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Nuclear Photoreactions At Intermediate Energies

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Abstract

We review the interaction of real photons with nuclei up to the GeV region. The common microscopic description of exchange effects below threshold and of the corresponding real photoproduction above, is emphasized. The theoretical problems connected with π photoproduction in Δ region and vector meson photoproduction are spelled out and solved. The gross features of the reaction mechanism are shown to explain both the low energy region, the bulk properties around the Δ resonance as well as the appearence of shadowing only above ρ threshold.

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1 Introduction

The possibility of using photons as a probe of the target structure has long been emphasized . Unlike in strong interactions where much is geometrically preordained by the short mean free path of the particle, the weakness of the interaction is indeed of much help in order to gain informations about the interaction mechanism.

In the present paper we review in a schematical way our knowledge of photonuclear reactions at intermediate energy, limiting ourselves to real photons.

We have first of all to define what we mean by intermediate energy.

In this connection let us recall that in the course of time the "nuclear" domain has gradually shifted to higher and higher energies. Indeed the now low lying giant dipole resonance provided the first evidence of a strong nuclear photoeffect, with an integrated cross section of the order of one classical sum rule [1], scaled by the ratio of the nucleon to the electron mass with respect to the atomic case. Its theoretical interpretation was given in 1948 by Goldhaber and Teller [2].

Subsequently it was realized by Bethe and Levinger [3] that an important modification to the electric dipole sum rule with respect to the atomic case was due to the presence of exchange potentials in nuclear physics. The most relevant part consisted of the well known Yukawa [4] one pion.

It was hence natural to regard the pion threshold as the upper limit of nuclear physics.

Finally Gell-Mann, Goldberger and Thirring [5] by applying dispersion relations to nuclei and by making an assumption about the asymptotic behaviour of the amplitude, were able to connect the total nuclear photoabsorption cross section below the pion threshold with a corresponding depletion (with respect to that of an incoherent assembly of nucleons) above.

In spite of the criticism as regards this assumption about asymptotia (see next chapter) this is of course our point of view.

Because of causality, virtual and real particles are intimately connected.

A consistent treatment necessarily cannot be restricted to the conventional low energy domain. Nuclear physics means physics on the nucleus at any energy. In this connection it is worth stressing that the new "discovery" known as EMC effect [6] that even at very high energies and momentum transfers the nucleus is not just made up of free nucleons was a well established fact, already known in photoabsorption some twenty years ago as "shadowing".

Of course to understand and disentangle the effects peculiar to nuclear physics a correct starting point, i.e. the elementary photoproduction amplitude in the various energy regions, is needed.

We will consider in this review laboratory photon energies ω up to the GeV region i.e.

from threshold to above ρ photoproduction as shown in Fig.1. The problems connected with the elementary photoabsorption process in the low energy, Δ and ρ region will be pointed out, discussed and, as much as possible, solved.

In particular the microscopic interpretation of exchange effects below pion threshold as virtual meson photoproduction and their connection with real photoproduction will be stressed. In the Δ region the much talked about problem of a unitary elementary amplitude and of its frame transformation will be addressed and solved, obtaining a formulation, in terms of physical quantities, appropriate for nuclear applications.

Finally in the ρ region the difficulties connected with the treatment of massive vector mesons will be recalled and the correct ρ photoproduction amplitude will be derived.

Such an amplitude has still to be implemented in nuclei.

As concerns nuclear effects we will not make a compilation of nuclear cases.

We will try to present in a schematic way how the physics of an interacting nucleon is different from that of a free one.

In this respect our treatment will not overlap with existing reviews [8,9,10,11,12]. In some of them finer details, regarding particular aspects, are extensively examined.

As regards the general level we have tried to make the present work as self contained as possible and accessible as regards the physical arguments, if not the details, to non specialists as well .

Its plan is as follows :

In chapter 2 generalities about the nuclear Compton scattering amplitude are spelled out so as to provide the framework for the treatment of photoreactions.

The analytic properties of the amplitude are briefly discussed in connection with dispersion relations, sum rules and low energy theorems.

Chapter 3 deals with the low energy region. In particular in §3.1 isovector exchange effects originating from the isovector π and ρ exchange potentials are recalled whereas in §3.2 isoscalar exchange effects due to the σ and ω are shown to appear on the same footing and their role is discussed. Finally in §3.3 the typical nuclear features of the giant dipole resonance in spherical and in deformed nuclei are reviewed.

Chapter 4 is concerned with the Δ region . In §4.1 the unitary and frame independent pion photoproduction amplitude, (where only $(3,3)$ final state interactions are considered) is presented. Its comparison with the Chew-Goldberger-Low-Nambou amplitude in the nucleon case clarifies the role of the rescattering mechanism and makes it possible to conclude that the Δ is an elementary (quark spin-flip) particle, thus confirming the preferential role of photons in probing the target structure.

As regards nuclei a simplified version (Δ dominance) is used to derive sum rules for the $M1$ part and to predict the total photoabsorption cross section on medium heavy nuclei exemplified by ^{208}Pb . In §4.4 the physical content of the quasi-deuteron model in photoabsorption is questioned and experimental tests of "three-body forces" causing the in-medium increase of the Δ width are suggested. Finally in §4.5, always in the Δ dominance language, elastic vs. inelastic Compton scattering is discussed and the related dispersive contributions of the Δ to the nuclear magnetic susceptibility are analyzed and compared with low energy nuclear effects.

Chapter 5 is concerned with the high energy domain. In §5.1 the photoproduction amplitude for two uncorrelated pions is recalled, whereas §5.2 deals with ρ photoproduction. It is shown that the requirement of invariance of the theory under local gauge transformations of the $SU(2) \times U(1)$ (isospin $\times$ hypercharge) group necessitates a Higgs mechanism to generate the ρ masses.

In such a scheme which intrinsically relies e.m. and strong interactions, linking part of the NN potential to the electromagnetic properties of hadrons, the vector meson dominance prescriptions automatically result, universality is explained and the desired ρ photoproduction amplitude (which differs from that obtained via the minimal e.m. substitution in the Proca Lagrangian) is obtained. In addition "genuine" three-body $\rho\rho\rho$ forces (as well as four-body ones) originating from the non abelian structure of the group are predicted.

As regards shadowing, i.e. the non linear behaviour in the atomic number A of the total photoabsorption cross section, treated in §5.3, one gets from the previous framework that a real photon is completely decoupled from ρs.

Therefore the appearance or not of shadowing is determined solely by the photoproduction mechanism relevant at that energy.

In particular shadowing is predicted not to happen in the yet unmeasured uncorrelated $\pi\pi$ region. Finally in §6 conclusions are drawn.

2 The Structure of the Nuclear Compton Scattering Amplitude

2.1 Generalities

In this chapter we will recall the formalism of the Compton scattering amplitude, in order to set up a convenient general framework for the discussion of photoreactions in the real photon case.

We can roughly summarize the following discussion, which may appear a bit too formal, simply by reminding that gauge invariance, especially in the interacting case (nucleons in a nucleus) is a very powerful tool to comply , thank to appropriate conterterms, with low energy theorems. In essence exchange effects cannot be introduced in a cavalier way only in the current. The fact that such counterterms naturally derive from dispersive effects when the Hilbert space is not arbitrarily restricted to conventional (low energy) nuclear physics, as in this § and in § 3.1 will be touched upon in § 4.2 and 4.5.

The S matrix for Compton scattering off a nucleus in an initial state $|0>$ to a final state $|f>$, of a photon of initial four momentum $k_\mu = (\omega, \vec{k})$ and polarization $\epsilon_\mu = (0, \vec{\epsilon})$ and final $k'_\nu \equiv (\omega, \vec{k}')\epsilon'_\nu \equiv (0, \vec{\epsilon}')$, according to standard reduction techniques, reads to $0(e^2)$ [13,14,15]

$$
\begin{aligned}
S_{fi} &= -i(2\pi)^4\delta^4(p_i + k - p_k - k')\varepsilon_\mu\varepsilon'_\nu T^{\mu\nu} = \\
&= 2\pi e^2 \varepsilon_\mu \varepsilon'_\nu \int d^4x d^4y e^{+ik.x} e^{-ik'.y} < N^*|T\{j^\mu(x) j^\nu(y)\}|0> \\
&= 2\pi e^2 \delta(E_0 + \hbar\omega - E_{N^*} - \hbar\omega')\varepsilon_\mu\varepsilon'_\nu \int d\vec{x} d\vec{y} e^{-i\vec{k}.\vec{x}} e^{+i\vec{k}'.\vec{y}} \\
&\cdot \sum_n \{ \frac{< f|j^\nu(0,\vec{y})|n>< n|j^\mu(0,\vec{x})|0>}{E_0 - E_n + \hbar\omega + i\Gamma_n/2} + \\
&+ \frac{< f|j^\mu(0,\vec{x})|n>< n|j^\nu(0,\vec{y})|0>}{E_0 - E_n + \hbar\omega'} \}
\end{aligned}
\tag{1}
$$

where T stands for the time ordered product and $E_n - i\Gamma_n/2$ for the complex energy of the intermediate state.

Here $j_\mu(x) \equiv (j_0(x), \vec{j}(x))$ is the full e.m. current operator satisfying the gauge condition

$$\partial_\mu j^\mu(x) = 0 \ , \tag{2}$$

and the sum $\sum_n$ runs over a complete set of states. It is, however, easy to prove that, despite eq.(2) $T_{\mu\nu}$, in principle, may not be gauge invariant. As a matter of fact :

$$
\begin{aligned}
& k'_\nu k_\mu \int d^4x d^4y \exp(ikx)\exp(-ik'y) < f|T(j^\mu(x), j^\nu(y))|0> \\
& = \int d^4x \int d^4y \exp(ikx)\exp(-ik'y)\delta(x_0 - y_0) < f|[j_0(y),[H, j_0(x)]]|0>
\end{aligned}
\tag{3}
$$

whence a counterterm $S^{\mu\nu}$ must be added to $T^{\mu\nu}$ such that

$$k_\mu k'_\nu [T^{\mu\nu} + S^{\mu\nu}] = 0 \; . \tag{4}$$

Eq.(4) determines $S_{\mu\nu}$ uniquely up to terms of first order in k, k'. The total amplitude in the radiation gauge is therefore [1] :

$$f \sim \epsilon_l \epsilon'_m \{T_{lm} + S_{lm}\} \; . \tag{5}$$

We remark that the value of the double commutator in eq.(3) depends on the explicit form of $J_\mu(x)$ and H. In particular it is different from zero when a non-relativistic limit is used for both. Since with this choice $J_\mu(x)$ has matrix elements only between positive energy states, this corresponds to a truncation in the sum of eq.(1) and as a consequence to a loss of its gauge invariance.

In the atomic case this can be regarded just as an alternative formulation, since the above procedure immediately yields the e^2 contact (seagull) term of the well known interaction Hamiltonian

$$H_{int} = \sum_{i=1}^{A} \frac{(1+\tau_i^3)}{2} \left[\frac{e}{2m} \vec{p}_i . \vec{A}(x_i) + \frac{e}{2m} \vec{A}(x_i) . \vec{p}_i - \frac{e^2}{2m} \vec{A}(x_i) . \vec{A}(x_i) \right] \tag{6}$$

Of course with respect to that, the peculiar feature of nuclear physics lies in the existence of exchange potentials, so as to violate current conservation eq.(2) if the current is simply written as the sum of non interacting nucleonic currents

The current is consequently modified and so is the seagull term, and in this connection the previous approach will prove useful.

Low energy theorems however stipulate that irrespective of the details of the interaction (and here we stress again that exchange currents originate from the standard nuclear physics approach of expressing everything in terms of nucleonic coordinates only) the general form of the low energy elastic amplitude is completely determined by the total charge Z (in the following N and A will denote the neutron and atomic number) and by the anomalous magnetic moment κ.

To the first order in ω, then [16,17]

$$\lim_{\omega \to 0} f(\omega, \theta) = -\frac{Z^2 e^2}{MA} \vec{\epsilon} . \vec{\epsilon}\,' + \frac{\kappa^2}{2M^2} e^2 \vec{S} . \vec{\epsilon} \times \vec{\epsilon}\,' \omega \tag{7}$$

$\vec{S}$ being the total spin of the system and $e^2 = \alpha = \frac{1}{137}$.

[1] Of course state normalization and hence phase space factor determines the proportionality coefficient. This will be taken care of, when giving explicit formulas in the following

The amplitude f is at all energies connected to the differential cross section in the laboratory frame by

$$\frac{d\sigma}{d\Omega} = \frac{\omega'}{\omega}|f^2| \tag{8}$$

where ω and ω' are also expressed in the lab. and connected by

$$\omega' = \frac{\omega - \Delta}{1 + \omega(1 - \cos\theta)/MA} \qquad \Delta = \frac{(E_{f_0})^2}{2MA} \tag{9}$$

and $\theta = \hat{k}.\hat{k}'$, in such a way that, in the elastic case ($E_{f_0} = 0$) ω' can be practically taken equal to ω at all angles (of course in the inelastic case, a corresponding inelastic amplitude will intervene).

2.2 Analytic properties of the amplitude

The previous form of the amplitude remains valid at all energies for the forward scattering amplitude $f(\omega) \equiv f(\omega, \theta = 0)$.
Hence, always in the laboratory system

$$f(\omega) = f_1(\omega)\vec{\epsilon}'.\vec{\epsilon} + if_2(\omega)\omega\vec{S}.\vec{\epsilon} \times \vec{\epsilon}' \tag{10}$$

Clearly, by averaging over nuclear spins in the amplitude we are left only with f_1, which is therefore denoted as the spin averaged amplitude. These amplitudes are separable if we can perform experiments either with both initial and final polarized photons or with one polarized photon (say the initial one) and with a polarized target.

In the first case f_1 is clearly selected when the two polarizations vectors are parallel.

In the second case, if the nuclear spin is aligned along (i.e parallel) the beam direction or antiparallel to the photon helicity λ (since $i\vec{S} \times \vec{\epsilon}^\lambda = \lambda S\vec{\epsilon}^\lambda$) we have

$$\begin{aligned} f_p(\omega) &= f_1(\omega) - f_2(\omega)\omega S \\ f_a(\omega) &= f_1(\omega) + f_2(\omega)\omega S \end{aligned} \tag{11}$$

where p = parallel and a = antiparallel.

The optical theorem for any of the four amplitudes stipulates

$$\Im m f(\omega) = \frac{\omega}{4\pi}\sigma(\omega) \tag{12}$$

As usual, postulating analiticity of the amplitude, we can calculate their real parts if we know their imaginary parts by means of dispersion relations. They read

$$\Re e f_1(\omega) - \Re e f_1(0) = \frac{\omega^2}{2\pi^2} P.V. \int_{\omega_0}^{\infty} \frac{d\omega'}{\omega'^2 - \omega^2}\sigma_T(\omega') \tag{13}$$

$$\Re e f_2(0) = \frac{1}{2\pi^2}\int_{\omega_0}^{\infty}\frac{d\omega'}{\omega'}[\sigma_p(\omega') - \sigma_a(\omega')] \tag{14}$$

where P.V. stands for principal value, and where $\sigma_T = \frac{1}{2}(\sigma_a + \sigma_p)$ represents the spin averaged total photoabsorption cross section and ω_0 the threshold energy for photoabsorption (equal to $m_\pi + m_\pi^2/2M$ in the nucleon case and to 0 for nuclei). The basic difference in the derivation of the previous relations lies in the fact that a subtraction is needed for f_1 (otherwise from the unsubtracted form $\Re e f_1(\omega) = \frac{1}{\pi}P.V.\int \frac{\Im m f_1(\omega')}{\omega' - \omega}d\omega'$, apart from convergence problems, a contradiction would result at $\omega = 0$, the r.h.s. representing the integral of the positive photoabsorption cross section, the l.h.s. being the negative Thomson limit) whereas no such a subtraction is needed for f_2.

In their first classical application [5], dispersion relations were applied to the difference between the nuclear amplitude and the incoherent sum over nucleons, $\Delta f(\omega) = f(\omega) - Zf_p(\omega) - Nf_n(\omega)$. The assumption $\Re e\Delta f(\infty) = 0$, together with $\lim_{\omega\to\infty}\Delta\sigma = 0$ sufficiently fast to allow for convergence, leads to (since $\omega_0 = m_\pi$ for the nucleonic cross section)

$$\int_0^{m_\pi}\sigma(\omega)d\omega - 2\pi^2 e^2\frac{ZN}{MA} = -\int_{m_\pi}^{\infty}\Delta\sigma(\omega)d\omega \tag{15}$$

This sum rule connects the enhancement due to exchange effects below the pion threshold to the difference between incoherent nucleons and total nuclear cross sections.

Because of the discussion of § 5, the previous hypothesis is however untenable, the very difference between nucleonic and nuclear photoabsorption lying in the possibility of interference irrespective e.g. of the existence of the ρ meson. Conclusions from Eq.(15) are still qualitatively correct.

It is however obvious that the only unbiased use of dispersion relations for a nucleus is a direct one as embodied by Eq.(13). They can then be used as a consistency check only, since $\Re e f(\omega)$ cannot be predicted or obtained experimentally (in the forward direction Delbrück scattering dominates by far). It may be argued [18,19] that at $\omega = \bar{\omega} \simeq m_\pi$ $\Re e f(\omega) = 0$ i.e. that the pion threshold has a sort of a universal meaning.

Using the experimental data for σ, this seems indeed to be the case as shown in Fig.2 for Be, and as checked over the whole periodic table by Ahrens [20]. This entails

$$2\pi^2\frac{Z^2e^2}{MA} = \quad P.V.\int_0^{\infty}\frac{\omega^2}{\omega'^2 - \bar{\omega}^2}\sigma(\omega')d\omega' \tag{16}$$

which can also be interpreted as if, by a proper weighting, the cross section above threshold (hence real pions) would correct the incoherent cross section by an amount which is strictly connected to the degree to which exchange effects influence absorption below threshold.

On the other hand in Eq.(14) it is assumed that the photoabsorption cross sections become spin independent so as to assure the convergence of the dispersive integral (and the disappearance of $-\Re e f_2(\infty)$ which is in principle present in the r.h.s.). It is obvious from the preceding equations that, under the previous hypothesis, a sum rule can be immediatly written down for f_2, by using the low energy prediction that $f_2(0)$ is proportional to the square of the anomalous magnetic moment of the system. One obtains

$$\alpha S(\frac{\mu}{S} - \frac{Z}{M})^2 = \frac{1}{4\pi^2} \int_{\omega_0}^{\infty} \frac{d\omega'}{\omega'} [\sigma_p(\omega') - \sigma_a(\omega')] \tag{17}$$

μ representing the total magnetic moment of the system of total mass MA and maximum spin component S. This represents the celebrated Drell-Hearn-Gerasimov [21,22] sum rule. Notice that the dispersive integral in (14) and of course in (17), in contradistinction with the one entering dispersion relations for f_1 is not positive definite. The interpretation of Eq.(17) is that the photon coupling in photoproduction is connected to and hence affects the nucleon magnetic moment. In contrast with Drell and Hearn, Gerasimov applied the sum rule to the nuclear case [22], to predict modifications of the nucleon magnetic moment by taking into account nuclear structure only through the Pauli principle incorporated in the Fermi gas model.

Since the Pauli principle obviously leads to a decrease in the cross section for the photoproduction of mesons by bound nucleons, this would result in a corresponding damping of the g factor of the bound nucleon. A decrease by 7-8% seemed in fair agreement with an alternative estimate of the isovector part of the anomalous magnetic moment obtained by Drell and Walecka [23] by summing up the most relevant Feynman diagrams.

This appealingly simple result was however marred by a sign mistake.

In a more thorough scrutiny it was realized [24], as confirmed by later isospin analyzes of the sum rule [25] that already on the nucleon the single pion photo-production region does not exhaust the sum rule, and that substantial contributions must come from the region $\omega > 1$ GeV where data were absent and where the theoretical analysis is not free from uncertainties.

Moreover, the low energy domain yields in the nuclear case the additional contribution

$$\int_0 \frac{\Delta\sigma_{M_1}(\omega)}{\omega} d\omega \simeq < J_Z = J | [\hat{M}_x, \hat{M}_y] | J_Z = J > \tag{18}$$

obtained in the long wavelength limit for the magnetic moment operator. In addition, to make reliable predictions, one should have a consistent treatment of nuclear effects both below and above pion threshold which represents indeed a formidable task.

In conclusion, although appealing, the D-H-G sum rule when applied to nuclei can hardly tell even the sign of the modification of the nucleon anomalous magnetic moment.

Also on the experimental side, polarized photons and targets are not presently avalaible in the whole energy domain of interest for the r.h.s. of Eqs.(14),(17).

As regards the dispersion relation for f_1, it can yield a sum rule only with an independent piece of information about its real part.

Hence, let alone the problem of the good causal properties of a non relativistic amplitude [26,27], which in principle question the very applicability of dispersion relations, we see that their predictive content in the nuclear case is indeed limited.

They can be viewed therefore as a sort of rough consistency constraint for our theoretical treatment in the various energy regions.

Their use to connect low and high energy properties rests on the knowledge of the photoproduction mechanism of all possible mesons.

Luckily, because of the $1/\omega$ and $1/\omega^2$ factors in the r.h.s., the relevant domain practically extends only up to the ρ photoproduction region.

The problem is further simplified by the fact that of all possible N and Δ resonances, only the $\Delta(1241)$ plays a significant role in this whole energy range. Details will be given in the subsequent sections. However the Compton scattering formalism outlined above allows the direct (i.e. without having to resort to dispersion relations) calculation of all the quantities entering the scattering amplitude.

One such example will be given in §4.5 by the calculation of the magnetic susceptibility.

In the following we will limit ourselves to the spin independent amplitude f_1 (which we will loosely denoted by f).

3 The low energy region

3.1 Exchange currents, sum rules, polarizabilities.

In this energy domain, our states $|n>$ correspond to " genuine" nuclear states i.e. they contain only positive energy nucleons.

In such a case the electromagnetic current $J_\mu \equiv (J_0, \vec{J}_0)$ reads to the lowest order in $\frac{p}{M}$ in the impulse approximation

$$
\begin{aligned}
J_0(\vec{x}) &= \sum_i \frac{1+\tau_i^3}{2}\delta(\vec{x}-\vec{x}_i) \\
\vec{J}_0(\vec{x}) &= \sum_i \frac{1+\tau_i^3}{2}\left[\frac{\vec{p}_i}{2M}\delta(\vec{x}-\vec{x}_i)+\delta(\vec{x}-\vec{x}_i)\frac{\vec{p}_i}{2M}\right] \\
&+\frac{ie}{2M}\sum_i\left[\mu_p\frac{1+\tau_i^3}{2}+\mu_n\frac{1+\tau_i^3}{2}\right]\vec{\sigma}_i\times\vec{k}\delta(\vec{x}-\vec{x}_i)
\end{aligned}
\tag{19}
$$

where τ_i^3 stands for the third component of the isospin-operator, $\vec{p}_i$ and M are the i^{th}-nucleon momentum and its mass and μ and σ_i are the magnetic moment and the i^{th}-nucleon spin, respectively and where the subscript in the three current is to indicate the impulse approximation.

By assuming no change in J_0 (since on the nucleon corrections to the charge are of $0(\frac{p^2}{M^2})$) and by taking $H = H_0 + V^{ex}$ where H_0 stands for the kinetic plus central part of the nuclear Hamiltonian and V^{ex} the spin isospin dependent part, $\partial_\mu J^\mu = 0$ reads in momentum space

$$
\begin{aligned}
\vec{k}.\vec{J}_0(\vec{k}) &= [H_0, J_0(\vec{k})] \\
\vec{k}.\Delta\vec{J}(\vec{k}) &= [V^{ex}, J_0(\vec{k})]
\end{aligned}
\tag{20}
$$

which determines to $0(\omega^0)$ the correction to the longitudinal part of the current.

For example, in the long wave length limit, in the Coulomb gauge the total current reads

$$
\vec{J}(\vec{k}) = \sum_i \frac{1+\tau_i^3}{2}\frac{\vec{p}_i}{M} + \sum_{i\neq j} i(\vec{\tau}_i\times\vec{\tau}_j)^3(\vec{x}_i-\vec{x}_j)V_{ij}^{ex}(x) \tag{21}
$$

where $\vec{x} = \vec{x}_i - \vec{x}_j$ and where the isospin structure $\vec{\tau}_i.\vec{\tau}_j$ of the potential has been separated out.

Exchange effects originate from the fact that the isovector part of the charge density does not commute with the potential.

They are therefore *isovector* and correspond to the fact that, since a nucleon can emit charged mesons (reabsorbed by other nucleons in the nucleus), *the nucleonic charge is locally not conserved.*

Of course non commutativity can arise also because of the momentum dependence of the potential. This happens for the *isoscalar* ω and σ exchange, through which one customarily parametrizes the short range part of the nucleon potential. They will be considered in the next section.

We will take here as reasonable representatives of $V^{ex}, V_\pi + V_\rho$ i.e. the long range one pion plus the intermediate ranger rho exchange potential

$$\begin{aligned} V_\pi(q) &= -\frac{f^2_{\pi NN}}{m^2_\pi}\frac{\vec{\sigma}_i.\vec{q}\,\vec{\sigma}_j.\vec{q}}{\vec{q}^2+m^2_\pi} \\ V_\rho(q) &= -\frac{f^2_{\rho NN}}{m^2_\rho}\frac{\vec{\sigma}_i\times\vec{q}.\vec{\sigma}_j\times\vec{q}}{\vec{q}^2+m^2_\rho} \end{aligned} \tag{22}$$

where $\frac{f_{\pi NN}}{m_\pi} = \frac{g_{\pi NN}}{2M}, \frac{f^2_{\pi NN}}{4\pi} \simeq 0.08$ and where $\frac{f^2_{\rho NN}}{4\pi} \sim 4.5 \div 5$. Only the total magnetic coupling (coming predominantly from the ρNN tensor vertex) has been considered in the rho case.

Notice that the rho potential comes as a N.R. reduction of the corresponding ρ exchange Feymann diagram where only the $g_{\mu\nu}$ part of the spin 1 propagator contributes.

Then

$$\begin{aligned} \vec{\epsilon}.\vec{x}V^{ex}_{ij}(x) &= i\int\frac{d^3q}{(2\pi)^3}e^{-i\vec{q}.\vec{x}}\epsilon_\alpha\frac{\partial}{\partial q_\alpha}V(q) \\ &= i\int\frac{d^3q}{(2\pi)^3}e^{-i\vec{q}.\vec{x}}\frac{f^2_{\pi NN}}{m^2_\pi}[\frac{\vec{\sigma}_i.\vec{q}\,\vec{\sigma}_j.\vec{\epsilon}+\vec{\sigma}_i.\vec{\epsilon}\,\vec{\sigma}_j.\vec{q}}{\vec{q}^2+m^2_\pi}] \\ &-\frac{f^2_{\rho NN}}{m^2_\rho}[\frac{\vec{\sigma}_i\times\vec{\epsilon}.\vec{\sigma}_j\times\vec{q}+\vec{\sigma}_i\times\vec{q}.\vec{\sigma}_j\times\vec{\epsilon}}{\vec{q}^2+m^2_\rho}-\vec{\sigma}_i\times\vec{q}.\frac{2\vec{q}.\vec{\epsilon}}{(\vec{q}^2+m^2_\rho)^2}\vec{\sigma}_j\times\vec{q}] \end{aligned} \tag{23}$$

The two terms have the immediate physical interpretation of Fig.3. Exchange effects originate from virtual charged pion [28] and rho electric photoproduction [29]. *Notice that the gauge condition automatically generates in both amplitudes the contact terms, coming fron a contact elementary interaction and or from $N\bar{N}$ intermediate states, although only positive energy states enter explicitly the non relativistic treatment.*

Neutral pions and rhos do not intervene since they cannot be photoproduced in the long wave length ($\omega \to 0$) limit.

The interpretation of Eqs.(20), (23) is straightforward especially if reexpressed, in the same long wavelength limit via the Siegert theorem [30]

$$e\vec{J} = i[H, \vec{D}] \tag{24}$$

where $\vec{D} = \sum\frac{1+\tau^3_i}{2}e(\vec{x}_i - \sum_j\frac{\vec{x}_j}{A})$ is the dipole operator (referred to the c.m.s.).

It is then obvious that in addition to dipole absorption off protons corresponding to the first term of Eq.(21) (since $\frac{e\vec{p}}{M} = e\vec{v} = \frac{d}{dt}e\vec{x}$), photons can be additionally absorbed because of the dipole moment of an $n-p$ pair (interacting via an isospin dependent exchange potential).

It is worth pointing out how the absence of a dipole moment for a pp and an nn couple (Eqs.(21), (24)) because of the isospin factor, is accompanied in the other picture (Eq. 23) by the absence of a neutral particle photoproduction.

In general (no long wavelength limit) an explicit expression for the exchange current obeying current conservation has been built for the pion in terms of the photoproduction amplitude [31].

Refinements to the previous treatment, due to center of mass effects in Siegert theorem [32], or the extra contributions to the current due to exchange modifications of J_0 [33] or the non static treatment of the current [34] (see however § 4.4) will not be considered.

From the previous expressions the $E1$ part of the scattering amplitude reads therefore in the long wavelength limit

$$\begin{aligned} f_{E1}(\omega,\theta) &= E_{n_0}^2 \frac{<0|\vec{D}.\vec{\epsilon}\,'|n> \sum_n <n|\vec{D}.\vec{\epsilon}|0>}{E_{n_0} - i\frac{\Gamma_n}{2} - \omega} + crossed \\ &- \frac{Ze^2}{M}\vec{\epsilon}.\vec{\epsilon}\,' + exch.seagull \end{aligned} \tag{25}$$

where the expression "crossed" in Eq.(25) denotes the so called crossed term which results from exchanging $\vec{\epsilon}$, $-\vec{k} \to \vec{\epsilon}\,', \vec{k}\,'$ and $-\omega \to +\omega$ in the first expression on the r.h.s.. The seagull term depends upon the form of the current and of the intermediate state $|n>$ and will be commented upon at length later on.

It is then immediate to obtain

$$\begin{aligned} \sigma_{E1}(\omega) = \frac{4\pi}{\omega}\Im m f_{E1}(\omega) = & \frac{4\pi}{\omega}\frac{\sum_\lambda}{2} <0|\vec{D}.\vec{\epsilon}^*|n> \sum_n <n|\vec{D}.\vec{\epsilon}|0> \\ & E_{n_0}^2 \frac{\Gamma_{n/2}}{(E_{n_0}-\omega)^2 + (\frac{\Gamma_n}{2})^2} \end{aligned} \tag{26}$$

and by the replacement $\frac{\Gamma/2}{(E_{n_0}-\omega)^2+(\Gamma_n/2)^2} \to \pi\delta(E_{n_0}-\omega)$ the corresponding Bethe-Levinger sum rule

$$\begin{aligned} \int d\omega\sigma_{E1}(\omega) &= 2\pi^2\frac{\sum_\lambda}{2} <0| \left[[\vec{D}.\vec{\epsilon}^*, H]\vec{D}.\vec{\epsilon}\right] |0> \\ &= 2\pi^2 e^2 \frac{ZN}{MA}(1+\kappa) \\ &= 2\pi^2\alpha\frac{ZN}{MA}(1 + \frac{\sum_\lambda}{2} <0|\sum_{ij}(\tau_i^+\tau_j^- + \tau_i^-\tau_j^+)\vec{\epsilon}.\vec{x}\vec{\epsilon}^*.\vec{x}V_{ij}^{ex}(x)|0>) \end{aligned} \tag{27}$$

The first term of Eq.(27) represents the famous "classical sum rule" i.e. the model independent contribution in the absence of exchange forces. It is traditionally measured in units of $\frac{2\pi^2\alpha}{M} = 60$ MeV mb.

Its main difference with respect to the atomic case (apart from the trivial nucleon-electron mass substitution) lies in the center of mass effect i.e. in that in the nuclear case roughly $(Z/4)$ (to 0 $(N-Z)/(A)$) protons contribute instead of Z in the atomic case.

In the exchange term polarizations have been retained (for spherical nuclei $\vec{\epsilon}.\vec{x}\vec{\epsilon}'.\vec{x} = \frac{1}{3}\vec{\epsilon}.\vec{\epsilon}\,'x^2$) to allow for the interpretation of Fig.4 i.e. virtual photoproduction at both vertices plus a genuine seagull (i.e. the two photons at the same point) off the exchanged particle [35,37,29].

The factor κ which represents the enhancement of the classical sum rule reads in our notation $\kappa_\pi + \kappa_\rho$. It is manifestly positive (for spin saturated nuclear matter) for an attractive potential such as the central part of both the pion and the ρ. On the other hand, since $\vec{\sigma}_1.\hat{q}\vec{\sigma}_2.\hat{q} = \frac{1}{3}\vec{\sigma}_1\vec{\sigma}_2 + \frac{1}{3}S_{12}(\hat{q})$ whereas $(\vec{\sigma}_1 \times \hat{q}).(\vec{\sigma}_2 \times \hat{q}) = \frac{2}{3}\vec{\sigma}_1.\vec{\sigma}_2 - \frac{1}{3}S_{12}(\hat{q})$ where $S_{12}(\hat{q})$ is the usual tensor operator, there is a partial cancellation of the pion tensor term, whose role has been greatly emphasized by Brown and co workers [39], due to the negative rho contribution [40].

Experimentally, the integrated cross section up to the pion threshold, with which people generally tend to compare the calculation of the double commutator, is of the order of 2 classical sum rules, for all nuclei [41,42].

Such a comparison is of course questionable no matter how good is the phenomenological potential one uses [43]. The previous use of a microscopical potential makes it quite clear. Whereas in the π case, although the amount of κ_π above pion threshold is of course not under control, one may argue that this assumption is not so unreasonable, in the ρ case this is completely meaningless.

