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ERBIUM ISOTOPES ($A = 158 - 170$) AND ISOTONES WITH
 $N = 94$ ($A = 158 - 164$) /AS WELL/

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REDUCED PROBABILITIES OF GAMMA TRANSITIONS IN EVEN-EVEN
ERBIUM ISOTOPES ($A = 158 - 170$) AND ISOTONES WITH
 $N = 94$ ($A = 158 - 164$) /AS WELL/

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The reduced probabilities of the E2 intraband and interband transitions in even-even erbium isotopes ($A = 158 - 170$) and $N = 94$ isotones ($A = 158 - 164$) are analyzed using perturbation theory and assuming the mixing wave functions of ground, beta- and gamma-vibrational states and different value of intrinsic quadrupole moments of their states.

Experimental values of $B(E2)$ gamma transitions between levels of rotational β -, γ -bands and the ground state in even-even deformed nuclei were originally analyzed only with the Bohr-Mottelson rotational model [1]. Ratios of $B(E2)$ of two interband transitions that deexcite a given level of the β - or the γ -band to two different levels of the ground state band should obey the Alaga rules [2], according to this model. Instances that violated this rule were observed [3] early.
Explanation was found in taking into account of Coriolis and other interactions in the framework of perturbation theory

[3-8]. Formulae for $B(E2)$ in these papers [3-8] are obtained under different assumptions. E.g. C.W. Reich and J.E. Cline [7] supposed that intrinsic quadrupole moment of the ground state and that of the γ -vibrational state have different values. P.O. Lipas [4] and L.L. Riedinger et al. [6] assumed mixing of wave functions of rotational levels of the β - and the γ -vibrational states and N. Rud et al. [8] took into account a term nonlinear in the parameter $\epsilon_{\gamma\gamma}$ of the type $\epsilon_{\gamma}\epsilon_{\beta\gamma}$ in addition. Besides this, the parameters Z_Q , which are usually obtained from experimental data, are not defined in the same way in [3-8]. It remains to remark that misprint have probably crept into the definitions of functions $F_{\gamma}(I_1, I_f)$, $F_{\beta\gamma}(I_1, I_f)$ and $F'_{\beta\gamma}(I_1, I_f)$ /see below/ in the paper [6], which authors of experimental work refer to very often. These circumstances made us for deriving of formulae for $B(E2)$ see [9] under general assumptions:

values of intrinsic quadrupole moments differ -

$Q_{gg} \neq Q_{\beta\beta} \neq Q_{\gamma\gamma}$ mixing of wave functions of rotational levels of β - and γ -vibrational states is nonzero

($\epsilon_{\beta\gamma} \neq 0$), and the term proportional to $\epsilon_{\beta}\epsilon_{\beta\gamma}$ or

$\epsilon_{\gamma}\epsilon_{\beta\gamma}$ is not neglected.

Adequacy and usefulness of inclusion of these effects is checked up below on available experimental data on deformed isotopes of erbium ($Z = 68$) and isotones with $N = 94$. We obtained considerable part of our experimental results from investigation of beta decay of relevant nuclei by anti-Compton spectrometer in INP Řež and in the framework of the program JASNAPP-1. As sufficiently exact experimental data from reaction (n, γ) or $(\alpha, xn\gamma)$ and $(p, xn\gamma)$ are known

we took them in the set of initial data.

Reduced E2 probabilities of γ -transition that depopulate levels of rotational bands of β - or γ -vibrational states to levels of the rotational band over the ground state could be written generally [9]

$$\begin{aligned}
 B(E2; I_1 0_\beta \rightarrow I_f 0_g) = & B_0(E2; I_1 0_\beta \rightarrow I_f 0_g) \times \{ 1 + Z_\beta(0_\beta) P_\beta(I_1, I_f) + \\
 & + [Z_\beta(0_\beta) - Z_\beta(0_g)] f_\beta(I_1) + \{ Z_{\beta\gamma} P_{\beta\gamma}(I_1, I_f) [1 + Z_\gamma(2_\gamma) P_\gamma(I_1, I_f) + \\
 & + \frac{1}{2} Z_\gamma(2_\gamma) P_{\beta\gamma}(I_1, I_f)] \}^2 \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 B(E2; I_1 2_\gamma \rightarrow I_f 0_g) = & B_0(E2; I_1 2_\gamma \rightarrow I_f 0_g) \times \{ 1 + Z_\gamma(2_\gamma) P_\gamma(I_1, I_f) + \\
 & + [\frac{1}{2} Z_\gamma(2_\gamma) - \frac{1}{2} Z_\gamma(0_g) + Z_{\beta\gamma} (1 + Z_\beta(0_\beta) f_\beta(I_f))] P_{\beta\gamma}(I_1, I_f) \}^2 \quad (2)
 \end{aligned}$$

We showed in the article [9] that the nonlinear correction by Rud et al. [8] /a term proportional to the product $\{ Z_{\beta\gamma} Z_\gamma(2_\gamma) \}$ in eq. (1) or to the product $Z_{\beta\gamma} Z_\beta(0_\beta)$ in (2)/ comes to manifest itself for transitions from $I_1 \geq 4$ for the β -band or from $I_1 \geq 6$ for the γ -band. The definite value ^{of} minimal spin I_1 depends also on the accuracy of determination of gamma transition intensities. In case we neglect the nonlinear correction then in virtue of (2) it is not possible to separate the contribution to $B(E2)$ resulting from inequality of intrinsic quadrupole moments $Q_{gg} \neq Q_{\gamma\gamma}$ proportional to matrix elements of their operator $\mathcal{M}(E2, 0)$ and the contribution to $B(E2)$ caused by mixing of wave functions of β - and γ -vibrational states characterized by the parameter $Z_{\beta\gamma}$. However, there is possible to separate

these contributions in the case of knowledge of $B(E2, 2_g \rightarrow 0_g)$, $B(E2, 2_\gamma \rightarrow 0_g)$ and intensities of intraband transitions too.

The notation used in eqs. (1) and (2) is defined by the following relations (where $q = \beta, \gamma$)

$$B_0(E2, I_i K_q \rightarrow I_f 0_g) \equiv \langle I_i K_q 2 -K_f | I_f 0 \rangle^2 Q_{qg}^2 (2 - \delta_{K_q, 0}) \quad (3)$$

$$F_\beta(I_i, I_f) \equiv f_\beta(I_f) - f_\beta(I_i) \quad (4)$$

$$F_\gamma(I_i, I_f) \equiv \frac{1}{\sqrt{48}} \left\{ \frac{\langle I_i 220 | I_f 2 \rangle}{\langle I_i 22-2 | I_f 0 \rangle} f_\gamma(I_f) - \frac{1}{2} [1 + (-1)^{I_i}] \frac{\langle I_i 020 | I_f 0 \rangle}{\langle I_i 22-2 | I_f 0 \rangle} f_\gamma(I_i) \right\} \quad (5)$$

$$F_{\beta\gamma}(I_i, I_f) \equiv \frac{1}{2} [1 + (-1)^{I_i}] \frac{1}{\sqrt{12}} \frac{\langle I_i 020 | I_f 0 \rangle}{\langle I_i 22-2 | I_f 0 \rangle} f_\gamma(I_i) \quad (6)$$

$$F'_{\beta\gamma}(I_i, I_f) \equiv -\sqrt{12} \frac{\langle I_i 22-2 | I_f 0 \rangle}{\langle I_i 020 | I_f 0 \rangle} f_\gamma(I_i) \quad (7)$$

$$f_\beta(I) \equiv I(I+1) \quad f_\gamma(I) \equiv \sqrt{2(I-1)I(I+1)(I+2)} \quad (8)$$

$$Z_\beta(0_g) \equiv -\frac{Q_{\beta K}}{Q_{\beta g}} \epsilon_\beta \quad Z_\beta(0_\beta) \equiv -\frac{Q_{\beta\beta}}{Q_{\beta g}} \epsilon_\beta \quad (9)$$

$$Z_\gamma(0_g) \equiv -\sqrt{24} \frac{Q_{\gamma K}}{Q_{\gamma g}} \epsilon_\gamma \quad Z_\gamma(2_\gamma) \equiv -\sqrt{24} \frac{Q_{\gamma\gamma}}{Q_{\gamma g}} \epsilon_\gamma \quad (10)$$

$$Z_{\beta\gamma} \equiv -\sqrt{6} \frac{Q_{\beta\gamma}}{Q_{\beta g}} \epsilon_{\beta\gamma} \quad \{_{\beta\gamma} \equiv -\frac{1}{\sqrt{6}} \frac{Q_{\beta\gamma}}{Q_{\beta g}} \epsilon_{\beta\gamma} \quad (11)$$

The coefficients of wave functions mixing are denoted by ϵ_β , ϵ_γ and $\epsilon_{\beta\gamma}$ in these formulae. If $Q_{gg} = Q_{\beta\beta}$, then $Z_\beta(0_g) = Z_\beta(0_\beta) = Z_\beta$ and similarly if $Q_{gg} = Q_{\gamma\gamma}$ then $Z_\gamma(0_g) = Z_\gamma(2_\gamma) = Z_\gamma$.

