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ABSTRACT

In the presence of finite plasma current, the axisymmetric $(m = 0)$ magnetoacoustic wave resonance exhibits a frequency splitting for finite toroidal mode number $(n \neq 0)$ between oppositely directed travelling waves. Calculations are presented which demonstrate that $\Delta(1/f)$, the difference between the inverse resonance frequencies, is directly proportional to known moments of the plasma current profile and, in the limit where the radial wavenumber is much larger than the parallel wavenumber, is independent of the plasma mass density and its profile. Some aspects of the implementation of a current profile diagnostic based on the excitation of these resonances are also considered.

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I. INTRODUCTION

Knowledge of the plasma current profile is important for the understanding of plasma confinement and stability. Several techniques now exist for its measurement. Each involves a radial profile of a parameter related to the plasma current density such as i) the poloidal magnetic field measured by the Zeeman splitting of a neutral lithium beam[1], ii) the Faraday rotation of a far-infrared laser beam[2] or iii) the density fluctuations of the Kinetic Alfvén wave at resonance layers whose radial locations depend on the safety factor[3], q. These techniques have the advantage that they measure the current profile itself.

Often however, it suffices to measure a parameter whose value represents a known average of the plasma current density. The simplest is the measure of $\Lambda = \beta + l_i/2$ provided by the vertical field. In this case the plasma internal inductance l_i can be obtained by a separate measurement of β_p, poloidal beta. Changes in l_i can be inferred indirectly by comparing the temporal evolution of Λ with that of $\beta_{p\perp}$ obtained from the diamagnetic loop.

The use of the fundamental and harmonics of the magnetoacoustic resonance impedance where (n,m)= (0,0) has been proposed to obtain unambiguous information on the current profile[4]. An experiment[5] restricted to the fundamental resonance, however, has shown little sensitivity of the magnetoacoustic resonance to the plasma current.

Eigenmode resonances of the global Alfvén eigenmode or the Discrete Alfvén wave (DAW) have also been considered[6] as a possible current profile diagnostic. To date, however, it is not known precisely what measure of the current profile is provided by the DAW eigenfrequency. In fact, more than one profile may lead to the same frequency. The DAW frequency does however lead to a fairly precise parameterisation of q(a/2) and l_i for a restricted set of assumed current profiles. A major disadvantage is that the frequency also depends critically on the plasma mass density and its profile, neither of which can be determined.

In this paper, we examine an interesting property of the axisymmetric magnetoacoustic wave (also referred to as fast or compressional wave) with finite toroidal mode number. We show that the splitting of the resonance frequencies of its various radial modes is directly related to known moments of the plasma current profile. In addition, for low n modes, the frequency splitting is independent of the plasma mass density and its profile.

The structure of this paper is as follows. In section II, the analytical theory of the magnetoacoustic resonance splitting in a homogeneous plasma is discussed and a simple formula is derived expressing the inverse frequency splitting $\Delta(1/f)$ in terms of the constant current density. In section III numerical calculations relevant to the experimental conditions of a small tokamak are presented which test the predictions of the simple formula, examine the effects of profiles and generalize the simple formula to the case of an inhomogeneous plasma. In section 4, some aspects concerning the implementation of a current profile diagnostic are briefly considered.

II. THE ANALYTICAL THEORY OF THE FREQUENCY SPLITTING OF THE MAGNETOACOUSTIC WAVE IN A HOMOGENEOUS PLASMA

A. Introduction

Splitting of the magnetoacoustic wave resonance has previously been observed using magnetic probes[7,8] and collective scattering[9]. In these cases, high power RF was employed at a single frequency while a density scan swept the spectrum. Calculations had shown that the density splitting is given by[8],

$$\frac{\Delta\rho}{\rho} \approx \frac{2nma^2}{3R^2\langle q\rangle} \qquad (1)$$

where a, R are the minor, major radius respectively, $\langle q\rangle$ the average value of the safety factor and $\Delta\rho/\rho$ the fractional density splitting at constant frequency. Terms of order n^2a^2/R^2 have been neglected with respect to unity in equation (1).

The absense of splitting for m = 0 is a result of the model used to derive equation (1). The splitting has two origins[10]. The first is the redefinition of k to $k_{||}$ when finite poloidal field is introduced into the MHD model. This effect is proportional to m and results in a k-shift of the dispersion curves. The second is a slight asymmetry resulting from the Jxb force due to plasma current density and wave magnetic field on the plasma. Dispersion curves of the magnetoacoustic waves can be found in the paper by Ballico et. al.[10].

B. Calculation of the Frequency Splitting for the Case of a Homogeneous Plasma

We now give two derivations of the frequency splitting for the case of a homogeneous plasma. These are based on the MHD equations including finite frequency and plasma current,

$$-i\omega\rho v = j \times B + J \times b \qquad \text{and} \qquad E + v \times B = \frac{1}{n_e e}(j \times B + J \times b)$$

Together with $\nabla \times E = -i\omega b$ and $\nabla \times b = \mu_0 j$ these relations can be solved for a homogeneous plasma[11] to obtain the following Bessel equation,

$$\frac{d^2 b_{||}}{dr^2} + \frac{1}{r}\frac{db_{||}}{dr} + (k_\perp^2 - \frac{m_\perp^2}{r^2})b_{||} = 0 \qquad (2)$$

and dispersion relation,

$$k_\perp^2 = \frac{(A - k_{||}^2)^2 - (fA - Dk_{||})^2}{A - k_{||}^2} \qquad (3)$$

$$A = \frac{\omega^2/V_A^2}{1 - f^2}, \quad V_A^2 = \frac{B^2}{\rho\mu_0}, \quad f = \frac{\omega}{\omega_{ci}}$$

$$D = \mu_0 J/B, \quad m_\perp = m - kD/2, \quad k_{||} = k + mD/2,$$

which is valid when B_θ the poloidal field, is much smaller than, B_z the toroidal field and the plasma mass and current density profiles are uniform.

The dispersion relation is a quadratic in $k_{||}^2$ and therefore has two solutions; the Alfvén wave and the magnetoacoustic wave. When there is no vacuum gap between plasma and wall, application of the boundary condition $b_r = 0$ leads to a spectrum of discrete radial modes of the magnetoacoustic wave for all values of (n,m). These radial modes have discrete values of $k_\perp$ and are labelled by the letter l in increasing order, according to the number of half wavelengths in the radial direction. These modes experience a waveguide cutoff. If there is a vacuum layer[11] then this conclusion is no longer valid for the $m \neq 0$, $l = 1$ modes. These waves propagate as surface waves, have a continuously varying $k_\perp$ and do not experience a waveguide cutoff. We are not concerned with the surface waves in this paper.