On the other hand it is straightforward to get

$$\Re e f(0,\theta) = -\frac{Z^2e^2}{MA}\vec{\epsilon}\,'.\vec{\epsilon} + \frac{ZN}{MA}e^2\kappa + exch.seagull \tag{28}$$

Hence the exchange seagull, necessary to yield the Thomson limit demanded by low energy theorems for the amplitude, is represented by (minus) the same enhancement factor of the dipole sum rule.

Its interpretation in terms of a genuine seagull off the charged exchanged particle and of the square of a photoproduction amplitude suggests [19] that the latter term comes from the low energy limit of dispersive effects (i.e. from the real part of the amplitude where mesonic degrees of freedom are explicitely allowed).[2] This overcomes in principle

[2]This means that the intermediate states $|n>$ now consists of nucleons plus pions. The current has

the problems connected with the extrapolation of such a counterterm to higher energies. This will be discussed in § 4.

Coming in the same long wavelength limit to the magnetic part we have

$$
\begin{aligned}
f_{M1}(\omega,\theta) &= \sum_n \frac{< 0|\vec{M}1.\vec{k}' \times \vec{\epsilon}'|n >< n|\vec{M}1.\vec{k} \times \vec{\epsilon}|0 >}{E_{n0} - \omega} \\
&+ \text{crossed}
\end{aligned} \tag{29}
$$

where $M1$, by neglecting exchange effects, is given by

$$
\vec{M}1 = \frac{e}{2M} \sum_i \left[\frac{1+\tau_i^3}{2} \vec{l}_i + \mu_p \frac{1+\tau_i^3}{2} \vec{\sigma}_i + \mu_n \frac{1-\tau_i^3}{2} \vec{\sigma}_i \right] \tag{30}
$$

with $\vec{l}$ being the orbital angular momentum operator. Therefore, from the optical theorem (by averaging over polarization)

$$
\begin{aligned}
\sigma_{M1}(\omega) &= \frac{4\pi}{\omega} \Im m f_{M1}(\omega, \theta = 0) \\
&= \frac{4\pi^2}{\omega} \sum_n | < n|\vec{M}1.\vec{k} \times \vec{\epsilon}|0 > |^2 \delta(E_{n0} - \omega).
\end{aligned} \tag{31}
$$

Whence remembering that $\vec{k} = \omega \hat{k}$ one obtains immediately the Kurath sum rule [44,45]

$$
\begin{aligned}
\int \sigma_{M1}(\omega) d\omega &= 4\pi^2 \textstyle\sum_n E_{n0} | < n|\vec{M}1.\hat{k} \times \vec{\epsilon}|0 > |^2 \\
&= \frac{2}{3}\pi^2 < 0|[[\vec{M}1, H], \vec{M}1]|0 >,
\end{aligned} \tag{32}
$$

Elementary considerations allow us to understand how the electric sum rule is by far more important than the magnetic one, the former being in any case dominated by the kinetic term in H, thus yielding one classical sum rule as compared to the much smaller spin orbit contribution in the latter. This is the reason why we have not considered the (even smaller) exchange terms for $M1$, apart from the discussion of next paragraph in connection with relativistic effects. On the other hand we have in the limit of $\omega \to 0$ the following expression for the Compton amplitude

$$
f_{M1}(\omega \to 0, \theta) = (\hat{k} \times \vec{\epsilon}).(\hat{k}' \times \vec{\epsilon}')\omega^2 \chi \tag{33}
$$

where the paramagnetic susceptibility is defined as

$$
\chi = \frac{2}{3} \sum_n \frac{| < n|\vec{M}1|0 > |^2}{E_n - E_0} \, . \tag{34}
$$

consequently different dimensions and $\sum_n$ has an additional integration over the pion momentum. The imaginary part of such an amplitude is connected to pion and rho photoproduction.

Hence one explicitly sees how from the $E1$ operator one gets contributions to $\Re e f(0,\omega)$ of the same (zeroth) order of the corresponding photoabsorption cross section upon which gauge invariance puts a constraint.

On the other hand the behaviour of the real and imaginary part of the amplitude are completely different for the $M1$ case. In contrast to an integrated $M1$ cross section of the same order as the corresponding $E1$, the real part now contributes at low energy to $0(\omega^2)$, representing an unconstrained dynamical feature of the system.

The general expression of the spin independent part of f reads, for $\omega \to 0$

$$\begin{aligned} f(\omega,\theta) &= \vec{\epsilon}\,'.\vec{\epsilon}[-\frac{Z^2e^2}{MA} + \alpha\omega^2] \\ &+(\vec{\epsilon}\,' \times \hat{k}').(\vec{\epsilon} \times \hat{k})\beta\omega^2 \end{aligned} \tag{35}$$

where $\alpha = \alpha_0 + \Delta\alpha_R$, α_0 representing the electric polarizability

$$\alpha_0 = \frac{2}{3}\sum_n \frac{|< n|\vec{D}|0 >|^2}{E_n - E_0} \tag{36}$$

and

$$\Delta\alpha_R = \frac{1}{3}\frac{Z^2e^2}{MA} < 0|\vec{r}^2|0 > \tag{37}$$

whereas $\beta = \chi + \Delta\beta$, χ standing for the paramagnetic susceptibility Eq.(34) and

$$\Delta\beta = -\frac{1}{6}\frac{Ze^2}{M} < 0|\vec{r}^2|0 > -\frac{1}{2MA} < 0|\vec{D}^2|0 > \tag{38}$$

for the diamagnetic susceptibility.

In the previous expressions both the mean-square radius $\vec{r}^2$ and the dipole operator $\vec{D}$ are referred to the center of mass, and exchange effects have been disregarded. The "corrective" terms $\Delta\alpha_R$ and $\Delta\beta$ originate from retardation and sum over excited states.

Notice that retardation corrections to the electric polarizability add, whereas diamagnetic and retardation effects subtract from the paramagnetic susceptibility.

The previous expressions are valid for any system (nucleon, nucleus, atom) with obvious meaning of the symbols.

Polarizatibilities were introduced and discussed in [46,47,48,49], whereas rigurous formulations can be found in [15,50] and reviews in [51,52]. Whereas for atoms $\Delta\alpha_R$ and $\Delta\beta$ are totally negligible, they play a significant role for the proton.

In this case it is easy to understand why, even in the presence of a dominant magnetic Δ excitation, α_0 and χ are comparable. This is due to the fact that the πN continuum, mainly induced by the electric $\vec{\sigma}.\vec{\epsilon}$ operator (see next chapter), peaks at lower energies.

However the structure constants which intervene in low energy Compton scattering are manifestly α and β. Calculations for $\Delta\alpha_R$ and $\Delta\beta$, have been performed in different quark models [53,54,55,56] with result (of less intuitive interpretation than for α_0 and χ) which depend sensitively on the bag radius or on the oscillator parameter.

The general conclusion is that diamagnetic and retardation effects tend to cancel the paramagnetic contribution. Consideration of the vacuum magnetic polarizability is even claimed [57] to possibly bring β to negative values. In any case α, by far, dominates.

As regards nuclei, the main thing to be stressed is that, because of the smallness of $\Delta\alpha$ and $\Delta\beta$ (0 ($\sim$ 5 – 10%)) which implies that practically $\underline{\alpha \sim \alpha_0 \text{ and } \beta = \chi}$, in the subthreshold region we are considering one expects α to be much bigger than χ since the $M1$ sum rule is much smaller than the $E1$. This is substantiated by a number of elaborate calculations of polarizabilities [58,59,60].

The evaluation of dispersive effects coming from explicit pionic degrees of freedom, and in particular the relative rôle of electric and magnetic virtual photoproduction with respect to the conventional low energy contributions, will be considered in next chapter.

Let us finally mention that from Eq.(13) a dispersion relation can be immediately written down for the ω^2 term of f , namely

$$\alpha + \beta = \frac{1}{2\pi^2} \int \frac{d\omega\sigma(\omega)}{\omega^2} \tag{39}$$

It has to be stressed that although the use of dispersion relations is in this case unbiased, still they cannot be used to obtain informations on the physical quantities i.e. on α and β separately.

3.2 Relativistic effects

In the preceding paragraph the standard isovector exchange currents stemming from current conservation Eq.(2,20), generated by the π and ρ isospin dependent potentials have been discussed.

It should be stressed that both potentials essentially arise form the $O(\frac{1}{M^2})$ reduction of the corresponding Feynman diagrams. This results respectively from the off-diagonal $O(\frac{1}{M})$ πNN coupling and from the $O(\frac{1}{M})$ dominant ρNN total magnetic moment. For consistency reasons, also the short and intermediate range isoscalar potentials are to be treated on the same footing, i.e., a non-relativistic reduction has to be performed to the same order.

For definiteness we will keep in the following to the simplified version of the Bonn potential [61], which we prefer to more phenomenological approaches because of its microscopical fundations.

The OBE parametrization of the full Bonn potential contains essentially, in addition to the π and to the ρ, an ω and a σ respectively of $m_\omega = 782$ MeV, $\frac{g_\omega^2}{4\pi} = 20$ and $m_\sigma =$ 550 MeV $\frac{g_\sigma^2}{4\pi} = 8.3$.

These coupling constants correspond to the values at the poles of the exchanged particles. They are multiplied by form factors which reduce these values at $|\vec{q}|^2 = 0$ (relevant for the analysis of the behaviour of low angular momentum partial waves) respectively to 10.6 and 7 and even more at $|\vec{q}|^2 \simeq m^2$, which corresponds to the typical value range of the potential.

In [61] also the δ and the η interve, although with a smaller role, and the σ parameters are different from the quoted ones in the total isospin $T = 1$ channel ; these refinements will be neglected. The form of these potentials in the energy independent form considered i.e. without retardation in the propagators is determined by the spinor structure of the scalar and vector meson NN vertices.

The central and spin-orbit parts, up to $O(1/M^2)$ included, read in an arbitrary frame in momentum space

$$\begin{aligned} V_\sigma(q) = & -\frac{g_\sigma^2}{\vec{q}^2+m_\sigma^2}[1 - \frac{q^2}{4M^2} - \frac{p_i^2}{2M^2} - \frac{p_j^2}{2M^2} - \frac{\vec{q}.(\vec{p}_i-\vec{p}_j)}{2M^2}] \\ & +\frac{i}{4M^2}\vec{\sigma}_i.(\vec{q} \times \vec{p}_i) - \frac{i}{4M^2}\vec{\sigma}_j.(\vec{q} \times \vec{p}_j) \end{aligned} \tag{40}$$

$$\begin{aligned} V_\omega(q) = & \frac{g_\omega^2}{\vec{q}^2+m_\omega^2}[1 - \frac{\vec{p}_i.\vec{p}_j}{M^2} + \frac{\vec{q}.(\vec{p}_i-\vec{p}_j)}{2M^2}] \\ & -\frac{i}{4M^2}\vec{\sigma}_i.(\vec{q} \times \vec{p}_i) + \frac{i}{4M^2}\vec{\sigma}_j.(\vec{q} \times \vec{p}_j) \\ & +\frac{i}{2M^2}\vec{\sigma}_i.(\vec{q} \times \vec{p}_j) - \frac{i}{2M^2}\vec{\sigma}_j.(\vec{q} \times \vec{p}_i) \end{aligned} \tag{41}$$

where $\vec{p}_i, \vec{p}_j$ are the absolute momenta of the two nucleons and $\vec{q}$ the momentum transfer.

The first term in both Eq's corresponds to the usual expression to be found in the literature, namely

$$V_\sigma(x) = -\frac{g_\sigma^2}{4\pi}\frac{e^{-m_\sigma x}}{x} \tag{42}$$

$$V_\omega(x) = \frac{g_\omega^2}{4\pi}\frac{e^{-m_\omega x}}{x} \quad \text{where } x = |\vec{x}_i - \vec{x}_j| \tag{43}$$

i.e. to the central attractive σ and repulsive ω potential.

If one averages over a constant one body density $\rho(x) = \rho_0 = .17 fm^{-3}$ i.e. without correlations, the previous expressions Eq's.(42, 43) with the above mentioned "effective" coupling constants $\frac{g_\sigma^2}{4\pi} = 7$ and $\frac{g_\omega^2}{4\pi} = 10.6$, one obtains $< V_\sigma > \simeq -$ 390 MeV, $< V_\omega > \simeq$ 320 MeV.

These values practically correspond to the ones of relativistic mean field theories [62,63].

As regards $\frac{1}{M^2}$ relativistic corrections to the central part in Eqs.(40,41) the term $\frac{q^2}{4M^2}$ simply changes the short range behaviour of the σ potential by a factor $1-\frac{m_\sigma^2}{4M^2}$ plus a δ function, and will be irrelevant as well as $\vec{q}.(\vec{p}_i - \vec{p}_j)$ to our final results, whereas the kinetic terms $\frac{p^2}{2M^2}$ and the dipole-dipole exchange $-\frac{\vec{p}_i \cdot \vec{p}_j}{M^2}$ have opposite sign again (as the central part).

It should be stressed that in a mean field approximation in the Hartree sense i.e. by averaging over the $j-th$ uncorrelated nucleon all $i-j$ terms in Eq.(41) coming from the 3-vector part of the ω exchange potential (usually denoted by $\vec{\omega}$) manifestly vanish.

In such an approximation, focussing only upon the central part, relativistic effects disappear in V_ω. On the contrary they survive in V_σ with the immediate physical interpretation that the kinetic energy in the non-relativistic nuclear Hamiltonian turns into $\frac{p^2}{2M}(1+\frac{<-V_\sigma>}{M})$ i.e. changes the mass into an effective one $M^* = \frac{M}{1+<-V_\sigma>/M}$.

Consequently one obtains in a standard way for the respective convective exchange currents $\Delta\vec{J}_i^{(\omega)}(\vec{k}) = 0$ $\Delta\vec{J}_i^{(\sigma)}(\vec{k}) = -\frac{<V_\sigma>}{M}\frac{\vec{p}_i}{M}$.

However such an averaging procedure causes a number of "spurious" relativistic effects.

The best known examples is that of the single particle magnetic moment [64]. Its orbital part $\vec{M}1_\ell = \frac{e}{2M}\vec{\ell}_1$ (see Eq.(30)) is obtained from the long wave length limit of the convective part of the e.m. current.

With the previous position one has for the contribution of the kinetic energy term

$$\vec{M}1'_\ell = \frac{e}{2}\vec{r}_i \times \frac{\vec{p}_i}{M^*} = \frac{M}{M^*}\vec{M}1_\ell \tag{44}$$

Compared with the conventional non-relativistic result, one has an enhancement of the orbital magnetic moment by a factor M/M^*. Even for a valence nucleon, such an enhancement is large and is undesirable as we known that the experimental magnetic momenta are very close to the Schmidt line in agreement with the standard non-relativistic description. Furthermore, it has been pointed out that also the contribution of the spin-orbit term to the magnetic moment is very large and makes things even worse [65].

The reason for such a failure is obvious since in averaging, thus priviledging the laboratory frame, one has lost Lorentz and consequently gauge invariance. Indeed there is no need of doing so.

One immediately obtains from the continuity equation, the convective exchange current contribution in momentum space of the σ and ω potentials

$$\Delta\vec{j}_{ij}^{(\sigma)}(\vec{k}) = \frac{g_\sigma^2}{4\pi}\frac{e^{-m_\sigma x_{ij}}}{x_{ij}}[\frac{\vec{p}_i + \vec{p}_j}{M^2}] \tag{45a}$$

$$\Delta \vec{j}_{ij}^{(\omega)}(\vec{k}) = -\frac{g_\omega^2}{4\pi}\frac{e^{-m_\omega x_{ij}}}{x_{ij}}[\frac{\vec{p}_j + \vec{p}_i}{M^2}] \tag{45b}$$

The reason for the interchange $i \leftrightarrow j$ in the r.h.s. of Eq.(45b) is due to the fact that the current for the $j-th$ nucleon (obtained by commuting with $e^{i\vec{k}.\vec{x}_j}$) is proportional to $\vec{p}_i$.

From the previous expressions, averaging over j one gets from the σ

$$\Delta \vec{M}1_\ell^{(\sigma)} = \frac{e}{2M}\vec{\ell}_i \frac{< -V_\sigma >}{M} \tag{46a}$$

and from the $\vec{\omega}$ component

$$\Delta \vec{M}1_\ell^{(\omega)} = -\frac{e}{2M}\vec{\ell}_i \frac{< V_\omega >}{M} \tag{46b}$$

It should be clear from the preceding observation that the ω contribution comes from the $j-th$ nucleon : it corresponds to "backflow".

These is a partial cancellation between Eq.(46a) and (46b) depending upon the more or less approximate equality between $< -V_\sigma >$ and $< V_\omega >$ [67].

In an analogous way, there is a a similar cancellation of the spin-orbit contributions to the magnetic moment [68]. We have from the $\vec{\ell}.\vec{\sigma}$ term

$$\Delta \vec{M}1_\ell^{(\sigma)} = \frac{e}{2M}\frac{1}{4}\vec{x}_i \times (\vec{\sigma}_i \times \vec{x}_i) < \frac{1}{M}\frac{g_\sigma^2}{4\pi}\frac{1}{x}\frac{\partial}{\partial x}\frac{e^{-m_\sigma x}}{x} > \tag{47a}$$

whereas from the ω

$$\Delta \vec{M}1_\ell^{(\omega)} = -\frac{e}{2M}\frac{1}{4}\vec{x}_i \times (\vec{\sigma}_i \times \vec{x}_i) < \frac{1}{M}\frac{g_\omega^2}{4\pi}\frac{1}{x}\frac{\partial}{\partial x}\frac{e^{-m_\omega x}}{x} > \tag{47b}$$

where in the average a sum $\sum_j$ over $x_{ij} \equiv x$ has to be understood, and where the factor -1/4 results from +1/4 from ω_0 and -1/2 from the $\vec{\omega}$ contribution (see Eq.(41)).

In a mean field approximation the last term would be absent.

Let us mention in passing that the role of the ω is also instrumental in getting for two particles in their center of mass the $\vec{L}.\vec{S}$ interaction

$$V_{LS} = \frac{1}{x}\frac{\partial}{\partial x}(V_\sigma - 3V_\omega)\frac{1}{4M^2}\vec{L}.\vec{S} \tag{48}$$

Due to the quantum numbers of the ω, the above mechanism of cancellation is effective only for isoscalar quantities.

Unlike the previous cases where spurious effects disappear, the Bethe-Levinger sum rule constitutes a good example for genuine ones. It is obvious that the current modification $\frac{\vec{p}}{M} \leftrightarrow \frac{\vec{p}}{M^*}$ according to the mean field approach, necessarily modifies the double commutator $[\vec{D}, H]\vec{D}]$ yielding for the integrated cross section $2\pi^2\alpha \frac{ZN}{M^* A}$. To ascertain whether this really holds true, we calculate explicitly the potential energy contribution.

Notice that only terms that are bilinear or quadratic in the momenta will be non vanishing. These are the terms $\vec{p}_i^2 + \vec{p}_j^2$ in V_σ and $\vec{p}_i + \vec{p}_j$ in V_ω. By direct commutation one gets from the ω contribution [68]

$$\begin{aligned} &\tfrac{1}{3} < 0|[[\vec{D}, V_\omega], \vec{D}]|0 >= -e^2 \textstyle\sum_{ij} \frac{g_\omega^2}{4\pi M^2} \\ &\times < 0|\frac{e^{-m_\omega x_{ij}}}{x_{ij}} \left(\frac{1+\tau_i^3}{2} - \frac{Z}{A}\right)\left(\frac{1+\tau_j^3}{2} - \frac{Z}{A}\right)|0 > \end{aligned} \tag{49}$$

Making the approximation that, in terms of the nucleon density $\rho(r)$, the proton density is given by $\rho_p = \frac{A}{Z}\rho$, it is easy to see that the above ω-exchange contribution vanishes.

In the case of the σ, the previous expression goes into the corresponding one, and in addition $\tau_j^3 \rightarrow \tau_i^3$.

Within the same approximation one has

$$\frac{2\pi^2}{3} < 0|[[\vec{D}, V_\sigma], \vec{D}]|0 >= \frac{2\pi^2\alpha}{M}\frac{ZN}{A}\frac{< -V_\sigma >}{M} \tag{50}$$

which is just the first order expansion of the classical result, when M is replaced by M^*. This means that for *the dipole sum rule, the relativistic effect is not cancelled by the* $\vec{\omega}$ *exchange mechanism*. Such a result might have been expected since the dipole operator is made up of an isoscalar and of an isovector component.

Indeed, by analyzing separately the two contributions one easily gets the usual almost complete cancellation in the former case, whereas potentials add up in the latter. The separate contributions to κ of the σ and ω are shown in Fig.5.

As usual, use of the continuity equation automatically takes into account negative energy states among the external probes and the meson-NN vertex, even if in the potentials only positive energy states appear.

In conclusion, aside from the very particular case of the orbital magnetic moment, the relativistic effect brought about by the effective mass M^* of a mean-field description will generally stand [69] (as it known also from the weak axial current case [70,71]).

Actually, this so-called relativistic effect is not something that is specific to a relativistic description : it appears as soon as one replaces the nucleon mass M by some effective $M^*(r)$, independently of the origin of the latter, as for example in the non-relativistic description with a Skyrme interaction [72].

Coming back to the sum rule enhancement, one has of course to add to the previous contribution the standard ones coming from the pion and the rho, unlike what has been done in relativistic mean field theories [73]. Since use of the quoted value of $< V_\sigma >$ in Eq.(50) yields an enhancement given by the σ alone of the order of half a classical sum rule, this might seem to be incompatible with the integrated experimental photoabsorption cross-section up to the pion threshold.

In this connection two comments are in order : the first stricly concerning the sum rule, the second the connection between microscopic approaches and relativistic mean field theories.

Indeed it has to be stressed once more that, in view of its microscopical interpretation as virtual meson photoproduction, the comparison of the double commutator with the experimental photoabsorption cross section up to the pion threshold is somewhat justified only for the pion part, whereas the virtual photoproduction of higher mass mesons lies surely above so as to make such a comparison totally non quantitative.

Moreover the microscopic foundations of the intermediate range part of the NN exchange potential and of the corresponding exchange currents are not totally satisfactory.

Indeed, by using the Lagrangian which generates the σ exchange, one does not reproduce the two uncorrelated pion photoproduction region. The basic mechanism at work there (see § 5.1), corresponds in a potential language to a two pion exchange with intermediate Δ excitation.

Finally the previous estimate of the isoscalar contribution to the sum rule enhancement is based on the neglect of correlations.

As regards relativistic mean field theories, in the mean field approximation the relativistic nucleon is supposed to obey a Dirac equation

$$[\not{p} - M - S - \gamma_0 V]u = 0 \tag{51}$$

S representing the attractive scalar and V the repulsive vector potential. The eigenvalues of such an equation are such that the ordinary mass $M \rightarrow M^* = M + S$ whereas the energy $E \rightarrow E^* = \sqrt{p^2 + M^{*2}} = E - V$. Hence the mass is decreased because of attraction, whereas the energy is shifted in an analogous way by V. In the non relativistic (Schrödinger) reduction of the Dirac equation the mass and energy appear in the combination $(M - E) \rightarrow (M^* - E^*) = (M - E) + (S + V)$.

On the other hand, in the small components of the Dirac equation M and E appear in the opposite combination i.e. $M + E$ $(u^\dagger \simeq (\chi^\dagger \ \frac{\vec{\sigma}.\vec{p}}{M+E}\chi^\dagger))$.

Therefore the relatively weak central and large spin-orbit potentials one observes can be accomodated in such a scheme with almost equally large and opposite values of S and V.

The bulk properties of nuclei are reproduced within a relativistic Hartree approximation (only σ and ω) or with a Hartree-Fock treatment (σ, ω *and* π, ρ) without significant differences in the coupling constants [63], although the success in the description of intermediate energy nucleon nucleus scattering rests on the use of the empirical NN potential i.e. in a not totally self-consistent recipe. However, in the case of an *external* probe as for the Bethe-Levinger sum rule discussed above, *the inclusion of the pion in going from* a Hartree to a Hartree-Fock approximation results in substantial differences.

Therefore there is no justification to identify the boson of relativistic mean field theories with those appearing in the one boson exchange potentials [74].

In conclusion even disregarding the open problems concerning the connection between the Dirac equation and its nonrelativistic reduction [75,76,77], it should be clear that the (too hasty) mean field replacement $M \rightarrow M^*$ in the non relativistic expressions should be explicitly checked in each case.

The Lorentz invariant microscopical formulation outlined above and the ensuring gauge invariant exchange currents seem to provide the natural framework to that aim.

Interesting places to investigate the role of relativistic exchange contributions might be high momentum transfer processes like e.g. low energy deuteron photodisintegration *in the forward direction, which has been shown to* be *very* sensitive to relativistic effects [78].

3.3 Collective effects : the giant dipole resonance and deformed nuclei

We briefly review what has represented for quite a long time the traditional domain of nuclear physics i.e. the low energy region. As " the" example of genuine nuclear effects we consider the giant dipole resonance (GDR) in particular in deformed nuclei.

It is well known experimentally that all nuclei show a bump in the experimental cross section with a shape that tends with increasing A to become more and more (since nuclear structure effects are smeared out) that of a Lorentz curve. Its peak energy roughly follows the law

$$E_{GDR} \sim 80 A^{-1/3} \text{ MeV} \tag{52}$$

hence passing from about 25 MeV in light nuclei to 13 in Pb whereas the width Γ_{GDR} remains roughly constant with a value of the order of 5 MeV.

Such an energy dependence can be explained in simple models like the hydrodynamic one [79].

In it the nucleus is schematized as being made up of interprenetrating proton and neutron fluids of constant total density confined within a rigid boundary. GDR is just the lowest oscillation mode where the restoring force is connected to the symmetry energy of the semiempirical mass formula.

This picture represents an alternative version of the original Goldhaber and Teller theory which just reflects the form of the dipole operator.

Indeed, since protons have a dipole moment with respect to the center of mass, one can alternatively attribute effective $(-\frac{N}{A})$ and $(\frac{Z}{A})$ charges to protons and neutrons respectively, so that one can think of protons and neutrons oscillating against each other. However the previous result can also be obtained in a standard single particle shell model [80], where the $A^{-1/3}$ dependence follows immediatly from the form of the potential energy $-\frac{1}{2}\alpha r^2$.

This in spite of the fact that the coefficient turns out to be roughly 1/2 that of Eq.(47) hence demanding the introduction of a residual interaction for a more realistic evaluation [81,82] of the GDR structure. The above two seemingly alternative pictures have been shown by Brink [83] to be equivalent.

The total integrated strength is at least (as in the previous models where there are no exchange forces) of one classical sum rule $\frac{2\pi^2\alpha}{M}\frac{ZN}{A}$. Hence it is giant because it is collective in contrast with other transitions to discrete levels. A straightforward consequence of the above prediction that the mean energy of the collective state excited by the dipole operator varies inversely as the nuclear radius follows in deformed nuclei.

It is well known that deformed nuclei exist with a permanent intrinsic quadrupole moment different from zero [84]. This means that the ordinary spherical shape of radius R, can turn into an ellipsoidal one either prolate or oblate.

Most nuclei are prolate, with a major axis b along the (intrinsic) symmetry axis z' and a minor axis a in the x', y' plane.

It is then obvious that an optical anysotropy can result according to along which axis neutron proton oscillations take place.

Therefore one expects that correspondingly the GDR energy be split into two denoted with obvious meaning by E_a and E_b such that

$$\frac{E_a}{E_b} = \frac{b}{a} \tag{53}$$

More sophisticated calculations of the eigenmodes in the hydrodynamic model [85] confirm the previous result to the percent.

It is also well established that these nuclei are characterized by rotational spectra and that the ground state is a member of rotational band. The radial parts of the matrix elements associated with transitions between the collective giant resonance and all the members of the ground state rotational band are the same. The physical reason for that lies in the possibility of applying the adiabatic approximation, since the nucleus changes its shape in a time $\sim 1/(1MeV)$ (typical energy of the rotational band, which is very long compared to the giant resonance oscillation time $\sim 1/(15MeV)$.

The scattering to this rotational band is known as Raman scattering. A phenomenological description of both photon scattering and photoabsorption is then obtained (specializing our formulas the simple 0^+ ground state case to avoid Clebsch - Gordanries) by

$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \frac{1+\cos^2\theta}{2}\left|\frac{\omega^2}{3}\left\{\frac{<0|[[D_{z'},H]D_{z'}]|0>}{2E_b}\cdot\right.\right.$$
$$\cdot|(E_b-\omega-i\Gamma_b/2)^{-1}+(E_b+\omega)^{-1}|+$$
$$\left.\left.+2\frac{<0|[[D_{x'}H]D_{x'}]|0>}{2E_a}|(E_a-\omega-i\Gamma_a/2)^{-1}+(E_a+\omega)^{-1}|\right\}-\frac{Z^2e^2}{AM}\right|^2, \quad (54)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ra} = \frac{13+\cos^2\theta}{90}\left|\frac{\omega^2}{3}\left\{\frac{<2^+|[[D_{z'},H]D_{z'}]|0>}{2E_b}\cdot\right.\right.$$
$$\cdot|(E_b-\omega-i\Gamma_b/2)^{-1}+(E_b+\omega)^{-1}|-$$
$$\left.\left.-\frac{<2^+|[[D_{x'}H]D_{x'}]|0>}{2E_a}|(E_a-\omega-i\Gamma_a/2)^{-1}+(E_a+\omega)^{-1}|\right\}\right|^2, \quad (55)$$

and

$$\sigma_b(\omega) = 2\pi\frac{\omega}{E_b}\frac{\Gamma/2}{(E_b-\omega)^2+(\Gamma_b/2)^2}<0|[[D_{z'},H]D_{z'}]|0> \quad (56)$$

$$\sigma_a(\omega) = 2\pi\frac{\omega}{E_a}\frac{\Gamma/2}{(E_a-\omega)^2+(\Gamma_a/2)^2}2<0|[[D_{x'},H]D_{x'}]|0> \quad (57)$$

The previous formulas are obtained from Eq.(25) by properly recombining seagull and time ordered part of the amplitude to separate out the c.m. effect [86].

The angular distributions for the scalar and tensor components of the scattering amplitudes are simply obtained from the corresponding $\vec{D}.\vec{\epsilon}\vec{D}.\vec{\epsilon}'$ by analyzing polarizations into irreducible tensor operators

$$g_0 = \frac{1}{3}(\epsilon.\vec{\epsilon}')^2 = \frac{1}{6}(1+\cos^2\theta)$$

$$g_1 = \frac{1}{2}[1-(\vec{\epsilon}.\vec{\epsilon}')^2] = \frac{1}{4}(2+\sin^2\theta)$$

$$g_2 = \frac{1}{2}[1+\frac{1}{3}(\vec{\epsilon}.\vec{\epsilon}')^2] = \frac{13+\cos^2\theta}{12} \quad (58)$$

where the second set of equality holds only for the unpolarized case we are considering.

Of course, in our 0^+ case the vector component g_1 is identically zero. Moreover, in $(\frac{d\sigma}{d\Omega})_{Ra}$ the relation $\omega' = \omega$ has been used.

Of course in poor resolution experiments the sum of elastic and Raman scattering and for unpolarized targets total photoabsorption cross section $\sigma_a + \sigma_b$ is actually measured. Note that these equations have a meaning only in this limited energy range and that data can be reasonably well accounted for by some exchange contribution of the order of .2 - .3 in addition to the classical sum rule.

In other words one cannot account at the same time for the total integrated cross section and for the GDR region of energy E_{GDR} with the same value of the enhancement factor κ.

Still in other words one sort of artificialy distinguises in going from $< 0|D|n >< n|D|0 >$ to the double commutator with the Hamiltonian between the GDR region and the other domain up to the pion threshold and higher up where additional contributions of dipole character (exchange discussed previously) but not of collective (in the sense of depending upon nuclear dimensions) origin, yield $\kappa \simeq 1$.

Of course this separation is arbitrary, ill defined (on top of the fact that the dipole sum rule itself is ill defined) and contrary to the very concept of a sum rule from which, by definition, no informations about the strength location can be gotten. For this reason we have refrained from any considerations about the presumed connection between exchange enhancements in photoabsorption and for magnetic moments $\kappa = 2\delta g_\ell$ [87]. In the previous equations one can envisage the possibility of different enhancement factors coming from the double commutator along the different axes, namely

$$< 0|(x_i^\prime - x_j^\prime)^2 V_{ij}|0 > \neq < 0|(z_i^\prime - z_j^\prime)^2 V_{ij}|0 > \tag{59}$$

This entails, contrary to the hydrodynamic model , $\int d\omega\sigma_a(\omega) / \int d\omega\sigma_b(\omega) \neq 2$ and corresponding differences in the photon scattering expressions.

Data seem to be better describes by allowing for anysotropic exchange forces [88]. As an illustration typical photoabsorption cross sections are shown in Fig.6.

Of course at this rough phenomenological level , all the much more sophisticated nuclear details entering the dynamic collective model [90], (which however disregards the above possibility) i.e. coupling of surface vibration with the GDR and explicit microscopical calculation of Γ are absent.

So in these sketchy considerations have we disregarded the whole interesting area of the isospin decomposition of the sum rules [91,92,93] as well as the consideration of higher multipoles. Likewise there is no attempt at explaining direct (γ, N) reactions as calculated in a realistic way by the Bologna [94] and Pavia [95] group. The previous formulas are only meant to suggest how one can exploit angular distributions, especially if polarized beams are available, in consistent measurements of photo-absorption and scattering to obtain nuclear structure informations [96,97].

