ON THE LIMIT OF ENERGY RESOLUTION IN RADIATION DETECTORS

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The limit of energy resolution in various radiation detectors is reviewed from the theoretical view-point. Fano-factors in gaseous, liquid and solid detector media for ionization and for scintillation are discussed and the limit of energy resolution in micro-calorimeters operated at low temperature is also discussed.

Particles passing through matter dissipate their energies by following three kinds of collisions; ionization, excitation and nuclear elastic collisions. Ionization collisions produce many electron-ion pairs and their recombination produces many excited atoms. These excited atoms and those produced by direct excitation become the origin of "scintillation" photons. The elastic collision with nucleus produces a recoil atom or the so-called "phonon" in solid matter. In general, the straggling of signal events is due to the fluctuation of the energy deposited by these different collisions. If we consider the ionization produced by perfect absorption of an ionizing radiation with fixed energy, its ionization straggling, that is, the r.m.s. value of fluctuation, is given by $\sqrt{F_i N_i}$, where F_i is the Fano-factor which was defined by Fano[1] and N_i is the number of the ion-elctron pairs produced by the ionizing radiation. This also gives the theoretical limit of energy resolution in ionization detectors. Such a consideration is also applicable to the fluctuation of the number of scintillation photons emitted from matter which absorbed ionizing radiation with a fixed energy and also to that of ionization in superconducting state. The micro-calorimeter recently developed for observation of cosmic X-

rays measures only phonons. So, there exists no fluctuation between different collisions, such as ionization or scintillation, and only the fluctuation of the kinetic energy in lattice vibrations, that is , thermal noise. This noise level gives the limit of the energy resolution. In this paper, only the limit of the energy resolution which can be theoretically treated, will be discussed.

First, let us consider "ionization" detectors. The limit of their energy resolution, caused by the fluctuation of the number of electron-ion pairs, is given by a following equation,

$$\delta E(\mathrm{keV}) = \{ F_i \cdot W_i(\mathrm{eV}) \cdot E(\mathrm{MeV}) \}^{1/2} \qquad (1),$$

where δE is the r.m.s. value of fluctuation, W_i is the average energy required to produce an electron-ion pair and E is the total energy of the ionizing radiation which is absorbed in the matter. The Fano-factor can be estimated by using the formula derived by Fano,[1)]

$$F_i = \Sigma_a \, (N_a/N_i) \{ n_a - (\varepsilon_a/W_i) \}^{1/2} \qquad (2),$$

where N_a is the average number of collisions "a", N_i is the average number of electron-ion pairs produced by the ionizing radiation, n_a is the number of eletron-ion pairs produced by individual "a"-type collision and ε_a is the energy lost in individual "a"-type collision. If the production mechanism of scintillation photons is known, we can estimate the Fano-factor for scintillation from formula (2) as well as the case of ionization. The estimsation of the Fano-factor in the special case of scintillation will be shown later.

The energy loss in individual ionizing collision ε_i fluctuates around the sum of the average energy lost in ionization E_i and the average energy of subexcitation electrons ε_s, $E_i + \varepsilon_s = W_i$. The energy loss in individual excitation collision, ε_{ex}, fluctuates around theaverage energy lost in excitation collisions, $E_{ex} = W_{ex}$. By changing the parameters to these new ones, we can

express F_i by

$$F_i = (N_{ex}/N_i)\{1 + N_{ex}/N_i\}(W_{ex}/W_i)^2 + \{(\varepsilon_i - W_i)^2/W^2\}$$

$$+ (N_{ex}/N_i)\{(\varepsilon_{ex} - W_{ex})^2/W^2\} \quad (3).$$

The first term, which is the largest in three terms, is caused by the fluctuation of ionization and excitaion in collision process and the second and third terms are due to the fluctuation of the energy loss in ionization or excitation collisions. The sum of the second and third terms is much smaller than the first term. As seen from the above equation, if all the excited atoms contribute to ionization, that is, N_{ex} becomes zero and N_i increases to $N_i + N_{ex}$, the first term becomes zero and only the second and third terms remains. Such a case is realized by mixing a small amount of Ar, Kr or Xe is into helium gas,[2,3] and the Fano-factor in the gas mixture remarkably reduces. This is the so-called Penning Effect.

Recently, the Fano-factors in rare gases, rare gas mixtures and rare gases conatining organic molecules were experimentally obtained by using gridded ionization chambers, proportional counters or proportional scintillation counters. In the gridded ionization chamber, α-particles were used as an ionizing radiation, and in the other two detectors, X-rays were used. These results are shown in Table 1 as well as the theoretical estimates. As seen from the results for Ar + 10%CH_4 and Xe, the Fano-factors obtained for α-particles are 30 ~ 50 % larger than those obtained for X-rays in the same gases. In the table, the theoretical estimations are made only for mono-energetic electrons and the Fano-factors obtained for α-particles are 20 ~ 30 % larger than theoretically estimated ones in He and Ar. Recently, Inokuchi pointed out that in general the Fano-factor for α-particles should be smaller than that for mono-energetic electrons and the differences in the Fano-factor could be explained by the difference in the ionizing radiation experimentally used.[15]

In the table, the remarkable reduction of the Fano-factor in

the mixture gas of He + (a small amount of Ar, Kr or Xe) is also shown. The value, determined experimentally for α -particles, is 0.1 , which is about twice as large as that theoretically estimated for mono-energetic electrons. This large discrepancy might be caused not only by the use of different particles as suggested by Inokuchi, but also by nuclear elastic collisions. Because the energy transfer from α -particle to helium nucleus must be very effectively.

