r_2)$ near the resonant surface then adjusts to a value close to $(\omega - \omega^*(r_2))/N$, to maintain the force exerted by the mode on the layer (the RHS in (18)) at the low level of the viscosity forces imposed from outside. The new value K'_2 applicable to (17) is then determined by the corresponding viscosity coefficient D_v, independently of the actual resonant process. In view of (4) that new value varies with the island width δ_2 as δ_2^{-4}. From (17) we obtain the following set of equations

$$[\Delta'_2 - R_2\gamma + iK_2\ (\omega - \tilde{\omega}_2)]\ \Psi_2 + C\Psi_w = 0$$

$$[\Delta'_w - R_w\gamma + iK_w\ \omega]\ \Psi_w + C\Psi_2 = 0$$

allowing the determination of the parameters ω, γ, $/\Psi_w/\Psi_2/$ and Arg (Ψ_w/Ψ_2). The resulting dispersion relation for the normalized frequency $\Omega = \omega/\tilde{\omega}_2$ (for $-\Delta'_2 - C << \Delta'_w$) is :

$$Y(\Omega) = \Omega\ (1-\Omega)\ [1 + \frac{g(\Omega_o)}{(\Omega-\Omega_o)^2}] = f\ (\Omega_o)$$

where $\quad \Omega_o = 1/(1 + R_2/K_2)$

$$f(\Omega_o) = C^2\ /\ (K_2 K_w\ \tilde{\omega}_2{}^2)$$

$$g(\Omega_o) = \Delta'^2_2\ \Omega_0^2\ /\ K_2^2\ \tilde{\omega}_2{}^2$$

The possible solutions are exhibited on fig (1), and the phase locking of the mode may be interpreted as a bifurcation from a diamagnetic driven mode at $r = r_2$ (A) to a quasistationary mode (B), when the width δ_2 determining K_2 is increased. The critical relation between the physical parameters is

$$4C^2/K_2R_w\tilde{\omega}_2{}^2 = 1$$

In JET the application of the above formula to the interaction of a tearing mode $m = 2$, $N = 1$ with the wall, leads to the following estimation of the critical values of δ_2:

$$\delta_2/\delta_w = 9$$

and of the parameters for the stationary mode taking place after the bifurcation :

$$\omega/\tilde{\omega}_2 = 1.5\ 10^{-4}$$

$$\gamma/\gamma_2 = 3$$

$$|\Psi_2/\Psi_w| = 0.7$$

$$\mathrm{Arg}\ (\Psi_2/\Psi_w) = 10^{-2}\ \mathrm{rad}$$

The above parameters are in agreement with the observations in JET (Snipes 1989). The plasma velocity $\Omega(r_2)$ takes a value close to $(\omega-\omega_2^*(r_2)/N$. However the observed slowing down of the plasma bulk (in cases where a plasma rotation is induced by neutral injection) cannot be explained by the viscosity forces resulting from that constraint. This slowing down should rather result from the component $m = 1$ of the mode freezing the plasma rotation $\Omega(r_1)$ on the surface $q = 1$.

IV. CONCLUSION

We dispose of a model allowing to calculate the parameters of a tearing mode : growth rate, frequency, ratio between poloidal components, forces exerted by the mode on each resonant layer, and for instance the bifurcation from a normal mode regime driven by diamagnetism with $\omega/\tilde{\omega}_2 = 1$ to a quasi stationary mode regime with $\omega/\tilde{\omega}_2 \ll 1$.

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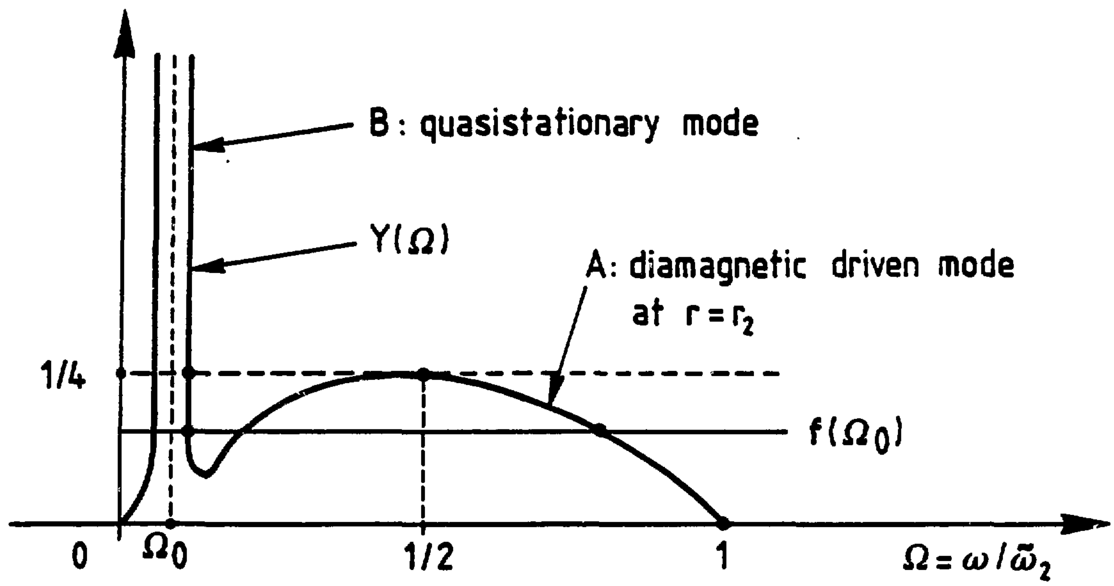

FIG. 1 Bifurcation diagram of the mode frequency
(Critical solution : $f(\Omega_o) = 1/4$)