4 The Δ region

4.1 The pion photoproduction amplitude

The two basic problems to be overcome in order to provide a satisfactory treatment of pion photoproduction are unitarity and frame independence.

The first problem exists already for the nucleon whereas the second is peculiar to nuclei in the sense that in this case nucleon momentum components orthogonal to the incoming photon are necessarily present in the nucleus, resulting in a non-trivial connection between the nucleonic transverse (i.e. $\vec{k}.\vec{\epsilon} = \vec{p}.\vec{\epsilon} = 0$ see below) and nuclear case.

Here the model of Ref's [98,99,100], constructed in terms of a background (Born) and of an elementary Δ, will be presented.

This picture is in contrast with the original Chew-Low one [107] where the Δ resonance is dynamicaly generated by a rescattering series. For the time being let us simply recall that Chew-Low equations predict the possibility of a resonance in the $(3,3)$ channel whose realization depends however on a number of approximations, in addition to the value of the πNN form factor cut-of Λ.

The relation to the fundamental works of the past, [101,102,103] using the previous picture, will be commented upon in the nucleon case after having obtained the explicit formulas.

The consideration of the resonant multipoles will sort of justify a posteriori the basic inputs of the model. The photoproduction amplitude, necessary for a non-relativistic treatment of nuclear physics, is constructed according to the following program.

$\gamma - \pi$ is intrinsically related [104] to πN whose dynamics in given partial waves is naturally expressed in the $\pi - N$ c.m.s. (hereafter all quantities in such a system being labelled by a tilda). This connection, known as Watson's theorem, states that, as a consequence of unitarity i.e. of probability conservation,

$$\mathcal{M}_{2J,2T} = |\mathcal{M}_{2J,2T}| e^{i\delta_{2J,2T}} \tag{60}$$

i.e. that below 2 π threshold *the photoproduction multipoles* $\mathcal{M}$ *in a given channel* (characterized by the total angular momentum J and isospin T *have the same phase of the corresponding* (unitary) πN *scattering amplitudes* $f_{2J,2T} = e^{i\delta_{2J,2T}} \sin\delta_{2J,2T}$. In a correct treatment unitarity is automatically satisfied. Therefore the starting point is a non relativistic reduction in the c.m. frame, followed by rescattering which makes such an amplitude unitary. Because of the smallness of the corresponding phase shifts ($\delta \leq 20^0$) rescattering in channels other than the resonant $(J = 3/2, T = 3/2) = (3,3)$ is neglected.

With nuclear applications in mind, the amplitude is expressed in an arbitrary Lorentz gauge, so as to yield both the transverse and longitudinal parts, necessary to prove its gauge invariance.

Finally the transformation from the $\pi - N$ c.m.s. to an arbitrary frame (in our case the nucleus rest frame) is made, by proper non relativistic transformations yielding a rearrangement between " old" longitudinal and transverse parts, resulting in the final $\gamma - \pi$ amplitude, which is frame independent to the order of the original non relativistic reduction.

We start from πN.

By considering the background alone and by projecting the crossed $\pi - N$ Born diagram onto the (3,3) channel we approximately have

$$f_B = e^{i\delta_B} sin\delta_B = \frac{M}{4\pi s^{1/2}}(\frac{g}{2M})^2 \frac{4}{3}\frac{\tilde{q}^3}{\omega_q} \simeq \delta_B \qquad (61)$$

Here M is the nucleon mass, g the pseudovector coupling constant ($g^2/4\pi = 14.5$), $\tilde{q} = \tilde{q}(s^{1/2})$ and $\omega_{\tilde{q}}$ the pion momentum and energy in the c.m. frame at total invariant mass $s^{1/2}$= W.

Eq.(61) gives the unitarized background amplitude in Born approximation i.e. to $O(\delta_B^2)$ which is a very good approximation in view of the smallness of δ_B (see Table I). It is worthwhile noting its meaning in terms of diagrams when the renormalized πNN coupling constant is used in the calculation. In fact for small δ_B one has $\exp(i\delta_B) \simeq 1 + \delta_B$ so that Eq.(61) corresponds to the sum of the first two diagrams of Fig.7c) (where the second, because of its smallness can be neglected at will) where only the imaginary part from the intermediate propagation is retained in the second one, the real part having been reabsorbed in the renormalized coupling constant.

At this level the one loop approximation is therefore totally justified, were it not for the possibility of dispersive effects which will be commented upon later on.

Now we come to the resonant part of the amplitude. To the lowest order in f_B we have to perform the sum of the last three diagrams of fig.7c), where the Δ propagator is to be understood as the dressed Δ propagator. Here again the real parts from the vertex correction given by π scattering through the background mechanism (diagrams (b) and (c) of fig.7) go into the definition of the renormalized $\pi N\Delta$ coupling constant. Then we need consider only the imaginary part from the intermediate π propagation, i.e. by denoting with $f_R(q)$ the resonant amplitude (third diagram of Fig.7c)), one has for the sum :

$$f_R(\tilde{q})(1 + 2i\delta_B) \simeq f_R(\tilde{q}) \exp(2i\delta_B) \qquad (62)$$

Here $f_R(\tilde{q})$ has been separately unitarized so as to be written in a unitary form i.e.

$$f_R(\tilde{q}) = e^{i\delta_R} \sin\delta_R \qquad (63)$$

where

$$f_R(\tilde{q}) = -\frac{M}{4\pi s^{1/2}}\frac{1}{3}\frac{f_\Delta^2}{m_\pi^2}\frac{2M_\Delta \tilde{q}^3 v_\Delta^2(\tilde{q})}{s - M_\Delta^2 + iM_\Delta \Gamma(\tilde{q})} \tag{64}$$

and

$$\Gamma(\tilde{q}) = \frac{2}{3}\frac{f_\Delta^2}{4\pi}\frac{M}{s_q^{1/2}}\frac{\tilde{q}^3 v_\Delta^2(\tilde{q})}{m_\pi^2} \tag{65}$$

which is the width that one would calculate from the Δ self-energy diagram with the renormalized coupling at both vertices.

By summing the unitarized background Eq.(61) to the Δ mechanism with distorted pions Eq.'s(62,63) one gets in terms of

$$\delta \equiv \delta_{33} = \delta_B + \delta_R \tag{66}$$

a *total unitarized amplitude*

$$e^{i\delta_B} \sin\delta_B + f_R(\tilde{q}) e^{2i\delta_B} \equiv e^{i\delta}(e^{-i\delta_R}\sin\delta_B + \sin\delta_R e^{i\delta_B}) = e^{i\delta}\sin\delta \tag{67}$$

This represents the so called phase addition rule [108], for combining a background and a resonance in the case of a single open channel. Notice how rescattering, i.e. pion distortion is essential in the Δ term Eqs.(62, 63) to make the amplitude automatically unitary.

It is worth stressing that this phase addition unitarization procedure to $O(\delta_B^2)$ in the presence of a background is the only physically correct one.

In particular, in contrast with other prescriptions only in this way Γ corresponds to the width one would calculate from the standard formulas for $3/2 \rightarrow (1/2, 0)$ decays, staying positive even away from the resonance. In this sense, given the πNN interaction, there is no arbitrariness (as in a general case) in the separation between background and resonance.

The best fit to the δ_{33} phase shift is reported in Table 1. It yields (for the πNN coupling constant the standard $f_{\pi NN}^2/4\pi = .08$ without form factor has been assumed) for the $\pi N\Delta$ form factor $v_\Delta(q) = (q_R^2 + \Lambda^2)/(\tilde{q}^2 + \Lambda^2)$ where $q_R \sim 236$ MeV is the momentum at resonance a cut-off $\Lambda = 705$ MeV, for the $\pi N\Delta$ renormalized coupling constant $f_\Delta^2/(4\pi)$ $= .43$ and for the renormalized Δ mass $M_\Delta = 1241$ MeV.

In this connection some comments are in order.

As regards the Δ mass, it is obvious that because of the presence of a background with a positive phase, and because of the phase addition rule *we have a Δ mass which is higher than the resonance energy* ($\delta = \pi/2$). Indeed, the often quoted value of 1232 corresponds to the no background (crossed Born) case and entails a cut-off of $\sim$ 300 MeV .

In that respect our Λ can be regarded as somewhat more " fundamental" . As regards the $\pi N \Delta$ coupling constant, *the value* $f_\Delta^2/(4\pi) = .43$ differs from the standard Chew-Low value .32 ; as before this outcome is just a consequence of the different dynamics of our model.

Likewise the Δ width, also reported in Table 1, is considerably bigger (at resonance $\Gamma/2 \sim m_\pi$) than the one usually quoted.

All these different effects combine to reproduce the only physical quantity i.e. the phase-shift.

The consistent implementation of such a scheme into photoproduction is immediate.

As before pions are allowed to rescatter, so that the usual photoproduction diagrams in the Born approximation, are followed in the (3,3) channel by the previous (3,3) black box.

The contribution of all loop diagrams is given by the dispersive integral $\frac{1}{\pi} \int \frac{ds'}{s' - s - i\varepsilon}$ of the proper projected amplitudes. If only the imaginary part of the loop, corresponding to on-shell pion propagation is retained (the dispersive part having been reabsorbed in the renormalized coupling constants) the total contribution in the resonant channel ($\delta = \delta_{33}$), is proportional to

$$1 + i \sin \delta e^{i\delta} \equiv \cos \delta e^{i\delta} \tag{68}$$

(where the first term comes from the diagram in the tree approximation) and has the same phase, as demanded by Watson theorem, of the corresponding πN channel. This happens for the crossed Born and for the pion in flight, whereas the Δ term gets a factor $1 + i\delta_B \sim e^{i\delta_B}$ because of rescattering.

It is therefore clear that the three contributions are separately unitary to the given order, irrespective of the photon coupling constants.

The dispersive contribution, relevant for the connection of renormalized to unrenormalized coupling constants, is separately unitary and proportional to $e^{i\delta} \sin \delta$. It will not be considered in the following formulae.

It then results very naturally, by projecting forth and back, that the whole amplitude is simply obtained by the addition of a rescattering contribution proportional to $i \sin \delta \exp[i\delta]$ to the terms in the tree approximation. This can be best obtained by the introduction of " projection operators" . The lowest order ones, necessary for the usual multipole analysis, are summarized in Table 2. All relevant operators can be expressed as a proper linear combination of them.

Notice that this alternative choice (with respect to the C.G.L.N. F1, F2, F3, F4) has the advantage that if such an operator is multiplied by an angle independent amplitude, the latter just provides the appropriate multipole. Of course crossed and t-channel diagrams intrinsically have such a dependence in the propagators, resulting in an infinite series of

Legendre polynomials. It has been checked [98,99], as expected, that the fundamental one is that obtained by neglecting the angular dependence of the denominator i.e. that corresponding to the pure projection operator. In practice this procedure yields a very good approximation for the lowest multipoles.

It is therefore clear that, in c.m. frame, rescattering is taken care of by simply dropping the $\tilde{y}$ dependence in the denominator ($\tilde{y} = \cos\theta$ being the angle between the photon and the pion momenta) of those terms whose numerator provides the relevant multipolarity, and by multiplying them, as mentioned before, by $i \sin\delta \exp[i\delta]$.

As regards the isospin analysis, we report the usual relations between physical and isospin amplitudes $M(0), M(1), M(3), 0, 1, 3$ standing respectively for isoscalar, isovector $T = 1/2$, isovector $T = 3/2$.

$$\begin{aligned} M(0) &= 1/2[M(\gamma p \to p\pi^0) - M(\gamma n \to n\pi^0)] \\ M(1/2) &= 3/2[M(\gamma n \to n\pi^0) + \sqrt{2}M(\gamma p \to n\pi^+)] \\ M(3/2) &= M(\gamma n \to n\pi^0) + 1/\sqrt{2}M(\gamma n \to p\pi^-) = \\ &= M(\gamma p \to p\pi^0) - 1/\sqrt{2}M(\gamma p \to n\pi^+) \end{aligned} \tag{69}$$

Here, the energy-momenta of the incoming, outgoing nucleon, photon and pion respectively are $p_\mu = (E, p); p'_\mu = (E', p'); k_\mu = (\omega, k); q_\mu = (\omega_q, q)$ and we define

$$\begin{aligned} DIR &= (p + k)^2_\mu - M^2 \\ DCR &= (p - q)^2_\mu - M^2 \\ D\pi &= (q - k)^2_\mu - m^2_\pi \end{aligned} \tag{70}$$

The relevant photoproduction amplitudes obtained from a pseudovector πNN interaction read in an arbitrary frame

$$\begin{aligned} T(\gamma p \to n\pi^+) &= \frac{ieg\sqrt{2}}{2M}\{-\vec{\sigma} + \frac{\omega\omega_q}{DIR}\vec{\sigma} - \frac{\vec{\sigma}\vec{q}}{DIR}(2\vec{p} + \vec{k} + i\mu_p\vec{\sigma} \times \vec{k}) - i\mu_n\vec{\sigma} \times \vec{k}\frac{\vec{\sigma}.\vec{q}}{DCR} \\ &+ \frac{\vec{\sigma}.(\vec{k} - \vec{q})}{D\pi}(2\vec{q} - \vec{k})\}.\vec{\epsilon} - \sqrt{2}/3[\Delta M_{1+}(3/2)] \end{aligned} \tag{71}$$

$$\begin{aligned} T(\gamma n \to p\pi^-) &= \frac{ieg\sqrt{2}}{2M}\{-\vec{\sigma} - \frac{\omega\omega_q}{DCR}\vec{\sigma} - i\mu_n\frac{\vec{\sigma}.\vec{q}}{DIR}\vec{\sigma} \times \vec{k} - (2\vec{p}' - \vec{k} + i\mu_p\vec{\sigma} \times \vec{k})\frac{\vec{\sigma}.\vec{q}}{DCR} \\ &- \frac{\vec{\sigma}.(\vec{k} - \vec{q})}{D\pi}(2\vec{q} - \vec{k})\}.\vec{\epsilon} + \sqrt{2}/3[\Delta M_{1+}(3/2)] \end{aligned} \tag{72}$$

$$T(\gamma p \to p\pi^0) \;= \frac{ieg}{2M}\{\omega\omega_q(\frac{1}{DIR} - \frac{1}{DCR})\vec{\sigma} - \frac{\vec{\sigma}\vec{q}}{DIR}(2\vec{p} + \vec{k} + i\mu_p\vec{\sigma}\times\vec{k}) - (2\vec{p}' - \vec{k} + i\mu_p\vec{\sigma}\times\vec{k})\frac{\vec{\sigma}.\vec{q}}{DCR}\}.\vec{\epsilon} + 2/3[\Delta M_{1+}(3/2)] \qquad (73)$$

where

$$\Delta M_{1+}(3/2) = \frac{1}{3}\left\{\frac{f_\Delta g_\Delta}{m_\pi^2}\frac{2M_\Delta v_\Delta(\tilde{q})}{s - M_\Delta^2 + iM_\Delta\Gamma(\tilde{q})}e^{i\delta_B} + \frac{eg}{2M}(\frac{2(\mu_p - \mu_n)}{DCR|_{\tilde{y}=0}} + \frac{1}{D\pi|_{\tilde{y}=0}})ie^{i\delta}\sin\delta\right\}\Lambda_M \qquad (74)$$

$\Lambda_M = 3\vec{S}^+.(\vec{q} - \frac{\omega_q}{W}(\vec{p} + \vec{k}))\vec{S}\times(\vec{k} - \frac{\omega}{W}(\vec{p} + \vec{k})).\vec{\epsilon}.$

In the given expressions, the choice, based on our previous arguments, has been made to divide the amplitude into the traditional unrescattered one and the rescattering part. This explains the notation ΔM in the sense that, of course, part of M is contained in the standard terms. As a matter of fact it is immediate to see that, if one does not allow neither πN nor $\gamma\pi$ Born terms to rescatter ΔM contains only the usual Δ-term with final plane wave pions. With respect to this unrealistic no rescattering limit, the main difference embodied in the previous expressions amounts to a reshuffling (e.g. the pion-in-flight contributes to neutral photoproduction as well).

Born terms are of immediate interpretation. They correspond (see fig.7a) to the contact term, to the pion in flight and to negative energy states as well as to photon absorption by the full e.m. current (i.e. convective plus magnetic) for positive energy states in the direct and crossed Born terms.

In these expressions absolute momenta intervene, as one obviously expects from Feymann diagram recipes. On the contrary in ΔM, relative momenta show up. This is manifest in the expression of ΔM, but happens of course also for v_Δ, Γ_Δ, δ_B and δ appearing in braces of Eq.(74) . Momenta are then transformed according to the general Lorentz transformation from the rest frame ($\sim$) of a system of total momentum $P_\mu = (P_0, \vec{P})$ and a large invariant mass $W = (P_0^2 - \vec{P}^2)^{1/2}$ [109]. For any four-vector $a_\mu = (a_0, \vec{a})$, to the leading order in $|P|/W$ one can write :

$$\tilde{\vec{a}} = \vec{a} - a_0\vec{P}/W \qquad (75)$$

$$\tilde{a}_0 = a_0 - (\vec{P}.\vec{a})/W + a_0\vec{P}^2/2W^2 \qquad (76)$$

where the last term in eq.(76) can be consistently neglected if also $a_0 << W$.

In our case one has $\vec{P} = \vec{p} + \vec{k}$ and $W \simeq M + \omega$. Accordingly one transforms also the denominators e.g. $D_\pi|_{\tilde{y}=0} \equiv -2\tilde{\omega}\tilde{\omega}_q$.

Only in the Δ denominator in the neighbourhood of the resonance terms of $O(p^2/M^2)$ have to be kept in W in order not to influence the peak position.

Since $\vec{\epsilon} = \vec{\epsilon}_\perp = \vec{\epsilon} - (\vec{\epsilon}.\hat{k})\hat{k}$ such that, as usual, $\sum_\lambda \epsilon_i^\lambda \epsilon_j^\lambda = \delta_{ij} - \hat{k}_i\hat{k}_j$, all terms along $\vec{k}$ in Eq's(71 - 74) necessary to prove the gauge invariance of the amplitude (see Appendix) automatically drop.

Notice also that because of the transformation the Δ as well, because of the term $\frac{\omega}{W}\vec{p}$ can get a longitudinal (i.e. along $\vec{k}$) component and hence contribute to the longitudinal response function in e, e'). The fact that the magnetic Δ excitation has no 4^{th} component, because of current conservation, in the c.m.s. then explains why in Eq.(74) in the general frame only momenta have been transformed.

Of course, for a real photon only the transverse part (in the Lorentz gauge) contributes in any frame. This can be immediatly seen by applying $k^\mu \epsilon_\mu = 0$ and $k^\mu J_\mu = 0$ in both the c.m.s. and in an arbitrary frame to get $\vec{\tilde{J}}_\perp \vec{\tilde{\epsilon}}_\perp = \vec{J}_\perp \vec{\epsilon}_\perp$, $\perp$ standing for transverse (where of course $\vec{J}(\vec{q}, \vec{k})$ is the transformed of $\vec{\tilde{J}}(\vec{\tilde{q}}, \vec{\tilde{k}})$ according to Eq.(75), both as regards $\vec{\tilde{J}}$ and its arguments $\vec{\tilde{q}}, \vec{\tilde{k}}$).

One can specialize to this transverse (Coulomb) gauge only in the final expression. Indeed e.g. recoil terms in $\vec{p}$ in the preceding equations which derive from $\tilde{J}_0$, would be identically zero had we (wrongly) imposed the (non covariant) Coulomb gauge in the c.m.s.. It is also easy to prove, given the expression of J_0 (see Appendix) that our non relativistic reduction is consistent with the frame transformation to $0(\frac{p}{M})$. (of course, since $g/2M = f_{\pi NN}/m_\pi$ the coupling constant in front has not to be confused, as done by some authors, with an expansion parameter).

To that aim our formulas somewhat differ from the usual ones, in the sense that the propagators of the Born terms have been kept in the original Lorentz invariant quadratic form . This introduces small differences of $0(p^2/M^2)$ which are beyond the accuracy of the present non relativistic reduction and makes anyhow easier the formal proof of gauge invariance and frame independence.

Let us conclude by stressing that a consistent N.R. reduction to $0(p^2/M^2)$ using as starting point the unitary description in the $\pi - N$ c.m.s. and a subsequent frame transformation would turn out exceedingly complicated with respect to the present treatment so as to really question its applicability.

The adequacy of our general elementary operator can be immediately tested by switching back to the c.m.s. ($\vec{p} = -\vec{k}, \vec{p}' = -\vec{q}$) and by comparing our predictions with experimental data.

We w:ll consider the differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{q}{k}| < f|\frac{M}{4\pi s^{1/2}}T|i > |^2 \tag{77}$$

and the resonant (3,3) multipoles i.e. the previously defined $\Delta M_{1+}(3/2)$ as well as the electric $\Delta E_{1+}(3/2)$ given by

$$\Delta E_{1+}(3/2) = \frac{eg}{2M}\frac{-1}{3D\pi|_{y=0}}\Lambda_E i e^{i\delta}\sin\delta \tag{78}$$

where

$$\Lambda_E = 3i[(\vec{q}.\vec{k})(\vec{\sigma}.\vec{\epsilon}) + (\vec{\sigma}.\vec{k})(\vec{q}.\vec{\epsilon})]$$

corresponds to the projection of Table 2.

Such a term had been omitted in Eq's(71 - 73) because of its little numerical relevance and because of its cumbersome expression in the general case.

It is however a crucial test of our theory. From the previous formulas the difference between our scheme and C.G.L.N.'s emerges quite clearly. Apart from their use of a πNN pseudoscalar interaction, in the case of *no direct* (renormalized) $\gamma N\Delta$ *magnetic coupling constant* g_Δ, Born terms only would appear in ΔM_{1+} (round bracket in Eq.(74)) and $M_{1}+(3/2)$ *and* $E_{1}+(3/2)$ *would have the same* $\cos e^{i\delta}$ *behaviour*. In the latter multipole only the pion-in-flight diagram contributes, whereas in the former both the pion in flight and Born magnetic terms (roughly in a 1 to 3 ratio) would generate the multipole. With respect to that, C.G.L.N.'s total non-pion-in-flight contribution to $M_{1+}(3/2)$ is, on the contrary, simply proportional to $e^{i\delta}\sin\delta$.

This comes about because of their *assumption* for the solution ("we do not understand at present how to justify their selection") of their dispersive equations. This can be explained by noting [98] that in the case g_Δ/m_π is equal to $\frac{e(\mu_p - \mu_n)}{2M}\frac{f_\Delta}{f_{\pi NN}}$ in such a way that in the $\gamma\pi$ and πN case the possibly different Born and resonance mechanisms are proportional, the first two terms of Eq.(74) combine with the Born terms to yield a total contribution related to the πN scattering amplitude i.e. to $e^{i\delta}\sin\delta$. This is not fortuitous since the previous relation corresponds to the quark model prediction.

In this case the shift of the zero of the real part of the multipole with respect to the maximum of the imagniarny part, observed experimentally, is entirely due to the "extra" pion-in-flight mechanism.

The fact that rescattering is essential, can be easily seen precisely from the pion in flight contribution to the electric resonant multipole. Indeed, *since* $E_{1+}(3/2) \simeq -(1 + i\sin\delta e^{i\delta}) = -\cos\delta e^{i\delta}$ (1 coming as usual from the properly projected Born part) one gets straightforwardly in a parameter free way a double zero at $\delta = \pi/2$ for $\Re e E_{1+} \simeq -\cos^2\delta$ as well as $\Im m E_{1+} \simeq -\sin\delta\cos\delta$, both reproducing pretty well the seemingly rather peculiar structure of this multipole.

This does not imply of course the absence of an elementary $\gamma N\Delta$ quadrupole coupling but only that the dominant mechanism (against which it has possibly to compete) is simply due to a " peripheral" rescattering induced mainly by the ordinary pion-in-flight diagram.

More than that, the simultaneous consideration of the two resonant multipoles (see Fig.8), by explicitly introducing dispersive effects our framework, makes it possible to draw conclusions, in an almost model independent way [100] about the nature of the Δ resonance.

Is it namely an elementary particle (quark model) or is it dynamically generated by πN multiple scattering [107] ? In other words the Δ which has been introduced in the model and is dressed by the πN continuum, does really represent an elementary particle or rather simulates a "Chew-Low resonance" build up by the infinite rescattering series ?

Calculations in the πN case indicate [105,106] that the Δ is an elementary particle, but due to the separable approximation and to crossing violation, are not fully conclusive.

In order to give a precise meaning to the previous question, let us recall that the $\gamma N\Delta$ vertex reads in a non relativistic form

$$\begin{aligned} \vec{J}_\Delta.\vec{\epsilon} \; &= g_M \vec{S}.\vec{k} \times \vec{\epsilon} + g_E i(\vec{S}.\vec{k}\vec{\sigma}.\vec{\epsilon}_\perp + \vec{S}.\vec{\epsilon}_\perp\vec{\sigma}.\vec{k}) + \\ &+ \tfrac{\omega}{M} g_C 2i(\vec{S}.\vec{k}\vec{\sigma}.\hat{k})\vec{\epsilon} \end{aligned} \tag{79}$$

where g_M g_E and g_C are respectively the magnetic transverse, electric quadrupole and Coulomb coupling constants. The $\pi N\Delta$ vertex $\vec{S}^+.\vec{q}$ combines with $\vec{J}_\Delta.\vec{\epsilon}$ to yield (apart from a conventional factor 3) the proper M_{1+}, E_{1+} and L_{1+} multipoles. It is then clear that Born mechanisms may simulate an elementary $\gamma N\Delta$ coupling.

Indeed, as mentioned before, the rescattering contribution following Born photoproduction has an on-shell $ie^{i\delta}\sin\delta$ part which combines with the original Born to yield $e^{i\delta}\cos\delta$ plus $e^{i\delta}\sin\delta$ times a real dispersive (off-shell) contribution.

In our previous scheme this terms has been considered to be constant and reabsorbed in the definition of the renormalized coupling constant $\gamma N\Delta$ $g_\Delta = g_\Delta^{(0)} - \frac{M}{4\pi W} B \frac{q^2 f_\Delta}{k} P.V.|_R$ i.e. the sum of the bare one plus the dispersive integral contribution (where $|_R \equiv$ "at resonance", the subscript Δ stands here for M, E and C, and B indicates the proper projected Born term).

In the case of a dynamically generated resonance, one has to put for all couplings $g_\Delta^{(0)} = 0$. The dominant $e^{i\delta}\sin\delta$ term, necessary to explain the M_{1+} resonant behaviour, could then come only from the principal value part.

However (apart from the fact that this is contradicted by explicit although model dependent calculations [100]) such a huge dispersive contribution would in parallel show up, due to the structure of the multipoles, in the E_{1+} case as well and should be cancelled by a direct bare coupling.

Therefore an elementary Δ

It is also worth mentioning the overall tendency for the ratio $R = g_E/g_M$ to be definitely bigger (apart from the different sign) in the unrenormalized than in the renormalized case. In other words, the pion cloud tends to bring to a symmetric situation an "elementary" rather deformed object.

In conclusion once care has been taken of rescattering i.e. once the background has been duely unitarized, from the experimental data of the resonant multipoles the elementary (renormalized) coupling constants can be extracted in a model independent way.

This means that at $\delta = \pi/2$ where the rescattering contribution vanishes one reads off from $\Im m\, M_{1^+}(3/2)$ $g_M \simeq 0.13$, and from the shift of the intercept of $\Im m\, E_{1^+}(3/2)$ (accompanied by two distinct zeros in $\Re e\, E_{1^+}(3/2)$) $-0.02 g_M \geq g_E \geq 0$ (which will be hence neglected so that in the formulas $g_\Delta \equiv g_M$).

The extraction of g_C from the longitudinal multipoles [110] might give additional pieces of evidence for the previous picture. Indeed the reason for having explicitly separated the transverse from the longitudinal part in Eq.(79), is to underline that (due to the number of independent covariants [111]) g_C and g_E are independent unlike for Born mechanisms, where longitudinal and electric multipoles are related [103].

Coming back to the amplitude, total cross sections and angular distributions are plotted respectively in Fig's 9, 10. Our results (full line) have as only free parameter $g_\Delta = .13$.

With the provision that new more accurate experimental data [112,113,114] would surely be most welcome, (in particular it is worth stressing that in the $\gamma n, p\pi^-$ case data are intrinsically model dependent because of their extraction from the deuteron which suggests considering the inverse process), we see that our treatment does rather well. The comparison with the less predictive (in the sense of more parameters) and less consistent treatments available shows that we are, at least, as good.

In particular the essential feature of the angular distribution, peak position and height do come out pretty well. The importance of unitarization has been tested by surreptitiously switching off rescattering ($\delta = 0$ dash dotted line).

We see in this case a sizeable effect on $M_{1^+}(3/2)$ and $E_{1^+}(3/2)$ and still a sensible effect on the differential cross section particularly in the charged case. For the neutral pion the effect is limited to the backward direction.

We can therefore summarize that both frame transformation (see also next §) and unitarity have sizeable effects.

Let us conclude by commenting on the dynamical inputs of the present theory. In the past, the role of the ω and of the crossed Δ has been much emphasized. One has for the crossed Δ

$$T_{\Delta cr} = \frac{f_\Delta g_\Delta}{m_\pi^2} \frac{2M_\Delta v_\Delta(q)}{(p-q)_\mu^2 - M_\Delta^2} (\vec{S} \times \vec{k}.\vec{\epsilon})(\vec{S}^+.\vec{q}) \tag{80}$$

(which has to be multiplied by the isospin coefficients $\pm\sqrt{2}/3$ for $\pi^\pm$ and $2/3$ for π^0). For the ω, it is immediate to reduce non relativistically the dominant vector coupling $\bar{u}\gamma^\mu u.\epsilon_{\mu\nu\rho\sigma}\epsilon^\nu k^\rho q^\sigma$ to get in addition to $\vec{q} \times \vec{k}.\vec{\epsilon}$ (from the $\mu = 0$ component), contributing a term $T_\omega = eg_{\gamma\pi\omega}(g_{\omega NN}/m_\pi)\vec{q} \times \vec{k}.\vec{\epsilon}/[(q-k)_\mu^2 - m_\omega^2]$, $(\omega_q\vec{k} - \omega\vec{q}) \times \vec{j}.\vec{\epsilon}$ where $\vec{j} = (\vec{q}+\vec{k})/2M + i\vec{\sigma} \times (\vec{q}-\vec{k})/2M$ (where all quantities in the previous expressions are to be understood in the c.m.s.).

In order to have a T matrix correct to $O(p/M)$ both contributions have to be included in principle. In practice however what really matters is the numerical relevance of the terms considered.

By the standard procedure the contribution of $T_{\Delta cr}$ and T_ω can be incorporated in $d\sigma/d\Omega$ and in the various multipoles. Concerning these latter ones, it can be seen [99] that the ω somewhat improves the $E_{0+}(3/2)$ and that the crossed Δ overshoots the $M_{1-}(1/2)$ multipole (the contribution of a $T = 3/2$ object to a $T = 1/2$ multipole comes naturally from the isospin reduction of the intermediate $1 \otimes 3/2$) state).

This last fact is not surprising since the $N^*(1470)$, which is an essential ingredient in the dynamics of the πN f_{11} amplitude [115], has been left out.

Therefore, in addition to the problems connected with the ambiguities inherent to the mere introduction of a crossed Δ (same coupling constant at $2m_\pi$ masses away from the resonance, off-shell behaviour) it is clear that if these candidates are to be considered, much more dynamics should be included on the same footing to account simultaneously for πN.

The recent claim [116] that the consideration of the crossed Δ alone is crucial for the reproduction of the $M_{1-}(1/2)$ multipole [117] lies in the misunderstood fact that in the second reference a choice has been made for describing the Δ propagator which contain " extra" (1,1) component which can be thought of as an effective $N^*(1470)$.

However, it can be seen that the inclusion of the ω and of the crossed Δ has practically no effect already on the differential cross section. The reason is twofold. In the first place part of these contributions, as explained above, is suppressed in the resonance region by the factor $\cos\delta$ because of rescattering in the (3,3) channel. In the second place the two contributions tend to cancel each other. This is demonstrated in Fig.9 for the π^0 case where a sizeable effect is shown at higher energies for the ω alone (without crossed Δ)

(dotted line). When the crossed Δ is included the curve cannot be distinguished from the full curve, in which both terms are disregarded.

Therefore in addition to the above mentioned reasons of consistency, we see that at this level the dropping of these contributions, contrary to the statements of some literature, is absolutely legitimate. This should obtain equally well in a nuclear context where the overall compensation between these different mechanisms should not be sizeably affected because of their short range nature.

4.2 Sum rules

The gross features of pion photoproduction on the nucleon, which are summarized in Fig's 11), 12), can be roughly summarized by saying that the integrated cross section over the resonance region is practically all $M1$ in the π^0 case (both on the proton and on the neutron) and half $M1$ and half $E1$ in the charged case.

Since neutral and charged cross sections are comparable we have roughly a ratio of 3 to 1 for the total isospin averaged cross section over the Δ region which turns out to be of the order of 90 MeV mb.

Hence of the order of 6 classical sum rules per nucleon with respect to 1 of the sub-threshold $E1$ exchange contribution ($\frac{ZN}{A}$ 60 $\simeq$ $\frac{A}{4}$ 60 MeV mb). With this in mind, we now turn to nuclei to see whether the gross features of photoproduction in the Δ region can be understood in simple terms.

Keeping to the $M1$ operator, one is indeed entitled to assume " Δ dominance" [118,119,120,121] which means to disregarded the (small) magnetic Born terms and hence consider the Δ rather than the πN system to provide the final state.

This allows a considerable simplification ; in particular sum rules can be immediately obtained along standard lines.