The parameters $Z_q, Z_{\beta\gamma}$ or $\{_{\beta\gamma}$ were found from known experimental values of $B(E2)$ or more often from their ratios by the least square method using the program "SOFT" [10]

Experimental values of intrinsic quadrupole moment of

the ground state Q_{20} , reduced probabilities $B(E2, 0_g^+ \rightarrow 2_g^+)$ and $B(E2, 0_g^+ \rightarrow 2_f^+)$ of even-even deformed nuclei found by various authors are presented in a review article [11]. Weight averages of these values for nuclei under question are listed in tab. 1. Unfortunately, the set of all three necessary physical quantities is not known for all nuclei. Levels of rotational bands of β -, γ -vibrational state and the ground state excited in beta decay are in fig. 1 for the isotopes of erbium and in fig. 2 for the isotones with $N = 94$. Mean lives (τ) of individual levels are shown in figs. 1 and 2, experimental values are marked by an asterisk. Summary of experimental values used and their character (i.e. number of transitions, number of levels that are de-excited via at least two transitions and could be used in our calculations, minimal and maximal spin of levels) is in tabs. 2 and 3. Relatively little data on the rotational β -band is noticeable from this summary. It is necessary to add that levels with higher spins, that are excited in nuclear reactions, are often de-excited via two transitions, one of which is an intraband and the other interband. In this case the ratio of their intensities is less sensitive to a change of parameters Z_q than in the case of two interband transitions.

Gamma transitions relating levels of the same spin or levels whose spins differ by one can be of mixed multipolarity $M1+E2$. The multipolarity $M1$ is forbidden in transitions $\beta \rightarrow g$ and $\gamma \rightarrow g$ according to the Bohr-Mottelson model [1]. Nevertheless, weak admixture of $M1$ multipolarity in $\gamma \rightarrow g$ transitions is observed, although it is usually less than 2% and it does not exceed 10% [12]. Experimental data on the $M1$ admixture

in interband transitions of even-even erbium isotopes and isotones with $N = 94$ does not exist or are marred up by great errors [12]. The exception is ^{166}Er , ^{168}Er and ^{160}Dy transitions $\gamma \rightarrow g$ (see below). This is why we made no correction due to the admixture of M1 multipolarity (except ^{160}Dy). The assumption that M1 multipolarity admixture in $\beta \rightarrow g$ transitions with spin $I \rightarrow I$ is small is supported by comparing the description of $B(E2)$ with values of Z_{β} obtained with and without elimination of $\beta \rightarrow g$ transitions with spin $I \rightarrow I$ from experimental data studied (see below). As usual, intraband transitions relating levels with spin $I \rightarrow I-1$ in a γ -band include such an M1 multipolarity admixture reaching as much as several tens percent. Therefore such transitions were not taken into account when analyzing reduced probabilities $B(E2)$, unless the M1 admixture has already been determined experimentally.

Spectra of gamma radiation registered during the de-excitation of excited levels in nuclear reactions or in even beta decay of nuclei far from the line of beta stability are rather complicated as usual. Relatively often, double gamma lines or a gamma transition are placed into a decay scheme on the evidence of energy balance, which could have been fulfilled by chance. This is the way in which arise systematical errors in analysis of $B(E2)$ probabilities or their ratios, whose inclusion is very difficult.

Experimental data were treated consecutively for each nucleus. First, data from beta decay were checked, whether they conform to the Alaga rules, then the parameter Z_q was evaluated by least square method and then the same for the

two parameters Z_q , Z_{qq} (or $Z_q(K_q)$, $Z_q(O_g)$) or three parameters $Z_q(K_q)$, $Z_q(O_g)$, $Z_{qq}(Z_{\beta\gamma}, \{\beta\gamma\})$ as the case may be. Eventually, influence of the nonlinear correction was examined. Each of the beta or gamma bands was treated independently. The results are also included in tabs 2 and 3. In all applications of the Alaga rules was $\chi^2(A) \geq 10$ and in some cases $\chi^2(A) > 100$. By introduction of the coefficient Z_q value of $\chi^2(1)$ is decreased many times. Further increase of number of parameters tends to change $\chi^2(2)$, $\chi^2(2')$ or $\chi^2(3)$ only little in comparison with $\chi^2(1)$.¹⁾

1) $-\chi^2(q)$ - is the sum of the squares of the deviations divided by the experimental error, which is normalized to the number of degrees of freedom. This function is determined for the different number q of parameters. Its value is rounded up to one, resp. two, valid digits when it is greater (or equal), resp. less, than 3.

The analysis is repeated with all input data from two (or more) different works, in which levels were excited in the same way, via beta decay or nuclear reactions, provided the values of Z_q overlap within 2σ . With values specified in such a way we carried out calculations of intensities of known gamma transitions and those not observed yet, as well. We evaluated mean lives of excited states, see figs 1, 2. Using the method described above we amalgamated input data from beta decay and from nuclear reactions. Results of these calculations are also presented in tabs 2 and 3.

If we know intensities of only three transitions de-exciting the same level, we determine a parameter Z_{β} (162,166 , ^{170}Er , ^{160}Dy). If three different ratios of $B(E2)$ are known,

it is possible to find two parameters $Z_{\beta}(0_{\beta})$, $Z_{\beta}(0_g)$ or Z_{β} , $\xi_{\beta\gamma}$ ($^{160,164}\text{Er}$, ^{164}Yb). Similarly, we determined only two parameters for the γ -band in nuclei $^{158,160,170}\text{Er}$, ^{160}Dy and ^{164}Yb . All three parameters are evaluated for the β -band $^{158,168}\text{Er}$, ^{158}Gd (see tab. 2) and for the γ -band $^{162,164,166,168}\text{Er}$, ^{158}Gd (see tab. 3).

If χ^2 does not change much when fitted with one or three parameters, then $Z_{\gamma} \approx \frac{1}{2} [Z_{\gamma}(2_{\gamma}) + Z_{\gamma}(0_g)]$ or $Z_{\beta} \approx \frac{1}{2} [Z_{\beta}(0_{\beta}) + Z_{\beta}(0_g)]$. Values of mixing parameters of wave functions for the bands β and γ in nuclei with $Z = 68$ and $N = 94$ are presented in figs 3 and 4. Relative difference of intrinsic quadrupole moments $(Q_{qq} - Q_{gg})/Q_{qq}$ is equal to relative difference of mixing parameters $(Z_q(K_q) - Z_q(0_g))/Z_q(K_q)$. We determined the relative difference of intrinsic quadrupole moments of all nuclei studied except ^{158}Er and ^{158}Gd under the assumption that $Z_{\beta\gamma} = \xi_{\beta\gamma} = 0$. We found out that the relative difference $(Q_{qq} - Q_{gg})/Q_{qq} = 0$ within 3% .

We evaluated the value of parameter $\xi_{\beta\gamma} = -0.0116(14)$ ($Z_{\beta\gamma} = -0.0128(19)$) when analyzing the rotational band of beta (gamma) vibrations in ^{158}Er (^{158}Gd). The mixing parameters $Z_{\beta\gamma}$ and $\xi_{\beta\gamma}$ are related for a given nucleus by equation $Z_{\beta\gamma} \xi_{\beta\gamma} = \epsilon_{\beta\gamma}^2$. Therefore we analyzed transitions relating levels of beta and gamma rotational bands of ^{158}Er or ^{158}Gd simultaneously under assumption $Q_{gg} \neq Q_{\beta\beta} \neq Q_{\gamma\gamma}$, $Z_{\beta\gamma} \neq 0$ and $\xi_{\beta\gamma} \neq 0$. It turned out that even in this case matrix elements of intrinsic quadrupole moments are equal within 2% , see figs 3 and 4. It should be remarked that the equality of the quadrupole moments $Q_{gg} \approx Q_{\gamma\gamma}$ for $^{164,166,168}\text{Er}$ and ^{158}Gd was established with accuracy of few percent. Measu-

rement of re-orientation effect in Coulomb excitation of 2^+_{γ} levels of some Er isotopes [13] made possible to find values of $Q_{\gamma\gamma}$ with lower accuracy, see fig. 4.