In a first attempt, we derive the density splitting $\Delta\rho/\rho$ using equation (3) and the definition of $k_{||}$ for the case of a homogeneous plasma. We neglect the effect of $J \times b$ on the plasma by dropping $Dk_{||}$, the only J-dependent term in equation (3).

For a given ω, m, l and $k_\perp$ there are two different resonant densities for each of +k and -k.. The variation in A between +k and -k is $A\Delta\rho/\rho$ so that,

$$\frac{\Delta\rho}{\rho} \approx \frac{2kmD(1+f^2)}{k_\perp^2} \tag{4}$$

Equation (4) demonstrates the proportionality to mD noted by Ballico et. al.[10]. In the cylindrical approximation of a tokamak $k = n/R$ and $D = 2/qR$, where q is the safety factor, so that equation (4) can be rewritten as,

$$\frac{\Delta\rho}{\rho} \approx \frac{4nm(1+f^2)}{qk_\perp^2R^2} \tag{5}$$

Apart from numerical factors, equation (5) has the same dependences as equation (1). We note however that the effect of **J**x**b** on the plasma has been neglected.

To find the frequency splitting for m = 0 we are forced to include the effect of **J**x**b** on the plasma by retaining $Dk_\parallel$ in equation (3). We use the same derivation as before except that, since we are interested in a current profile diagnostic, we keep the mass density constant and solve for Δf with $k_\parallel = k$. The following formula is obtained;

$$\Delta f \approx \frac{2\mu_0 nJf^2}{RB_\phi k_\perp^2} = \frac{4nf^2}{qR^2k_\perp^2}$$

where $f \geq 1$ and $k_\perp \gg k$ have been assumed. If Δf is sufficiently small with respect to unity then,

$$|\Delta(1/f)| \approx \frac{2\mu_0 nJ}{RB_\phi k_\perp^2} = \frac{4n}{qR^2k_\perp^2} \tag{6}$$

Equation (6) shows that there is a frequency splitting for the case of m = 0. We note the following important properties.

(i) $\Delta(1/f)$ is linearly proportional to J. This linearity is intuitively expected to remain valid in the case of an inhomogeneous plasma provided the current profile remains constant. Similarly, division of $\Delta(1/f)$ by the plasma current Rogowski signal provides a measure of the current profile form. The dependence of $\Delta(1/f)$ on the current profile will be derived in the next section.

(ii) Since the radial wavenumber, $k_\perp$, is a constant for a given vessel geometry, $\Delta(1/f)$ is not dependent on plasma mass density. On the other hand, a dependence of $\Delta(1/f)$ on the mass density profile is not excluded in the above derivation. This will also be discussed in the next section.

For the m = 0 magnetoacoustic wave in a homogeneous cylindrical plasma bounded by a conducting wall, $b_r \sim J_1(k_\perp r)$ where J_1 is the Bessel function of order unity. The condition $b_r(a)$ thus yields $k_\perp a$ = 3.83, 7.02, 10.2, 13.3 etc. where a is the minor radius. Substituting the TCA[13] values a = 0.18 m, R = 0.61 m into equation (6) and taking $q \sim 2$ we obtain $\Delta(1/f) = 0.012n$ and $0.0035n$ respectively for $l = 1$ and 2. The condition $k_\perp^2 \gg k^2$ is satisfied for $|n| < 4$ and $|n| < 7$ in each case. The splitting is therefore a tiny effect (0.3 - 5%) and requires a sensitive frequency measurement.

No previous attempt has been made to compare equation (6) with experimental results. In addition, to the best of the authors' knowledge, no experimental result on the splitting of the m = 0, n ≠ 0 magnetoacoustic resonance in a tokamak has ever been published.

III. NUMERICAL CALCULATIONS OF Δ(I/f) FOR THE CASE OF AN INHOMOGENEOUS PLASMA

A. Introduction

A numerical study was performed using the code ISMENE[12] in order to investigate the dependence of the splitting on the plasma current and mass density profiles and hence to generalise equation (6). ISMENE is a cylindrical kinetic code that includes Landau damping and transit time magnetic pumping (TTMP). Under conditions where damping due to a varying toroidal field can be ignored, the code should therefore provide a reasonable estimate of the resonance Q and the measurability of Δ(1/f).

Unless otherwise stated, the TCA parameters[13] of Table I were used in the study.

Table I

Plasma Current, I_p	130 kA
Toroidal Field, B	1.51 T
Major Radius, R	0.61 m
Minor Radius, a	0.18 m
Wall Radius, n	0.24 m
T_i (0), T_e (0)	500, 800 eV
Electron density, $n_e(0)$	$7.5 \times 10^{19} m^{-3}$
Electron density profile, $n_e(r)$	$(1-0.853\ (r/a)^2)^{\alpha_n}$
Density profile exponent (α_n) with Deuterium filling gas.	1.2 (typical)
Current density profile,	$(1-(r/a)^2)^2$

In Figure 1, the spectrum of calculated resonances for m = 0 , |n| = 1, 2, 4, 8 and 16, l = 1, 2 and 3 are plotted for the above conditions as a function of frequency between 10 and 30 MHz. The fundamental cyclotron frequency at 11.6 MHz and its first harmonic are shown as vertical broken lines. The frequency splitting for the different signs of n and the increase in the magnitude of the splitting with n is clearly evident in the figure; the n < 0 resonance having the lower frequency in each case. The broken curves connect the resonances associated with a given radial mode. The Q's of the resonances (≈ 10^5) are generally quite large. ISMENE was also run with an anomalous collision frequency of 100 times the Spitzer value to account for possible turbulence enhanced collisions[14], however no effect on wave damping was observed. The calculated Q values are nonetheless unrealistic and experimental values lower than 100 would make $\Delta(1/f)$ difficult to measure since the +n and -n resonances would not be resolvable

B. Effect of Plasma Current

The dependence of $\Delta(1/f)$ on plasma current for fixed profiles and conditions otherwise as in Table I, is shown in Figure 2 for $|n| = 1, 2$ and 4 and $l = 2$. It was a general result of the simulations that regardless of the current profile, as long as it is held fixed, $\Delta(1/f)$ is a very linear function of plasma current. Current profile changes are therefore immediately evident by comparison with the plasma current Rogowski signal.