Indeed, the total integrated cross section takes naturally the form of an energy weighted sum rule, whose most important contribution stems form the $N\Delta$ mass difference, and is hence linear in A [122]. To prove it, let us recall that from the previous form of the pion photoproduction amplitude, one has for the $\gamma N\Delta$ Hamiltonian in the laboratory

$$\begin{aligned} H_{\gamma N\Delta} &= \frac{g_\Delta}{m_\pi}\vec{S}.(\vec{k} - \frac{\vec{p}+\vec{k}}{W}\omega) \times \vec{\epsilon}T^3 + h.c. \\ &= \frac{g_\Delta}{m_\pi}(1-\frac{\omega}{W})\vec{S}.\vec{k}\times\vec{\epsilon}T_3 - \frac{g_\Delta}{m_\pi}\omega\vec{S}.\frac{\vec{p}\times\vec{\epsilon}}{W}T^3 + h.c. \end{aligned} \tag{81}$$

h.c. standing for the hermitian conjugate, which transform a Δ into a nucleon and which will be dropped since we assume no Δ components in the ground state (in line with explicit calculations [123] which estimate the Δ admixture at the percent levels) and T

standing for the standard $N\Delta$ isospin transition matrix. It is then clear that, due to the appearence of the relative momentum in the $\gamma N\Delta$ coupling, and neglecting for the moment the term proportional to the nucleon momentum $\vec{p}$, in the laboratory the effective Δ coupling constant g^*_Δ over the resonance region gets decreased with an average value $< g^*_\Delta >\sim g_\Delta(1-\frac{2m_\pi}{M+2m_\pi})\simeq 0.1$.

The photoabsorption cross section then reads

$$\begin{aligned}\sigma(\omega) &= \frac{1}{2}\sum_{\lambda=\pm1}\sum_n \frac{\pi}{\omega}| < n|\sum_i \frac{g^*_\Delta}{m_\pi}\vec{S}_i.\vec{k}\times\vec{\epsilon}T^3_i \\ &\times e^{-i\vec{k}.\vec{x}_i}|0>|^2\delta(E_{n_{||}}-\omega)\end{aligned} \tag{82}$$

and remembering that $\vec{k}=\hat{k}\omega$ it is straightforward to realize that $\int d\omega\sigma(\omega)$ can be immediately cast in the usual double commutator (of the nuclear Hamiltonian H with $H_{\gamma N\Delta}$) form. Taking $H = T_N + T_\Delta + U_\Delta + V_{NN}$, where T_N and T_Δ are the (nonrelativistic) nucleon and Δ kinetic energies, U_Δ is the excitation energy, and V_{NN} the nucleon-nucleon potential, the various terms of the sum rule are then easily calculated with the result that

$$\int\sigma(\omega)d\omega = \frac{4}{9}\pi\frac{g^{*2}_\Delta}{m^2_\pi}[\delta MA + \frac{\omega^2_R}{2M_\Delta}A + \frac{M}{M_\Delta} < T_N > - < T_N > -2 < V_{NN} >], \tag{83}$$

where the first dominant term comes from the $N\Delta$ mass difference and the standard approximation $< E_{n_{||}} >= \omega_R$ has been made to obtain the second. It is worth stressing that the contribution from the nucleon-nucleon potential is always the same, no matter whether the potential is local or not, in contrast with the situation met in the Bethe-Levinger sum rule. This happens because the Δ is a distinguishable particle from the nucleon and it must be destroyed at the same point where it is created by the current operator. The corrections to the main term δM are of the order of 10-15%.

Alternatively [124] one can directly use closure in Eq.(82), thus obtaining in $\sum_i\sum_j S^\dagger_j S_i$, $i=j$ terms, obviously proportional to A again, plus $i\neq j$ terms which vanish if no Δ are present in the nuclear ground state, due to the obvious fact that one cannot create a Δ at x_i and annihilate it at x_j. These corrections will, of course, depend on the details of the wave function.

In conclusion in this alternative approach we see that possible deviations from a linear dependence on A, are proportional to n_Δ (also because of the h.c. term) i.e. to the probability of finding a Δ in the ground state and hence again small.

At this point a quick estimate of the neglected term of Eq.(81) shows that ($|\vec{k}|=\omega$) it is $0(\frac{p}{W})$ with respect to the first. Its contribution is hence smaller than $A\frac{<K>}{M}$, K standing for the kinetic energy, whereas the $i=j$ interference term identically vanishes. It can be hence neglected.

Numerically by taking $\omega_R = \delta M = 2m_\pi$, we obtain a value of 100 MeV mb per nucleon for the sum rule, which compares favourably with the experimental outcome [125] of 90 MeV mb for the cross section integrated from the pion threshold to 440 MeV. By considering that the tail of pion photoproduction extends higher up but that we have at the same time neglected the $E1$ contribution we see that one is not that badly off. Of course one should not insist too much on these figures.

What should be emphasized is the relevance of the sum rule in predicting with very little dynamical assumptions, an overall gross behaviour of the integrated cross section proportional to A In spite of the strong spreading with respect to the proton, (see Fig.14), experimental results form $A = 9$ to $A = 208$ do indeed show a linear dependence on A. This is in complete disagreement with a shadowing hypothesis [126], since even a mild assumption like $\sigma_{nuclear}/\sigma_{nucleon} = A^{0.9}$ yields a factor of 0.80 for $A = 9$ but of 0.59 for $A = 208$.

In this connection the assumption underlying Eq.(81) will be critically examined in chapter 5.

Let us now come to the $E1$ part.

Here a sum rule cannot be obtained in such simple terms. However, using explicitly the operator introduced in § 4.1, one can essentially obtain for the integrated nuclear photoabsorption cross section [36]

$$\int_{m_\pi} d\omega \sigma_{E1}(\omega) \simeq A(1 - \kappa'_\pi(< \omega >)) \tag{84}$$

where the proportionality constant is of course the nucleon integrated cross section and where κ'_π in the long wave length limit can be identified with the enhancement factor of the dipole sum rule without the seagull contribution .

The minus sign is due to the Pauli principle that allows less phase space for a nucleon in the nucleus with respect to the free one.

One therefore explicitly obtains a partial realization of the G.G.T. mechanism, i.e. a strength shift from above to below threshold. It is threfore appealing, just as a consequence of the Pauli principle, within the same approximation in the derivation of the Bethe-Leving sum rule and in the treatment of photoproduction, to view part of $E1$ exchange effects below pion threshold as originating from a hindrance of the photoproduction cross section above it.

Of course, these considerations are highly non quantitative not only because of their obvious dependence on the nuclear model but also because of the repeated remark that the very use of a sum rule should prevent from idle discussions about strenght location.

However, in that spirit, part of κ_π (low energy nuclear physics pion exchange) is likely to contribute in the Δ region, as well as part of the above depletion will come from the higher energy domain.

4.3 Medium effects

Obviously the first place to spot some dynamical features which go beyond the gross properties embodied by the sum rule, is the total integrated cross section. Interestingly enough, although experimental measurements span the whole periodic table, there is only one existing calculation for medium heavy nuclei, which specializes to the case of ^{208}Pb.

Let us recall the essential ingredients of the model which because of its simplicity allows a simple understanding of the nuclear Δ dynamics.

First of all, Δ dominance is again assumed, with an effective coupling $g'_\Delta = .116$, somewhat bigger than the " mere effective" Δ coupling constant used for the $M1$ sum rule calculation, to account in some way in $\sigma(\omega)$ for the neglected $E1$ part.

A uniform medium is then assumed which should be well suited for large nuclei and which trivially provides a cross section proportional to A.

Let us start with the simple Fermi gas model in which a nucleon can be excited to a free Δ. From the optical theorem :

$$\sigma(\omega)/A = -(g'^2_\Delta/m_\pi^2)(3\pi^2/2k_F^3)\omega Im\Pi^0_\Delta(\omega, k=\omega) \tag{85}$$

with

$$\Pi^0_\Delta(\omega, k) = \frac{16}{9}\int \frac{d^3p}{(2\pi)^3}\frac{\theta(k_F - p)}{\omega + p^2/2M - \delta M - (\vec{p}+\vec{k})^2/2M_\Delta + \frac{1}{2}i\Gamma(\omega)} \tag{86}$$

From eqs.(85), (86) the leading term of the sum rules is immediately recovered by neglecting the Δ width as well as nucleon and kinetic energies. The result of eqs.(85), (86) is plotted in Fig.13 (dashed line). (Note that in eq.(86) in the energy dependence of the Δ-width, kinetic energies have been neglected and the standard form for $\Gamma(\omega)$ has been used). It is apparent that the peak position is almost reproduced whereas the shape of the curve is inadequate. More realistically we have to introduce a Δ-hole interaction, which in the channel of the photon is usually schematized as a ρ-exchange plus repulsive short range correlations embodied in a Landau-Migdal parameter g' [115]. The cross section the reads

$$\frac{\sigma(\omega)}{A} = \frac{g'^2_\Delta}{m_\pi^2}\frac{3\pi^2}{2k_F^3}\omega Im\frac{\Pi^0_\Delta(\omega, k=\omega)}{1 - g' - C_\rho\omega^2/m_\rho^2)(f_\Delta^2/m_\pi^2)\Pi^0_\Delta(\omega, k=\omega)} \tag{87}$$

where f_Δ is the $\pi N\Delta$ coupling constant[3]

[3] In the actual calculation the contribution of the particle-hole Lindhard function in the polarization propagator is also included and form factors of monopole type with cut-off momenta of 1.3 GeV/c and 2.5 GeV/c for the π and ρ vertices respectively are introduced.

By using strong ρ-coupling $C_\rho = 2.3$ and the fashionable $g' = 0.5$ we obtain the dot-dashed curve of Fig.13. The peak position is now shifted to higher energies. To restore it to the right place it is therefore necessary either to reduce g' (the dotted curve of Fig.13 refers to $g' = 0.3$) or to admit an $N\Delta$ binding energy difference by roughly the same amount. Note that the use of weak ρ-coupling requires even larger corrections. Nevertheless in this case too the shape is badly reproduced. For this reason we recall that the Δ inside the nucleus is strongly affected by the surrounding medium. Here we shall apply the results of the model of ref. [128].. In this model care is taken in treating properly the long range part of the Δ interaction due to the infinitely many rescattering of the real pion. This is achieved by a self-consistent solution of two coupled Dyson equations. On the other hand the effects of short range processes are controlled by few parameters which include besides the already mentioned g', a cut-off momentum Λ together with a parameter $\alpha(0 < \alpha < 1)$, which modify the $\pi N\Delta$ vertex for the nucleon above the fermi sea (a correction not accounted for by g') and a Hartree potential V_0 for the Δ. Since the self-consistent solution is obtained in the quasi-particle approximation for the pion propagator, V_0 has to take partly into account off-shell effects from short range processes which involve the π exchange itself. These include also absorptive contributions like the annihilation process $\Delta N \rightarrow NN$ so that V_0 has an imaginary part. For further detail we refer the reader to the quoted paper. The output of this calculation provides a Δ propagator in the medium to be used in calculating $\Pi^0_\Delta(\omega, k)$ in eq.(87). The result is given by the full line of Fig.13 for the following set of parameters : $g' = 0.3, \Lambda = 750$ MeV/c, $\alpha = 0.7$, $V_0 =$ (-100 - i40) MeV and a binding of ρ/ρ_0 60 MeV together with an effective mass $M_{eff} =$ (1 - 0.25 $\rho/\rho_0)M$ for the nucleons below the Fermi momentum. It must be stressed that the parameters g', Λ, α, and V_0 act all in a common way in modifying the peak position, leaving the shape of the curve practically unchanged. So, it is possible to vary them within wide ranges to get substantially the same fit. For instance a larger g' implies a larger $N\Delta$ binding energy difference and/or a smaller value for Λ. For this reason, together with the uncertainties brought about by the above mentioned approximation, it is not possible from the present analysis to infer separate informations on these parameters.

In conclusion we stress once more that only part of the width originates from ImV_0. The net result is due to the cooperative effect of these short range parts, which yield smooth contributions to the Δ self-energy, and of its energy and momentum dependence which is explicitly calculated in the model of ref.[128]. In this respect the analysis of the total photoabsorption cross section alone provides only a poor test of the model, since one might think that experimental data might be mocked up by an arbitrary ImV_0.

In addition, one should of course remark that the agreement is less satisfactory at low energy. This is due to the fact that the $E1$ contribution (dominant $\vec{\sigma}.\vec{\epsilon}$) which is known to be more relevant just in that region has been accounted for by simply rescaling the $M1$ contribution.

More stringent constraints should come as usual from exclusive measurements. In this connection let us discuss the medium modifications of the previous amplitude. First, as regards the pion momentum, if we work for simplicity in infinite nuclear matter, so that a simple expression in momentum space can be used for the pion self-energy $\Pi(q_0, q)$, one has to replace the free space momentum q satisfying the free pion dispersion condition ($\omega_q = q_0$) with q_R defined as the root of the in medium dispersion relation

$$q_0^2 - q_R^2 - m_\pi^2 - Re\Pi(q_0, q_R) = 0$$

i.e. by the in-medium pion momentum.

Such a replacement at all places in the amplitude keeps the gauge invariance of the current and is also totally consistent with all our on-shell propagation scheme in rescattering : the pion in the loops now propagates on-shell in nuclear matter with modified momentum.

As a matter of fact, by using for simplicity the γ_5 pion-nucleon coupling (and by omitting unnecessary details : the Δ is automatically gauge invariant) we have :

$$\epsilon_\mu M^\mu \simeq \bar{u}'\gamma_5 \left(\frac{1}{\not{p} + \not{k} - M} \not{\epsilon} + \frac{2q\epsilon}{(q-k)^2 - m_\pi^2} \right) u \tag{88}$$

and its divergence

$$k_\mu M^\mu \simeq \bar{u}'\gamma_5 \left(1 + \frac{2qk}{-2qk + q_\mu^2 - m_\pi^2} \right) u \ , \tag{89}$$

which is again equal to zero in the presence of the medium i.e. $\Pi(q_0, q)$ once we renormalize the photoproduced pion momentum by the substitution mentioned above [130].

Second, additional modifications of the amplitude stem from the Δ-propagator [121,131], which is known to be strongly modified in the medium. Such a propagator intervenes in J_Δ and, if one wants to take into account rescattering as well, in J_{resc} (in the $e^{i\delta} \sin\delta$ factor). In particular the Δ-width should be consequently modified.

In our model the free Δ-width is the width that one would calculate from the Δ-self-energy of a microscopical model which starts from an elementary Δ, pions and nucleons so that a many-body theory, which studies the Δ-self-energy in the medium can be employed in a coherent way.

As an illustration of the previous point it is clear that, if in the analytic expression of the free Δ-width $\Gamma(\tilde{q}) = 2/3(M/4\pi s^{1/2})(f_\Delta/m_\pi)^2\tilde{q}^3 v_\Delta^2(\tilde{q})$ one substitutes, as a first approximation the pion momentum q with the in-medium momentum q_R, a certain class of many body effects coming from the dressing of the π-propagator in the Δ-self-energy diagram is taken into account. However if the same substitution is made, rather than in a well defined microscopical theory with a cut-off for the $\pi N\Delta$ form factor $\Lambda \sim 700$ MeV/c, in a semiphenomenological description with a typical Λ of the order of 100 - 300 MeV/c numerical differences may result.

This latter case corresponds to the two presently mostly used amplitudes. The first is the Δhole approach [131], where this results for an inconsistent treatment of πN and $\gamma\pi$ dynamics (absence of the Born term in the former case). In the use of the other one [129] the final pion distortion is taken into account through an optical potential. However no possible meaningful medium modification of the operator is possible, given the lack of physical interpretation of intervening parameters, in addition to the problems of self consistency and unitarity already on the nucleon.

Of course it is difficult to assess, a priori, the numerical relevance of these effects. As a matter of fact, except in a limited number of cases, angular distributions and partial transitions measurements which we will not review here, are rather successfully reproduced, considering the usual uncertainties connected with nuclear wave functions, by the standard treatments.

A possible test might come from coherent (γ, π^0) where, since the direct and crossed Born terms practically cancel out, only the Δ survives (as in the analogous πN case for spin-isospin saturated nuclear matter [132]). This is under investigation [189].

4.4 The reaction mechanism : quasideuteron ?

The previous discussion has clarified quantitatively both the relevance of $M1$ excitation in the Δ region and the degree at which different dynamical details intervene. Indeed one can see e.g. how a dip at higher energies in the $(\gamma p, n\pi^+)$ cross section due to the $E1 - M1$ interference disappears in the total cross section on the proton because of the overwhelming role of the Δ, so as to allow, at this level, for a Δ dominance approximation.

It is therefore no great achievement just to reproduce cross sections. In particular the fact that various models have been able to parametrize part of or the total nuclear photoabsorption cross section in terms of that of the deuteron does not seem to us to be particularly exciting or illuminating. Indeed, below threshold the quasideuteron model has a physical basis because of the predominant $E1$ np photoabsorption mechanism.

However, above, this is absolutely not so. As a matter of fact $M1$ $\gamma p \rightarrow p\pi^0$ is comparable to $E1 + M1$ $\gamma p \rightarrow n\pi^+$ in such a way that, due to the isospin coefficients for π^0 and $\pi^\pm$ reabsorbtion on a second nucleon, one would naively expect a comparable ratio of pp to np pairs.

Now the intriguing experimental feature is that there is a strong depression (although probably not so quantitatively definite as generally quoted) of pp couples.

Therefore the " success" of the quasideuteron model results from the absence of a channel manifestly absent in the deuteron and in principle quite important in real nuclei. The total cross section is then made up of a quasi-free part (where a pion is emitted with various $(0, 1, \ldots)$ charged multiplicities as regards nucleons) and a quasi-deuteron (np only) part when the pion does not escape the nucleus.

Can this be explained at all ?

The problem does not seem to have bothered too much quasideuteron partisans and to our knowledge has been addressed only by Wakamatsu and Matsumoto [133]. They have indeed shown, in the Fermi gas model, the hindrance of the pp channel.

Their arguments essentially runs as follows. In the pp case the only relevant graphs are (see Fig.15) the above mentioned direct and crossed $M1$ s. Because of their near equality and of the antisymmetry of the pp wave function they lead to a vanishing contribution. On the other hand in the np case the $E1$ part is necessarily only of exchange type. Therefore, its interference with the direct and crossed (equal) $M1$ terms yields a non null result.

This essentially explains Homma's [134] as well as the old experimental results quoted in Ref.[8]

Of course, as stressed by the authors themselves, many points (final state interactions, ρ inclusion etc...) in the calculation should be treated more realistically. However, their main achievement i.e. the (pp) suppression, for which it should be probably worth looking for a more " fundamental" explanation (i.e. non necessarily tied to the FGM since it appears to be due to symmetry considerations), is a straightforward consequence of treating (γ, np) and (γ, pp) on the same footing.

This applies for energies $\omega > 200$ MeV. One may of course wonder whether this is so even around threshold.

The sense of this question can be understood by looking at Fig.(16) where the cross section per nucleon has been reported for D and for Pb . It significant dip in the first case is apparent.

This feature and its possible explanation have been discussed at length in Ref's [135,130].

Total photoabsorption cross section has been schematized as pion photoproduction in which the pion undergoes a final state interaction with the nucleus, described by the pion optical potential V_{opt}. In this way, via the many body effects incorporated in V_{opt}, a unified picture of photopion physics is given. Exchange effects below threshold appear naturally as an off-shell continuation of real (distorted) pion production above threshold.

The basic assumption in this phenomenological approach lies in the use of threshold values for the optical potential parameters and in a common off-shell extrapolation. In such a way the relevant term of the imaginary part of V_{opt} to which the cross section is proportional, is the one quadratic in the density.

In other words, the nonstatic pion, photoproduced on one nucleon, is absorbed by another nucleon pair. This is in contrast with the usual microscopic calculations of exchange effects [136,137,138,139,140] in which the γ is assumed to produce on one nucleon a static pion which is absorbed (because of off shellness) by another one only.

The picture of the underlying mechanism for photoabsorption is therefore totally at variace in the two approaches, namely, three body versus two body exchange effects.

The mechanism has been extended, in a parameter free way to (e,e') and found to account semiquantitatively for the analoguous dip observed in all nuclei between the quasi-elastic and the Δ peak [141].

The point to be stressed is, that also in this picture there are not two different mechanisms at work ref.[136], namely, exchange effects and Δ production but just the production of more or less virtual pions, described by the usual elementary amplitude which of course contains the Δ. As a matter of fact we explicitly have as decay products of our intermediate Δ resonance a final nucleon which obeys the Pauli principle and the pion whose propagation and absorption are " realistically" described by the optical potential. Its role is again the same as in photoabsorption, i.e., that with respect to the free case in which electroproduction starts off at a given threshold, virtual pions are present below it once V_{opt} is taken into account. In the literature on the opposite,exchange effects in which a pion is created and reabsorbed by another nucleon, have been distinguished in terms of the asymptotic final state (two nucleons above the Fermis sea, only one with possibility of interference with the one body quasi-elastic contribution, etc.) and added incoherently to the quasi-free Δ production.

A simple way to consider part of this presumed effect is to consider only the Δ term in the " Δ dominance" approximation as in Eq.(82)).

In such a way the complicated structure of the final two body state can be disregarded as well as the problem of the modification of the operator ($q \rightarrow q_R$) and of the pion distortion.

Indeed, it has been shown that in inclusive reactions, final state interactions can be treated consistently with the exclusive case through the introduction of the imaginary part of the optical potential [142,143].

This simply amounts then to the substitution $\delta(E_{n_{\shortparallel}} - \omega) \rightarrow \frac{1}{\pi} Im \frac{1}{E_{n_{\shortparallel}} - \omega - i\Gamma n/2}$ where in our case Γ pertains to the Δ in the medium.

The common feature to all treatments of many body π nucleus dynamics is the recognition that although a collective Δ nucleus state does not develop in the usual (low energy) sense, Δ properties are sufficiently modified by the surrounding medium. In particular, whereas its position remains practically unchanged its width increases because of different mechanism.

Probably the most striking difference with respect to the free case is that at threshold Γ is different from zero, due to the fact that the emitted (static $\vec{q} \sim 0$) pion can now be absorbed because of energy momentum conservation by a couple of nucleons.

It is then straightforward to obtain, due to such a mechanism, a photoabsorption cross section at $\omega \simeq m_\pi$ of 30 μ barn per nucleon with Γ of the order of 50 MeV.

This possible contribution, in addition to those of the electric $\vec{\sigma}.\vec{\epsilon}$ term, would then result, in an increased pp and (nn) yield.

Of course, the kinematic reconstruction of the (in the c.m.s.) back-to-back pp couple would be necessary to ascertain the existence of this mechanism against final state interaction effects.

Difficult but clean exclusive experiments are therefore awaited, not only in this connection, to really test our theoretical models and prejudices.

4.5 Compton scattering and dispersive effects

Two experiments [144,145] have recently revived interest in the possibility of using Compton scattering to extract information about nuclear structure. They were performed on ^{12}C and ^{208}Pb at fixed angles by elastically scattering photons both at intermediate energies ($\omega \simeq m_\pi$) and in the Δ region. The main results are that the differential cross section approximately reproduces in shape the photoabsorption cross section, i.e. a flattish plateau at intermediate energies and a resonance shape at the Δ, and that in the latter region experimental results are greatly at variance with respect to a simplistic treatment based on a cross section governed by the nuclear form factor squared (see below). On the theoretical side, the complementarity of Compton scattering to the other photoreactions (γ and (e,e')) had already been suggested (ref.[146]) stressing the features of this two-current process, where, as in electron scattering but with a different structure of the operator, energy and momentum transfer can be varied independently.

Let us start from the Δ region. The gross features of the process can be understand in the Δ dominance approximation and by assuming no Δ to be present in the ground state.

At this level we also neglect the effect of the spreading potential in the energy dominator (or in other words the possibility that a Δ created at x_i can decay via the Δ-nucleon interaction and hence re-emit the photon at x_j), so that a sort of closure can be applied to the sum over intermediate state $(|n> \sum_n <n|)/(E_{n_0} - \omega - i\Gamma n/2)$ with the introduction

of a mean (Δ) energy $\vec{E}$ and width $\vec{\Gamma}$. Therefore, the elastic cross section reads

$$\begin{aligned} \left.\frac{d\sigma}{d\Omega}\right|_{el} &= \tfrac{1}{2}\textstyle\sum_\lambda |\frac{1}{4\pi} < 0|(\frac{g_\Delta}{m_\pi})^2 \sum_i(\vec{S}_i.\vec{k}' \times \vec{\epsilon}\,'T_i^3)^+(\vec{S}_i.\vec{k} \times \vec{\epsilon}\; T_i^3)/\frac{1}{(E-\omega-i\Gamma/2)} \\ &.e^{i(\vec{k}'-\vec{k}).\vec{x}_i}|0> +cr.|^2 \end{aligned} \tag{90}$$

i.e. the sum of one body operators (of spin independent part $\frac{2}{3}(\vec{k}' \times \vec{\epsilon}').(\vec{k} \times \vec{\epsilon})$ with the obvious exponential behaviour proportional to the elastic form factor $F(q^2)$ where $\vec{q} = \vec{k}' - \vec{k}$.

By neglecting the crossed term

$$\left.\frac{d\sigma}{d\Omega}\right|_{el} = \frac{1+\cos^2\theta}{2}\left(\frac{1}{4\pi}\frac{4}{9}\frac{g_\Delta^2}{m_\pi^2}\right)^2 \frac{\omega^4}{(\bar{E}-\omega)^2+(\bar{\Gamma}/2)^2}A^2F^2(q^2) \tag{91}$$

The previous expression is partially constrained, because of the optical theorem, to account for the experimental photoabsorption cross section. Of course in the absence of a model the position $\overline{E}, \overline{\Gamma}$ = constant, i.e. no momentum dependence, cannot correctly reproduce the spread, i.e. $\sigma(\omega)$, of the peak. It is nevertheless enough for a semi-quantitative estimate of the scattering behaviour with $\overline{E}$ = 300 MeV, $\overline{\Gamma}$ = 140 MeV.

Its most relevant features (especially in Pb) is the dramatic depression ($O(10^{\circ})$) brought about by the elastic form factor. This is totally at variance with experimental results. The reason has been given by Arenhövel et al [147] and a refined treatment in the $\Delta - h$ model made by Vesper et al. [148]. Indeed one must not neglect the possibility to reach, with the same operator of Eq.(90), because of energy resolution, slightly inelastic state $< f|$. In such a case the cross section is proportional to the response function and, by neglecting the small inelasticity for the final photon, one gets

$$\left.\frac{d\sigma}{d\Omega}\right|_{an} = \frac{1+\cos^2\theta}{2}\left(\frac{1}{4\pi}\frac{g_\Delta^2}{m_\pi^2}\right)^2 \frac{\omega^4}{(\bar{E}_n-\omega)^2+(\bar{\Gamma}/2)^2}A\int_0^{\Delta E} R(\omega',q)d\omega' \tag{92}$$

where ΔE is the final energy resolution. (Experimentally $\Delta E/E \sim .1$).

In spite of the linear dependence on A (*vs* A^2 of the elastic term) and of the small phase space allowed, inelastic transitions can actually (as can be shown straightforwardly e.g. in the FGM) turn out to be more than competitive.

Below threshold, the first formulation in the intermediate-energy region was based on the (questionable) extrapolation of low-energy theorems [146], [13,14] which guarantee the low-energy behaviour of the amplitude.

The elastic cross section then reads

$$\begin{aligned} \left.\frac{d\sigma}{d\Omega}\right|_{el} = \; & \tfrac{1+\cos^2\theta}{2}| - \tfrac{Ze^2}{M}F(q^2) - < 0|\textstyle\sum_{i<j}(\tau_i^3 - \tau_j^3)^2 \\ & \tfrac{\vec{x}^2}{3}V^{ex}(x)e^{i\vec{q}(\vec{x}_i+\vec{x}_j)/2}|0> |^2 \end{aligned} \tag{93}$$

where the first term is due to the low energy limit scattering off protons whereas the second to scattering off np couples.

It would correspond in a non " low energy theorems" language to dispersive effects coming from the electric part of pion photoproduction.

The same dispersive effects originating from the Δ, Eq.(90) have to be necessarily included in the comparison with experimental data.

In this region, on the contrary, one has not to worry about inelasticity, due to the fact that, given a constant $\Delta E/E$, one has at the same time a smaller momentum transfer which depresses much less the elastic form factor, and smaller phase space for inelastic transitions.

However, the possibility of actually " measuring" the exchange form factor, although the extrapolation of low lying nuclear contributions of the time ordered product seems to the harmless,is probably not so straightforward due to all mentioned effects.

In general the problem of the energy resolution or of the detection of the final state is really a severe one and is paramount before Compton scattering can really serve as a complementary tool to pinpoint nuclear dynamics as first expected.

In this respect it is worth mentioning that $(e, e'\gamma)$, only apparently more complicated because of extra diagrams (which are however under control because of the kinematics), is in reality gaining ground as a bridge between (e, e') and Compton scattering by allowing at the same time for final state detection as well as for the study of nuclear distributions through the momentum transfer variation [149].

Dispersive effects from the Δ do not influence only intermediate energy Compton scattering but also its low energy properties [150]. As a matter of fact one immediatly obtains for the Compton amplitude for the nucleon (N)

$$\begin{aligned} f_{M1}(\omega \to 0, \theta) = & \frac{1}{4\pi}\frac{8}{9}\left(\frac{g_\Delta}{m_\pi}\right)^2 \frac{\omega^2}{2m_\pi}(\hat{k} \times \vec{\epsilon}).(\hat{k}' \times \vec{\epsilon}\,') \\ & = \chi_N \omega^2 (\hat{k} \times \vec{\epsilon}).(\hat{k}' \times \vec{\epsilon}\,') \end{aligned} \tag{94}$$

The obvious advantage of using the Compton amplitude rather than dispersion relation Eq.(39) is that a direct expression for all intervening quantity can be explicitly obtained.

In the latter case on the contrary, as already mentioned, electric magnetic and retardation effects are all mixted up.

The paramagnetic susceptibility χ_N defined in Eq.(34) strongly depends on g_Δ.

By using $g_\Delta = .13$ as from chapter 4.1 one obtains $\chi_N = 17\ 10^{-4}$ fm^3 , at variance with the quark model estimate of 8 10^{-4} fm^3 [51,52,53,54,55,56,57].

In this connection some comments are in order. The first is that finite size and retardation effects which are negligible for atoms and small for nuclei, are paramount for the nucleon.

They indeed contribute so as almost to cancel the paramagnetic contribution to yield the final magnetic structure constant β which is the only measurable quantity.

Hence these effects should be consistently evaluated in our model which yields

$$\frac{g_\Delta}{f_\Delta} = .05 \tag{95}$$

in good agreement with the quark model, but f_Δ and g_Δ separately at variance, (let us recall that in the quark model $g_\Delta^{(1)} = \frac{em_\pi}{2M}(\mu_p - \mu_n) = .1055$ or $g_\Delta^{(2)} = \frac{em_\pi}{2M}\sqrt{2}\mu_p = .0885$). This is no surprise because of the presence of background (Born) contributions. If one regards this picture as more physical, then because of current conservation $\{\vec{J}_\Delta \sim g_\Delta(\vec{k} - \frac{\omega}{M+\omega}(\vec{k} + \vec{p}))\}$ one cannot have the same coupling constant e.g. at $\omega = \omega_R \simeq 2m_\pi$ and at zero energy in the laboratory (as e.g. in the current non conserving formulation of Ref.[129]).

This proviso is simply meant as a warning against too hasty conclusions from the preceding numerical estimate. One should indeed consistently work out the contributions from crossed Born and pion - in flight (remember e.g. the markedly different numerical role of the crossed Born for πN at resonance and at threshold).

Within these limitations, a more " realistic" estimate of χ_N can be got by considering not the Δ but the nucleon plus a pion of momentum q to constitute the intermediate state $|n>$.

Correspondingly, the electromagnetic current has different dimensions due to the fact that it connects a nucleon to a nucleon plus a pion. It is therefore given by the photoproduction amplitude of the pion.

Moreover the sum $\sum_n$ over intermediate states is replaced by a proper integration $\frac{d^3q}{(2\pi)^3 2\omega_q}$. This will result in an extra information on the off-shell behaviour of the $\pi N\Delta$-vertex and in the possibility of calculating corrections (although in an admittled crude model as the Fermi gas model) in nuclei. This has been done in ref.[150] by using both a monopole and dipole $\pi N\Delta$ form factor v_Δ.

Two main points emerge. The first is a large sensitivity of χ_N to the cut-off parameter Λ. This is expected because one is dealing with a non-convergent integral where the integration extends over all possible momenta. One should remember , however, that the cut-off form factors $v(\Lambda)$ are derived from a fit of the πN-phase shifts in a limited momentum range. Different forms which lead to essentially the same results there thus yield for χ_N different predictions.

This can also be understood rather easily by noticing that the paramagnetism, which is totally an off-shell effect can be thought of as a much more sensitive probe of the off-shell properties of different formulations of the photoproduction amplitude which are apparently almost equivalent in the Δ region.

The second point concerns the behaviour of the susceptibility $\Delta\chi_N$ of a nucleon in a nucleus where $\Delta\chi_N$ stands for the quenching of the paramagnetic susceptibility (per nucleon) due to the Pauli principle, i.e. to the fact that in the simple FGM of ref.[150] only the states where the final nucleon momentum lies above the Fermi sea are allowed . As well known, this entails a corresponding modification with respect to the free nucleon due to the momentum range $q < 2k_F$.

This explains the insensitivity of $\Delta\chi_N$ to the cut-off and to the particular form factor used.