In the following part of this paper we deal with more thorough analysis of mixing parameters of particular erbium isotopes ($A = 158-170$) and isotones with $N = 94$ (^{158}Gd , ^{160}Dy , ^{162}Er and ^{164}Yb).

^{158}Er

There were identified three levels of the rotational band of beta vibrations during investigation of excited levels of ^{158}Er via beta decay [14]. It is possible to use seven gamma transitions that de-excite the two levels with spin $I^{\pi} = 2^+$ and 4^+ for analyzing of $B(E2)$. Values of $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ for interband transitions are shown in fig. 5. They were obtained under the assumption that (i) only one parameter $Z_{\beta} = 0.030(16)$ is non-zero (empty circles) or (ii) two parameters $Z_{\beta} = 0.056(4)$ and $\beta_{\gamma} = -0.0111(11)$ (full circles). In the former case ($\chi^2(1) = 30$) we observe considerable differences between calculated and experimental values of reduced probabilities, which exceeds three times the value of experimental error for the transition $2_{\beta} \rightarrow 4_g$. In the latter case $\chi^2(2')$ drops considerably (to 0.2). Within one σ the ratio $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2} \approx 1$. It is interesting that with fitting using two parameters $Z_{\beta}(0_{\beta})$ and $Z_{\beta}(0_g)$ we have higher value $\chi^2(2) = 50$. The elimination of transition with spin $I \rightarrow I$ from experimental data doesn't affect the value of Z_{β} ($Z_{\beta} = 0.032(4)$, $\chi^2 = 3$). Comparison of calculated inten-

sity $I_{\gamma}(E2)$ and experimental $I_{\gamma}(E2+M1)$ of this transition with spin $I \rightarrow I$ leads to the value of $I_{\gamma}(M1)$ less than experimental error of $I_{\gamma}(E2+M1)$.

Two intraband transitions $2_{\beta}^{+} \rightarrow 0_{\beta}^{+}$ and $4_{\beta}^{+} \rightarrow 2_{\beta}^{+}$, whose ratio is $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}] \approx 1$ within 1.3 $\%$ for all version of calculations, were also included into analysis of β -band. Knowledge of an intraband transition intensity enables us to find matrix element Q_{β} or Q_{γ} as the case may be.

It is known only five interband transitions de-exciting three levels of γ -rotational band ($I^{\pi} = 2^{+}, 3^{+}, (4^{+})$) and two intraband transitions within the band: $3_{\gamma}^{+} \rightarrow 2_{\gamma}^{+}$, $4_{\gamma}^{+} \rightarrow 2_{\gamma}^{+}$ for this nucleus. Analysis of reduced probabilities with one parameter $Z_{\gamma} = 0.23(6)$ gives relatively low value $\chi^2 = 12$ and $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2} \approx 1$ within 2 $\%$ for all the seven transitions. If we increase number of parameters to two, the value of $\chi^2(2)$ increases to 14. This increase is caused by decrease of number of degrees of freedom. Despite this agreement of calculated and experimental values became better, see fig. 5.

Calculated intensities of gamma transitions, not observed yet, de-exciting rotational levels of the β - and the γ -bands are in tab. 4. Calculation of intensities was carried out under assumption $Z_{\beta} = 0.030(16)$ and $Z_{\gamma} = 0.23(6)$.

^{160}Er

Investigation of excited levels of ^{160}Er was performed in beta decay of the ground [15] and isomeric [16] states of ^{160}Tm . Transitions $2_{\beta}^{+} \rightarrow 2_{g}^{+}$, $2_{\beta}^{+} \rightarrow 0_{g}^{+}$, $3_{\gamma}^{+} \rightarrow 2_{g}^{+}$ and $3_{\gamma}^{+} \rightarrow 4_{g}^{+}$ were ob-

served both in decay of the ground and the isomeric states and we took weighted mean average for establishing of the corresponding $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$, see fig. 6. We can get $\chi^2(A) = 7$ and $\chi^2(A) = 27$ if we use the Alaga rules and compare them with experimental values when calculating $B(E2)$ for transitions de-exciting levels of β - and γ -bands.

The one-parametric fit in each band with values $Z_\beta = 0.029(5)$ and $Z_\gamma = 0.133(14)$ gives values $\chi^2(1) = 0.5$ and $\chi^2(1) = 0.9$, and $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2} \approx 1$ within 1σ . If two parameters are employed, $\chi^2(2)$ increases a bit in the β -band and decreases in the γ -band. The elimination of transitions with spin $I \rightarrow I$ from experimental data of β -band doesn't affect the value of Z_β ($Z_\beta = 0.029(7)$, $\chi^2 = -$). Comparison of calculated intensities $I_\gamma(E2)$ and experimental $I_\gamma(E2+M1)$ of these transitions with spin $I \rightarrow I$ leads to the values of $I_\gamma(M1)$ less than experimental errors of $I_\gamma(E2+M1)$. Calculated intensities of not observed transitions are listed in tab. 5; the above given values of the parameters Z_β and Z_γ were used. Because there are not experimental data known on $B(E2, 0_g^+ \rightarrow 0_\beta^+)$ and $B(E2, 0_g^+ \rightarrow 2_\gamma^+)$, see tab. 1, it is not possible to calculate mean lives of decay for rotational levels of β - and γ -bands and intensities of intraband transitions.

^{162}Er

Two (three) levels of the β - (γ -) band were excited in beta decay of the ground state ^{162}Tm [15,17]. In virtue of coincidence measurement it was established that the transition $2_\beta^+ \rightarrow 0_g^+$ is double and all observed transitions de-ex-

citing rotational levels with $I^\pi = 2^+, 3^+$ and 4^+ of the γ -band are double as well. Intensities of double transitions were determined in [15,17] in accord with quantitative analysis of γ - γ coincidences and are bound to be inaccurate and probably marred by systematic errors (angular correlations, accidental coincidences, changes in efficiency of registration of the coincidences with energy), which are difficult of inclusion. We could employ only intensities of the two transitions from [15] and the three transitions given in [17], which de-excite the level 2^+ . The one-parametric fit $Z_\beta = 0.007(11)$, $\chi^2(1) = 16$ resulted in moderate increase of χ^2 when compared with calculations using the Alaga rules ($\chi^2(A) = 13$).

We analysed B(E2) transitions de-exciting levels of the γ -band excited via beta decay with 12 values of I_γ . Using $Z_\beta = 0.104(14)$, $\chi^2(1) = 5$, i.e. it dropped eight times when compared with the value found with the Alaga rules ($\chi^2(A) = 40$).

Rotational levels of the γ -band with the higher spin $I^\pi = 11^+$ and $I^\pi = 8^+$ were studied in reactions $(p,4n\gamma)$ [18] and $(\alpha,2n\gamma)$ [19]. Same as in case of beta decay, most transitions de-exciting levels $2^+, 3^+$ and 4^+ are double, but splitting into two components was not carried out in [18,19] and therefore we did not employ these transitions. Likewise, even the double transitions with energy 212.6, 1037.1 and 1250.2 keV observed in the reaction $(p,4n\gamma)$, which partly de-excited the levels 9^+ and 10^+ , by analogy, were not splitted into components and therefore were not included. At last we did not use the transition 269.6 keV ($8_\gamma^+ \rightarrow 10_g^+$) either, because its intensity is higher by order in comparison with the

calculated one with $Z_{\gamma} = 0.042$ and we have a suspicion that this transition is also double. We included intensities of twenty transitions in all observed in reactions [18,19] into calculations. Comparison of experimental values of $B(E2)$ and those evaluated using the Alaga rules gave $\chi^2(A) = 70$, with parameter $Z_{\gamma} = 0.042(4)$, $\chi^2(1) = 8$. And two-parametric fit gave $Z_{\gamma}(2_{\gamma}) = 0.048(5)$, $Z_{\gamma}(0_g) = 0.044(4)$ or $Z_{\gamma} = 0.051(7)$, $Z_{\beta\gamma} = 0.0027(20)$ with a certain decrease of $\chi^2(2) = 6$ or 7.

The values of Z_{γ} obtained through intensities of transitions observed in beta decay and in reactions overlap only within 3%. To draw conclusions of physical relevance is premature in our opinion, because data from decays are not particularly reliable (see above). Calculated intensities of not observed transitions are in tab. 6. Comparison of values of $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ is presented in fig. 7.