C. Effect of Mass Density

The dependence of $\Delta(1/f)$ on equilibrium mass density is demonstrated in Figure 3. Curves are shown for $n = 1, 2, 4$ and $l = 2$ as a function of central density from 2.5 to 15×10^{19} m^{-3} and with the standard density profile of Table I.

Despite the obvious large variations in the individual frequencies f of each resonance, required to satisfy the resonance condition with a changing density, $\Delta(1/f)$ varies negligibly for n = 1, 2 and slightly for n = 4. In the latter case $k/k_\perp \approx 0.17$ and the $k \ll k_\perp$ approximation is brought into question.

For the case of $|n| = 1$, the effect of a mass density profile change has also been examined. In Figure 3 the effect of varying the exponent, α_n, of the density profile from 0.40 to 1.60 away from its standard value of 1.2 has been examined for the case of a fixed central density of 7.5×10^{19} m^{-3} and a fixed average density of 5.11×10^{19} m^{-3}. For n = 1 there is clearly no discernible variation in $\Delta(1/f)$. This surprising result is attributable to the fact that $k_\perp$ is not sensitive to the nature of the density profile. Indeed, the magnetoacoustic wave fields within the plasma are not very different for the standard profile with a 6 cm vacuum gap than they are for the case of a homogeneous plasma without a vacuum gap. Hence one can imagine a close fitting Bessel function which defines a fairly precise $k_\perp$. This observation ties in with the well known property that the magnetoacoustic wave propagates at an average Alfvén speed and therefore, to a first approximation, "sees" the plasma as a homogeneous medium. On the other hand, the magnetoacoustic wave with $n > 0$ is affected differently by plasma current to that with $n < 0$, since the wavefields, **b** and hence the forces **J**x**b** are differently phased with respect to the wavefields **j** and their forces **j**x**B**.

The insensitivity of $\Delta(1/f)$ to the mass density and its profile is an important asset for a current profile diagnostic.

D. The Effect of Plasma Current Profile

Several examples of extreme profile variations are shown in Figure 4 for $|n| = 1$, $l = 2$. Curve a) shows $\Delta(1/f)$ at constant current profile as in Figure 2. Curve b) shows the variation in $\Delta(1/f)$ for q(0) varying from 0.50 to 3.85 with $I_p = 100$ kA. In this case the exponent in the parabolic profile of Table I was varied to keep I_p fixed. Curve c) shows the variation in $\Delta(1/f)$ versus I_p for $q(0) = 1$. As expected, the variation is less than linear and therefore weaker than that of curve a). This is emphasized in curve d) where curve c) is divided by I_p. The flattening of the current profile at high plasma current is revealed by a decrease in $\Delta(1/f)/I_p$. This sort of behaviour would be expected for example in TCA where q(0) approaches unity at high plasma current[3].

E. Generalising the Expression for $\Delta(1/f)$

Having established that $\Delta(1/f)/I_p$ contains information about the current profile we now attempt to derive the profile transformation on which $\Delta(1/f)$ depends linearly. It must be emphasized however that, because $\Delta(1/f)/I_p$ is closely constant for a fixed current profile, current profile changes are already detectable. In situations where the current profile can be measured, $\Delta(1/f)/I_p$ can also be calibrated. Hence, for practical purposes knowledge of the transformation is of secondary interest.

One possibility for the transformation of the current profile is suggested by calculating the speed of the low frequency axisymmetric magnetoacoustic wave in a zero current plasma with a non-uniform mass density. If one considers the ideal MHD equations,

$$-i\omega\rho v = j \times B \qquad E + v \times B = 0 \tag{7}$$

and Maxwell's equations then the equation for a propagating magnetoacoustic wave is given by,

$$\nabla^2 b + \frac{\omega^2}{V_A^2} b = \mu_0 \nabla \times j_z + \frac{i\omega\mu_0}{B^2}(\nabla\rho \times E) \tag{8}$$

For the axisymmetric wave, the radial component of equation (8) is

$$\left(\nabla^2 - \frac{1}{r^2}\right) b_r + \frac{\omega^2}{V_A^2} b_r = 0 \tag{9}$$

Taking $\int_0^a dr\, 2\pi r b_r^*$ on both sides one obtains:

$$\left\langle b_r^*\left(\nabla^2 - \frac{1}{r^2}\right)b_r\right\rangle + \omega^2\left\langle\frac{|b_r|^2}{V_A^2}\right\rangle = 0 \qquad (10)$$

where $\langle X\rangle = \int_0^a dr\, 2\pi r X$. As noted previously the m = 0 magnetoacoustic wave within the plasma has approximately a Bessel function character even when the density profile is not uniform. Hence equation (10) and equation (2) with $m_\perp = m = 0$ can be combined to yield;

$$\frac{\omega^2}{V_A^{*2}} = k^2 + k_\perp^2 \qquad (11)$$

where

$$1/V_A^{*2} = \frac{\langle |b_r|^2/V_A^2\rangle}{\langle |b_r|^2\rangle}. \qquad (12)$$

Note that b_r (approximately a J_1- Bessel function) is involved in the moment integral for the definition of $V_A{}^*$ and hence a plasma mass density moment ρ^*. This is natural because the force responsible for the wave is the jxB force of equation (7) and the only component of j for the axisymmetric magnetoacoustic wave (j_θ in this limit) is propositional to b_r. Only the plasma in the region of non-zero j_θ provides the inertia restored by jxB. Equation (11) defines precisely the average density "seen" by the magnetoacoustic wave.

One could use the same procedure to calculate analytically the moment of the plasma current profile described by $\Delta(1/f)$, however the algebra is not so tractable. A moment of the plasma current can be derived heuristically by rewriting the dispersion relation of equation (3) in the $k_\perp \gg k$ limit;

$$\frac{\omega^2}{V_A^2} + 2fkD = k_\perp^2 + k^2(1+f^2) \qquad (12).$$

Since the term in D is linear like the term in ρ, we conclude by analogy with equation (11) that the moment D^* of D must be defined in the same way as the moment ρ^* of ρ. On the basis of equation (6) we therefore define the moment of the current profile as follows;

$$J^* = \frac{\langle J|b_r|^2\rangle}{\langle |b_r|^2\rangle} \qquad (13)$$

which for a uniform J gives $J^* = I_p/\pi a^2$. We can now rewrite equation (4) as

$$|\Delta(1/f)| = \frac{2\mu_0 n J^*}{R B_\phi k_\perp^2} \qquad (14)$$

By taking b_r to be simply a J_1 - Bessel function the data of Figure 4. have been replotted as $\Delta(1/f)$ versus J^* in Figure 5.