The values between 0.20 and 0.25 times 10^{-4} fm^3, obtained by using $g_\Delta^{(2)}$as in [150] indicate that the paramagnetic susceptibility of a nucleon embedded in a nucleus is quenched by a few percent depending upon which values is employed for the free nucleon. The use of a more realistic nuclear model with dynamical correlations would increase this value. Apart from its exact value the essential point, is that the quenching of the static magnetic polarizability is a direct consequence of a corresponding quenching of the photoproduction amplitude in the Δ-region. It is based on the idea discussed in chapter 2 in connection the anomalous magnetic moment [22,23], and taken up again recently [151] for the quenching of $\alpha + \beta$.

We finally remark on the relative importance of the different parts of the paramagnetic susceptibility coming from the excitation of nuclear states below the pion threshold and above. If we denote the former by χ_n we have as typical average value resulting from experiment [152] $\chi_n \simeq 2.4.10^{-2} fm^3$

This value is grossly overestimated by independent particle shell model calculations indicating a sizable quenching. Taking lead we obtain with χ_N =8.10^{-4} fm^3 $A\chi_N$ = 16.$10^{-1} fm^3$ i.e. a value which is roughly an order of magnitude larger than χ_n. This result, of course, is not peculiar to lead but is obtained also for other nuclei. It emphazises the strong influence of the Δ-resonance on the magnetic susceptibility . As pointed out above, the value χ_N of a free nucleon is altered by the value $\Delta\chi_N$ for a nucleon inside the nucleus. We can therefore write the total susceptibility as

$$\chi = \chi_n + A(\chi_N + \Delta\chi_N) \tag{96}$$

(Note that $\Delta\chi_N$ carries a negative sign). Comparing χ_n with $A\Delta\chi_N$ we stress that there are strong shell effects in the former and none in the latter quantity and that in particular cases, also depending on a more realistic evaluation of $\Delta\chi_N$ they can be of the same order of magnitude.

Finally, let us confront the situation of the magnetic dipole polarizability with the corresponding electric dipole polarizability. We thus have in analogy to Eq.(96)

$$\alpha = \alpha_n + A(\alpha_N + \Delta\alpha_N). \tag{97}$$

Given the experimental $E1$-photoabsorption one sees immediately that $\alpha \gg A(\alpha_N + \Delta\alpha_N)$, i.e. the nuclear $E1$-polarizability is mainly determined by excitations below the pion threshold (giant resonance, quasi-deuteron absorption, ...), in complete contrast to the magnetic case treated here. This holds true in spite of the fact that χ_N and α_N are of the same magnitude as known from photoabsorption in the Δ-region.

5 $\pi\pi$ and above

5.1 Two uncorrelated pions

As can be seen from experimental data Fig.17a) $\gamma p \rightarrow p\pi\pi$, whose threshold occurs at $\omega \simeq 300$ MeV, begins to be sizeable at $\omega \simeq 400$ MeV and reaches a broad maximum at $\omega \simeq 600$ MeV extending higher up with a value of 70-80 μb.

The general situation and can be roughly summarized as follows [153].

Out of the three main possible contributions to the final two pion states i.e. a) $\Delta\pi$ b) $\pi\pi$ (P wave proceeding via the $J = T = 1$ ρ) c) $\pi\pi$ (S wave, through $J = T = 0$ σ) it seems to be well established that the Δ mechanism, i.e. the one in which one pion resonates with the nucleon, is by far the dominant one from threshold up to $\omega \sim 700$ MeV. Mechanism b) and c) which are of course possible are found to be much smaller.

With increasing energy they play a bigger role. The overall treatment is complicated by the fact that nucleon resonances in the intermediate state may of course add up.

Their (ir) relevance for the $\pi^-\Delta^{++}$ mechanism is shown in Fig.17a). Their inclusion does not seem to alter the conclusions about the role of mechanism b) and c).

Mechanism a) is constructed, in complete analogy with single pion photoproduction, in terms of the elementary $\pi N\Delta$ coupling.

The photon is attached to the charged pion, Δ and nucleon (whose convective term vanishes in the Coulomb gauge in the c.m.s.) to make a gauge invariant amplitude together with the seagull $\gamma\pi N\Delta$ interaction. This term, which is obtained from the principle of minimal e.m. coupling in the $\pi N\Delta$ coupling $\frac{f_\Delta}{m_\pi}\psi^\mu\psi q_\mu\phi_\pi$, reads in the non-relativistic limit $\vec{S}.\vec{\epsilon}$ (analog of $\vec{\sigma}.\vec{\epsilon}$ of (γ,π)) and dominates at threshold.

This isotropic S wave real contribution, yielding a rapidly rising cross section *constitutes the essential test for the success of the model* [178,179,180] against the possible but too low background of S wave resonances. The Δ then decays via the standard $\vec{S}^\dagger.\vec{q}$ interaction.

Numerically it turns out that one has essentially to add only the pion-in flight term ($\sim 25\%$ of the cross section) to the dominant contact as shown in Fig.17c). The elementary amplitude then takes the form

$$T_{\gamma N\rightarrow\pi\pi N} = -C\frac{f_\Delta^2}{R^0 - E_R + i\Gamma/2}\left(\vec{S}^\dagger.(\vec{q}\,' - \frac{q^0}{M_\Delta}\vec{R})\vec{S}.(\vec{\epsilon} + \frac{2\vec{\mu}.\vec{\epsilon}}{(\mu-k)^2 - m_\pi^2}(\vec{\mu} - \vec{k}))\right) \quad (98)$$

where $R^0 = k + E - \mu^0$ is the actual energy of the intermediate Δ and $E_R = [M_\Delta^2 + (\vec{k} + \vec{p} - \vec{\mu})^2]^{1/2}$ its on-shell energy. The momenta and the energies of each particle are labelled in Fig.17b). The coefficient C due to isospin determines the weight of the different $N\pi\pi$

channels. It is obtained from the $\Delta N\pi$ Clebsch-Gordan $G(\Delta^{++} \rightarrow p\pi^{+}) = (\Delta^{-} \rightarrow n\pi^{-})$ $= \sqrt{3}G(\Delta^{+} \rightarrow n\pi^{+})$ etc .. which appear at the two emission vertices. They are usually used to predict e.g. a ratio of $\sigma(\gamma p \rightarrow \pi^{-}\Delta^{++})$ to $\sigma(\gamma p \rightarrow \pi^{+}\Delta^{0}(p\pi^{-}))$ of 9 and to $\gamma n \rightarrow \pi^{-}\Delta^{+}(\pi^{0}p)$ of 9/2. Whereas the agreement of the first figure with reported data [153] is fair that of the second is less satisfactory. Obviously Clebsch-Gordanries do not summarize the dynamics entering the amplitudes, which may drastically alter the previous estimates (for instance $\sigma(\gamma p \rightarrow p\pi^{0})/\sigma(\gamma p \rightarrow n\pi^{+})$ is well different from the C.G. ratio of 2).

The values of the coupling constant f_{Δ}^{2}, of the mass M_{Δ} and of the width Γ used in the quoted references are not consistent with the corresponding quantities entering the photoproduction amplitude of the previous chapter. It should be stressed in this connection that in this energy region single and double pion photoproduction cross sections are comparable and that the Δ plays a dominant role in both of them : its consistent treatment in the $\pi\pi$ case might be desirable in the future.

It is also worth stressing that because of the dominant electric $\vec{S}.\vec{\epsilon}$ term the Δ *in the* $\pi\pi$ *region is not transverse*. Hence a substantial contribution of the Δ in the longitudinal response function in (e, e') may be expected.

5.2 Vector meson photoproduction

Photoproduction above the ρ threshold can come from three different mechanisms : a) diffractive dissociation, b) resonance production, c) Born production.

The first, as can be seen from Fig.18a), corresponds to the coupling of the photon to two pions one of which scatters on the nucleon (where the black box probably contains mainly a Δ) and eventually recombines with the other one. This resonant $(\pi\pi)$ final state interaction of the mechanism of §5.1 might increase at high energy the previous cross section which decreases as $(\frac{1}{\omega^{2}})$.

However this process has never been quantitatively estimated and will be ignored in the following discussion.

As regards mechanism b), the substitution of the $\gamma N\Delta$ vertex $f_{\Delta}\vec{S}.\vec{q}$ with $g_{\rho N\Delta}\vec{S}.\vec{q}\times\vec{p}$ where $g_{\rho N\Delta} \simeq f_{\Delta}$ and a rule of the thumb estimate with respect to Born production (see the following), show that this term may not be negligible.

Since we are away from the resonance, both direct and crossed Δ terms must be considered (remember that N.R. the propagators go like $\frac{1}{\omega-\omega_{R}}$ and $\frac{1}{\omega+\omega_{R}}$ respectively) with a corresponding partial cancellation in the spin and non spin-flip terms according to the ρ charge. However, as mentioned in Chapt.4.1, the treatment of the crossed Δ introduces additional theoretical ambiguities.

Let us finally discuss mechanism c) [29].

The basic question to be answered amounts to whether the principle of minimal e.m. coupling in the free ρ Lagrangian

$$\mathcal{L}_\rho^* = -\frac{1}{4}\vec{\rho}^*_{\mu\nu}\cdot\vec{\rho}^{*\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\cdot\vec{\rho}^\mu \tag{99}$$

where $\vec{\rho}^*_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$, the arrow denoting isospin, yields the correct photoproduction amplitude. Such a prescription results in the standard $\gamma\rho\rho$ vertex [154]

$$V^*_{\sigma\mu\nu} = (2q_\mu - k_\mu)g_{\sigma\nu} - q_\sigma g_{\mu\nu} - (q-k)g_{\sigma\mu} \tag{100}$$

with the symbols as in Fig.(19).

In addition, because of the ρNN tensor coupling in

$$\mathcal{L}_{\rho NN} = g_{\rho NN}\bar{\psi}(\gamma^\mu + \frac{i\kappa^\rho}{2M}\sigma^{\mu\nu}k_\nu)\frac{\vec{\tau}}{2}\psi\cdot\vec{\rho}_\mu \tag{101}$$

a $\gamma NN\rho$ seagull term appears, so that the total amplitude (if the momentum variation of form factor is neglected) is by construction gauge invariant.

In the previous expression the tensor coupling has not to be regarded as fundamental, in total analogy with the nucleonic e.m. current

$$\mathcal{L}_{\gamma NN} = e\bar{\psi}(\frac{1+\tau_3}{2}\gamma_\mu + \frac{i\sigma_{\mu\nu}}{2M}k^\nu\ \frac{\kappa^S + \tau^3\kappa^V}{2})\psi A^\mu \tag{102}$$

of general use (as for instance in pion photoproduction). However if one imposes *invariance under a local isospin transformation* [155] for the interacting ρs and nucleons (i.e. for Eq.(99) (101) and for the free nucleon Dirac equation), which amounts to assess the freedom of choosing the phase independently from spectators, Eq.(99) is inadequate in two respects. The first is that $\vec{\rho}^*_{\mu\nu}$ must go into

$$\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu + g[\vec{\rho}_\mu, \vec{\rho}_\nu] \tag{103}$$

where $g = g_{\rho NN}$ and where the last term is due to the non abelian structure of the group. Second the rho must be massless, because the mass term is not invariant under such a transformation.

Even forgetting about the mass problem, of course the minimal substitution by using $\vec{\rho}_{\mu\nu}$ Eq.(103), modifies the $\rho\rho\gamma$ vertex prescription of Eq.(100). In addition in such a scheme *vector meson dominance* (VMD) enters as an *assumption* i.e. that for *virtual photons* the γ does not couple directly to hadrons but is mediated by the ρ.

The satisfactory solution to all these problems comes by demanding invariance of the theory under a local $SU(2)_I \times U(1)_Y$ (I = isospin, Y = hypercharge) symmetry in total analogy, mutatis mutandis, with the corresponding electro-weak case. Hence only the basics will be recalled.

An isospin triplet of massless spin one vector bosons V_μ is introduced in addition to the e.m. potential B_μ.

The covariant derivative for the fermion field is thus

$$D_\mu \psi = (\partial_\mu + i g_0 \vec{V}_\mu . \frac{\vec{\tau}}{2} - \frac{i}{2} e_0 B_\mu)\psi \tag{104}$$

The boson mass is generated in a standard way through the spontaneous symmetry breaking due to Higgs bosons. In the simplest way (consistent with the global $SU(2)$ symmetry of our problem) they are a scalar complex isodoublet i.e. four fields, three of which go into the longitudinal components of the now massive gauge bosons, the surviving fourth (of non-zero vacuum expectation value $< H >= \phi_0$) realizing the spontaneous symmetry breaking mode. Its interaction with $\vec{V}_\mu$ and B_μ yields a mass term

$$\frac{1}{4}\phi_0^2 g_0^2 [V_\mu^{(1)^2} + V_\mu^{(2)^2}] + \frac{1}{4}\phi_0^2 (g_0 V_\mu^3 - e_0 B_\mu)^2 \tag{105}$$

which can be diagonalized through a rotation in terms of the physical ρ_μ^0 and A_μ

$$\begin{aligned} V_\mu^3 &= \frac{1}{\xi}\rho_\mu^0 + \frac{e_0}{g_0}\frac{1}{\xi}A_\mu \\ B_\mu &= \frac{1}{\xi}A_\mu - \frac{e_0}{g_0}\rho_\mu^0 \end{aligned} \tag{106}$$

where

$$\xi = \frac{1}{\sqrt{1+\frac{e_0^2}{g_0^2}}} \qquad g = \frac{g_0}{\sqrt{1+\frac{e_0^2}{g_0^2}}} \qquad e = \frac{e_0}{\sqrt{1+\frac{e_0^2}{g_0^2}}}$$

Hence the physical photon is massless as due, and there is a mass splitting between charged and neutral ρs

$$2\,\frac{m_{\rho^0} - m_{\rho^\pm}}{m_\rho} = (\frac{e}{g})^2 \tag{107}$$

which yields for $g^2/4\pi = 2.3$ (see below), $\Delta m = 1.22$ MeV.

This prediction has to be compared with the weighted experimental average of 0.3 ± 2.2 MeV [157]. New and consistent measurements would be welcome.

This relation has been originally obtained by Bando et al [158,159,160] in a non linear theory of pions where nucleons appear as solutions of the corresponding Skyrme Lagrangian.

In this electro-strong coupling, the new free parameter of the theory is given by the Higgs mass m_H. Since the Higgs couples to ρs with a coupling $g_0 m_\rho$ (times isospin coefficients), it might be observed in the ρ photoproduction process $\gamma N \rightarrow N \rho H$ as well as in $e^+e^- \rightarrow \rho^0 H$.

As in the analogous electro-weak case, its confirmation is essentially an experimental problem.

However due to the strong coupling ($\frac{1}{\pi}\frac{g^2}{4\pi} \leq 1$) the phenomenological analysis relevant in that framework [161] cannot be simply taken over to our case.

As regards its mass the only handwaving consideration one can make at present is that because of the coupling $H \rightarrow (\rho\rho)NN$, a light Higgs mass is forbiddent by the analysis of the nucleon-nucleon potential [162].

Whether the Higgs is a reality or simply an artifact to account for more fondamental things is not clear [163]. In order to further check the present picture one should concentrate rather than on the Higgs itself on possible tests of the non abelian structure of the theory.

In this connection it has to be stressed first that *the previous diagonalization is possible at all k^2-values* i.e. also for virtual particles. It entails that in considering the propagators of the physical ρ^0_μ and A_μ, the off-diagonal term $V^3_\mu B_\mu$ is absent.

To see the consequences in the interaction with matter fields let us first introduce the coupling with electrons

$$\begin{aligned} \mathcal{L}_{e,int} &= -e_0 \bar{\psi}_e \gamma^\mu B_\mu \psi_e = \\ &= -e\bar{\psi}_e \gamma^\mu (A_\mu - \tfrac{e}{g}\rho^0_\mu)\psi_e \end{aligned} \tag{108}$$

One has therefore the standard γee coupling, plus a *direct* $\rho^0 ee$ interaction of strength e^2/g.

In all electromagnetic processes i.e. $e^+e^- \rightarrow e^+e^-$ or Bhabha scattering, propagators combine so as to yield

$$e^2\left(\frac{1}{k^2} + \frac{e^2}{g^2}\frac{1}{k^2 - m_\rho^2}\right) \tag{109.a}$$

the momenta of both virtual particles being labelled by k as from Fig.19. The following comments regarding Eq.(109.a) are in order. At low momentum transfer ($k^2 << m_\rho^2$) the photon contribution dominates and one measures, as due, the physical electric charge e. This is clearly seen by considering that at high momentum transfer the photon propagator just gets multiplied by $e^2(1+\frac{e^2}{g^2}) \equiv e_0^2$ explicitly showing the strong interaction contribution to the electric charge renormalization. This is obvious since in the latter limit masses can be neglected and there is no mixing so that only B_μ, because of quantum numbers, can couple to electrons. The result of Eq.(108) is consistent with the experimental limits on strong interaction modifications of the above mentioned electromagnetic processes as well as with the standard treatment of the strong interaction vacuum polarization in the $g-2$ of the muon [164].

When considering $e^+e^- \rightarrow \rho^+\rho^-$, if and only if, as due from the rotation in the 3-linear coupling in the analog of $\rho_{\mu\nu}\ \rho^{\mu\nu}$ where V_μ fields, enter,

$$\frac{V^{\gamma\rho^+\rho^-}_{\sigma\mu\nu}}{e} = \frac{V^{\rho^0\rho^+\rho^-}_{\sigma\mu\nu}}{g} = (2q_\mu - k_\mu)g_{\sigma\nu} - (q+k)_\sigma g_{\mu\nu} - (q-2k)_\nu g_{\sigma\mu} \tag{110}$$

the fields appearing at the vertices Eq.(108)(110) naturally combine to yield the propagators

$$e\frac{1}{k^2}e - \frac{e^2}{g}\frac{1}{k^2 - m_\rho^2}g \equiv \frac{e^2}{k^2}\frac{m_\rho^2}{k^2 - m_\rho^2} \tag{109.b}$$

i.e. the VMD recipe [156].

As a consequence of Eq.(110), the $\gamma\rho\rho$ vertex has additional contributions with respect to the prescriptions of Eq.(100). The possibilities of using e^+e^- as an additional test of the present framework by predicting in a parameter free way (because of the fixed $\rho\rho\rho$ and $\rho\rho\rho\rho$ couplings) the large 4π and 6π cross sections [165] will be examined elsewhere.

Additional hadrons can be introduced in the present scheme, by requiring their total interaction to be invariant under the total $SU(2)\times U(1)$ group under consideration.

Let us consider pions.

In this case the only possible (universal) coupling between the gauge fields and pions obeying the symmetry group requirements is

$$\mathcal{L}_{\pi,int} = g_0\vec{V}_\mu.(\vec{\pi}\times\partial_\mu\vec{\pi}) \tag{111.a}$$

(in elementary terms since the pion current is an isovector it can only be coupled to the isovector gauge field in order to form an isoscalar interaction).

By reexpressing V_μ in terms of the physical fields one has for the third component

$$\mathcal{L}_{\pi,int^{(3)}} = g(\rho^0_\mu + \frac{e}{g}A_\mu)[\vec{\pi}\times\partial_\mu\vec{\pi}]^3 \tag{111.b}$$

i.e. both the $\rho^0\pi^+\pi^-$ and the standard direct isovector coupling of the photon *(irrespective of whether it is real or virtual)*.

It is worth stressing the general validity of the previous result which has also been obtained in the non linear sigma model of Ref.[158] with a particular choice ($a=2$ reproducing the KSFR relation) for a sort of a Lagrange multiplier a in the total Hamiltonian. In $ee \rightarrow \pi\pi$ scattering (in all channels) the fields entering Eq.'s(108,111.b) combine as in $\rho^+\rho^-$ case Eq.(109.b) predicting the ρ dominance of the e.m. pion form factor .

In the present approach the whole axial sector as well as the interesting speculations about the description of baryons in terms of effective mesonic degrees of freedom only [166], has been left out. It is more economical for our purposes to regard the nucleon as well as an explicit degree of freedom for low-energy phenomena.

It is then clear, by taking for simplicity only the vector part, that $\mathcal{L}_{\gamma NN}$ and $\mathcal{L}_{\rho NN}$ Eq's (101,102) derive by reexpressing again the gauge fields appearing in the covariant derivative entering the VNN interactions as

$$\begin{aligned} \mathcal{L} &= g_0\bar{\psi}\gamma^{\mu}\left(\frac{\rho_\mu}{\sqrt{2}}\tau^- + \frac{\rho^*_\mu}{\sqrt{2}}\tau^+ + A_\mu\left(\frac{1+\tau^3}{2}\right)\frac{\xi}{\sqrt{1+\varsigma^2}}\right. \\ &\left. + \frac{\tau^3-\varsigma^2}{2}\frac{1}{\sqrt{1+\varsigma^2}}\rho^0_\mu\right)\psi \end{aligned} \tag{112}$$

One gets therefore at the same time the usual coupling constant $g = g_{\rho NN}$ (apart from small $O(\frac{e^2}{g^2})$ isospin breaking effects) of the ρ to nucleons and a corresponding e for the e.m. field.

As before, for the isovector part A_μ and ρ^0_μ appear in the right combination because they are stemming from the V^0_μ coupling, which makes the propagators subtract, thus reproducing also in $eN \to eN$ the VMD result.

The direct coupling of the photon and of the ρ both to (e, e') and to (N, N') makes clear that, automatically, *such a combination is possible also for the induced tensor part only if*

$$\kappa^V = \kappa^\rho = 3.7 \tag{113}$$

This sort of generalized VMD results quite naturally in our scheme since also induced tensors must come from the original VNN Lagrangian through the same rotation which makes A_μ and ρ^0_μ appear in the e/g ratio for the vector interaction Eq.(112), necessary to reproduce the VDM recipe without assumptions. The prediction of Eq.(113) may seem in contradiction with the value usually quoted $\kappa^\rho \simeq 6.6$ [167]. In this connection it has to be observed that the extraction of the tensor coupling from experimental data is a model dependent procedure.

In particular the result by Höhler and Pietarinen is based on the neglect of the intermediate Δ in the iteration of the "background" one pion exchange potential.

Indeed these numbers should not be taken at their face value. In spite of their importance as regards for instance the sign of the net (pion + ρ) NN tensor force, they have a significance only in a given context i.e. depend on the other ingredients present in the model and in the wave equation used (let us remark in passing that $\kappa^\rho \simeq 6.6$ for the Bonn potential [61] whereas $\kappa^\rho \simeq 1$ for the Paris potential [168].

As regards the isoscalar part one has to enlarge the previous $SU(2)$ group to accomodate the ω in order to have the proper orthogonal combinations.

Whence one understands the equality between the isoscalar photon- and $\omega - NN$ tensor coupling constants.

It is therefore worth stressing that from the knowledge of the nucleon anomalous magnetic moments Eq.(102) and from the experimental $\rho, \omega \rightarrow e^+e^-$ widths ($\Gamma \sim \frac{\alpha^2}{g^2}m$), i.e. from purely e.m. processes, one can predict quantitatively the vector boson part of the nucleon nucleon potential, namely the near equality of the repulsive ω term due to the vector coupling and of the attractive ρ determined by the tensor coupling. This predictivity holds up to form factors included (as discussed below).

The same mechanism we are advocating shows up also in C.V.C. [169,170] revisited. In other words the "weak magnetism" term also derives from a field rotation in the induced tensor part of the interaction. It is amusing to realize that the e.m. value of κ^V =3.7 determines the corresponding weak as well as strong interaction coupling constants.

The previous results combine straightforwardly to derive the ρ photoproduction amplitude in the simplified case of no momentum transfer variation of the form factors at the vertices. Pedagogically one can separately discuss neutral and charged photoproduction, without and with induced tensor coupling.

Denoting the general amplitude by $\epsilon^\mu \rho^\nu M_{\mu\nu}$, one can immediately verify in the neutral case that both

$$\kappa^\mu M_{\mu\nu} = 0 \tag{114.a}$$

and

$$q^\nu M_{\mu\nu} = 0 \tag{114.b}$$

i.e. that the amplitude is both gauge invariant and ρ transverse . In the charged case without tensor terms, provided ones uses the final on-shell rho transversality condition $q^\nu \rho_\nu = 0$ (ρ_ν denoting the rho polarization), both prescriptions Eq.(100) and Eq.(110) lead to a gauge invariant amplitude.

This is obvious since the two differ by

$$\Delta V_{\sigma\mu\nu} = -k_\sigma g_{\mu\nu} + k_\nu g_{\sigma\mu} \tag{115}$$

which corresponds to a (k divergenceless) unit anomalous magnetic moment for the ρ.

However only the second prescription leads to a ρ-transverse amplitude, in accord with the general principles of Yang-Mills fields [171].

The introduction of tensor terms shows, again, that only if $\kappa^V = \kappa^\rho$ ρ transversality is satisfied. If one considers in addition electroproduction, VMD is obtained automatically only thanks to the above-mentioned proportionality among $\gamma NN - \rho NN$ and $\gamma\rho\rho - \rho\rho\rho$ vertices. Also in such a case the amplitude is transverse, proving the strict connection between universality and transversality. It is parenthetically worth stresssing that the shadowing properties for real photons (see next §) have to be determined by a mechanism where only the γ enters (lim. $k^2 \rightarrow 0$ in Eq.(109.b)) and moreover that γ and (e, e') (where both particles intervene) do not have to have, a priori, the same shadowing behaviour .

In conclusion the requirement of invariance of the theory under a local $SU(2) \times U(1)$ group provides a consistent framework to accomodate predictively and consistently for a number of physical properties which were separately known or assumed.

One should note in particular, because of the link between e.m. and strong interactions, that the presence of genuine (i.e. non arising from iteration) three-body parameter free forces due to the $\rho\rho\rho$ vertex (in addition to those coming from $\rho\pi\pi$) follows naturally, although their numerical relevance is at present far from settled.

In addition, there is a parallel link among e.m and strong form factors.

Let us first recall that the isovector electromagnetic form factor of the nucleon is only measured in the time-like region for $s \geq 4M^2$ and in the space-like ($t \leq 0$) region where it is reasonably well reproduced by a dipole fit $(\frac{\Lambda^2}{\Lambda^2-t})^2$ with $\Lambda \sim 800$ MeV.

Hence $g_{\rho NN}(t) \simeq g_{\rho NN}(0)\ \frac{\Lambda^2}{\Lambda^2-t}$, this additional intrinsic form factor with respect to the one naturally provided by VMD being simply understood as originating from vertex corrections due to intermediate pions (and ρs).

The usual parametrizations of the ρNN vertex from the NN potential as $\frac{\Lambda'^2-m_\rho^2}{\Lambda'^2-t}$ with $\Lambda' \sim 1300$ MeV reproduce the variation of the previous form in the relevant $-m_\rho^2 \leq t \leq 0$ region rather well .

As regards the pion, its form factor is measured for all t and s values. The ρ coupling leading to Eq.(109.b), with the proper introduction of the ρ width, essentially accounts for all of its structure i.e. $g_{\rho\pi\pi}(t) \simeq \text{const} = g_{\rho\pi\pi}(0)$.

Deviations from a strict constancy can be appreciated from the comparison of the measured $\frac{g^2_{\rho e^+e^-}}{4\pi} = 2.3 = \frac{g^2}{4\pi}$ with $\frac{g^2_{\rho\pi\pi}(m_\rho^2)}{4\pi} \simeq 2.8$

Because of its manifest lack of structure, the first process yields the universal g coupling constant in agreement with the measured $g_{\rho\pi\pi}$ and $g_{\rho NN}$ at $t = 0$. Of course such a universality should not hold at $s = m_\rho^2$ since, whereas $\frac{g_{\rho\pi\pi}(m_\rho^2)}{g_{\rho\pi\pi}(0)} \simeq 1.1$ the information from e.m. form factors guarantees that $\frac{g_{\rho NN}(m_\rho^2)}{g_{\rho NN}(0)} \geq 2$, according to the extrapolation.

The only place where one can measure $g_{\rho NN}(m_\rho^2)$ is represented by ρ photoproduction (in the Born graphs of fig.18). Once form factors are allowed however, the photoproduction amplitude is, as well known, no longer gauge invariant nor ρ transverse. Because of the previous discussion, both these properties have to be always obeyed by adding counterterms not uniquely determined by low energy theorems [192,193,194]. To get an idea about the orders of magnitude involved one immediately obtains for charged ρs at threshold, the analog of the Kroll-Rudermann

$$\begin{aligned} |\frac{k}{q}|\frac{d\sigma}{d\Omega} &= |\frac{M}{4\pi s^{1/2}}\sum_{pol}\frac{ef'_{\rho NN}}{m_\rho}\vec{\sigma}.\vec{\rho}\times\vec{\epsilon}|^2 \\ &\simeq \frac{\alpha}{4\pi}f'^2_{\rho NN}\frac{1}{(M+m_\rho)^2} \end{aligned} \tag{116}$$

coming from the seagull plus the direct Born with ρNN and γNN vector vertices. Notice that $\frac{f'_{\rho NN}}{m_\rho} = (1+\kappa^\rho)\frac{g_{\rho NN}}{2M}$ so that $f_{\rho NN} = \frac{1}{2}f'_{\rho NN}$.

The previous term is not as dominant as in the pion case since corrections in the former case are $0(\frac{m_\pi}{M})$ whereas here $0(\frac{m_\rho}{M})$. This underlines the importance of the correct form for $V_{\sigma\nu\mu}$ since, as can be immediately seen e.g. by a non relativistic reduction at threshold, ρ-in-flight terms are a substantial correction to the seagull.

An estimate of Eq.(116) with the quoted value for the coupling constants at $t = 0$ yields $\simeq 12\mu b/Sr$ for charged ρs i.e. roughly the same magnitude than for charged pions. To have the final correct amplitude already on the nucleon, of later use in a nuclear context, all mentioned problems connected with form factor variation (with some possible additional ambiguities concerning the momentum transfer at the seagull vertex) have to be explicitly worked out.

Let us finally come to ρ exchange effects, and let us first establish the connection between the commutator of the dipole operator with the ρ exchange potential and virtual ρ photoproduction in the unphysical long wavelength and static limit as used in the derivation of the Bethe-Levinger sum rule.

In this limit no differences arise (see Eq.115) from the use of the correct prescriptions for the $\gamma\rho\rho$ vertex intervening in the ρ-in-flight diagram.

As regards the other terms it is immediate to check that in Eq.(23) the term $\vec{\sigma}.\vec{\rho}\times\vec{\epsilon}$, corresponds to the previous N.R. limit and $(\vec{\sigma}\times\vec{q}.\vec{\rho}2\vec{q}\vec{\epsilon})/(\vec{q}^2+m_\pi^2)$ to the N.R. limit of the first term in $V_{\sigma\mu\nu}$ of the ρ in flight diagram (i.e. the pedestrian attitude of treating the rho in flight in total analogy with the pion in flight).

The term $q_\nu g_{\sigma\mu}$, even if present in principle because $q_\nu\rho^\nu = 0$ does not hold for off-shell particles, does not contribute when the virtual ρ is attached to a conserved current i.e. to the second nucleon. Therefore the only additional contribution comes from the non contact ρ photoproduction off the nucleon.

Of it, only the part coming from positive energy states can be dropped if ρ correlated ground state wave functions are used. As usual a potential formalism inherently yields an approximate form of virtual photoproduction. Consequently also as regards nuclear Compton scattering the cancellation (necessary to ensure the low energy Thomson limit, outlined in Ref.[19] for the pion) between κ_ρ and the negative dispersive effects due to real rhos plus the genuine seagull (i.e. two photons at the same point) off the exchanged ρ is not exact. This can be regarded as a first approximation and as proof of the limits implicit in a Hamiltonian formulation at low energy.

The above considerations explain in any case the preference for a microscopical potential rather than for a phenomenological one, since in the former case the double commutator can be naturally connected to a genuine photoproduction process.

Therefore the standard attitude of identifying κ_ρ with the integral up to the pion mass of the experimental photoabsorption cross-section which was already shaky in the π case, is totally non quantitative in connection with virtual rho photoproduction.

In the closely related and much more studied [172,173,174,175] (e,e') process, because of the higher momentum transfer e.m. form factors must be introduced and the amplitude obeying Eq.(114) can be reconstructed in a non unique way.

As regards the interaction a non linear $\rho\rho\rho$ coupling had already been used in Ref.[174] together with the VMD assumption of attaching the virtual γ to the ρ with a coupling $\frac{e}{g}m_\rho^2$.

In the light of our previous considerations it corresponds in the end to the correct formulation provided one uses at the same time $\kappa^\rho = \kappa^V$.

Such an approach obviously and correctly differs by a factor of two in the transverse exchange current due to the extra anomalous magnetic moment Eq.(100) with respect to the recipe Eq.(103) used in Ref.[176] and in Ref.[177].

5.3 Shadowing

The fact that reactions on nuclei initiated by non strongly interacting particles might show a departure from a linear dependence on the atomic number A, has been pointed out a long time ago by Bell [181] and by Stodolsky [182] and extensively considered in the literature [183,184].

These authors have explained the apparent paradox that a particle with a very long mean free path can be substantially shadowed by other nucleons in traversing a nucleus.

Indeed, according to classical arguments whereas it is easy to understand shadowing for a strongly interacting particle, since the beam intensity, due to the large cross section, decreases in traversing the nucleus, with a corresponding geometrical $A^{2/3}$ behaviour, the appearance of such a phenomenon in photoreactions may seem puzzling.

As well known one already talks about shadowing in the classical scattering of light [185]. In such a case the forward propagation through an infinite slab of material of the unscattered beam is changed, because of the interference with the coherently scattered waves (secondary wavelets) by the amount $\exp(i2\pi d\, f\, z/k)$, (where d is the nuclear density,

k the photon momentum, f the forward scattering amplitude and z the distance travelled in the medium). This leads to a complex index of refraction and to a corresponding damping of the transmitted wave proportional to the total electromagnetic cross section (optical theorem). Hence generally speaking this quasielastic mechanism can provide an effect of the order of the percent.