^{164}Er

Rotational levels with $I^{\pi} = 0^+, 2^+, 4^+$ are excited in the β -band via beta decay [20]. Only intensities of three transitions de-exciting levels 2^+ could be employed and besides this we suspect that the transition 1015.02 keV ($2^+ \rightarrow 0^+$) is double. Comparison of values of $B(E2)_{\text{calc}}$ calculated using the Alaga rules and the parameter $Z_{\beta} = -0.010(9)$ with experimental values gave $\chi^2(A) = 25$ and $\chi^2(1) = 23$. If we exclude the transition 1015.02 keV from our analysis, we get $Z_{\beta} = 0.036(5)$. The levels $2^+, 4^+, 6^+$, of the β -band are excited in the reaction (n, n'_{γ}) [23]. Six transitions de-exciting these levels could be used and we got $\chi^2(A) = 14$

or $Z_\beta = 0.034(7)$ and $\chi^2(1) = 3$. Despite the fact that levels 10^+ , 8^+ , 6^+ of the β -band are excited in the reaction $(\alpha, 2n\gamma)$ [18], we could make use only two transitions related with the level 8^+ in the calculations.

Comparison of experimental values of $B(E2)$ with those calculated with the Alaga rules gave $\chi^2(A) = 30$, while with $Z_\beta = -0.001(10)$ we got $\chi^2(1) = 40$, using all data measured in beta decay and α reactions $(\alpha, 2n\gamma)$ and $(n, n\gamma)$. Without using data measured in beta decay we got $\chi^2(A) = 26$ or $Z_\beta = 0.028(4)$ and $\chi^2(1) = 3$. The corresponding ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ are in fig. 8.

Intensities of the seven transition de-exciting levels 2_γ^+ , 3_γ^+ and 4_γ^+ excited in beta decay of the ground state of ^{164}Tm [20] together with intensities of two transitions related with the level 5_γ^+ , which is excited in beta decay of the isomeric state $^{164}\text{Tm}^m$ [21] were compared with intensities evaluated with the Alaga rules ($\chi^2(A) = 14$) and using the value $Z_\gamma = 0.059(8)$ ($\chi^2(1) = 1.6$). Further drop of $\chi^2(2)$ to 0.8 could be reached for two-parametric fit with $Z_\gamma(2_\gamma) = 0.068(8)$ and $Z_\gamma(0_g) = 0.045(9)$ or $Z_\gamma = 0.068(8)$ and $Z_{3_\gamma} = 0.011(5)$.

We can use data on only four transitions for analysis of reduced probabilities of gamma transitions de-exciting rotational levels of the γ -band excited in the reaction $(\alpha, 2n\gamma)$ in the article [19], while in the article [22] were observed 21 such transitions (de-exciting levels with spin 2_γ^+ to 11_γ^+). Intensities of the above mentioned transitions were analysed together. Comparison of experimental values of $B(E2)_{\text{exp}}$ with those calculated with the Alaga rules gave $\chi^2(A) = 80$, while

with $Z_{\gamma} = 0.063(5)$ we got $\chi^2(1) = 9$.

With regard to the agreement of values Z_{γ} calculated from data measured in beta decay and in the reaction $(\alpha, 2n\gamma)$ in the framework of the experimental errors further analysis was carried out with all data together. The one-parametric fit gave $Z_{\gamma} = 0.063(4)$ ($\chi^2(1) = 7$); with two parameters $Z_{\gamma} = 0.067(4)$ and $Z_{\beta\gamma} = 0.0048(21)$ ($\chi^2(2) = 6$) and finally with three parameters we got values $Z_{\gamma}(2_{\gamma}) = 0.068(5)$, $Z_{\gamma}(0_{\gamma}) = 0.068(8)$, $Z_{\beta\gamma} = 0.005(5)$ with $\chi^2(3) = 6$. Comparison of $B(E2)$ calculated in such a way with experimental values is presented in fig. 6. For most transitions the ratio $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ approximately equals one within 1σ , except seven transitions where the difference is greater than 1σ but less than 2σ and in one case the difference is greater than 2σ but less than 3σ . We do not find any systematical deviations depending on the spin of the level which is de-excited. The levels with $I \geq 8$ are always de-excited via one interband transition whereas second transition that de-excites them is an intraband transition. Calculated intensities of not observed transitions are listed in tabs 7a, 7b and 7c.

166_{Er}

Very little is known about properties of the β -band of this nucleus. In beta decay the level 2_{β}^{+} is excited. It is depopulated via three transitions, whose intensities were established in [24]. By comparison of $B(E2)_{\text{exp}}$ with calculated values we have $\chi^2(A) = 600$, or $\chi^2(1) = 400$ with $Z_{\beta} =$

= 0.05(4). Above all this unsatisfactory situation must be dealt with by more thorough experimental study of excited levels of ^{166}Er .

The rotational γ -band is excited in beta decay of ^{166}Tm to the level with spin 5^+ [24] and of $^{166\text{m}}\text{Ho}$ to 8^+ [25,26], in the reaction $(\alpha, 2n\gamma)$ [27] then to the level with spin 10^+ . Data on intensities of gamma transitions [25,26] are characterized by completeness and accuracy. E.g. levels of the γ -band related to beta decay of $^{166\text{m}}\text{Ho}$ are de-excited by 21 transitions and their intensities are determined with 2% accuracy [25], while K. Kato [26] observed only 16 transitions, but he determined intensities of some these transitions with remarkable high accuracy $\approx 0.5\%$. We performed analysis of each of these four independent experiments [24-26] separately and the results are in tab. 3. One can see mutual agreement within the framework of 3σ . Particularly noticeable change of χ^2 manifests itself when we compare the most accurate experimental data [26] with computed ones: with the Alaga rules $\chi^2(A) = 4000$, using $Z_\gamma = 0.0432(5) - \chi^2(1) = 8$, with two parameters $Z_\gamma(2_\gamma) = 0.0448(5)$, $Z_\gamma(0_g) = 0.0435(3) - \chi^2(2) = 4$ or $Z_\gamma = 0.0447(6)$, $Z_{3\gamma} = 0.00060(19) - \chi^2(2') = 4$. χ^2 does not change if we employ three parameters and also changes of the parameters $Z_\gamma(2_\gamma)$, $Z_\gamma(0_g)$ and $Z_{3\gamma}$ are small, see tab. 3. Calculated I_γ of not observed transitions are presented in tab. 8. Values of $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ for transitions studied in beta decay and in the reaction $(\alpha, 2n\gamma)$ in ^{166}Er are compared in fig. 9.

In view of the fact that the most accurate data are available for the rotational band of gamma vibrations in

¹⁶⁶Er, we studied influence of the nonlinear correction in more detail. Since the parameter Z_{β} is marred by great error, we took the nonlinear correction into account for several values of the parameter Z_{β} : 0.02, 0.04 and 0.06. The results are listed in tab. 9. Average values of $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ based on experimental data [24-27] and on one-, two- and three-parametric fit are shown in fig. 10. The nonlinear corrections were included in three-parametric fit. It turns out that the nonlinear correction is recognizable for transitions de-exciting levels with $I^{\pi} \geq 4^+$ and agreement between experimental and calculated values of $B(E2)$ improves.

After including of a small admixture of M1 multipolarity in the interband transitions determined in [32] the change of parameters (see tab. 9) is very small.

¹⁶⁸Er

We observed five transitions related to the levels 2^+ , 4^+ of the rotational β -band [28] in beta decay. Experimental values of $B(E2)$ for these transitions when compared with calculated with the Alaga rules give the value $\chi^2(A) = 19$. If we employ one parameter Z_{β} we get $\chi^2(1) = 1.8$, with two parameters $Z_{\beta}(O_1)$ and $Z_{\beta}(O_g)$ we have $\chi^2(2) = 1.2$ or Z_{β} , $\chi^2(3) = 0.4$. The corresponding values are in tab. 2.

The β -band is excited to spin $6^+(8^+)$ in reactions (n, γ) [29]. Identification of the level with spin 8^+ is preliminary according to authors [29]. Analysis of $B(E2)$ for the thirteen transitions that de-excite levels 2^+ to 8^+ showed that χ^2 varies little. If we use the Alaga rules, then

$\chi^2(A) = 30$, with Z_{β} we get $\chi^2(1) = 17$ and with two parameters $Z_{\beta}(0_{\beta})$ and $Z_{\beta}(0_{\beta})$ $\chi^2(2)$ increases to 26. Far greatest deviation of $B(E2)_{\text{exp}}$ from $B(E2)_{\text{calc}}$ (evaluated with one parameter Z_{β}) was found for the transition 1341.577 keV ($8_{\beta}^+ \rightarrow 6_{\beta}^+$), thus we excluded transitions related to the level 8^+ from further analysis. The situation improved by change of χ^2 , see tab. 2, but quite adequate description of experimental data has not been achieved.