In Figure 5 calculations have also been made for a class of flat top Gaussian current profiles of the form,

$$J(r) = \begin{cases} 1 & 0<r<r_0 \\ \exp\{-(r-r_0)^2/\Delta^2\} & r_0<r<a \end{cases}$$

where Δ is the Gaussian width. These profiles are typical of those observed in TCA[3]. Points are shown for Δ values from 0.05 to 0.50 m at Ip = 130 kA with associated changes in q(0) from .98 to 3.00.

From Figure 5 there is a clear clustering of points around the line described by equation (13). This indicates that there is some truth in the notion that the inverse splitting is uniquely dependent on a moment of the current profile as a result of the average current profile seen by the magnetoacoustic wave. The scatter in the points from one profile to another however, is sufficient to render the determination of the current profile moment from $\Delta(1/f)$ inconclusive, especially in cases where small profile changes are to be measured.

IV. DISCUSSION

In this section we superficially consider the practical aspects and problems in the use of the inverse frequency splitting $\Delta(1/f)$ as a current profile diagnostic. A detailed discussion is not possible until physical properties such as resonance Q are known. Notwithstanding, it is clear that the relative independence of the parameter $\Delta(1/f)$ on the mass density and its profile is both interesting from the physical point of view and advantageous for application to current profile measurement. It has also been shown that $\Delta(1/f)$ can be crudely interpreted in terms of a known moment of the current profile.

In Figure 6 we show the $\Delta(1/f)$ expected for the variation in q(0) and the variation of the density profile that might be obtained during ICRF heating in JET[15]. The typical plasma conditions, I_p = 5 MA, B_ϕ = 3.15 T, $n_e(0)$ = 2- 4 x10^{19} m^{-3}, R = 2.96 m, an equivalent cylindrical minor radius, a = $\sqrt{(2.10 \times 1.25\ m^2)}$ = 1.62 m and Deuterium filling gas were used in the calculation. The magnetoacoustic wave has

$n = 10$ ($k = 3.38$ m^{-1}) and $l = 10$ ($k_\perp = 19.9$ m^{-1}). The variation in $\Delta(1/f)$ is shown for line averaged n_e constant with q(0) varying and for both n_e and q(0) varying as in a typical JET heating scenario. It is clear from the figure that $\Delta(1/f)$ depends more strongly on the current profile change than on the density variation as previously stated, even though $k/k_\perp = 0.17$. The resonance Q factors are again in the range 10^5 for the equivalent JET cylindrical plasma with T_e= 8 KeV, T_i= 5 KeV and electron-ion collisions 100 times anomolously high.

The diagnostic is not without problems. We now state briefly certain problems likely to arise in practice.

The first is that $\Delta(1/f)$ does not provide a simply interpretable measurement of q(0). Only in the case where many magnetoacoustic resonances could be resolved would the different moments J^* provide detailed information about the profile structure. In addition, the variation in $\Delta(1/f)$ at fixed q(0) and q(a) is too small to carry information about the detailed structure of the current profile such as the internal current distribution or the edge value of current density when the value of q(0) is assumed to be known.

A second problem is the resolution of the two resonances at low values of quality factor Q. Experimental results for m = 1 magnetoacoustic resonances show a significant discrepancy between the theoretical $Q \sim 10^5$ and the experimentally observed values $Q \sim 10^3$. The mechanism for the discrepancy is poorly understood, however we have already shown that anomolous collisions due to MHD turbulence are not responsible. An additional proposed mechanism is damping at the ion plasma frequency near the edge[16]. This effect is compounded by the possibility of cyclotron damping in the ion cyclotron range of frequencies. Experimental results[7] indicate that fundamental (f = 1) harmonic damping, which occurs when the ion cyclotron layer is in the plasma, completely damps the magnetoacoustic wave and eliminates the resonance. In addition, the resonance peaks are not recovered when the whole plasma is below the ion cyclotron frequency. This excludes the possibility of using the first radial mode in TCA (Figure 1) except at low densities. Normally no problem is expected at the harmonic, f = 2, in a pure deuterium plasma. A small amount of hydrogen impurity however, causes significant damping of the resonance when the hydrogen cyclotron layer is present in the plasma[7,16]. Once again, one is limited to the choice of resonances above the hydrogen cyclotron frequency. From a practical point of view, it is to be noted that the desirable properties of the $\Delta(1/f)$ diagnostic stem from the proximity of the two resonant frequencies. These frequencies may be so close ($\Delta f/f$ = .003 in JET) that a very careful measurement with a slow scan over the peaks is necessitated; especially if the resonance quality factor Q is lower than 10^3. Under these conditions mode tracking by a servocontrolled generator can be used for increased frequency resolution.

A third disadvantage is that at low values of $k/k_\perp$ all values of k for a given $k_\perp$ occur at almost the same frequency. In practice therefore, a large number of probes or antennas are needed to distinguish the different resonance peaks for each value of k.

V. CONCLUSION

All methods of measuring parameters derived from the current profile present difficulties, either in the complexity of the diagnostic, the reliability, the resolution or the interpretation of the measurement. A method has been proposed using the inverse frequency splitting $\Delta(1/f)$ of the axisymmetric magnetoacoustic wave resonance which is simple to implement and interpret. The diagnostic has several elegant properties such as relative independence of its output on the mass density and its profile and an approximate physical interpretation in terms of a moment of the plasma current profile. It also appears that the diagnostic has several disadvantages such as resolution and a tenuous link with the standard current profile parameters, q(0) and the inversion radius. In addition, the diagnostic would be completely unusable in the presence of ICRF damping.

An important question not adressed in this paper is the nature of the frequency splitting when toroidal effects are included. In particular, what is the two dimensional analogue of the current profile moment J^*. One may for example speculate that the integral over b_r^2 should be taken in the plasma cross section.

A simple experiment to explore the spectrum of axisymmetric magnetoacoustic waves would soon reveal which modes provide a suitably resolvable frequency splitting for application to a current diagnostic.