The substantial shadowing observed in the region $\omega > 2$ GeV (see Fig.20), has been explained either in the vector meson dominance scheme or by using a description where photons are decoupled from rhos.

Although " the question of whether the photon changes into a ρ before or after reaching the nucleus is a purely matter of taste" [184], we prefer to keep the second (less misleading) picture for two distinct reasons.

The first is that in a correct treatment of the e.m. interaction of rhos and nucleons, ρs are decoupled from photons as shown in the previous §. The second is that one must explain at the same time why shadowing is not observed, on the contrary, in the Δ region(see e.g. Ref.[7]) where ρs play manifestly no role.

A common framework is then required. This is provided by the obvious observation that photoabsorption at a given energy is simply given by photoproduction of the real allowed particles eventually followed by final state interactions.

As regards the Δ region, sum rules have been derived for the magnetic part of photoabsorption [122,124] predicting a linear dependence on A in this region in contradiction with earlier results obtained from dispersion relations [126]. However both results are unreliable.

The first because is based on the assumption of a final Δ and hence misses the possibility of interference (see below). The second because both the theoretical knowledge and the experimental photoabsorption strength entering dispersion relations in the " asymptotic" region considered ($\sim$ 20 GeV) prevent from any definite conclusions.

Shadowing is a simple quantum mechanical interference effect between one and two-step processes, whose presence or absence is governed by the A dependence of the coherent photoproduction mechanism [187].

As a matter of fact the amplitude for the general $|0> \rightarrow |n>$ transition is given in the plane wave Born approximation for the final particle by

$$\begin{aligned} M \simeq &< n|\textstyle\sum_i e^{i(\vec{q}-\vec{k}).\vec{z}_i}(\vec{L}.\vec{\sigma}_i + K)|0> \\ &+ \int < n|f(q,q')|0> \frac{d^3q'}{(2\pi)^3 2\omega_{q'}} \frac{<0|\sum_i e^{i(\vec{q}'-\vec{k}).\vec{z}_i} K|0>}{\omega - \omega_q - i\epsilon} \end{aligned} \tag{117}$$

where $f(q,q')$ is the amplitude for inelastic scattering on the nucleus of the intermediate particle of momentum q' and energy $\omega_{q'}$ produced by a photon of momentum $\vec{k}$ to a final particle of momentum $\vec{q}$. In standard notation L stands for the spin flip part of the photoproduction amplitude and K for the spin independent.

For simplicity the small nucleus recoil has been neglected. We are then reaching the same nuclear final state $|n>$ via the direct process plus the one in which a neutral particle (π^0, ρ^0) is coherently photoproduced (spin independent part of the Hamiltonian) on the nucleus, followed by inelastic rescattering in all three charged states.

By assuming on shell propagation to be dominant (or at least indicative), such a mechanism does indeed reduce the initial wave (in fact the spin independent part K if we neglect the spin structure of f which is known to play a small role) since we get $+i$ from the energy denominator and $+i$ from the imaginary part of f. The one and two-step mechanism of Eq.(117) are respectively reported in Fig.21a) and b).

In writing down Eq.(117) several approximations have been made.

The first concerns the crossed term which yields manifestly always only an off-shell contribution and has hence been neglected in accord with our previous position.

The second regards the fact that in the two step mechanism it is possible to reach the final state $< n|$ via any intermediate state $|n' >$.

Whether it is legitimate for our purposes to drop it, will be discussed in the following.

All the usual quantum mechanical considerations appropriate to two-step processes can be applied to the second term of Eq(117). In particular (see e.g. [195]) the second step represents the probability amplitude for reaching $|n>$ via the final strongly interacting particle.

Its evaluation which is essential for our arguments, is intrinsically difficult for a number of reasons. The first is that estimates are generally based on a geometrical " eikonal" model of mean free path.

This is questionable not only because of its probable excessive simplicity, but also because of the fact that when considering the problem with correct boundary conditions, the modification of the in medium momentum introduces residues [188], which can be easily evaluated only for a uniform medium. Hence A independent. Whereas on the contrary a finite geometry is necessarily used for the disappearence probability.

With this proviso in mind, the probability for the second step, i.e. that a particle with mean free path $\lambda = \frac{1}{\rho\sigma}$ survives after having quasielastically scattered through a nucleus of radius R, is given by the ratio of the mean free path times the scattering cross section over the nuclear volume i.e. by the well known expression $P = \frac{\lambda\sigma}{4/3\pi R^3} = \frac{3}{4}\frac{\lambda}{R}\frac{\sigma}{\pi R^2}$ where $\sigma = \pi R^2[1 - (1/2y^2) + (1/2y^2)(1+2y)e^{-2y}]$, $y = \frac{R}{\lambda}$.

Already for $\lambda = \frac{R}{2}$ the term in brackets differs from 1 by about 10% so that the cross section practically corresponds to the geometrical (black disc) one. Therefore the amplitude probability of the second step is given by $\sqrt{3/4(\lambda/R_0)}A^{-1/6} \equiv \sqrt{P'}A^{-1/6}$.

In the one step part, of course all final states $|n>$ are allowed, so that one has to consider coherent photoproduction in addition to all quasielastic transitions ($|n>\neq|0>$). Keeping to these latter ones, which are generally thought to represent the bulk of the process it is obvious that their contribution to σ goes like A (hence $A^{1/2}$ in the matrix element).

Therefore shadowing can happen only if the two step contribution has a different A dependence.

From the previous arguments, for all photoproduced particles the sum over excited intermediate states $|n'>$ would modify the one step part amplitude by a term proportional to $A^{-1/6}$, hence resulting in a small antishadowing common to all energy domains, which in view of our necessarily very simplified evaluation, we regard as consistent with constancy.

Therefore shadowing can come only from the coherent effect of an elastic first step, and explicitely depend on the form of the photoproduction amplitude. In this connection it is worth stressing that shadowing cannot be counterfeited just by using an optical potential for the outgoing particle ; this approach misses the essential point i.e. interference .

Let us now consider the different energy regions.

i) Δ region and pion photoproduction.

Experiments on γ,π^0 [190] exist, but since the (small) coherent part of the process has not been separated out, they are inconclusive in this respect.

Theoretically, K is given by the well known $\vec{q}.\vec{\epsilon}\times\vec{k}$ term, and the amplitude by

$$\int d^3r\rho(r)e^{-i\vec{k}.\vec{r}}(\vec{\nabla}\phi_\pi.\vec{\epsilon}\times\vec{k})$$

where $\rho(r)$ is the spherical nuclear density and ϕ_π the pion wave function. By assuming for simplicity a plane wave for the pion one obtains $\vec{q}.\vec{k}\times\vec{\epsilon}AF(|\vec{k}-\vec{q}|)$ where F is the nuclear elastic form factor, with the usual normalization $F(0)=1$.

The fact that the coherent amplitude does <u>not</u> go like A has been somewhat recognized in the literature.

Indeed, in the $\Delta-h$ model the possible shadowing effect we are talking about [131] (which is needless to say completely general and not peculiar to that model) has been considered and it has been argued that " the A^2 factor associated with a coherent process (in the cross section) is more than compensated by damping of the pion wave function and by the nuclear ground state form factor" . However, for technical reasons calculations do not go beyond ^{16}O so that reliable predictions about the A dependence cannot be made.

Some heuristic considerations have been recently made [188] observing that application of Green's theorem to the previous expression yields

$$-\int d^3r\rho(r)\phi_\pi(\vec{\epsilon}\times\vec{k}.\vec{\nabla}\rho)e^{i\vec{k}.\vec{r}}$$

This suggests that the production mechanism is probably more sensitive to the surface, rather than to the bulk of the nucleus, with a corresponding $A^{2/3}$ dependence. In addition also a realistic treatment of the distortion of the pion wave function favours a surface peaking. In this extreme case the A dependence of the second term of eq.(117) is the same $A^{1/2}$ of the direct term ($\sigma \sim |M|^2 \sim A$). Therefore even an increasing σ_{coh} (i.e. $\alpha = 4/3 > 1$), provided it does not exceed a typical 10% at most (i.e. in heavy nuclei) of the quasi free process to which it must be added, does the job, since its additional effect as a shadowing agency in the two step process is null.

The previous study case can therefore be considered as an upper limit for the non-occurrence of shadowing. Of course, if σ_{coh} goes like A^α with $\alpha < 1$, shadowing necessarily does not take place.

With the warning that of course in all these considerations there is no universal A dependence but that it may differ as a function of ω, preliminary results of a detailed calculation all over the nuclear table [189] show a decrease in A of coherent (γ, π^0).

This substantiates the simple idea that shadowing does not occur in the Δ region because of the surface nature ($\vec{q}.\vec{k} \times \vec{\epsilon}$ which acts against collectivity) of the coherent π^0 photoproduction amplitude.

ii) 500 Mev $< \omega <$ 1000 MeV

In this region the dominant mechanism has been shown to be the two pion production via the Δ contact term $\vec{S}^+.\vec{q}\vec{S}.\vec{\epsilon}$. In this case, as well, the non spin flip part i.e. $2/3\vec{q}.\vec{\epsilon}$ is a surface interaction,, depressed in the forward direction $(\sum_\lambda (\vec{q}.\vec{\epsilon}_\lambda)^2 = q^2 \sin^2\theta)$. Hence we predict no shadowing in this region either.

iii) ρ production ($\omega > 1$ GeV)

Here, we can indeed have coherent ρ^0 production with the desired properties (i.e. volume effect) via $\rho^\mu \epsilon_\mu$ terms (ρ standing here for the rho polarization), as confirmed e.g. by an explicit calculation of Born diagrams.

This can be immediately understood in more physical and simple terms by recalling that since the ρ has the same quantum numbers of the photon one can think of the above term as deriving from the corresponding $\epsilon^\mu \epsilon'_\mu$ of Compton scattering.

In such a case the one and two step processes give a contribution to quasielastic scattering $\sigma \sim A(1 - xA^{1/3})^2$ where $x = \sqrt{P'}.\alpha$ stands for the probability amplitude mentioned above times the ratio of the coherent to quasielastic amplitude. To this we must of course add $\sigma_{coh} \simeq \alpha^2 A^2$. It is then easy to see semiquantitatively that an increasing coherent cross section can nevertheless indeed cause shadowing.

Let us also mention that the rho has been considered as a stable particle and that the relaxation of this assumption results in smaller effects because of a reduced mean free path due to decay. Of course with increasing energy this effect will disappear following the relativistic time dilatation.

To get an idea about the order of magnitude of the effect let us recall that the percentage of cross section per nucleon at $\omega \sim 2630$ MeV, respectively for D, O, Ca and Pb is .96, .89 $\pm$ .02, .83 $\pm$ .02, .83 $\pm$ .02.

By recalling that $\sigma(\rho N) \sim 40$ mb yields $\lambda \sim 1.4$ fm, we see that this is somewhat accounted for by $\alpha \sim 1 \div 2\%$, showing that even a small interference can cause appreciable shadowing effects.

The present considerations are applicable to (e, e'), with the usual conclusion, as regards the virtual γ, that at fixed ω (or invariant mass s) $k^2 \neq 0$ implies more damping of the nuclear form factor (hence smaller x) with respect to the real case.

As regards the experimental situation, in the same energy loss region we are considering for real photons, there seems to be some evidence at moderate momentum transfer for a slightly smaller shadowing than for real photons [191].

It is worth mentioning in this connection the possibility, to be explored with future machines, of a different behaviour of the longitudinal and transverse response function. As regards, the Δ region it is immediate to realize from the form of the photoproduction operator that no coherent longitudinal π^0 electroproduction is possible. Hence no shadowing will be observed in the longitudinal response function either.

As regards the ρ region, because of the foregoing arguments, one can have on the contrary coherent longitudinal ρ^0 electroproduction. Its ratio to quasielastic processes would then determine the independent longitudinal (e, e') shadowing (or antishadowing) behaviour.

The experimental confirmation of the absence of shadowing in the region of photoproduction of two uncorrelated pions and the investigation of the above mentioned possible different shadowing behaviour in electron scattering are awaited.

6 Conclusions

As a first general remark, one may provocatively question altogether the advantages and the extra informations one can get from the electromagnetic probe over strong interactions. Indeed the fact that photoproduced particles must undergo the same final state interactions of the corresponding strong process, drastically alters the naive picture of the weak probe. However in the latter case, due to the different multipolarities of the e.m. field, different mechanisms of excitation of the same final state are possible first, and second one is in the presence in the nuclear case, in general, of a volume interaction with the ensuing possibility of exploring different nuclear density regions.

Therefore photoreactions really furnish complementary pieces of information.

In the present paper photoreactions on nuclei up to the GeV region have been reviewed.

The main features of the three intrinsically intertwined regions i.e. i) low energy up to the pion threshold ii) Δ, and iii) vector meson region, into which we have nevertheless schematically subdivided our treatment are the following. As regards the operators to be used in the different energy regions, processes in i) are satisfactorily described in terms of virtual photoproduction, consistently with the corresponding operators used above threshold .

As regards ii) the problems connected with unitarity and frame transformation have been overcome in a consistent and parameter free way, providing an amplitude of unambiguous extension to nuclei.

In region iii) the problem of the photoproduction amplitude of vector mesons has been solved thanks to a Higgs mechanism, similar to the one operative in electroweak interactions. However, it has still to be put in a form appropriate for nuclear physics.

As regards the total photoabsorption cross section it is well understood why in region i) the integrated cross section is linear in A with one classical sum rule contribution from the kinetic part and a comparable one of dipole ($E1$) exchange character. Possible magnetic ($M1$) contributions are on the opposite strongly depressed.

In the Δ region, the total integrated cross section is again linear in A due to the particular form of coherent π^0 photoproduction which might, in principle, cause shadowing.

It is made up both of $M1$ (magnetic Δ excitations and of $E1$ (contact Kroll-Rudermann term) contributions, roughly in the ratio 3 to 1. As a consequence whereas the electric polarizability is determined by low energy nuclear effects, the magnetic susceptibility is mainly affected by real Δ photoproduction.

This picture provides a natural framework to understand the physical foundation of the quasideuteron model at low energy since the dipole exchange contribution necessarily produces a pair of particles with dipole moment different form zero.

The corresponding suppression of (pp) in the Δ region, is on the other hand a priori unexpected and sort of accidental.

At higher energy shadowing for ρs emerges naturally from the ρ photoproduction amplitude which allows direct and coherent two-step processes to interfere and to reduce consequently the intensity of the photon beam.

Of course, this overall seemingly satisfactory portrait, cannot hide all the difficulties inherent to nuclear physics when one asks for more detailed informations i.e. " true" many body dynamics, wave functions etc.

It goes without saying however that a progress in our understanding can only come by refraining as much as possible from a wide-spread dull fitting habit.

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A Appendix

We report here the expressions of the current in an arbitrary frame, entering the scalar response function of (e, e') in a more explicit form, particularly as regards rescattering, than in Ref.[99].

This is relevant for the proof of the gauge invariance and frame independence of the amplitude used in chapter 4. Moreover, it shows explicitly, in contradiction with some of the existing literature, the presence of the Δ resonance in such a response function, even without an elementary Coulomb coupling, because of two distinct reasons i.e. rescattering and frame transformation. The Born terms read

$$J^{\pi^+}_{0,B} = \frac{ig\sqrt{2}}{2M} F(k^2)[\frac{\vec{\sigma}.\vec{k}}{DIR}\omega_q - \frac{\vec{\sigma}.\vec{q}}{DIR}(2E+\omega) + \frac{\vec{\sigma}.(\vec{k}-\vec{q})}{D\pi}(2\omega_q - \omega)] \qquad (A.1)$$

$$J^{\pi^-}_{0,B} = \frac{ig\sqrt{2}}{2M} F(k^2)[-\frac{\vec{\sigma}.\vec{k}}{DCR}\omega_q - \frac{\vec{\sigma}.\vec{q}}{DCR}(2E'-\omega) - \frac{\vec{\sigma}.(\vec{k}-\vec{q})}{D\pi}(2\omega_q - \omega)] \qquad (A.2)$$

$$J^{\pi^0}_{0,B} = \frac{ig}{2M} F(k^2)\{\vec{\sigma}.\vec{k}(\frac{1}{DIR} - \frac{1}{DCR})\omega_q - \vec{\sigma}.\vec{q}(\frac{2E+\omega}{DIR} + \frac{2E'-\omega}{DCR})\} \qquad (A.3)$$

A common form factor $F(k^2_\mu)$ of the usual dipole form is assumed for all virtual γ couplings.

The reason for such a choice is that, coherent with our $0(p/M)$ N.R. reduction the amplitude has a range of validity limited to $k \leq 400$ MeV/c. Hence the possibility of allowing for different form factors will be neglected because of its little numerical influence. On the other hand the determination of the gauge invariant amplitude in the latter case, by adding counterterms proportional to k_μ is beyond the validity granted by low energy theorems [192,193,194]. Therefore such a reconstruction is an open problem.

From the previous equations it is immediate to realize and the corresponding ones for $\vec{J}$ of chapter 4.1, $\vec{J}$ has a dominant zeroth order term $(\vec{\sigma}.\vec{\epsilon})$ plus all remaining $0(p/M)$ pieces, whereas correspondingly J_0 has $\frac{2E}{DIR}, \frac{2E'}{DCR}$ versus the rest, plus the pion in flight in both places to be placed in the first category.

The frame independence of the amplitude $(\tilde{J}_\mu \rightarrow J_\mu)$ can be proved using Eq.'s (75, 76) consistently with our nonrelativistic reduction up to $0(p/M)$ included only.

Rescattering of interest for (e, e') can be expressed in terms of the standard resonant multipole

$$S_{1+}(3/2) = \frac{ig}{2M} F(k^2) \left[\frac{\vec{\sigma}.\tilde{\vec{k}}}{2} \int_{-1}^{1} \frac{(3\tilde{y}^2 - 1)}{2D_\pi} d\tilde{y} - \frac{\vec{\sigma}.\tilde{\vec{q}}}{2} \int_{-1}^{1} \frac{\tilde{y}}{D_\pi} d\tilde{y} \right] (2\tilde{\omega}_q - \tilde{\omega}) \qquad (A.4)$$

and of the longitudinal Δ current

$$J_{0,\Delta} = \frac{f_\Delta g_\Delta}{em_\pi^2} \frac{2M_\Delta v_\Delta(\tilde{q})F(k^2)}{s - M_\Delta^2 + iM_\Delta\Gamma(\tilde{q})} e^{i\delta_B} \frac{(\vec{S}^+(\vec{q})(\vec{S} \times \vec{k}).\vec{p}}{W} \tag{A.5}$$

This latter term is immediatly seen to satisfy gauge invariance with the longitudinal part of the Δ current Eq.(74). We note in passing the obvious current non conservation (except at $\omega = M_\Delta - M$) when the form of the Δ current of Ref.[129] is used. As before the phase δ_B, Γ, and V_Δ are function of $\tilde{q}$ i.e. the c.m.s. relative momentum.

The total contribution reads therefore
for π^+

$$J_0^{\pi^+} = J_{0,B}^{\pi^+} - \frac{\sqrt{2}}{3} J_{0,\Delta} + \frac{\sqrt{2}}{3} S_{1+}(3/2) i e^{i\delta} \sin\delta \tag{A.6}$$

for π^-

$$J_0^{\pi^-} = J_{0,B}^{\pi^-} + \frac{\sqrt{2}}{3} J_{0,\Delta} - \frac{\sqrt{2}}{3} S_{1+}(3/2) i e^{i\delta} \sin\delta \tag{A.7}$$

for π^0

$$J_0^{\pi^0} = J_{0,B}^{\pi^0} + \frac{\sqrt{2}}{3} J_{0,\Delta} - \frac{2}{3} S_{1+}(3/2) i e^{i\delta} \sin\delta \tag{A.8}$$

where to the leading order c.m.s. and laboratory quantities can be interchanged in S_{1+}. The reason to prefer to work with J_0 rather than with the longitudinal component stems from two distincts reasons.

The first is that the particle physicists' attitude to these the gauge condition in order to avoid the explicit calculation of J_0 is no great gain.

The second is that the extraction of the contributions to the transverse resonant multipoles via the projection technique, cannot be applied with the same very good accuracy to the longitudinal ones.

This is due to the fact, which can be best seen at $\omega = 0$ as explained in [99], that for the longitudinal part, these is a competing mechanism concerning the angular dependence between numerator and denominator. We therefore prefer the correct and simple expressions for the transverse part given in the text, and the explicit more cumbersone one given here for the charge.

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Table Captions

Table 1 Quantities pertaining to πN in the Δ region. $\tilde{q}$ stands for the pion c.m. momentum, T_π for the lab. kinetic energy, W for the total invariant mass, δ_B for the (3,3) Born and, $\delta_{\Delta cr}$ for the neglected crossed Δ phase shift, $\delta_{33} = \delta_B + \delta_R$ for the theoretical prediction, undistinguishable from the experimental one and Γ for the Δ width. Finally ω (lab.) and $\tilde{\omega}$ (c.m.s.) are the corresponding energies in the photoproduction process. All energies in MeV have been rounded off.

Table 2 Projection operators for the lowest order photoproduction (transverse) multipoles. Second column refers to the multipolarity of the e.m. field, ℓ to the orbital J to the total $\pi - N$ angular momentum. Pion and photon momenta are denoted by $\vec{q}$ and $\vec{k}$ and $\epsilon = \epsilon_\perp$ is the transverse photon polarization. All quantities in the c.m.s. .

$\tilde{q}$	T_π	W	δ_B	$\delta_{\Delta cr}$	δ_{33}	Γ	ω	$\tilde{\omega}$
53	13	1089	0.27	0.02	0.72	2	162	140
77	27	1101	0.75	0.06	2.15	7	176	150
96	40	1113	1.33	0.11	4.21	13	190	160
112	54	1124	1.98	0.17	6.97	21	204	170
127	68	1136	2.69	0.23	10.58	29	218	180
142	83	1148	3.44	0.30	15.27	39	232	190
155	98	1160	4.22	0.38	21.34	50	247	200
168	113	1172	5.02	0.47	29.20	63	262	210
181	128	1184	5.84	0.55	39.22	76	277	220
193	144	1197	6.68	0.65	51.52	90	293	230
205	160	1209	7.53	0.74	65.52	105	309	240
217	176	1221	8.40	0.83	79.82	121	325	250
229	192	1234	9.26	0.93	92.91	137	342	260
240	209	1247	10.13	1.03	103.91	154	359	270
252	226	1260	11.01	1.13	112.78	172	376	280
263	244	1273	11.88	1.22	119.85	190	393	290
274	261	1286	12.76	1.32	125.54	209	411	300
285	280	1299	13.63	1.42	130.20	228	429	310
296	298	1312	14.50	1.51	134.09	247	447	320
307	316	1325	15.36	1.60	137.41	266	466	330
318	335	1339	16.22	1.69	140.29	286	485	340

Table 1:

Multipole	e.m. multip.	ℓ	J	Operator
E_{0+}	$E1$	0	1/2	$i\vec{\sigma}.\vec{\epsilon}$
M_{1-}	$M1$	1	1/2	$\vec{\sigma}.\hat{q}.\vec{\sigma}.\hat{k} \times \vec{\epsilon}$
E_{2-}	$E1$	2	3/2	$i\vec{\sigma}.\vec{\epsilon} - 3i\vec{\sigma}.\hat{q}\hat{q}.\vec{\epsilon}$
M_{1+}	$M1$	1	3/2	$3i(\vec{\sigma}.\vec{\epsilon}\hat{q}.\hat{k} - \vec{\sigma}.\hat{k}\hat{q}.\vec{\epsilon}) + 2\vec{\sigma}.\hat{q}\vec{\sigma}.\hat{k} \times \vec{\epsilon}$
E_{1+}	$E2$	1	3/2	$3i(\vec{\sigma}.\vec{\epsilon}\hat{q}.\hat{k} + \vec{\sigma}.\hat{k}\hat{q}.\vec{\epsilon})$

Table 2:

Figure captions

Fig.1 Total photoabsorption cross section per nucleon as a function of the photon laboratory energy ω. From ref.[7].

Fig.2 Real and imaginary parts of the forward scattering amplitude on Be in MeV^{-1} as a function of the energy ω.

Fig.3 Two body $E1$ exchange current due to pions and rhos (dashed lines. Full and wavy lines for nucleons and photon).

Fig.4 Contribution to Compton scattering. Symbols as before. Diagrams (a)-(d) represent at the same time the enhancement factor κ to the dipole sum rule. Diagram (f), with an intermediate π^0 only, numerically negligible.

Fig.5 Compton scattering and dipole sum rule enhancement contributions due to σ (a) and ω (b) mesons.

Fig.6 Photoabsorption cross-section for the even isotopes of neodymium. Note the progressive broadening of the dipole resonance in going from the spherical Nd^{142} to the statically deformed Nd^{150} where the mode is split into two peaks. Taken from Ref. [89]

Fig.7 a) Born (lowest order) graphs contributiong to pion photoproduction. Solid lines stand for nucleons, wavy for photons, dashed for pions and hatched for Δ.
b,c) Rescattering terms in the (3,3) channel up to the first order in the background phase shift δ_B.

Fig.8 T = 3/2 multipoles vs. the c.m. energy $\tilde{\omega}$. Solid line result of the present unitary calculation without ω and crossed Δ contribution. Dash-dotted line no rescattering.

Fig.9 Total $\gamma p \rightarrow n\pi^+$, $\gamma n \rightarrow p\pi^-$, $\gamma p \rightarrow p\pi^0$ photoabsorption cross section vs. invariant mass W. Full line result cf the present unitary calculation. Dash-dotted line (undistinguishable in the π^0 case) without rescattering. Dotted line contribution of the ω term alone. In all three cases ω and crossed Δ neglected in the full curve.

Fig.10 Typical angular distributions at various values of W with (full line) and without (dashed dotted) rescattering for γ, π^+ (a) $\gamma\pi^-$ (b) γ, π^0 (c). In the last case also asymmetry at $\theta = 90^0$ vs energy ω. All quantities in the c.m.s..

Fig.11 Total pion photoproduction cross sections (full curve) function of the c.m. energy $\tilde{\omega}$ (MeV). Dashed and dot dashed without the Δ and without the total magnetic contributions.

Fig.12 Total photoabsorption cross sections on the proton and on the neutron. Symbols as in Fig.11.

Fig.13 Photoabsorption cross section per nucleon (data from [125], dots refer to Pb and crosses to U. The dashed line represents the predictions of the free Fermi gas model ($k_F = 1.3 fm^{-1}$; dotted and dash-dotted lines refer to the Fermis gas model with ρ exchange and short range correlations ($g' = 0.3$ and $g' = 0.5$ respectively). The solid line is the result of the self-consistent calculation with $g' = 0.3, \alpha = 0.7, \Lambda = 750$ MeV/c, $V_0 = (-100 - i40)$ MeV.

Fig.14 Total photoabsorption cross section per nucleon in the Δ region.

Fig.15 Relevant one pion exchange diagrams in the Δ region.

Fig.16 Photoabsorption per nucleon as a function of ω. Full and broken line are guides to the eye for the experimental data on Pb and D, respectively.

Fig.17 a) s channel resonance contribution to $\sigma(\gamma p \rightarrow \pi^- \Delta^{++})$
b) Most relevant Feynman graphs for $\gamma N \rightarrow \pi \Delta$.
c) Contribution of the electric Born diagrams (calculated in the Coulomb gauge in the c.m.s.) to $\sigma(\gamma p \rightarrow \pi^- \Delta^{++})$. From ref. [153].

Fig.18 Vector meson photoproduction mechanisms. a) diffractive dissociation, b) resonance production, c) - f) Born mechanism. Dashed lines stand for pions, double dashed for ρ (and possibly for ω and ϕ). Thick solid line in (e) for the Higgs boson of unknown mass.

Fig.19 Vertex $V_{\sigma\mu\nu}$ for $\gamma\rho^+\rho^-$ and for $\rho^0\rho^+\rho^-$ (see Eq.(110))

Fig.20 Total photoabsorption cross section per nucleon in the GeV region. From Ref.[7].

Fig.21 One and two-step processes leading to the same nuclear final state. In the intermediate state in b) only neutral particles can be coherently photoproduced via a non spin flip interactions.

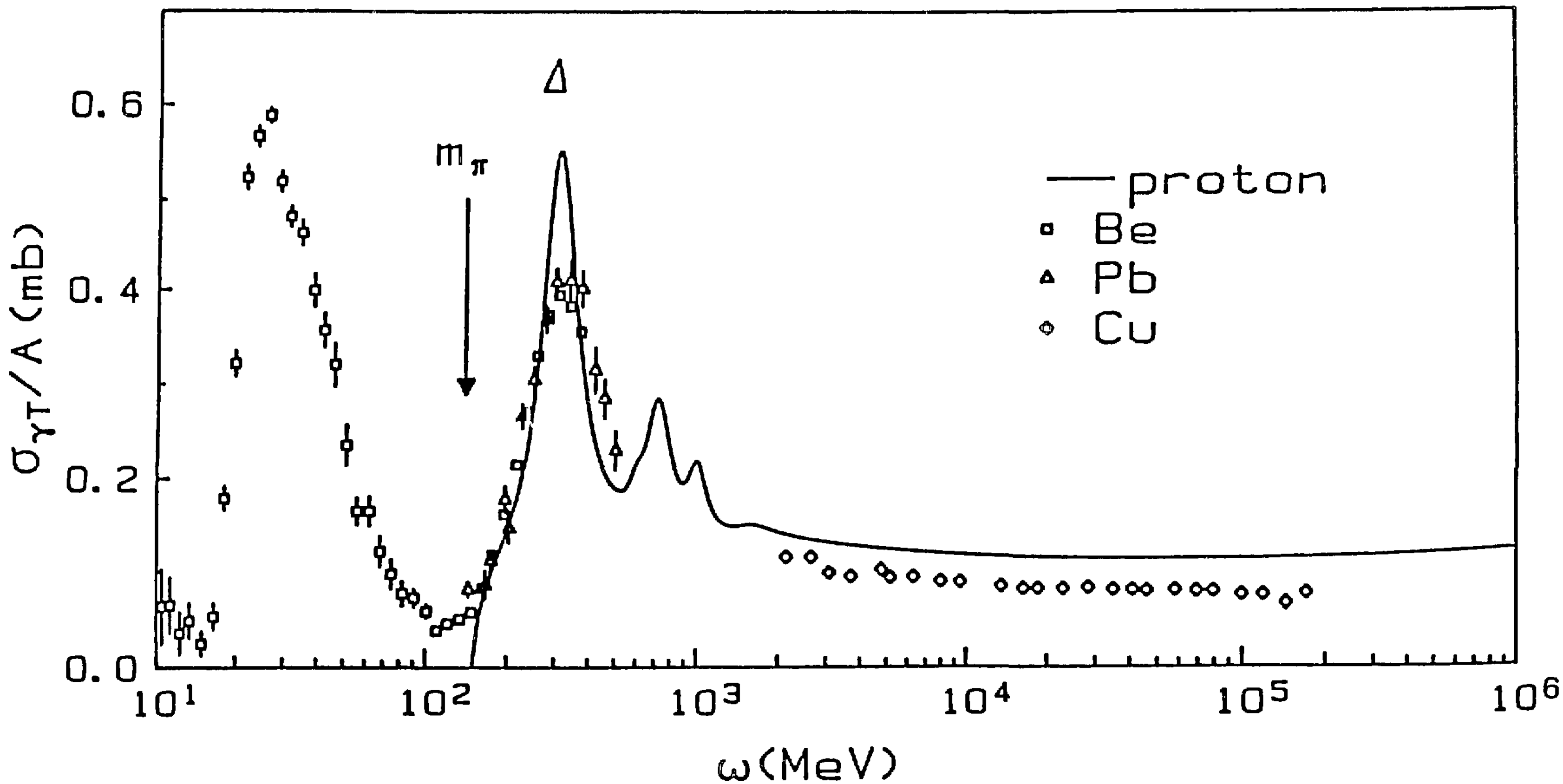

Δ
m_π
proton
Be
Pb
Cu
0.0
0.2
0.4
0.6
$\sigma_{\gamma T}/A$ (mb)
10^1
10^2
10^3
10^4
10^5
10^6
ω(MeV)