We performed calculations for total data both from beta decay and from reactions (n, γ) . The results we got are presented in tab. 2; comparison of experimental and calculated $B(E2)$ values with $Z_{\beta} = 0.025(5)$, $\chi^2(1) = 9$ is in fig. 11. The elimination of transitions with spin $I \rightarrow I$ from experimental data does not affect the value of Z_{β} ($Z_{\beta} = 0.025(3)$, $\chi^2 = 5$). Comparison of calculated intensities $I_{\gamma}(E2)$ and experimental $I_{\gamma}(E2+M1)$ of these transitions with spin $I \rightarrow I$ leads to the value of $I_{\gamma}(M1)$ less than experimental errors of $I_{\gamma}(E2+M1)$.

Levels of the γ -band excited in beta decay [28] are from $I^{\pi} = 2^+$ to $I^{\pi} = 5^-$. We established intensities of ten gamma transitions de-exciting these levels with accuracy reading $\approx 2\%$ in [28]. Comparison of experimental and calculated values of $B(E2)$ gives $\chi^2(A) = 500$ (Alaga), $\chi^2(1) = 0.6$ (Z_{γ}), $\chi^2(2) = 0.4$ ($Z_{\gamma}(Z_{\gamma}), Z_{\gamma}(0_{\beta})$), $\chi^2(2') = 0.4$ ($Z_{\gamma}, Z_{\beta}\gamma$) and with three parameters $Z_{\gamma}(Z_{\gamma}) = 0.0366(9)$, $Z_{\gamma}(0_{\beta}) = 0.0366(20)$, $Z_{\beta}\gamma = -0.0004(10)$ we have $\chi^2(3) = 0.5$.

Having excluded the intraband transitions with M1 multipolarity admixture, we carried out analysis of $B(E2)_{\text{exp}}$ for the transitions observed in reaction (n, γ) [29] that de-exci-

te levels with spin from 2^+ to 8^+ . Results of this calculations are in tab. 2. We found fairly good agreement between values of Z_{γ} and Z_{β} evaluated with data from beta decay and from reaction (n, γ) . Comparison of all 31 values $B(E2)_{\text{exp}}$ and $B(E2)_{\text{calc}}$ evaluated with parameter $Z_{\gamma} = 0.0374(6)$ is presented in fig. 11. This value of Z_{γ} was obtained in simultaneous analysis of intensities of all 31 transitions, $\chi^2(1) = 0.9$. The inclusion of a small admixture of M1 multipolarity in the interband transitions determined in [52] doesn't affect the value of Z_{γ} ($Z_{\gamma} = 0.0374(7)$, $\chi^2 = 1.0$). Calculated intensities of not observed transitions are in tab. 10.

^{170}Er

Beta decay of two states of ^{170}Ho with $T_{1/2} = 2.8(2)$ min, $I^{\pi} = (4^{\pm})$ and with $T_{1/2} = 43(2)$ sec., $I^{\pi} = (1^{\pm})$ was investigated in [30]. Decay of the state with $T_{1/2} = 2.76(5)$ min, $I^{\pi} = (6^{\pm})$ was studied in later work [31].

Rotational levels 0^+ and 2^+ of the β -vibrational band are excited in decay of the state with $T_{1/2} = 43(2)$ sec., but only three transitions de-exciting the level with $I^{\pi} = 2^+$ [30] can be used in analysis of $B(E2)$. If we compare experimental values and those calculated using the Alaga rules or $Z_{\beta} = 0.011(9)$, we get $\chi^2(A) = 22$ or $\chi^2(1) = 16$ see fig. 12.

The eight transitions de-exciting levels with $I = 2^+$, 4^+ and 6^+ of the β -band were observed in reaction $(n, n\gamma)$ [23]. We performed calculation for total data both from beta decay and from reactions $(n, n\gamma)$ and got $\chi^2(A) = 20$ or $Z_{\beta} = 0.013(5)$ and $\chi^2(1) = 12$. The elimination of transitions with spin

$I \rightarrow I$ from experimental data does not affect the value of Z_β ($Z_\beta = 0.0143(24)$, $\chi^2 = 1.9$).

Only four transitions de-exciting levels with $I^\pi = 3^+$ and 4^+ of the rotational γ -band were observed in beta decay [30,31]. Data on intensities are not particularly accurate in either paper. Six levels with spin and parity $I^\pi = 2^+ \div 7^+$ of the γ -rotational band are investigated in reactions $(n, n\gamma)$ [23]. Independent analysis of these experiments results in rather different values of Z_γ , see tab. 2, however, with great errors. Therefore we compared experimental [30,31,23] and calculated B(32) with third sets of initial data simultaneously. We obtained $\chi^2(A) = 20$, $Z_\gamma = 0.040(10)$ with $\chi^2(1) = 9$ and the two-parametric fit gave $Z_\gamma(2_\gamma) = 0.025(7)$, $Z_\gamma(0_g) = 0.058(7)$ and $\chi^2(2) = 4$, or $Z_\gamma = 0.020(6)$, $Z_{\beta\gamma} = -0.019$ (3) with $\chi^2(2') = 2.5$. Values of I_γ of not observed transitions are in tab. 11.

$^{158}_{64}\text{Gd}_{94}$

Two levels with spin and parity 0^+ and 2^+ of the β -rotational band are excited in beta decay of ^{158}Eu ($T_{1/2} = 46$ min), $I^\pi = 1^-$ [33]. We could employ only three transitions de-exciting the state 2^+ in analysis of reduced probabilities. Levels 0_β^+ , 2_β^+ and 4_β^+ are excited in reaction (n, γ) [34], the last of them is depopulated by interband transitions to rotational levels of the ground state and by one intraband transition. The ratio of intensities $I_\gamma(2_\beta^+ \rightarrow 0_g^+) : I_\gamma(2_\beta^+ \rightarrow 2_g^+) : I_\gamma(2_\beta^+ \rightarrow 4_g^+)$ observed in reaction (n, γ) [34] equals 181(11):

: 100(8) : 251(14), whereas in beta decay [33] this ratio is 118(18) : 100(18) : 115(13). The intensity of the transition $2_{\beta}^{+} \rightarrow 2_{\beta}^{+}$ presented in [34] is probably wrong; one can draw this conclusion even from our calculations.

If we compare experimental values [33,34] and those computed using the Alaga rules, we have $\chi^2(A) = 30$ and if we use the parameter $Z_{\beta} = 0.010(6)$, we have $\chi^2(1) = 27$. Having excluded intensity of $2_{\beta}^{+} \rightarrow 2_{\beta}^{+}$ transition mentioned in [34], χ^2 with $Z_{\beta} = 0.009(4)$ dropped more than twice ($\chi^2(1) = 12$). The two-parametric fit results in further decrease of χ^2 : $\chi^2(2) = 11$ for $Z_{\beta}(0_{\beta}) = 0.008(3)$, $Z_{\beta}(0_{\beta}) = 0.010(5)$ and $\chi^2(2) = 7$ for $Z_{\beta} = 0.011(3)$, $\beta_{\beta} = -0.0020(10)$. If three parameters are used $Z_{\beta}(0_{\beta}) = 0.0094(21)$, $Z_{\beta}(0_{\beta}) = 0.013(3)$, $\beta_{\beta} = -0.0018(7)$ without or with the correction by Rud et al. [8], we have $\chi^2 = 3.9$ or $\chi^2 = 4.2$, respectively. The elimination of transitions with spin $I \rightarrow I$ from experimental data does not affect the value of Z_{β} ($Z_{\beta} = 0.005(4)$, $\chi^2 = 12$). Comparison of calculated intensities $I_{\gamma}(E2)$ and experimental $I_{\gamma}(E2+M1)$ of these transitions with spin $I \rightarrow I$ leads to the value of $I_{\gamma}(M1)$ less than experimental errors of $I_{\gamma}(E2+M1)$. Table 12 shows measured values of I_{γ} and calculated transition intensities with $Z_{\beta} = 0.009(4)$. Comparison of experimental $B(E2)$ and values calculated with three parameters is in fig. 13.

Levels 2^{+} and 3^{+} of the rotational γ -band are depopulated by five gamma transitions in beta decay [33]. Nine interband transitions and two E2 multipolarity transitions within the rotational γ -band excited to the level 5^{+} were observed in reaction (n,γ) [34]. Intensities of all these transitions

were analysed simultaneously. Values of $\chi^2 \leq 1$ could be reached only if mixing of wave functions of β - and γ -vibrations are taken into account ($\epsilon_{\beta\gamma} \neq 0$). If we include the correction by Rud et al. ($Z_\beta = 0.01$), we get minimal values $\chi^2(3) = 0.8$, if $Z_\gamma(2_\gamma) = 0.020(3)$, $Z_\gamma(0_g) = 0.019(3)$ and $Z_{\beta\gamma} = -0.0115(17)$. Without this correction we arrive at $\chi^2(3) = 0.9$ and the values of mixing parameters are not changed substantially, see tab. 3. Intensities of not observed transitions are in tab. 12.