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Figure Captions

1. Spectrum of axisymmetric magnetoacoustic waves in TCA.

2. Dependence of $\Delta(1/f)$ on plasma current at fixed profile.

3. Dependence of $\Delta(1/f)$ on plasma mass density and its profile. The plasma mass density profile was varied by changing the exponent α_n whilst keeping either $n_e(0)$ or line averaged n_e constant.

4. Dependence of $\Delta(1/f)$ for various plasma currents and current profiles.

5. Dependence of $\Delta(1/f)$ on J^* calculated for the profiles of Figure 5. using a J_1-Bessel function as an approximation to the wavefield **b**, the weighting function in the moment integral.

6. $\Delta(1/f)$ versus q(0) at constant line averaged $n_e = 3.00 \times 10^{19}$ m^{-3} and versus both n_e and q(0) in JET.

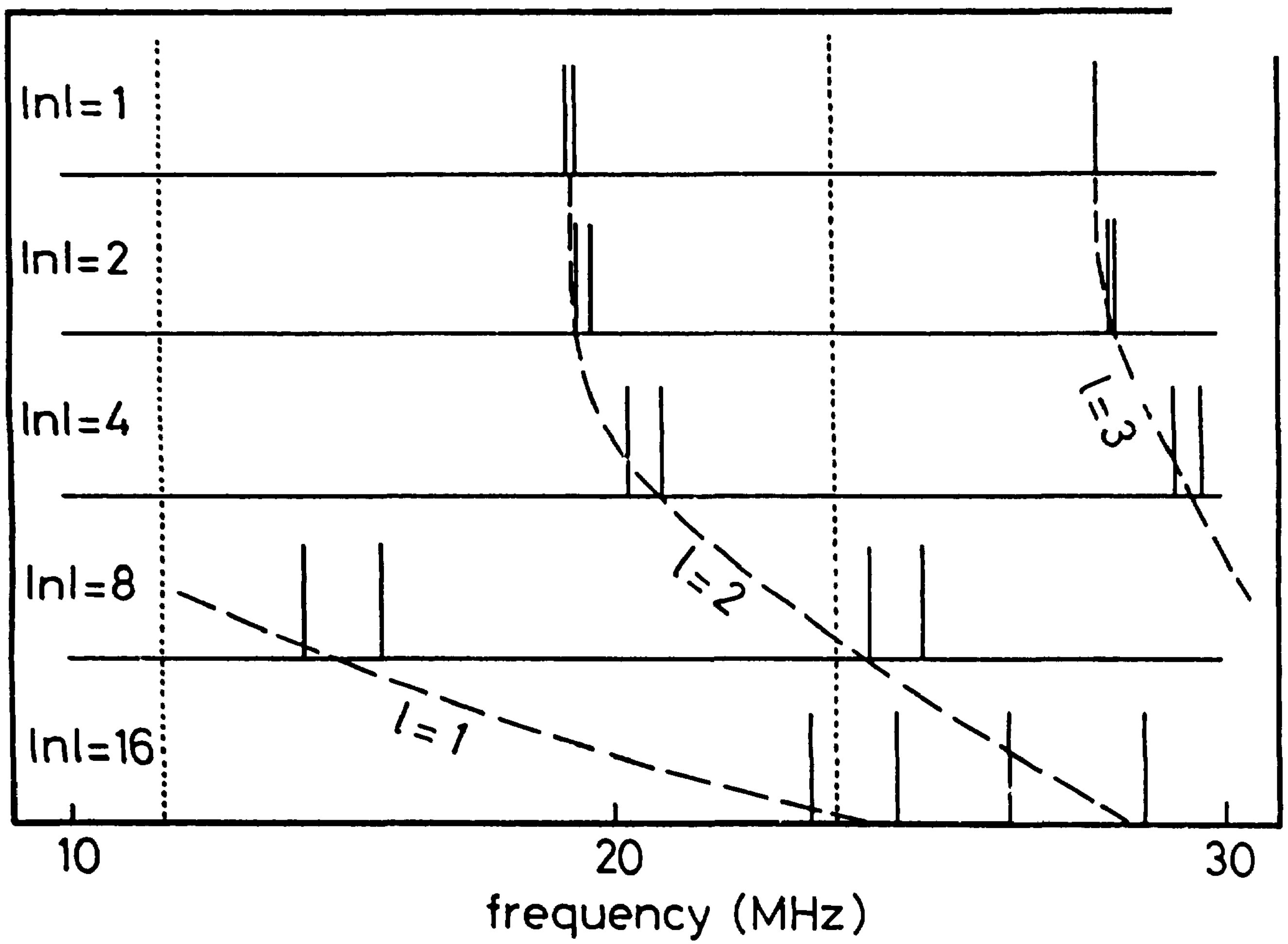


FIG. 1

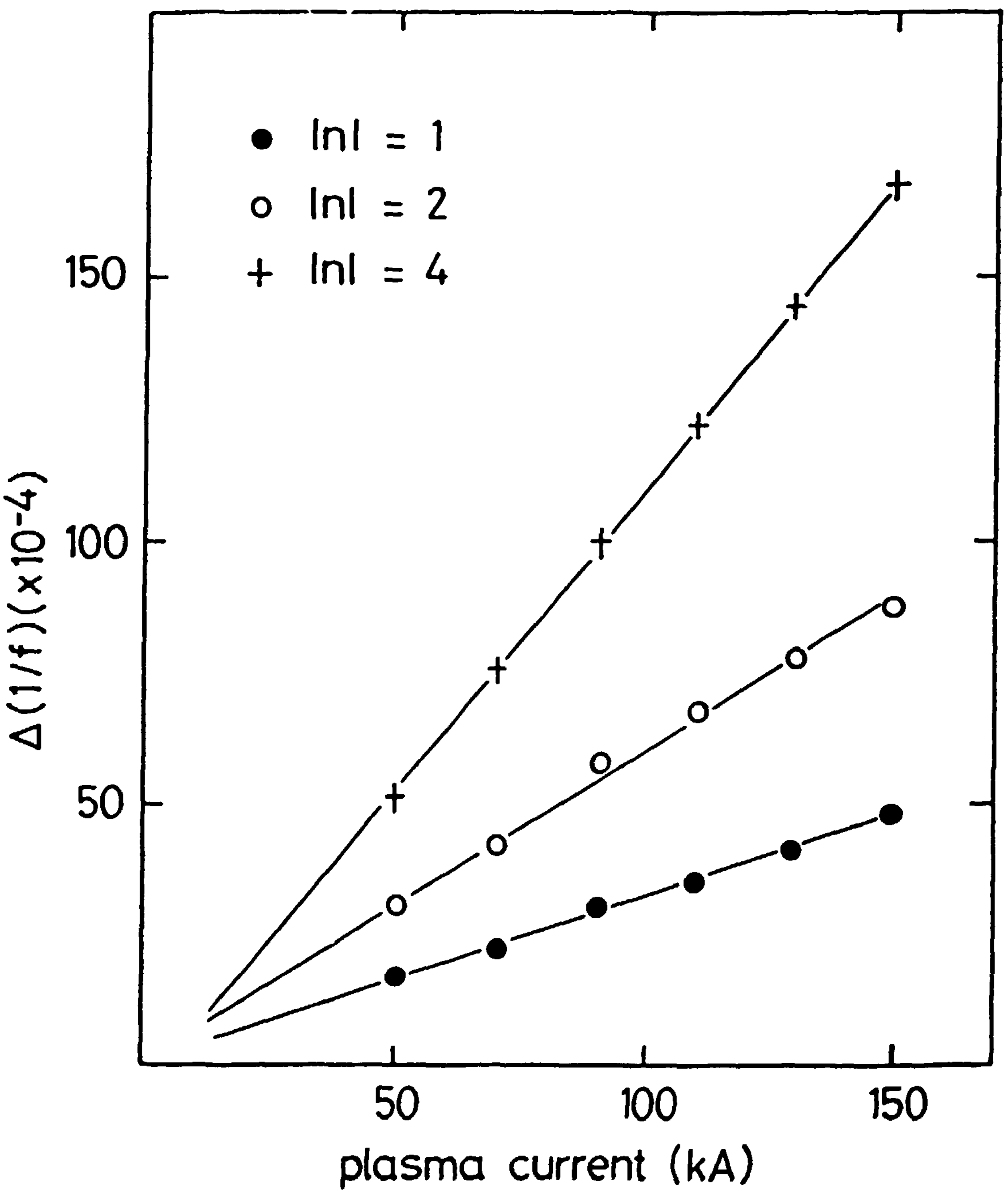
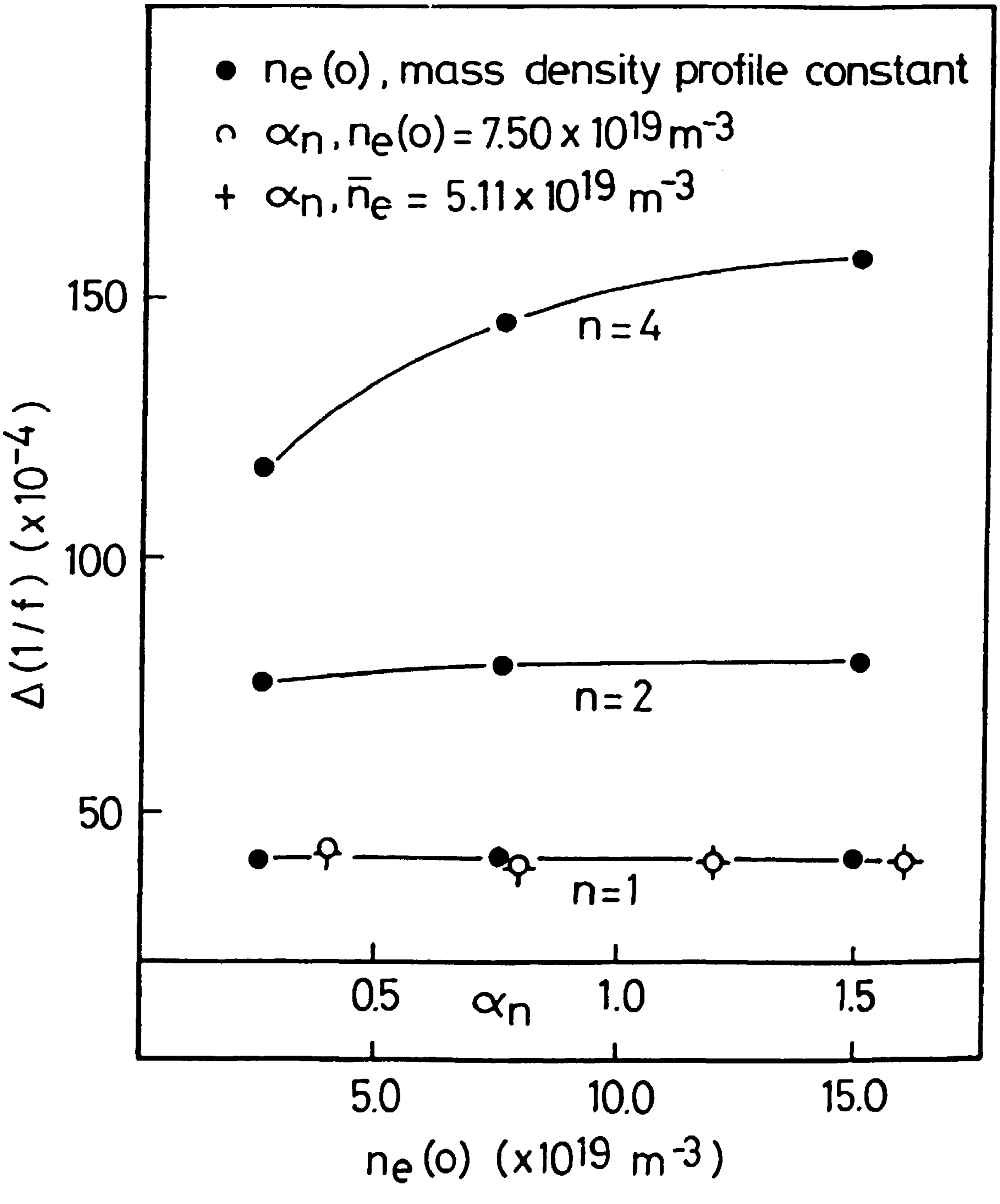


FIG. 2



● ne (o), mass density profile constant
○ αn, ne(o) = 7.50 x 10^19 m^-3
+ αn, n̄e = 5.11 x 10^19 m^-3
150
100
50
Δ(1/f) (x10^-4)
n=4
n=2
n=1
0.5
αn
1.0
1.5
5.0
10.0
15.0
ne (o) (x10^19 m^-3)