Fig.1

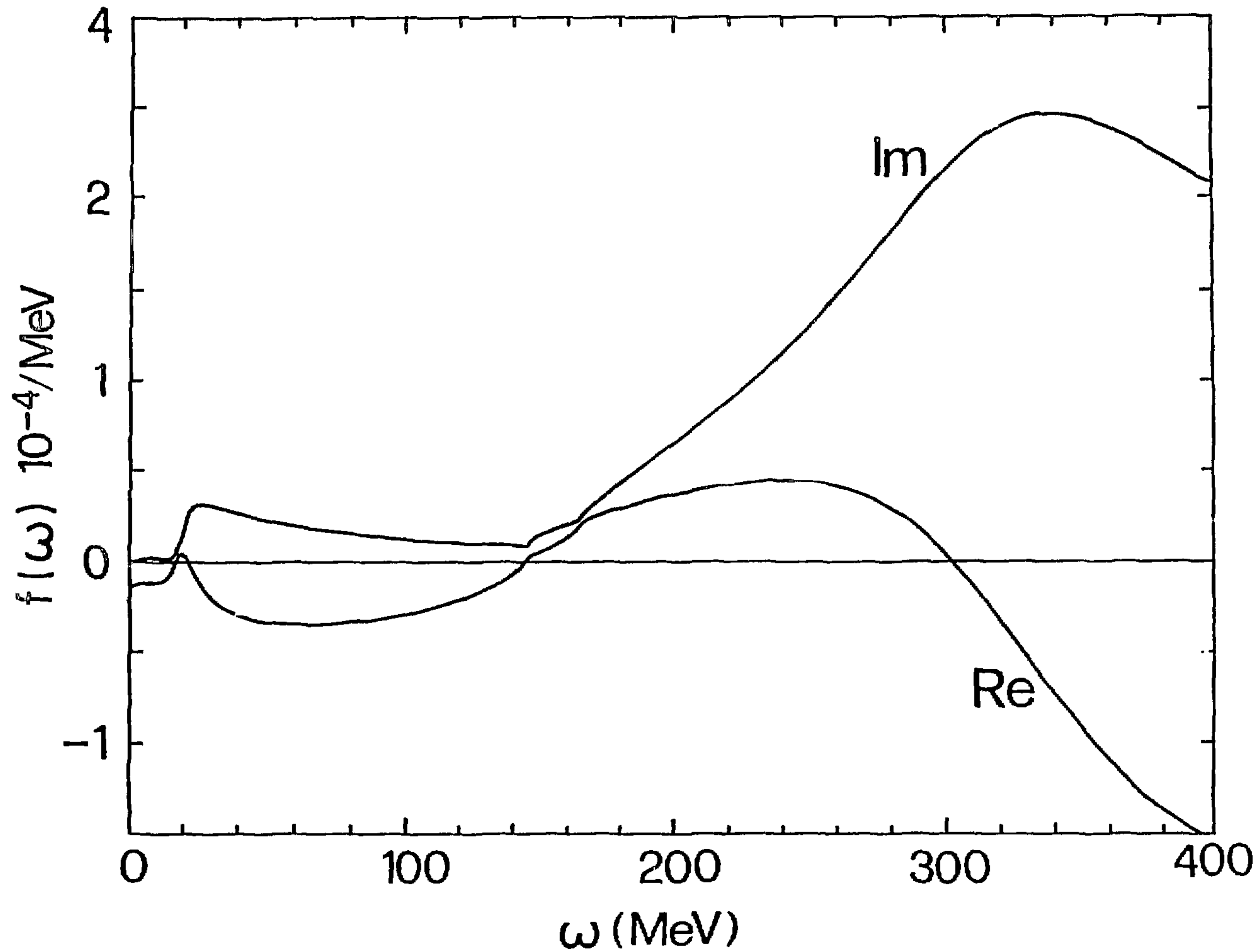


Fig.2

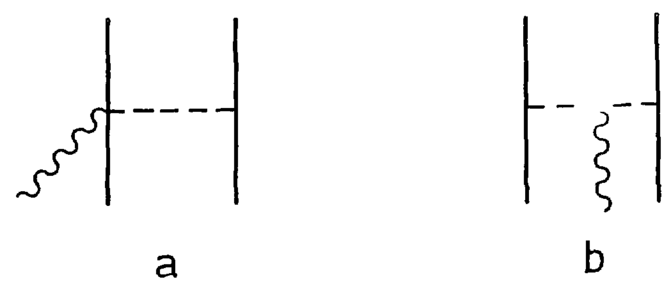


Fig.3

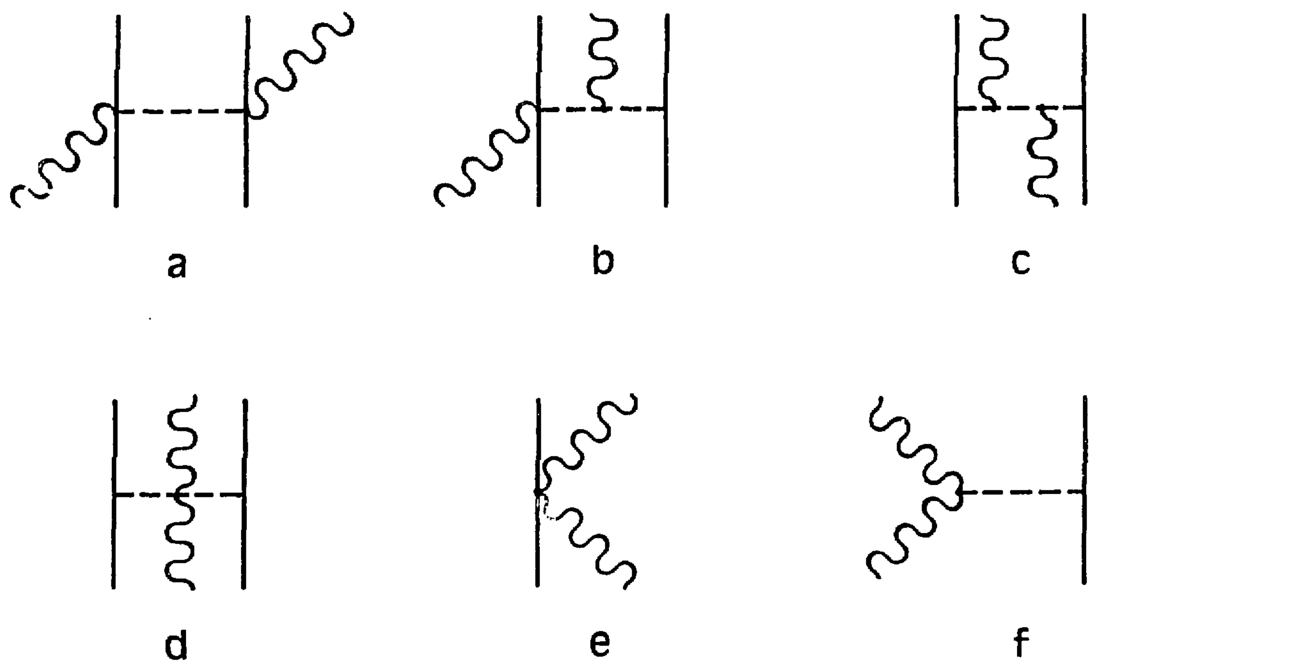


Fig.4

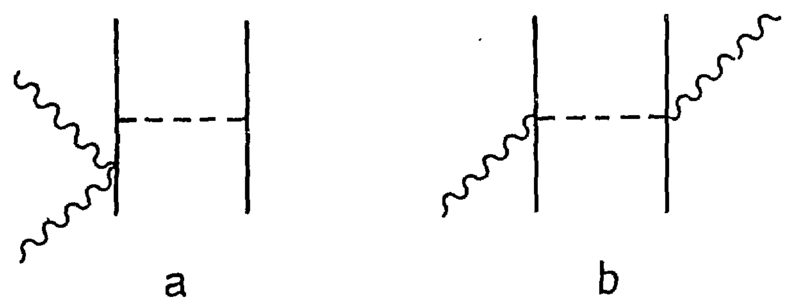
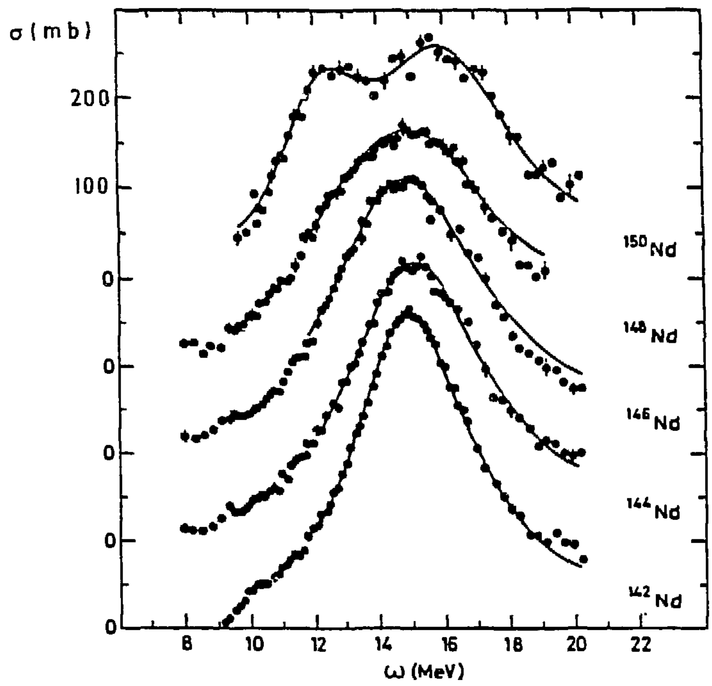


Fig.5



σ (mb)
200
100
0
0
0
0
0
^{150}Nd
^{148}Nd
^{146}Nd
^{144}Nd
^{142}Nd
8
10
12
14
16
18
20
22
ω (MeV)

Fig.6

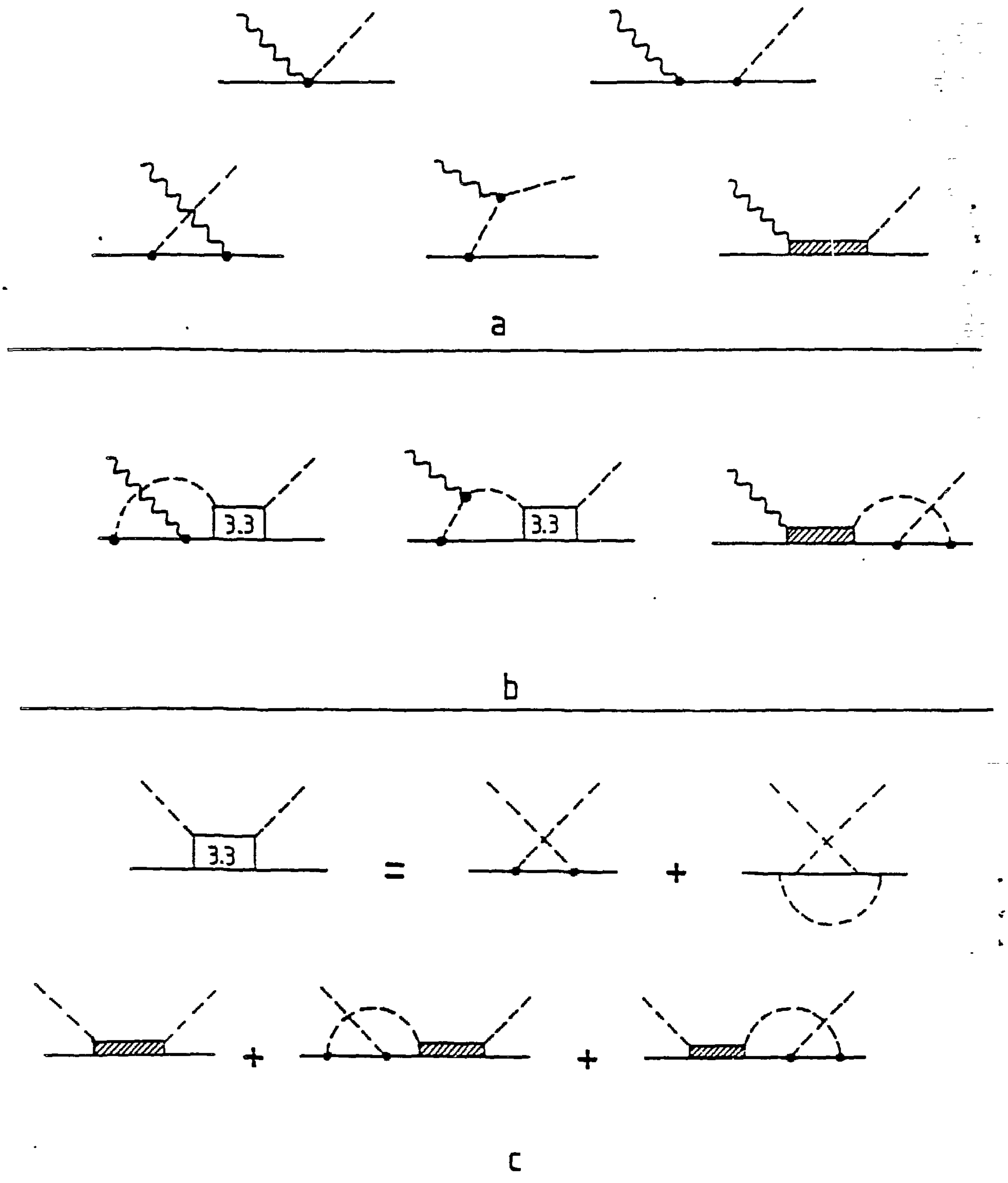
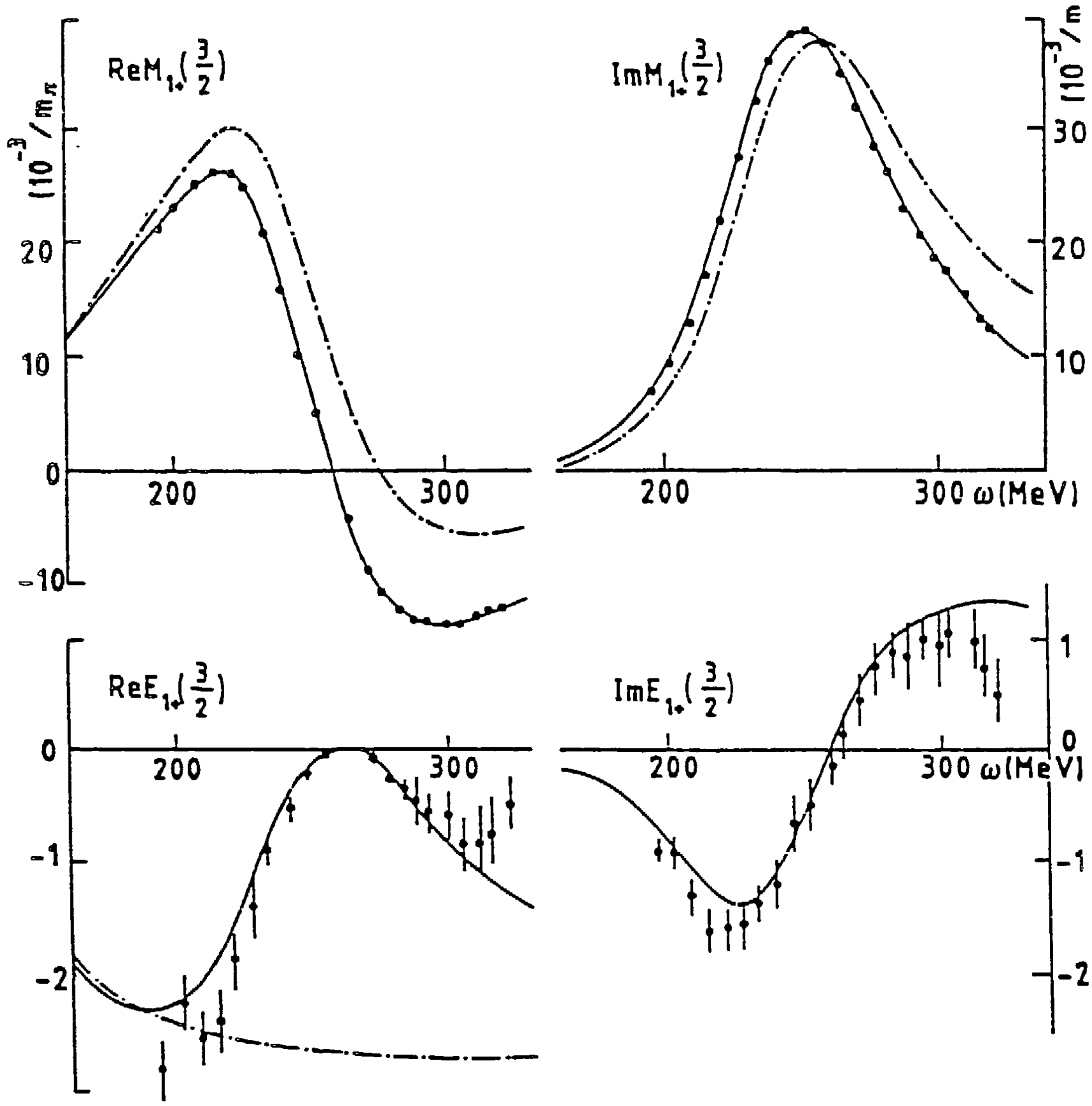


Fig.7



$ReM_{1+}(\frac{3}{2})$
$(10^{-3}/m_\pi)$
$ImM_{1+}(\frac{3}{2})$
$ReE_{1+}(\frac{3}{2})$
$ImE_{1+}(\frac{3}{2})$
ω(MeV)

Fig 8

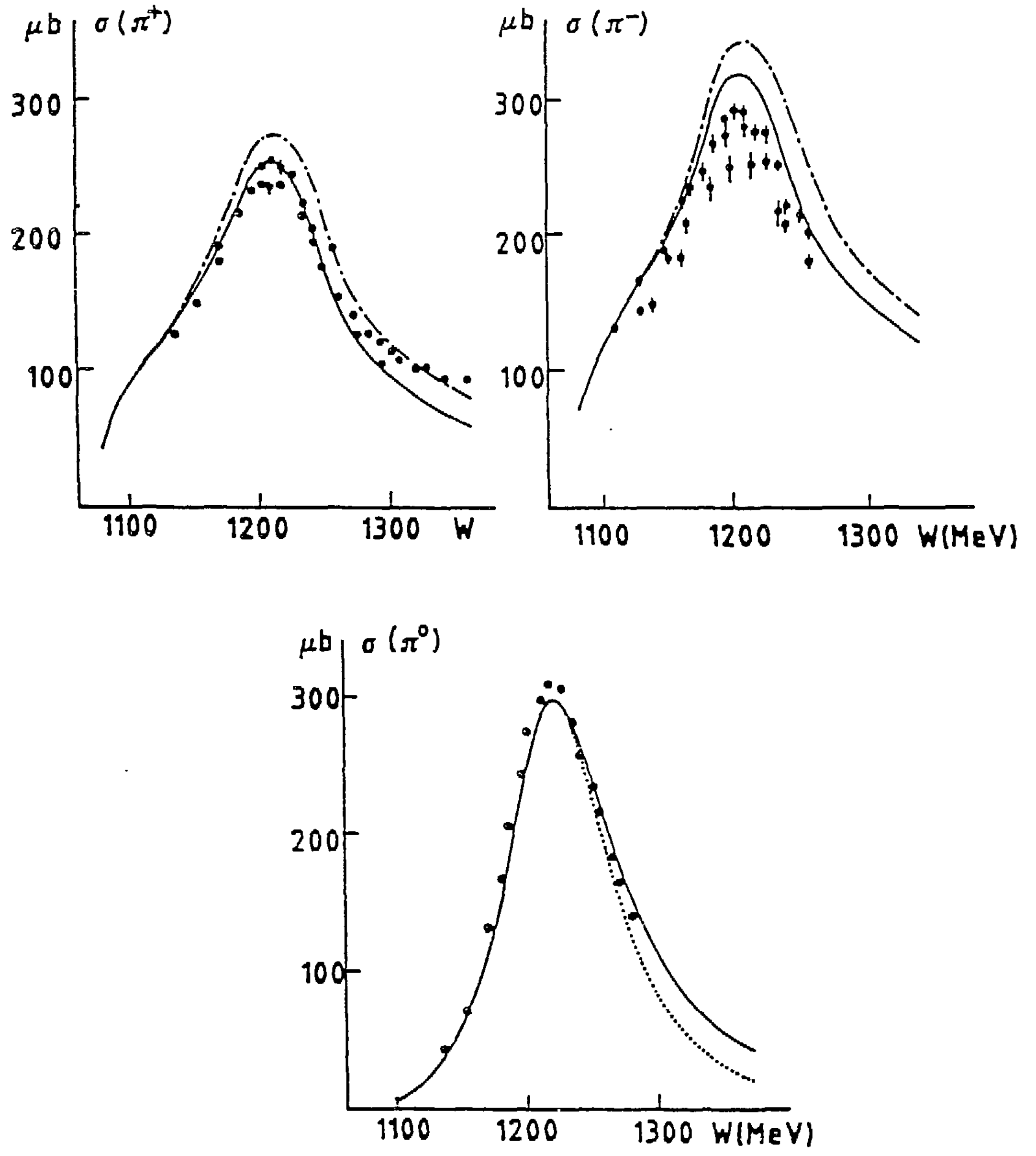
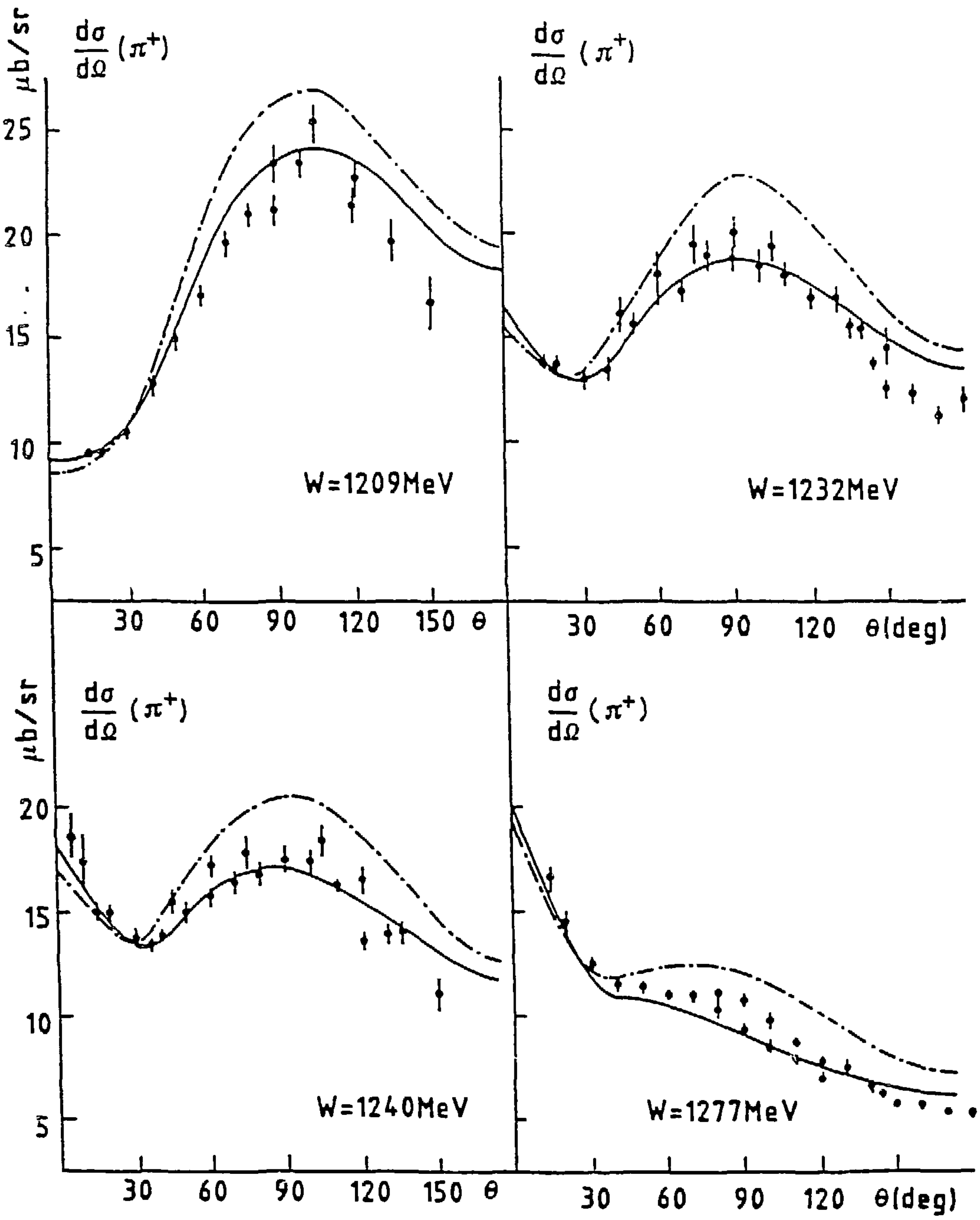
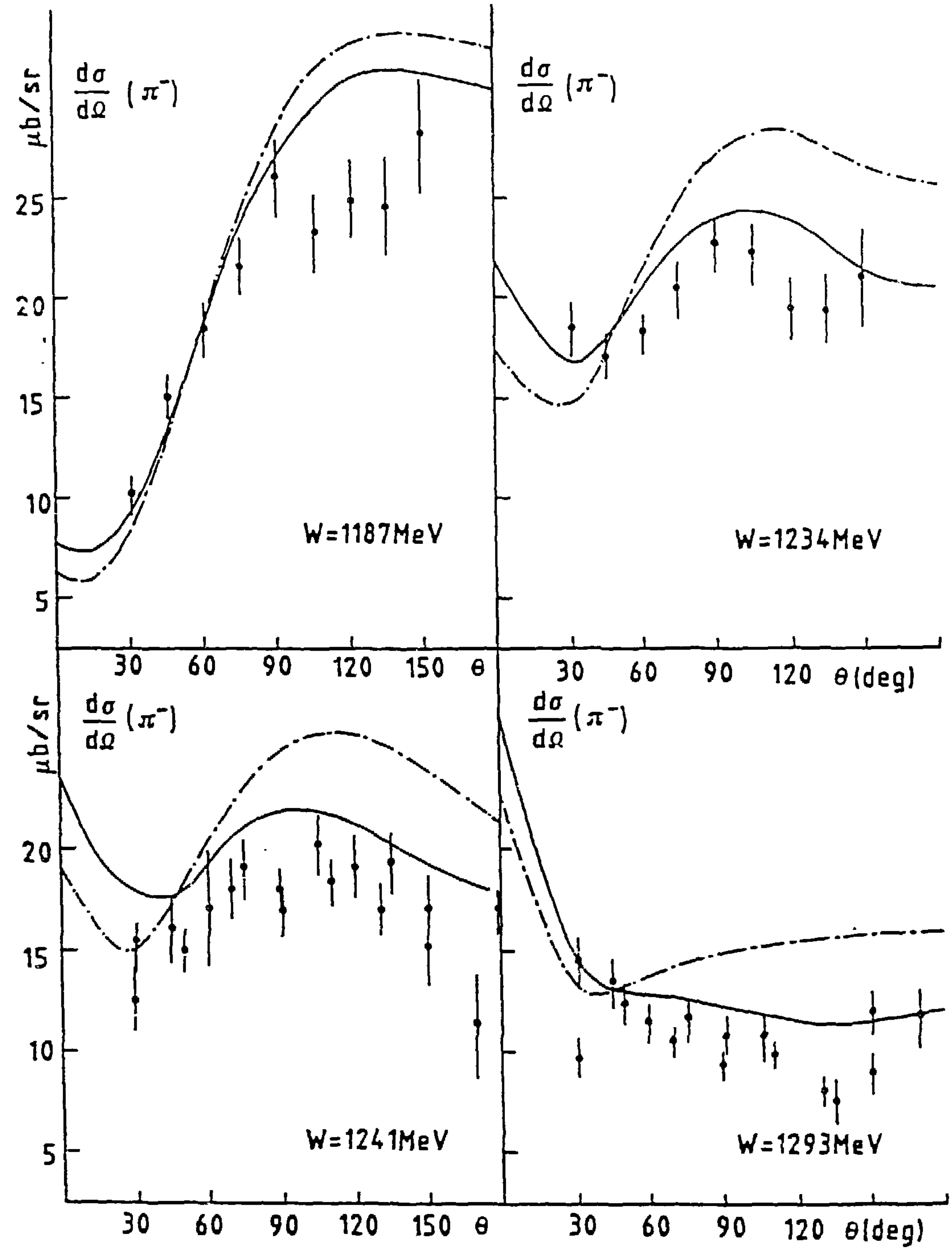

μb σ (π⁺)
300
200
100
1100
1200
1300
W
μb σ (π⁻)
300
200
100
1100
1200
1300
W(MeV)
μb σ (π⁰)
300
200
100
1100
1200
1300
W(MeV)

Fig.9



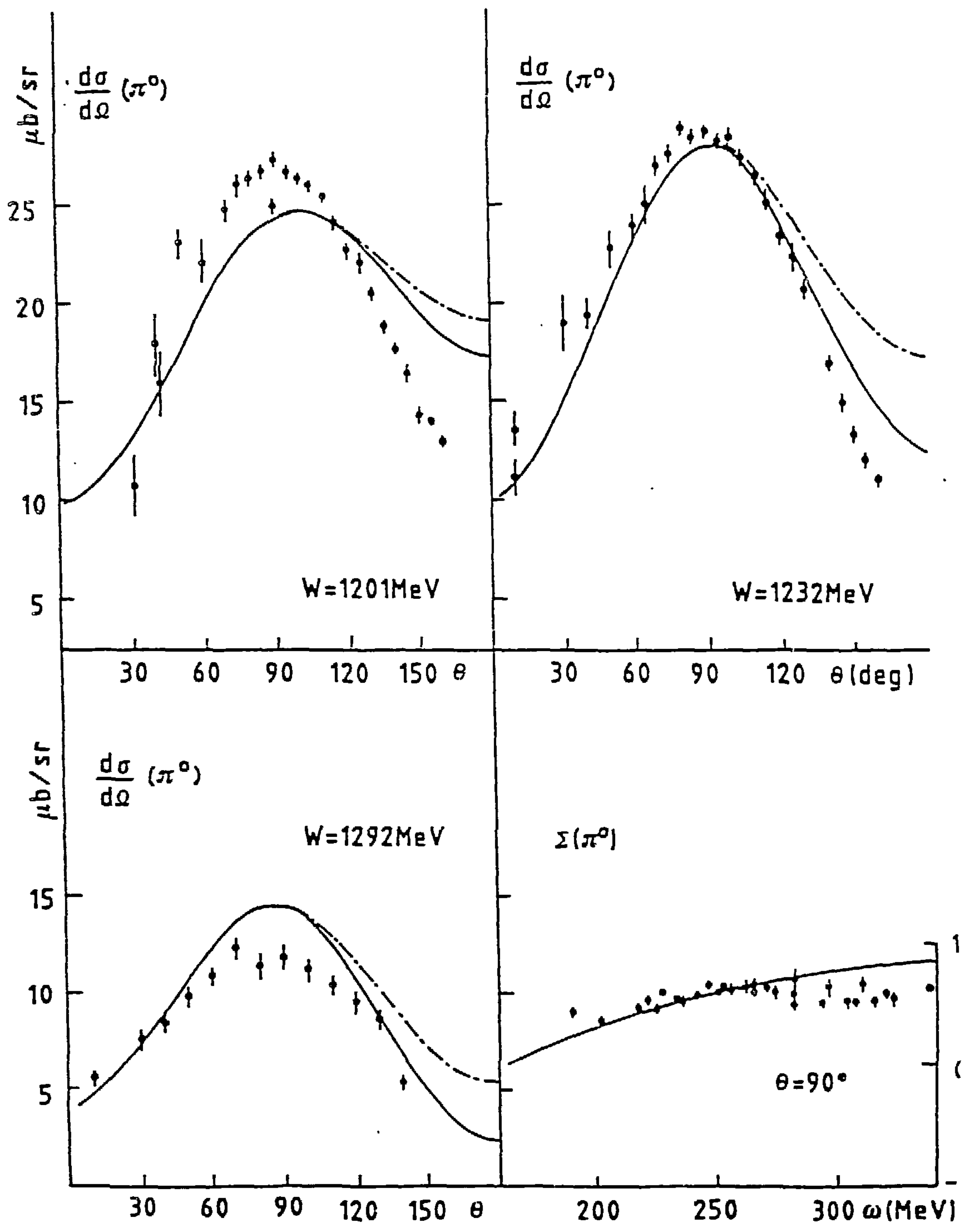
μb/sr
dσ/dΩ (π⁺)
25
20
15
10
5
W=1209MeV
30 60 90 120 150 θ
dσ/dΩ (π⁺)
W=1232MeV
30 60 90 120 θ(deg)
μb/sr
dσ/dΩ (π⁺)
20
15
10
5
W=1240MeV
30 60 90 120 150 θ
dσ/dΩ (π⁺)
W=1277MeV
30 60 90 120 θ(deg)

Fig.10(a)



dσ/dΩ (π⁻)
μb/sr
25
20
15
10
5
W=1187MeV
30
60
90
120
150
θ
dσ/dΩ (π⁻)
W=1234MeV
30
60
90
120
θ(deg)
dσ/dΩ (π⁻)
μb/sr
20
15
10
5
W=1241MeV
30
60
90
120
150
θ
dσ/dΩ (π⁻)
W=1293MeV
30
60
90
120
θ(deg)

Fig.10(b)



μb/sr
dσ/dΩ (π°)
W=1201MeV
25
20
15
10
5
30
60
90
120
150
θ
dσ/dΩ (π°)
W=1232MeV
30
60
90
120
θ(deg)
μb/sr
dσ/dΩ (π°)
W=1292MeV
15
10
5
30
60
90
120
150
θ
Σ(π°)
θ=90°
200
250
300
ω(MeV)

Fig.10(c)