$^{160}_{56}\text{Dy}_{94}$

The 2^+ level of the β -band was observed in beta decay of ^{160}Ho [35], where it was depopulated by two gamma transitions $2^+_{\beta} \rightarrow 0^+_g$ and $2^+_{\beta} \rightarrow 2^+_g$. Study of conversion electrons spectra [36] made possible to identify the 1065.8 keV transition, which could be placed between levels $2^+_{\beta} \rightarrow 4^+_g$ drawing on the energy balance. Its multipolarity is E2 under these assumptions and the intensity of gamma quanta is evaluated from measured intensities of K conversion electrons. Comparison of experimental $B(E2)$ of the three transitions from 2^+_{β} with calculated ones using the Alaga rules gives $\chi^2(A) = 8$ or $\chi^2(1) = 5$ if $Z_\beta = -0.014(9)$.

Intensities of the nine gamma transitions depopulating levels of the γ -band with spin and parity 2^+ to 5^+ were used in analysis on reduced probabilities. Parameters $\mathcal{F}(E2/M1)$ were determined for three above-mentioned transitions ($2^+_{\gamma} \rightarrow 2^+_g$, $3^+_{\gamma} \rightarrow 2^+_g$, $3^+_{\gamma} \rightarrow 4^+_g$) with sufficient accuracy. Correction of in-

tensities of these transitions $1/(1+\delta^2)$ does not exceed 3%. Comparison of $B(E2)_{\text{exp}}$ and $B(E2)_{\text{calc}}$ calculated with the Alaga rules gives $\chi^2(A) = 100$, if the calculation is performed with the parameter Z_γ we have $\chi^2(1) = 2.2$ or $\chi^2(1) = 1.5$ for $B(E2)_{\text{exp}}$ derived from I_γ not corrected or corrected by the M1 admixture, respectively. With two-parametrical fit we arrive at $Z_\gamma(2_\gamma) = 0.0438(20)$, $Z_\gamma(0_g) = 0.051(3)$ and $\chi^2(2) = 0.04$; the correction of experimental values has been performed. The ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ for the transitions depopulating rotational levels of the gamma and beta bands are in fig. 14 and calculated intensities are in tab. 13.

$^{164}_{70}\text{Yb}_{94}$

Levels of the β -band (0_β^+ , 2_β^+ , 4_β^+) and the γ -band (2_γ^+ ÷ 5_γ^+) were observed in beta decay of ^{164}Lu [39]. Adequacy of calculated values $B(E2)_{\text{calc}}$ and experimental $B(E2)_{\text{exp}}$ is characterized by $\chi^2(A) = 80$ (or 230) if the calculations were performed with the Alaga rules for the β -band (or γ -band). One-parametric fit gives $\chi^2(1) = 17$ (or 13) and with two parameters Z_β, Z_γ we got $\chi^2(2) = 18$, or with $Z_\beta(0_\beta)$, $Z_\beta(0_g)$ - $\chi^2(2) = 30$. An increase of χ^2 takes place for the γ -band, too (with two parameters Z_γ, Z_β we arrive at $\chi^2(2) = 20$). The ratio of calculated (with one parameter) and experimental reduced probabilities is shown in fig. 15. Values of $B(E2)_{\text{exp}}$ coincide within 2σ with the calculated ones. Both experimental and calculated (with $Z_\beta = 0.023(6)$ or $Z_\gamma = 0.122(15)$) intensities of gamma transitions depopulating the β -

or γ -bands are in tab. 14.

Resulting values of the physical constants (Q_{gg} , Q_{β} , Q_{γ} , ϵ_{β} , ϵ_{γ} or $\epsilon_{\beta\gamma}$) for nuclei studied (except for ^{160}Er and ^{164}Yb , where neither experimental data on $B(E2, 0_g^+ \rightarrow 2_q^+)$ or intensities of intraband transitions are known) were obtained from experimental data analysed in tab. 2 and tab. 3. The transitions depopulating rotational bands of β - and γ -vibrations were studied simultaneously under the assumption that $Q_{gg} = Q_{\beta\beta} = Q_{\gamma\gamma}$ and $\epsilon_{\beta\gamma} = 0$ (first version of calculations) or $\epsilon_{\beta\gamma} \neq 0$ (second version). The correction by the nonlinear term could be done only in the second version. As we already verified, taking into account of this nonlinear term influences changes of χ^2 considerably only for ^{166}Er . This is why we carried out calculations paying regard to the nonlinear correction (third version) only for this nucleus. The results are summarized in tab. 15 and displayed on fig. 16, where matrix elements of the Coriolis interaction $h_{qq} \equiv \epsilon_{qq} (E_q - E_q)$ are also entered. Experimental values Q_{gg} are compared with the calculated ones in fig. 16. Evaluation of Q_{gg} has been carried out by I.A. Mitropolskij [40] using the superconducting nuclear model taking into account the selfconsistency condition in the channel particle-particle, which influences vibrational, rotational and translational invariance broken in the traditional superconducting model. We found very good agreement between calculated and experimental Q_{gg} 's.

Knowledge of the physical constants enables us to evaluate also parameters α_{β} , α_{γ} (see tab. 15), defined by relations

$$\alpha_{\beta} = - \frac{Q_{\beta}}{Q_{\beta\beta}} \epsilon_{\beta} \quad \alpha_{\gamma} = - \frac{Q_{\gamma}}{Q_{\beta\beta}} \epsilon_{\gamma} \quad (12)$$

These parameters determine the deviation of reduced probabilities of intraband E2 -transitions found with mixing of wave function for the ground, beta and gamma states from values evaluated with plain rotational model. E.g. for intraband transitions over the ground band holds (see e.g. [9])

$$B(E2, I+2, 0_g \rightarrow I, 0_g) = B_{\text{exp}}(E2, 2, 0_g \rightarrow 0, 0_g) \frac{\langle I+2, 020 | I0 \rangle^2}{\langle 2020 | 00 \rangle^2} \times \\ \times \left\{ 1 + (\alpha_{\beta} + \alpha_{\gamma}) [(I+2)(I+3) + I(I+1) - 6] \right\}^2 \quad (13)$$

We analysed experimental $B_{\text{exp}}(E2, I+2, 0_g \rightarrow I, 0_g)$ mentioned in references (see tab. 16) for erbium isotopes and isotones with $N = 94$. We determined the sum $\alpha_{\beta} + \alpha_{\gamma}$ using the program "ZORKA" [10]. The results of it are in figs 17 and 18. We can state that mixing of wave functions does not improve agreement of experiment and calculations considerable when compared with plain rotational model. This conclusions can be confirmed by comparison of $\chi^2(A)$ and $\chi^2(1)$ for ^{166}Er , ^{168}Er and ^{164}Yb in tab. 16. Values of $\alpha_{\beta} + \alpha_{\gamma}$ from analysis of interband transitions (see tab. 15 - second version) are also included in this table. The agreement of $(\alpha_{\beta} + \alpha_{\gamma})_{\text{Er. b.}}$ with $(\alpha_{\beta} + \alpha_{\gamma})_{\text{Yb. b.}}$ is not particularly good, although such an agreement is reached within 3%.

Study of transitions between rotational levels of beta and gamma vibrational states can lead to interesting knowledge about nuclear structure. According to the rotational model such transitions are forbidden ($Q_{\beta\gamma} = 0$) because the change of vibrational quantum numbers ($\Delta n_{\gamma} + \Delta n_{\beta} = 2$) nevertheless

they are observed experimentally. For the set of nuclei we study, 9, 2 and 2 $\beta \rightarrow \gamma$ transitions were observed ^{168}Er , ^{158}Gd and ^{164}Er , respectively. All transitions of this type are not easily identifiable due to their low energy and intensity as well. We analysed $\beta \rightarrow \gamma$ transitions for ^{168}Er and ^{158}Gd under the assumption that the matrix element $Q_{\beta\gamma}$ was sufficiently large; see the relation (21) in [9]. Results for Q_{gg} , Q_{β} , Q_{γ} , $Q_{\beta\gamma}$, ϵ_{β} , ϵ_{γ} and $\epsilon_{\beta\gamma}$ obtained by the program "ZORKA" [10] are in tab. 15. As for ^{168}Er these physical constants were evaluated also in the framework of the model of interacting bosons IBA-1 [50]. While Q_{gg} , Q_{β} , Q_{γ} and $Q_{\beta\gamma}$ evaluated by these methods agree well, ϵ_{β} and ϵ_{γ} found using IBA-1 are about four times lower than ours and $\epsilon_{\beta\gamma}$ have even opposite sign. Using the new (XQ) formalism [50] better agreement is obtained for the values of ϵ_{β} and ϵ_{γ} but the sign of $\epsilon_{\beta\gamma}$ is wrong again.