FIG. 3

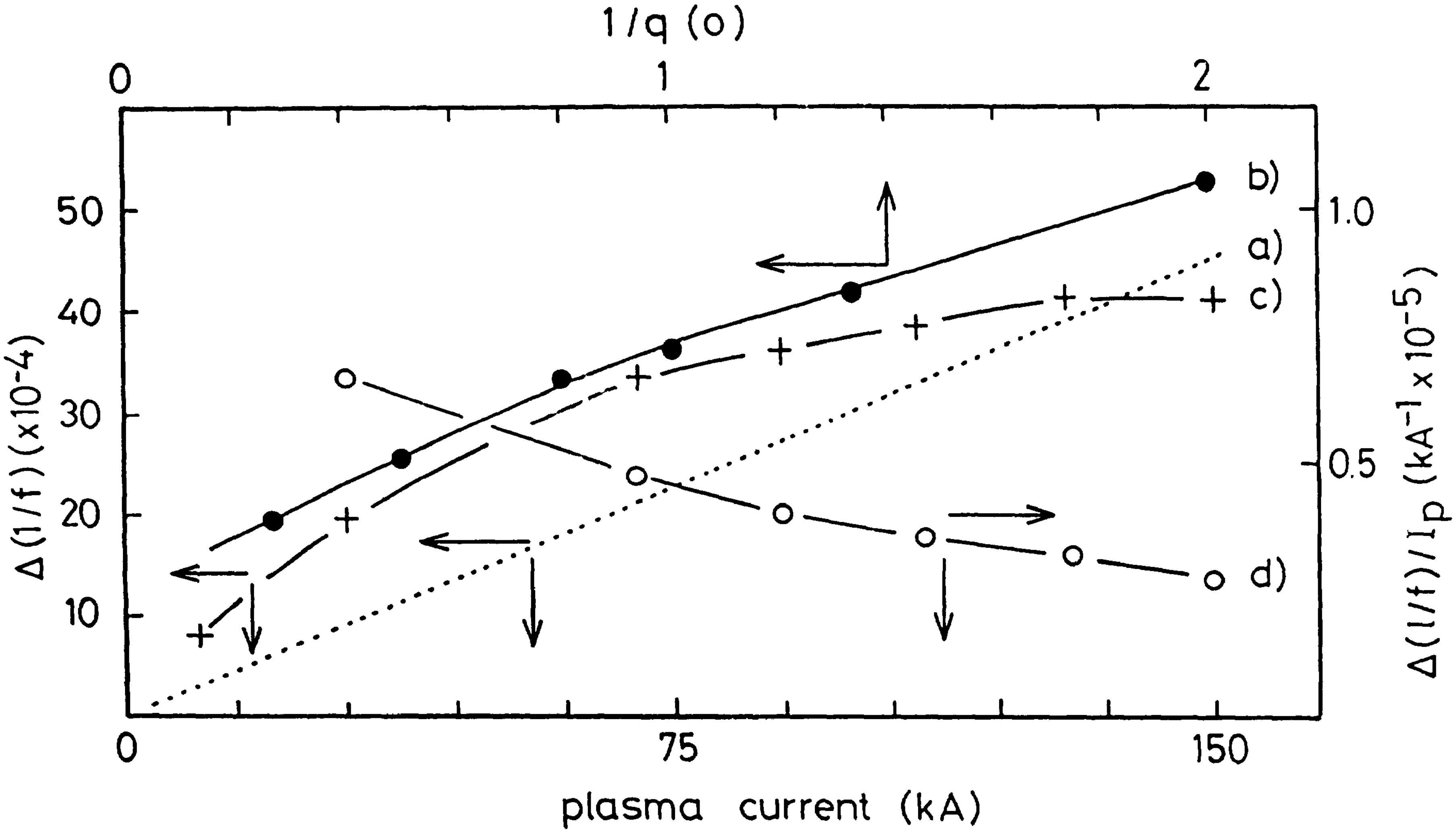
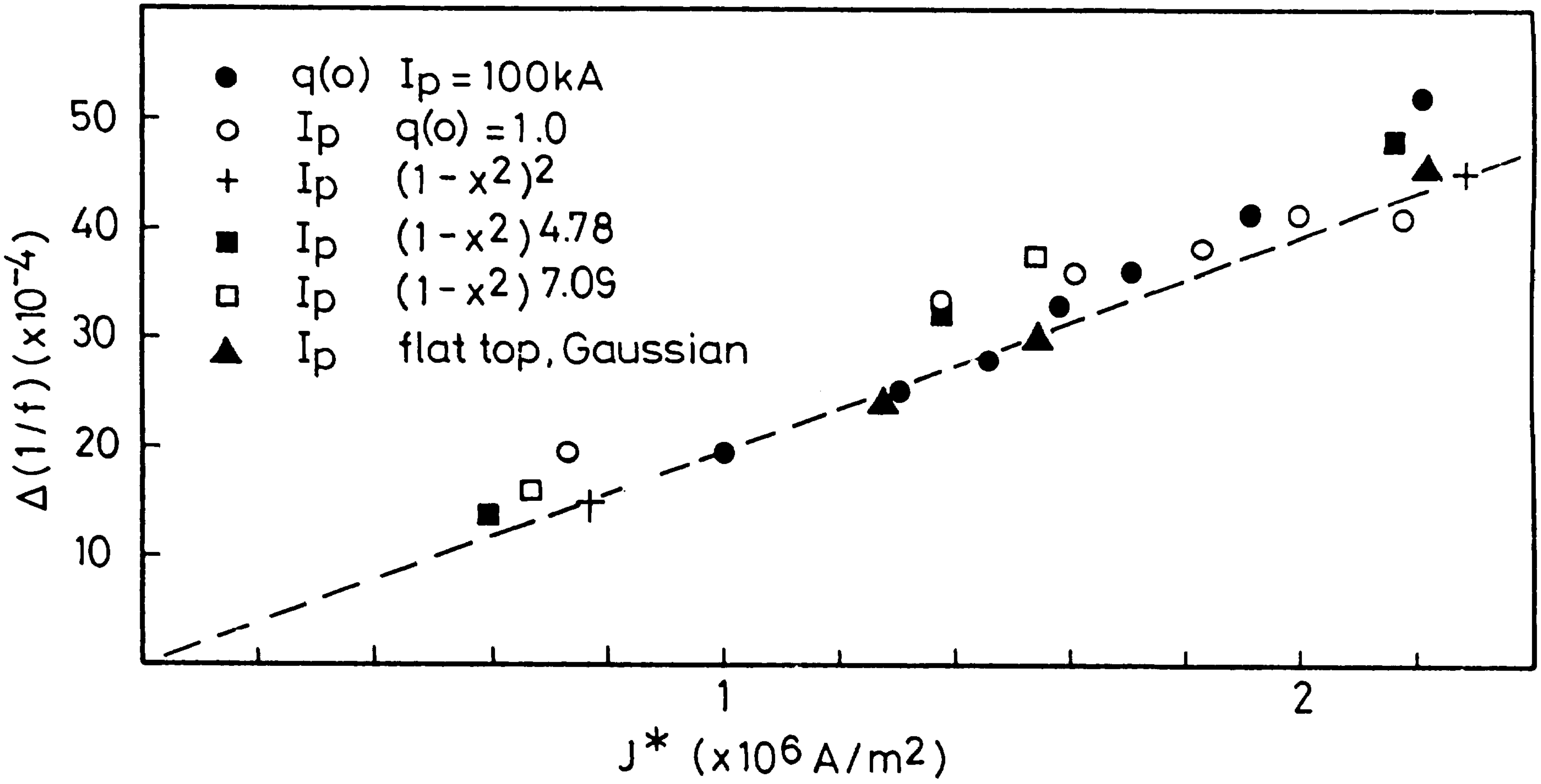