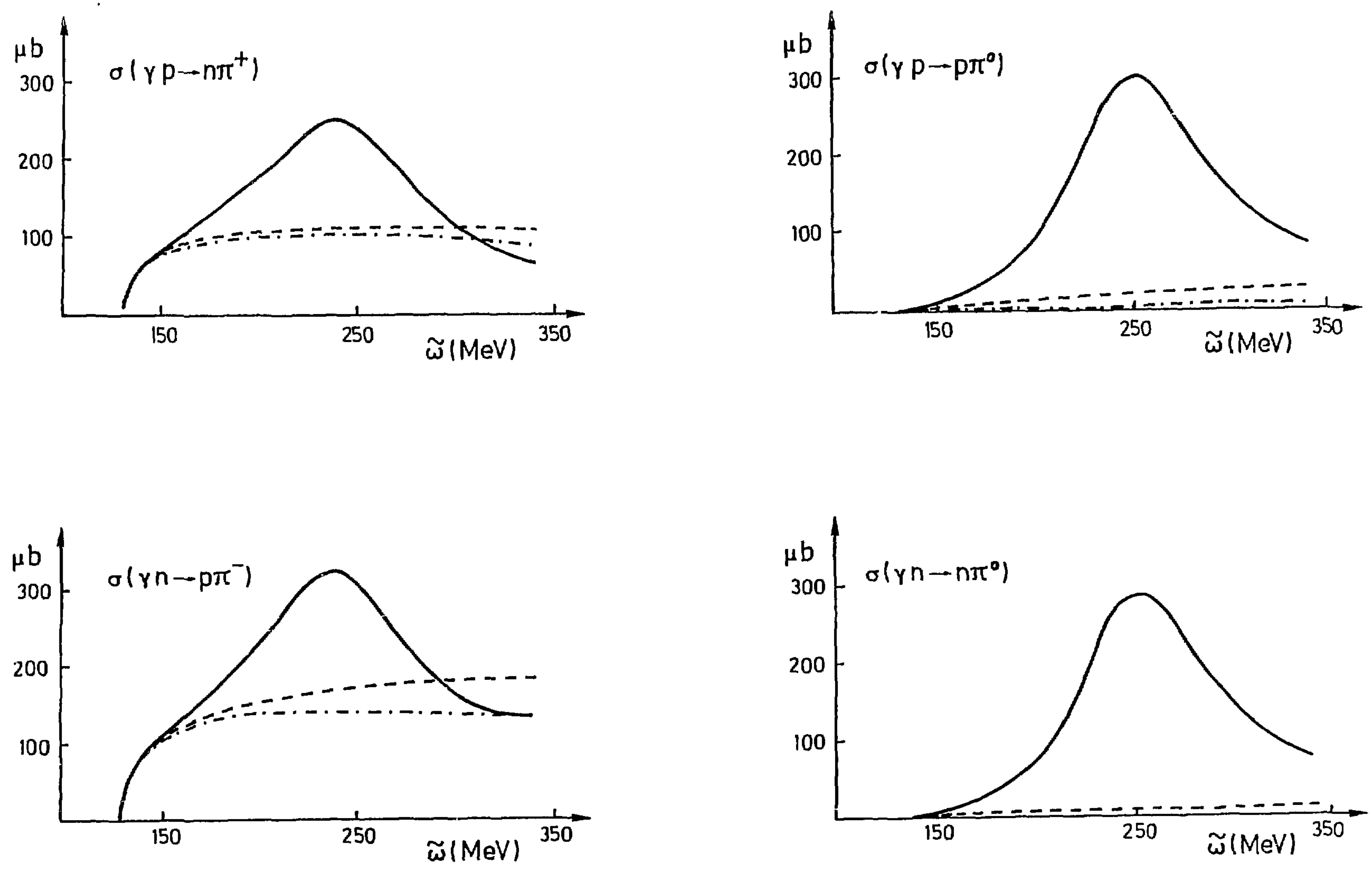
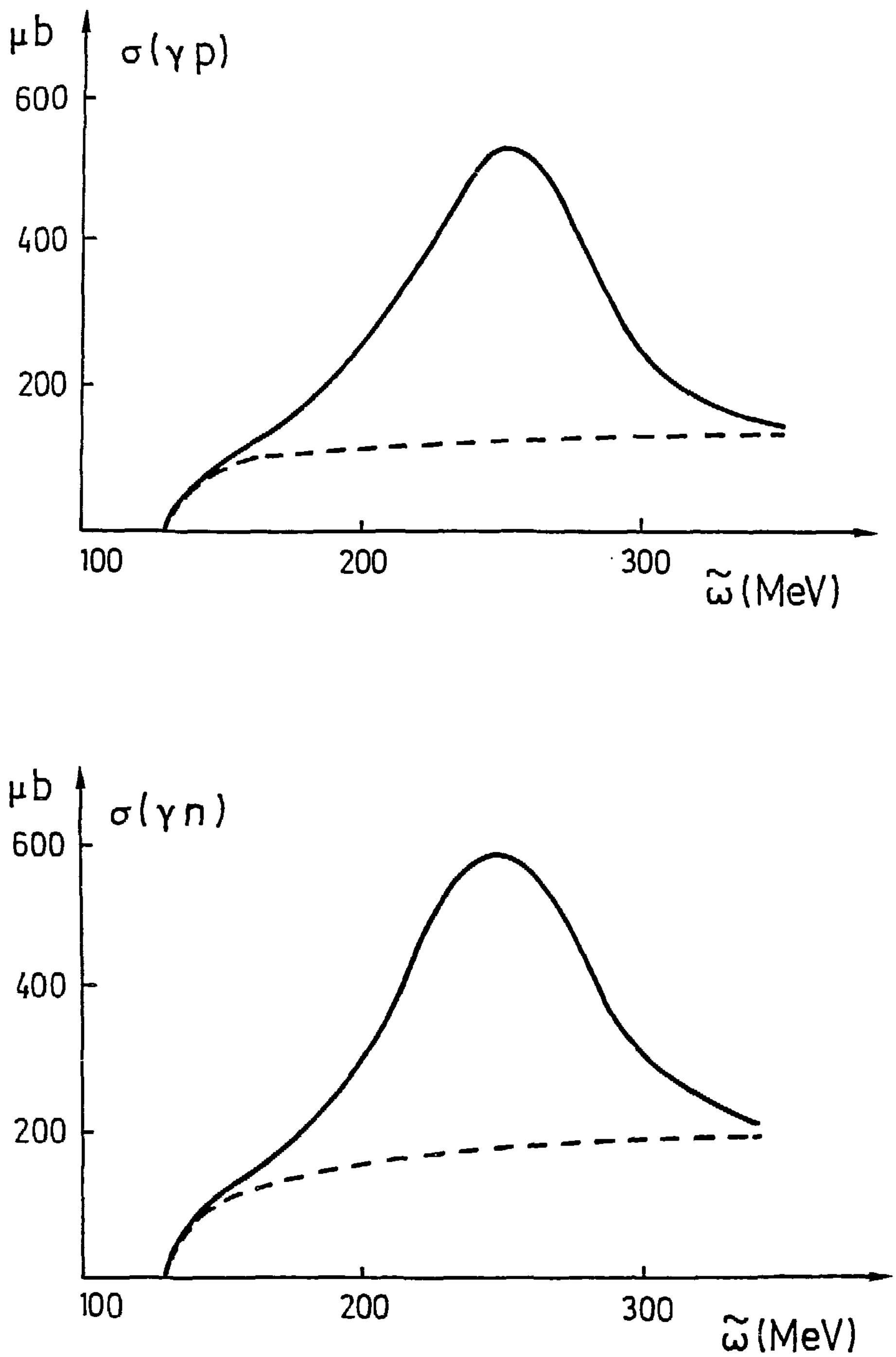

μb
300
200
100
σ(γp→nπ⁺)
150
250
350
ω̃(MeV)
μb
300
200
100
σ(γp→pπ⁰)
150
250
350
ω̃(MeV)
μb
300
200
100
σ(γn→pπ⁻)
150
250
350
ω̃(MeV)
μb
300
200
100
σ(γn→nπ⁰)
150
250
350
ω̃(MeV)

Fig.11



μb
σ(γp)
600
400
200
100
200
300
ω̃ (MeV)
μb
σ(γn)
600
400
200
100
200
300
ω̃ (MeV)

Fig .12

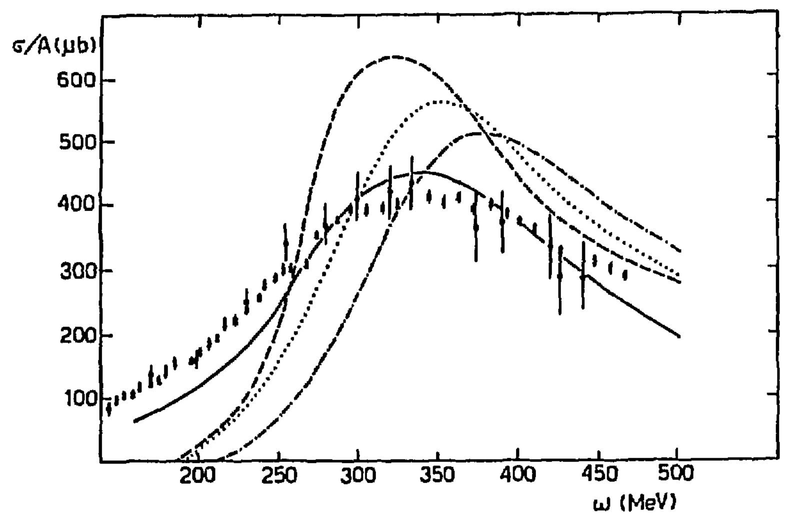
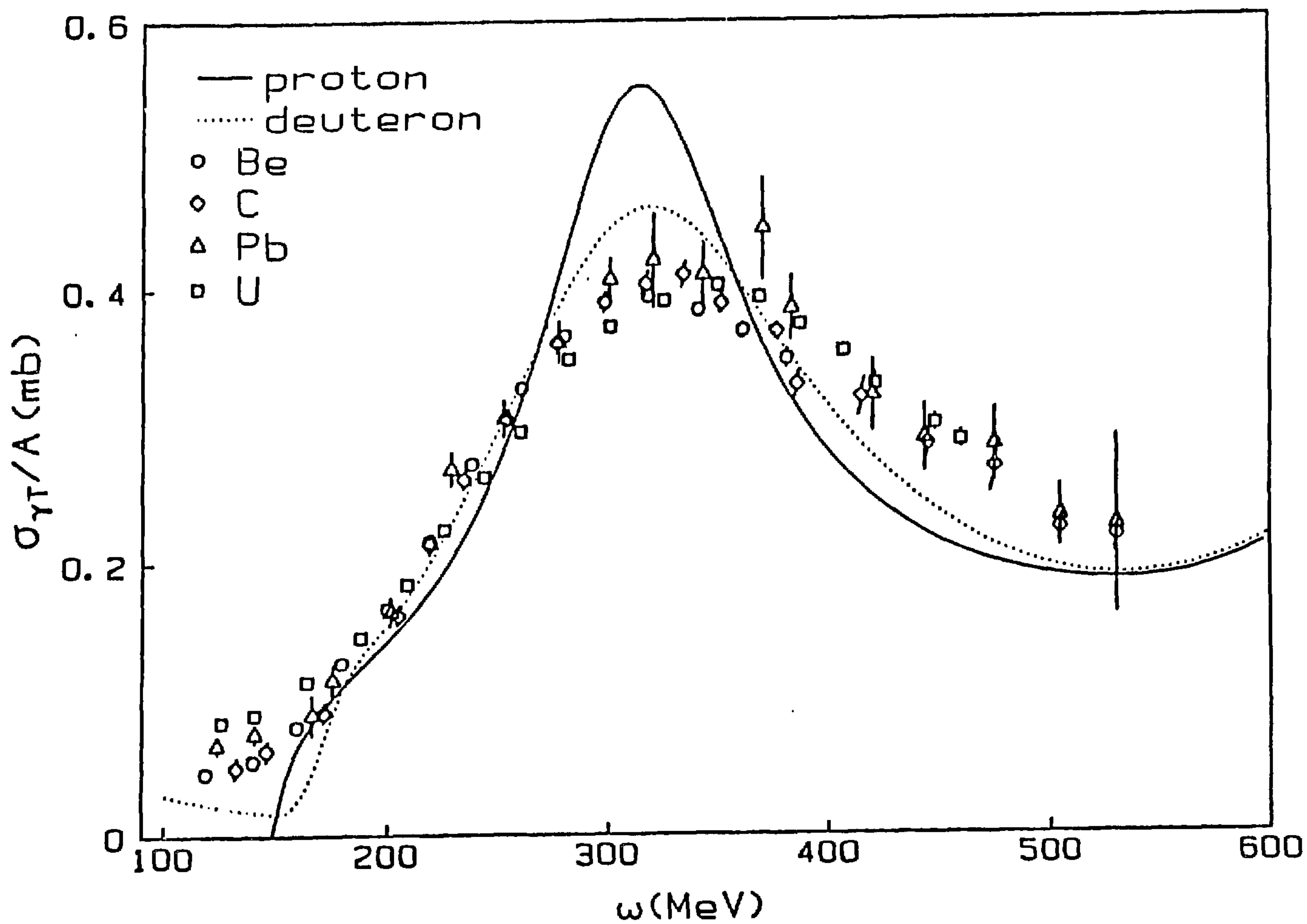


Fig.13



proton
deuteron
Be
C
Pb
U
0.6
0.4
0.2
0
100
200
300
400
500
600
$\sigma_{\gamma T}/A$ (mb)
ω(MeV)

Fig.14

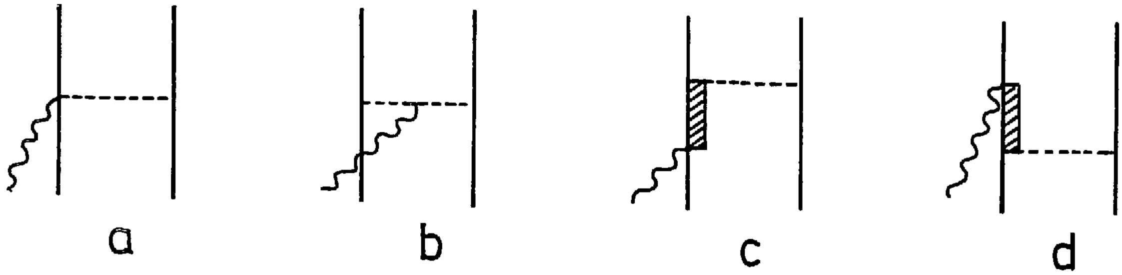


Fig.15

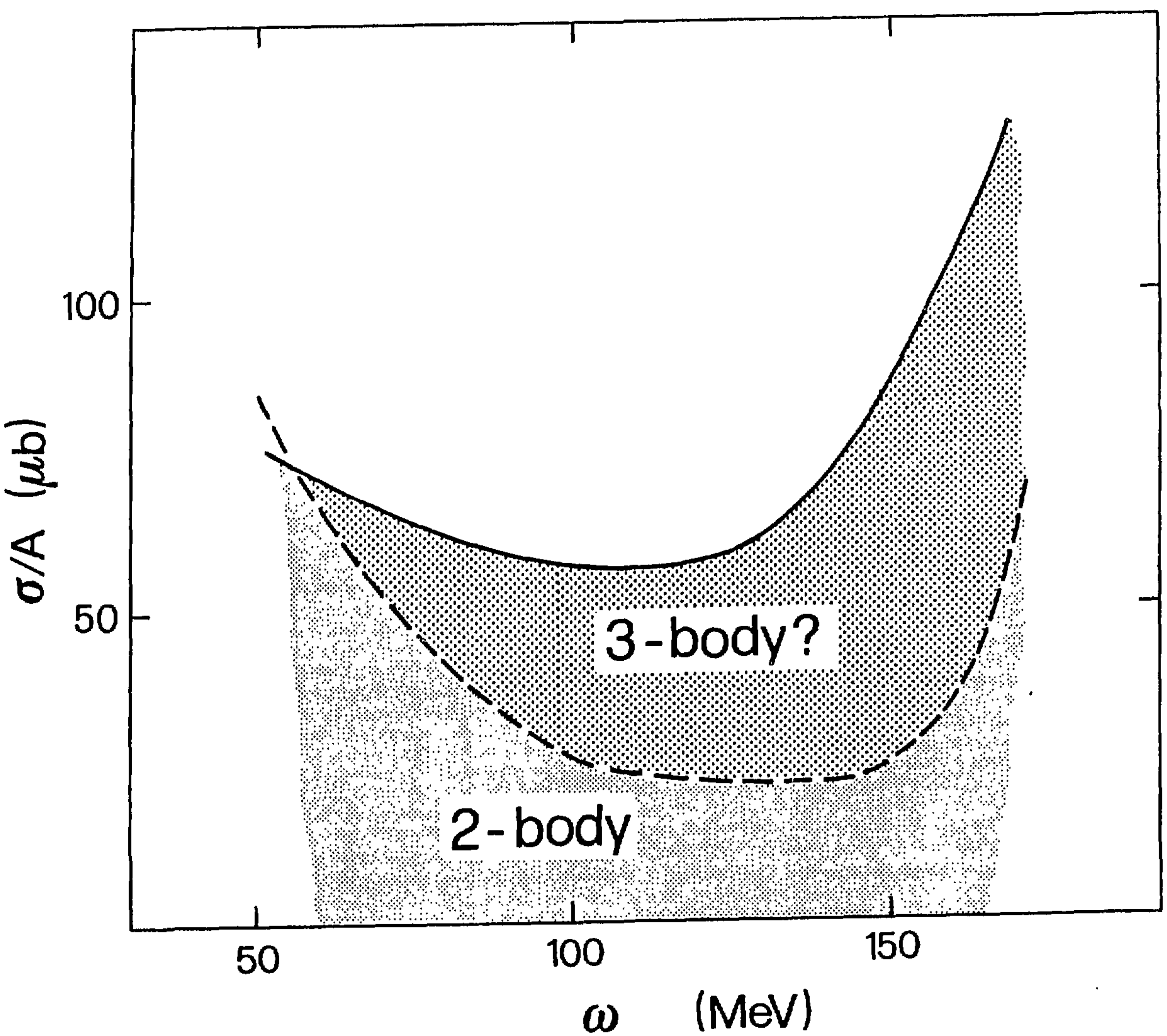
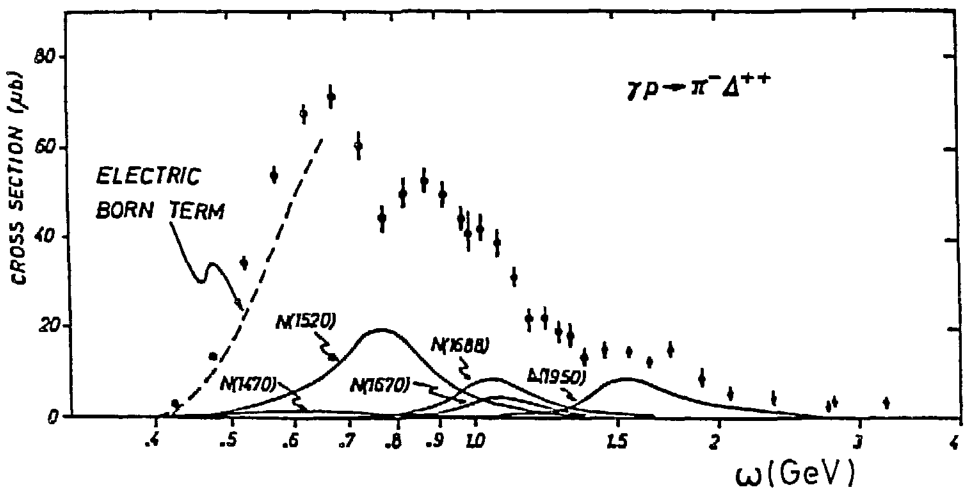


Fig.16



CROSS SECTION (μb)
ELECTRIC
BORN TERM
γp → π⁻Δ⁺⁺
N(1520)
N(1688)
N(1470)
N(1670)
Δ(1950)
ω(GeV)

Fig.17 a)

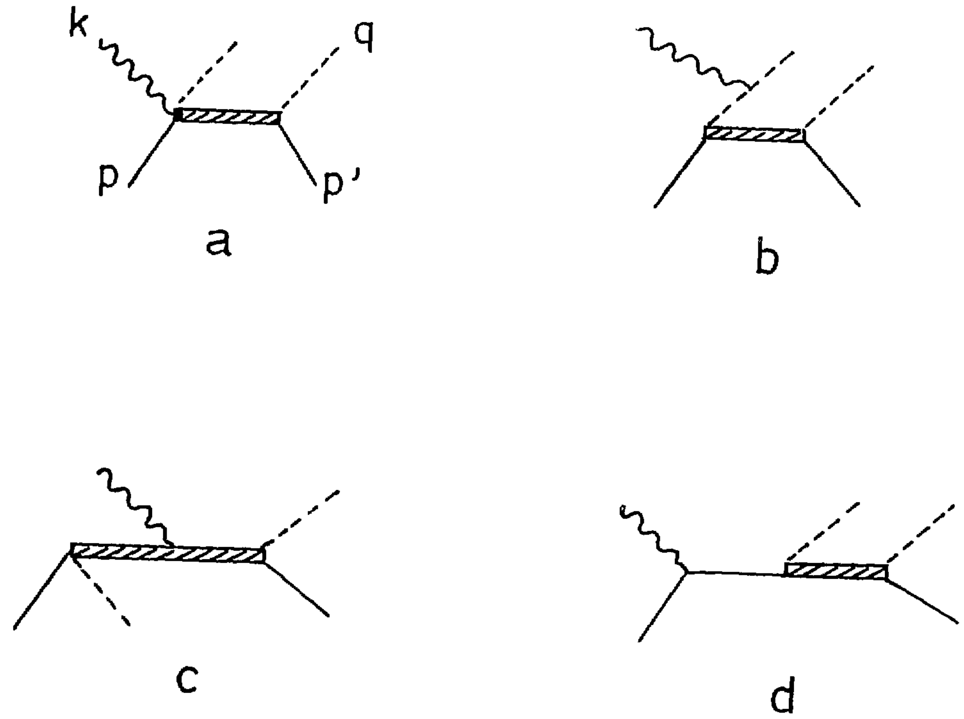
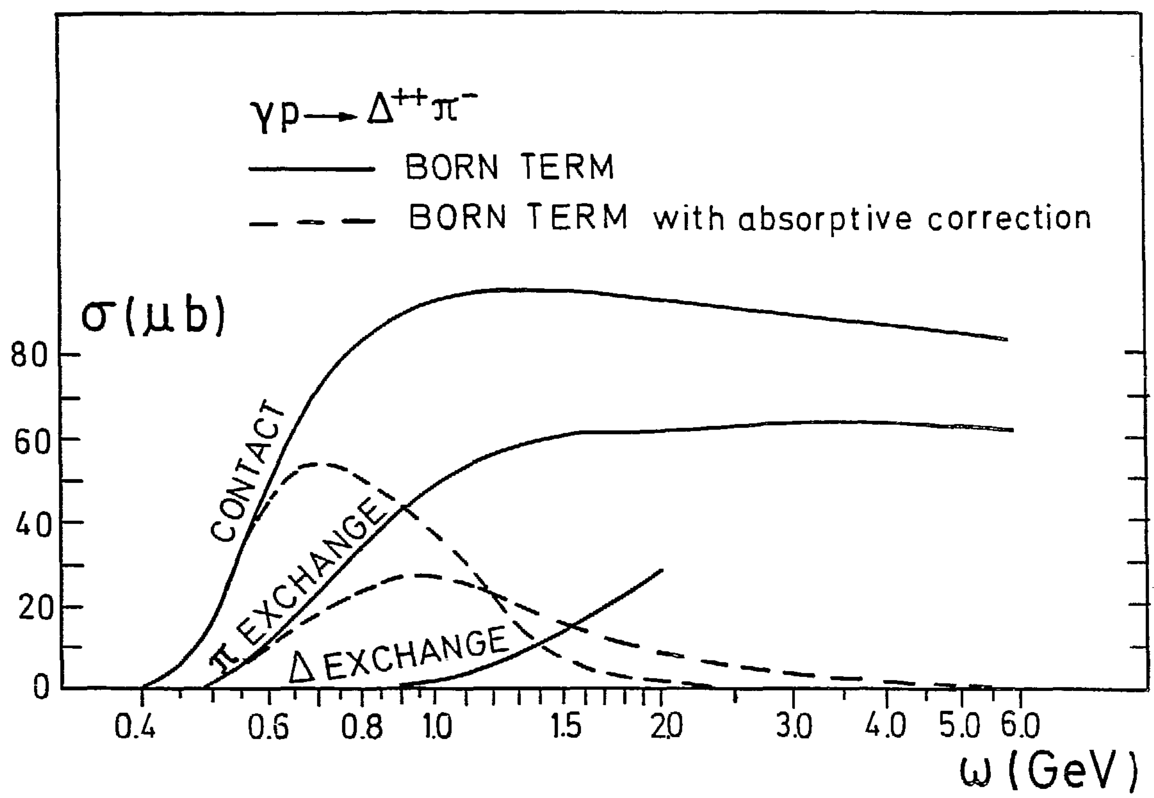


Fig. 17 b)



γp → Δ++π−
BORN TERM
BORN TERM with absorptive correction
σ (μb)
80
60
40
20
0
CONTACT
π EXCHANGE
Δ EXCHANGE
0.4
0.6
0.8
1.0
1.5
2.0
3.0
4.0
5.0
6.0
ω (GeV)

Fig.17 c)

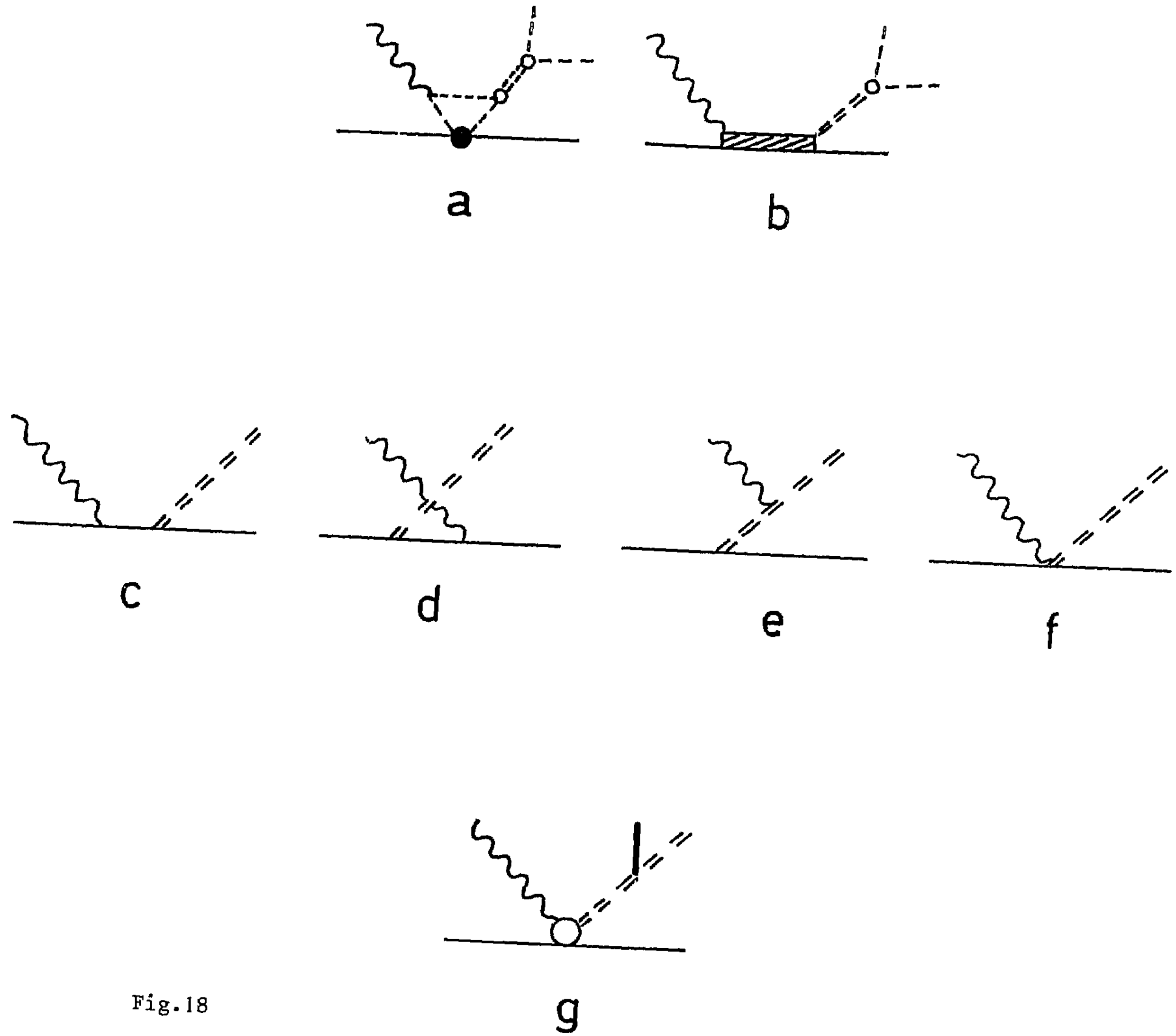


Fig.18

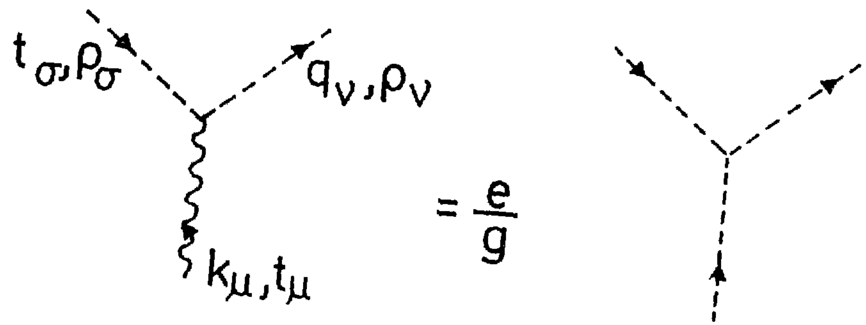


Fig. 19

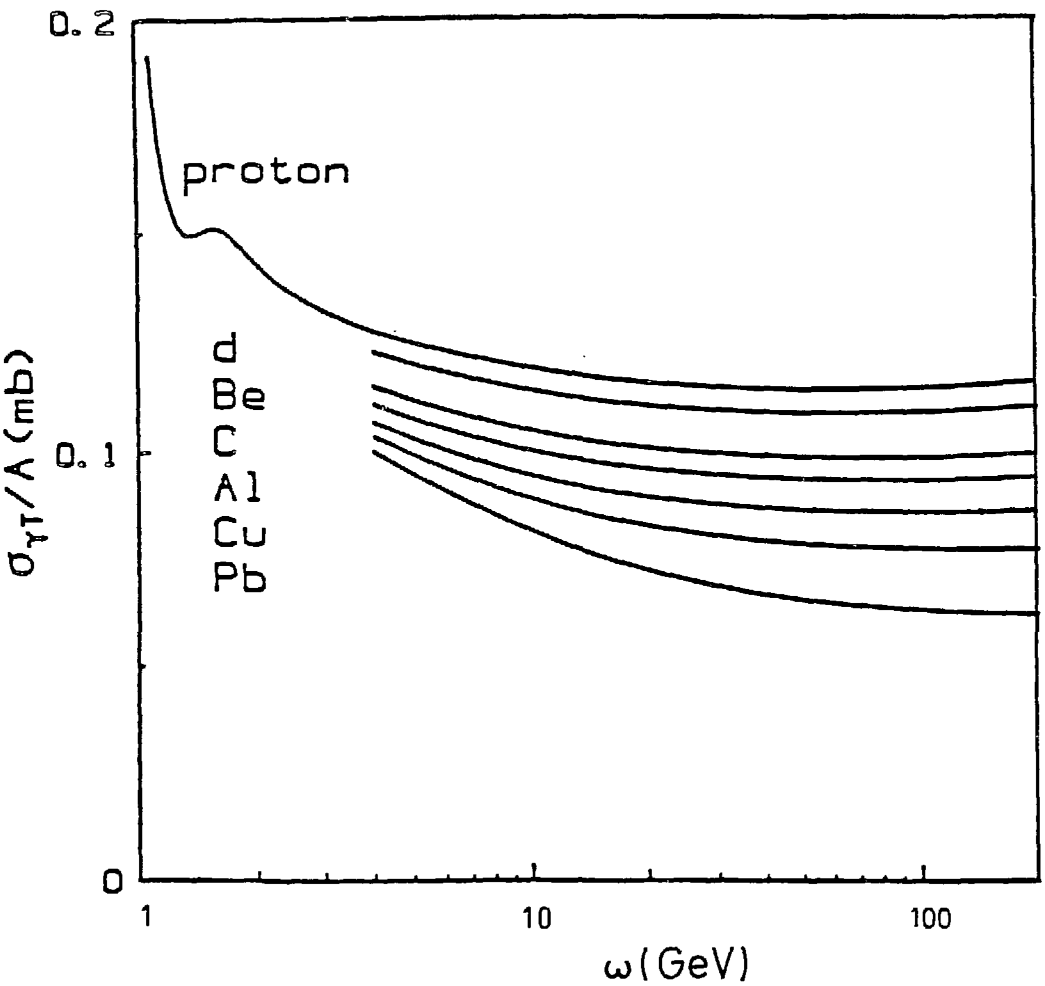


Fig.20

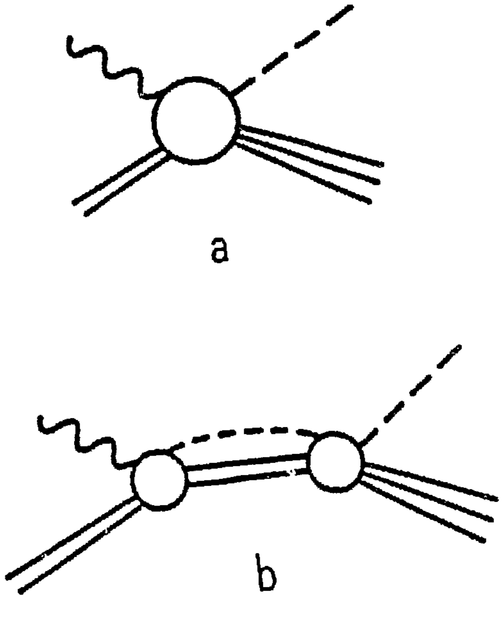


Fig.21