The mixing parameters Z_q were evaluated using microscopical models (see e.g. [47,48,49]) or using IBA [50]. The most complete results for the biggest number of nuclei under study are in [51], where a microscopical model with both pair and quadrupole-quadrupole interaction in Woods-Saxon mean field was used.

In conclusion we should say that taking into account of wave function mixing between β - or γ -vibrations and the ground state (parameters Z_{β} or Z_{γ}) improves agreement of experimental and calculated reduced probabilities of E2 inter-band transitions considerably. Wave function mixing β - and γ -vibrational state (parameters $Z_{\beta\gamma}$ or $\{_{\beta\gamma}$) manifested itself only in ^{158}Er and ^{158}Gd . Whether the assumption about

unequality of quadrupole moments $Q_{qq} \neq Q_{gg}$ and the assumption about nonzero correction by Rud et al. [8] is justified could be investigated on sufficiently rich ensemble of very accurate experimental data on gamma transition intensities (ΔI_γ should be $\approx 1\%$). Such a high accuracy of I_γ was achieved only in beta decay of some nuclei with sufficiently long lifetime, while especially in the reactions $(\alpha, xn\gamma)$ or $(p, xn\gamma)$ the ratio $\Delta I_\gamma / I_\gamma$ is usually about 10% or even worse. It would be desirable that we have experimental data on M1 multipolarity admixture with accuracy $\approx 1\%$ or better for transitions $|I_i - I_f| = 1$ or 0, whereas for transitions where $|I_i - I_f| = 0$, $I_i \neq 0$ is also desirable to know the E0 multipolarity admixture with the above mentioned accuracy. Parameters Z_q evaluated with the microscopic model [51] describe experimental values obtained for nuclei under study not quite adequately.

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REFERENCES

- 1 Bohr A., Mottelson B.R.: *Mat. Fys. Medd. Dan. Vid. Selsk.* 27 (1953), No. 16.
- 2 Alaga A., Alder K., Bohr A., Mottelson B.R.: *Mat. Fys. Medd. Dan. Vid. Selsk.* 29 (1955), No. 9.
- 3 Gregers Hansen P., Nielsen O.B., Shelton R.K.: *Nucl. Phys.* 12 (1959) 389.
- 4 Lipas P.O.: *Nucl. Phys.* 39 (1962) 468.
- 5 Michajlov V.M.: *Izv. AN SSSR (ser.fiz.)* 28 (1964) 308.
- 6 Riedinger L.L., Johnson N.R., Hamilton J.H.: *Phys. Rev.* 179 (1969) 1214.
- 7 Reich C.W., Cline J.E.: *Nucl. Phys.* A159 (1970) 181.
- 8 Rud N., Nielsen H.L., Wilsky K.: *Nucl. Phys.* A167 (1971) 401.
- 9 Adam J., Honusek M., Wagner V., Krivopustov M.I., Morozov V.A.: Preprint JINR R6-87-202, Dubna, 1987.
- 10 Adam J., Wagner V., Honusek M., Krivopustov M.I., Morozov V.A.: Preprint JINR R6-87-203, Dubna, 1987.
- 11 Begzhanov R.B., Belenkij B.M.: *Gamma-spektroskopiya atomnykh yader.* Izd. FAN, Tashkent, 1980.
- 12 Lange J., Kumar K., Hamilton J.H.; *Rev. Mod. Phys.* 54 (1982) 119.

- 13 Humanic T.J., Saladin J.X., Alessi J.G., Hussein A.:
Phys. Rev. C27 (1983) 550.
- 14 Auger P., Liang C.F., Libert J., Paris P., Peghaire A.,
Charvet A., Duffait R., Marguier G.: Nucl. Phys. A249
(1975) 239.
- 15 Strusny H., Tyrroff H., Hermann E., Musiol G., Molnar F.,
Abdurazakov A.A., Beyer G., Gromov K.Ya., Islamov T.A.,
Jachim M., Siebert H.U., Usmanova S.A.: Czech. J. Phys.
B25 (1975) 626.
- 16 Singh B., Kogan A., Mark S.K.: Phys. Rev. C28 (1983)
2118.
- 17 Boer F.W.N. de, Goudsmit P.F.A., Meijer B.J., Koldewijn
P., Konijn J., Beetz R.: Nucl. Phys. A236 (1974) 349.
- 18 Janssens R., Masri Y.El., Ferté J.M., Michel C., Steyaert
J., Vervier J.: Nucl. Phys. A283 (1977) 493.
- 19 West R.L., Funk E.G., Visvanathan A., Adams J.P., Mihe-
lich J.W.: Nucl. Phys. A270 (1976) 300.
- 20 Adam J., Hons Z., Gromov K.Ya., Kononenko G.A., Islamov
T.A., Kholmatov A.Kh.: Preprint JINR R6-87-141, Dubna,
1987.
- 21 Adam J., Hons Z., Gromov K.Ya., Dobeš J., Fér J.: Izv.
AN SSSR (ser.fiz.) 51 (1987) 834.
- 22 Fields C.A., Hicks K.H., Ristinen R.A., Boer F.W.N. de,
Peker L.K., Peterson N.J., Walker P.M.: Nucl. Phys. A422

(1984) 215.

23 Bondarenko V.A., Grigorev E.P., Prokofev P.T.: Izv. AN SSSR (ser.fiz.) 45 (1981) 2142.

Bondarenko V.A., Grigorev E.P., Prokofev P.T.: Izv. AN SSSR (ser.fiz.) 46 (1982) 2080.

24 Adam J., Honusek M., Gromov K.Ya., Frána J.: Czech. J. Phys. B25 (1975) 504.

Adam J., Wagner V., Zvolská V., Zvolský J., Kracík B., Fišer M.: Izv. AN SSSR (ser.fiz.) - will be published.

25 Adam J., Wagner V., Zvolská V., Zvolský J., Kracík B., Fišer M.: Izv. AN SSSR (ser.fiz.) 52 (1988) 18.

26 Kato K., Hoshi M., Yoshizawa Y.: J. Phys. Soc. Japan 50 (1981) 2810.

27 Fields C.A., Hicks K.H., Peterson R.J.: Nucl. Phys. A440 (1985) 301.

28 Adam J., Hnatowicz V., Dobeš J., Zvolská V., Zvolský J., Kracík B., Fišer M.: Izv. AN SSSR (ser.fiz.) 49 (1985) 6.

29 Davidson W.F., Warner D.D., Casten R.F., Schreckenbach K., Börner H.G., Simić J., Stojanović M., Bogdanović M., Koićki S., Gellethy W., Orr G.B., Stelts M.L.: J. Phys. G : Nucl. Phys. 7 (1981) 455.

30 Kawade K., Hiei A., Yamamoto H., Katoh T.: J. Phys. Soc. Japan 36 (1974) 1221.

- 31 Katajanheimo R., Tuurnala T., Hammarén E.: Z. Phys. A286 (1978) 57.
- 32 Krane K.S., Moses J.D.: Phys. Rev. C24 (1981) 654.
- 33 Kluk A.F., Johnson N.R., Hamilton J.H.: Phys. Rev. C10 (1974) 1966.
- 34 Greenwood R.C., Reich C.W., Baader H.A., Koch H.R., Breitig D., Schult O.W.B., Fogelberg B., Bäcklin A., Mampe W., Egidy T. von, Schreckenbach K.: Nucl. Phys. A304 (1978) 327.
- 35 Aleksandrov A.A., Butcev V.S., Vylov C., Grigorev E.P., Gromov K.Ya., Kalinnikov V.G., Lebedev N.A.: Izv. AN SSSR (ser.fiz.) 38 (1974) 2096.
- 36 Aleksandrov A.A., Vinogradov V.M., Grigorev E.P., Gromov K.Ya., Zolotavin A.V., Kalinnikov V.G., Makarov V.M.: Prizmennye beta-spektrometry i ikh primeneniye. Vilnyus, 1974, p.66.
- 37 Adam J., Vinogradov V.M., Grigorev E.P., Kracík B., Kugler A.: Tezisy dokladov 36 soveshchaniya po yadernoj spektroskopii i strukture yadra. Nauka, Leningrad, 1986, p.119.
- 38 Gromova I.I., Dupák J., Koníček J., Kracíková T.I., Lebedev N.A., Neganov B.S., Pavlov V.N., Procházka I., Finger M., Tsupko-Sitnikov V.M., Shchus A.F., Machová A., Hamilton W.D., Fox R.A.: Izv. AN SSSR (ser.fiz.) 43 (1979) 26.