1/q (o)
0
1
2
50
40
30
20
10
Δ(1/f) (x10-4)
1.0
0.5
Δ(1/f)/Ip (kA-1 x 10-5)
b)
a)
c)
d)
0
75
150
plasma current (kA)

FIG. 4



q(o) Ip = 100kA
Ip q(o) = 1.0
Ip (1 - x2)2
Ip (1 - x2) 4.78
Ip (1 - x2) 7.09
Ip flat top, Gaussian
50
40
30
20
10
Δ (1/f) (x10-4)
1
2
J* (x106 A/m2)

FIG. 5

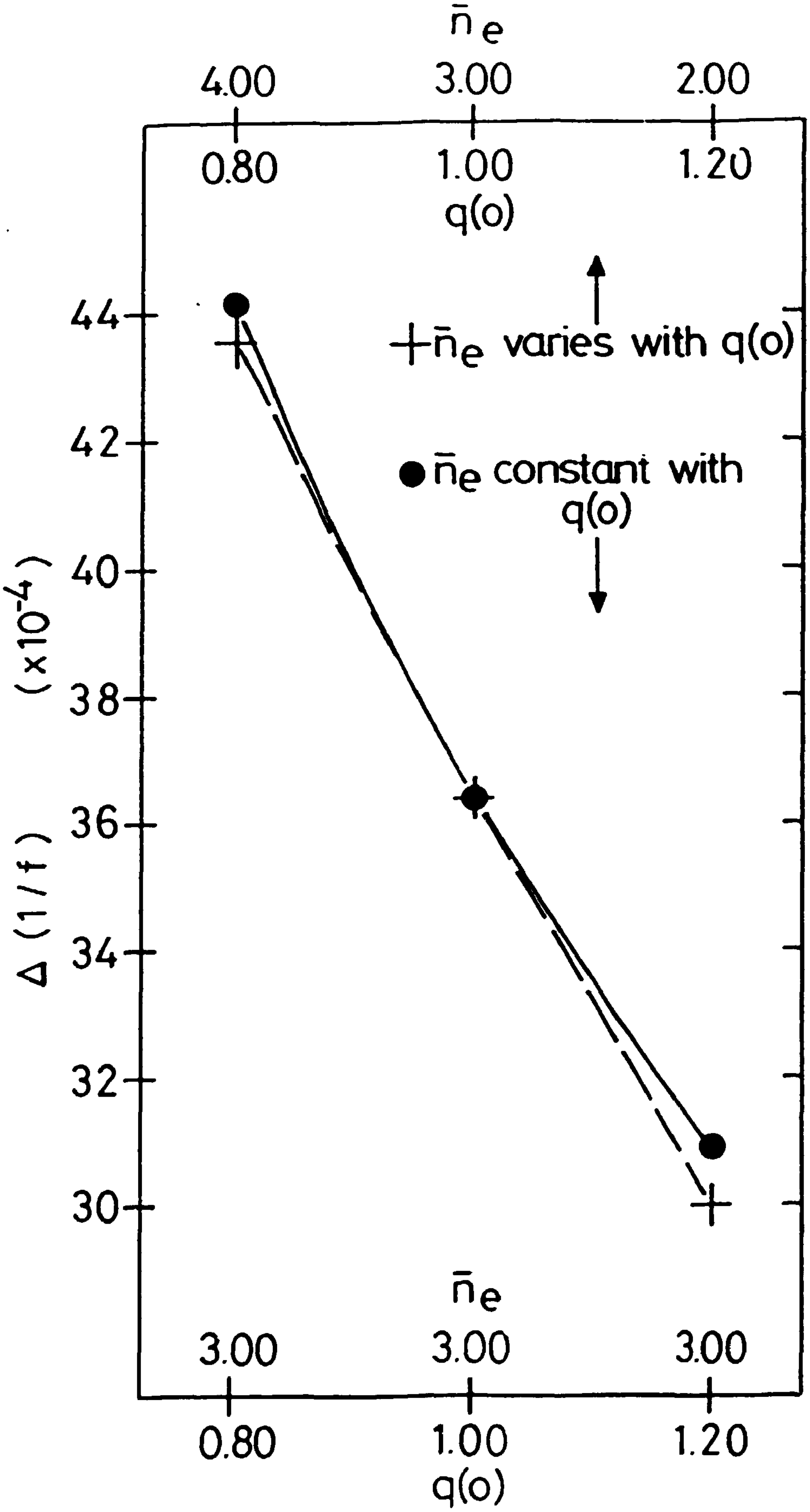

$\bar{n}_e$
4.00
3.00
2.00
0.80
1.00
1.20
q(o)
44
42
40
38
36
34
32
30
Δ (1/f) (×10^{-4})
+ $\bar{n}_e$ varies with q(o)
● $\bar{n}_e$ constant with q(o)
$\bar{n}_e$
3.00
3.00
3.00
0.80
1.00
1.20
q(o)

FIG. 6