Using (3), we can estimate the Fano factor for scintillation phonons for a following special case. Let us consider the number of scintillation photons emitted from a rare gas or a liquid rare gas. Scintillation photons from this rare gas or liquid rare gas are emitted through two processes: (1) atoms directly excited by incident particles collide with surrounding atoms and form excited dimers, which emit photons with a energy spread, and (2) atoms directly ionized by incident particles collide with surround atoms and form ionized dimers. These ionized dimers dissociatively recombine with electrons to form excited dimers, which then emit scintillation photons. If we can assume that each excited dimer emits one photon, the total scintillation photons is $N_i + N_{ex}$, that is, the Fano-factor is just equal to that in He + (a small amount of Ar, Kr and Xe). In equation (3), the first term casued by the fluctuation of N_i or N_{ex} disappears and the Fano-factor is given only by the sum of the second and the third terms. The scintillation from liquid argon or xenon excited by relativistic heavy ions which are lighter than gold ions is considered to be just this case.[16] So, if we can detect these photons with 100 % efficiency, the Fano-factor should be ~ 0.05, as seen from the table. If liquid xenon doped with a small amount(several ten ppm) of TMA(trimethylamine) is used as detector medium for detection of photons from liquid argon or xenon,the photons can be converted to ion-pairs with an efficiency of almost 100 %.[17] Thus, we can almost perfectly detect the photons emitted in liquid xenon by using liquid xenon doped with TMA and in such a detector the Fano-factor of ~ 0.05 might be realized.

Next, let us consider the Fano-factor in a semiconductor or an

insulator. The energy loss process of an ionizing particle passing through a semiconductor or an insulator crystal includes the electron-hole pair production and the collision with the crystal lattice. There are two kinds of collisions with the crystal lattice: one occurs between successive ionization collisions and the other occurs after the energies of the secondary electrons or holes produced by the ionizing radiation drop below the gap energy of the crystal. Using the "Shockley model",[18] the Fano-factor is given by (3),

$$F_1 = (r \cdot E_R/W_1)^2 + \delta \varepsilon_1^2/W_1^2 \qquad (4),$$

where r is the number of collisions with crystal lattice between a successive ionization collision, and E_R is the average energy loss per collision with lattice, which is assumed to be approximately the Raman energy. The first term corresponds to the first term of equation (3) and the second term to the second and third terms of equation (3). Using (4), we can explain the Fano-factor of ~ 0.05 in germanium which is the experimentally obtained value,[19] although its theoretical value is model dependent.

At present, the application of superconductors to nuclear radiation detectors is being tried because the average energy required to produce an excess quasiparticle is ~ 10^{-3} (that is, ~ 1 meV) of that needed in germanium detector[20] and the energy resolution(FWHM) is expected to be much better than that of a semiconductor detector. Using the Monte Carlo simulations, Kurakado estimated the Fano-factor in bulk superconducting Sn at 0 K to be 0.195 ± 0.01.[21] This gives the FWHM-value of a few eV for keV X-rays. However, the best resolution experimentally obtained so far is about 20 times larger than the theoretically estimated one,[22] because other contributions to the resolution are still dominant. So , we can not justify Kurakado's simulation by comparison with experiments.

Energy deposited due to ionizing radiation passing through matter finally becomes "heat". If the heat capacity of a material

is very small and its temperature rise can be measured by a high sensitivity thermometer, we can know the energy deposited in the material. This type of detector is called a "micro-calorimeter" or "bolometer". In the case, there is no fluctuation of excitation and ionization energies, and all the energy deposited becomes heat, that is, phonon energy. The limit of the energy resolution in this type of detector will be given by the thermodynamic energy fluctuation in detector. The spontaneous energy fluctuation in the detector is [23]

$$\langle \Delta U^2 \rangle = k_B T^2 C \qquad (5),$$

where K_B is Boltzmann's constant, T is the temperature of the detector and C is the heat capacity of the detector. One can understand this in a handwaving way by saying that the effective number of photon modes in the detector is $N = C/k_B$, the typical phonon mode has quantum occupation number 1, rms fluctuation of 1 photon, and mean energy of $k_B T$. Then, the mean energy square energy fluctuation is $(k_B T)^2 N = k_B T^2 C$. The energy resolution achievable in practice is about twice as large as the value given above. Namely,

$$\Delta E(\mathrm{rms}) = \xi \, (k_B T^2 C)^{1/2} \qquad (6),$$ [23]

where ξ is a parameter depending on thermal characteristics(~ 2). If there is no noise in the thermalization of the X-ray, a resolution better than ~ 1 eV(FWHM) should be possible for a detector operating at 0.1 K. At present, however, the best energy resolution for soft X-rays is still worse than 10 eV [23]. Such a difference between theory and experiment should be investigated.

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Table 1

Gases	F		Ref. No.
He	0.26 ± 0.02	for α -particles	1
	(0.21)	for electrons	2
He + Ar(1%)	0.1	for α -particles	1
He + (Ar, Kr, Xe)	(~0.05)	for electrons	2, 3
Ne	(0.13)		5
Ar	0.20 ± 0.02	for α -particles	1, 6
	(0.16)	for electrons	5, 7
Ar + 10%CH_4	0.18 ± 0.01	for α -particles	8
	0.14	for X-rays	9
Ar + 0.8%CH_4	0.19	for α -particles	10
Ar + 0.8%C_2H_2	0.09	for α -particles	11
Kr	0.19	for X-rays	12
Xe	0.22 ± 0.02	for α -particles	1, 13
	0.15	for X-rays	12
	0.13	for X-rays	14

( ) is the theoretical estimate.