- 39 Adam J., Hons Z., Gromov K.Ya., Pražák Fr., Jáchim M.:
Izv. AN SSSR (ser.fiz.) 48 (1984) 1819.
- 40 Mitropolskij I.A.: Yad. Fiz. 29 (1979) 1466.
- 41 Shurshikov E.N.: Nuclear Data Sheets 47 (1986) 433.
- 42 Lederer C.M., Shirley V.S.: Table of Isotopes (7th ed.).
Wiley and Sons, 1978.
- 43 Kearns F., Varley G., Dracoulis G.D., Inamura T., Lisle
J.C., Willmott J.C.: Nucl. Phys. A278 (1977) 109.
- 44 Kurfess J.D., Sharenberg R.P.: Phys. Rev. 161 (1967) 1185.
- 45 Lee M.A.: Nuclear Data Sheets 31 (1980) 381.
- 46 Lee M.A., Bunting R.L.: Nuclear Data Sheets 46 (1985) 187.
- 47 Pavlichenko I.M.: Nucl. Phys. 55 (1964) 225.
- 48 Bés D.R., Federman P., Magueda E., Zuker A.: Nucl. Phys.
65 (1965) 1.
- 49 Marshalek E.R.: Phys. Rev. 158 (1967) 993.
- 50 Warner D.D., Casten R.F.: Phys. Rev. Lett. 48 (1982) 1385.
- 51 Karadjov D., Mikhailov I.N., Nadjakov E., Piperova J.:
Nucl. Phys. A305 (1978) 78.
- 52 Gellethy W., Isacker P. van, Warner D.D., Colvin G.,
Schreckenbach K.: Phys. Lett. B191 (1987) 240.

Table 1.

Values of intrinsic quadrupole moment Q_{20} of the ground state and reduced probabilities $B(E2)$ of interband transitions. [11]

	Q_{20} [eb] ^{a)}	$B(E2, 0_g^+ \rightarrow 2_\beta^+)$ [e ² b ²]	$B(E2, 0_g^+ \rightarrow 2_f^+)$ [e ² b ²]
¹⁵⁸ Er	5.25(14)		
¹⁶⁰ Er	6.52(9)		
¹⁶² Er	7.09(4)	0.042(7)	0.165(8)
¹⁶⁴ Er	7.40(4)	0.037(18)	0.148(6)
¹⁶⁶ Er	7.52(5)	0.018(2)	0.144(7)
¹⁶⁸ Er	7.57(7)		0.127(6)
¹⁷⁰ Er	7.52(5)	0.0079(9)	0.097(4)
¹⁵⁸ Gd	7.07(3)	0.0080(6)	0.086(4)
¹⁶⁰ Dy	7.21(10)	0.0184(15)	0.114(8)
¹⁶⁴ Yb	6.79(13)		

a) 1 barn $b \equiv 10^{-24}$ cm²

Table 2 :

Determination of mixing parameters for the beta band.

Nucleus N	N'	n	I _{min}	I _{max}	Refs.	Alaga X ² (A)	Q _{ββ} = Q _{βA} , ε _{ββ} = 0			Q _{ββ} ≠ Q _{βA} , ε _{ββ} = 0			Q _{ββ} = Q _{βA} , ε _{ββ} ≠ 0			Q _{ββ} ≠ Q _{βA} , ε _{ββ} ≠ 0			
							X ² (1)	10 ³ Z _A	X ² (2)	10 ³ Z _A (0 _A)	10 ³ Z _A (0 _B)	X ² (2')	10 ³ Z _B	10 ³ ε _{ββ}	X ² (3)	10 ³ Z _A (0 _A)	10 ³ Z _A (0 _B)	10 ³ ε _{ββ}	
¹⁵⁸ Er	7	7	2	2 ⁺	4 ⁺	[14]	40	30	50(16)	50	50(60)	10(50)	0.2	56(4)	-11.1(11)	0.03	60(8)	52(9)	-11.6(14)
¹⁶⁰ Er	7	7	2	2 ⁺	4 ⁺	[15,16]	7	0.5	29(5)	0.6	26(9)	33(11)	0.7	31(8)	-0.4(11)				
¹⁶² Er	5	3	1	2 ⁺	2 ⁺	[16,17]	13	16	7(11)										
¹⁶⁴ Er	3	3	1	2 ⁺	2 ⁺	[20]	25	23	-10(9)										
	2	2	1	2 ⁺	2 ⁺	[20]	15	-	36(5)										
	6	6	3	2 ⁺	6 ⁺	[23]	14	3	34(7)	0.8	68(17)	39(8)	0.4	107(13)	-8.5(21)				
	11	9	4	2 ⁺	8 ⁺	[20-23]	30	40	-1(10)	30	-8(8)	22(5)	3	-17(3)	6.8(13)				
	6	8	4	2 ⁺	8 ⁺	[22,23]	26	3	28(4)	0.6	63(13)	41(6)	0.22	108(16)	-8.7(16)				
¹⁶⁶ Er	3	3	1	2 ⁺	2 ⁺	[24]	600	400	50(40)										
¹⁶⁸ Er	5	5	2	2 ⁺	4 ⁺	[28]	19	1.8	34(6)	1.2	59(27)	11(29)	0.4	37(5)	-1.7(9)				
	13	13	4	2 ⁺	8 ⁺	[29]	30	17	23(7)	26	19(7)	37(6)	19	23(8)	0.0(14)	29	17(7)	34(6)	0.7(10)
	11	11	3	2 ⁺	6 ⁺	[29]	30	11	23(6)	12	32(16)	16(18)	14	23(7)	0.0(11)	15	32(19)	16(21)	0.1(17)
	16	11	4	2 ⁺	6 ⁺	[28,29]	29	9	25(5)	8	35(14)	15(16)	10	25(5)	-0.2(9)	9	36(16)	16(17)	-0.4(13)
¹⁷⁰ Er	3	3	1	2 ⁺	2 ⁺	[30]	22	16	11(9)				-						
	11	8	3	2 ⁺	+	[30,23]	20	12	13(5)	11	9(4)	29(24)	1.3	96(17)	3.4(5)				
¹⁵⁸ Gd	10	7	2	2 ⁺	4 ⁺	[33,34]	30	27	10(6)	29	8(6)	11(8)	17	13(5)	-3.1(15)				
	9	7	2	2 ⁺	4 ⁺	[33,34]	21	12	9(4)	11	8(3)	10(5)	7	11(3)	-2.0(10)	4	9.4(21)	13(3)	-1.8(7)
¹⁶⁰ Dy	3	3	1	2 ⁺	2 ⁺	[35,36]	8	5	-14(9)										
¹⁶⁴ Yb	5	5	2	2 ⁺	4 ⁺	[39]	80	17	23(6)	30	28(23)	10(60)	18	30(10)	-1.3(13)				

N : number of gamma transitions

N' : number of different gamma transitions

n : number of levels

beta decay : [14,15,16,20,24,25,26,28,30,33,35,36,39]

reaction (H,I, m) : [18,22,27]

reaction (n,r) or (n,n'r) : [23,29,31,34]

[20] : the transition 1015.02 keV is excluded from input data of [20]

[29] : the transitions 1341.577 and 273.458 keV are excluded from input data of [29]

[34] : the transition 1180.31 keV is excluded from input data [34]

Table 3. Determination of mixing parameters for the g_{em} band.

Nucleus	M	N	n	I _{min}	I _{max}	Refs.	Alaga			$Q_{E0} = Q_{E1}, \delta_{E1} = 0$			$Q_{E0} = Q_{E1}, \delta_{E1} = 0$			$Q_{E0} = Q_{E1}, \delta_{E1} = 0$			$Q_{E0} = Q_{E1}, \delta_{E1} = 0$		
							$\chi^2(4)$	$\chi^2(1)$	$10^3 Z_{E1}$	$\chi^2(2)$	$10^3 Z_{E1} (2_{E1})$	$10^3 Z_{E1} (0_{E1})$	$\chi^2(2')$	$10^3 Z_{E1}$	$10^3 Z_{E1}$	$\chi^2(2)$	$10^3 Z_{E1} (0_{E1})$	$10^3 Z_{E1} (0_{E1})$	$10^3 Z_{E1}$		
¹⁵⁸ Er	7	7	3	2 ⁺	4 ⁺	[14]	60	12	230(60)	14	160(120)	300(100)	14	160(120)	-70(90)						
¹⁶⁰ Er	6	4	2	2 ⁺	3 ⁺	[15,16]	27	0.9	133(14)	-	119(16)	170(30)	-	119(16)	-28(21)						
¹⁶² Er	12	7	3	2 ⁺	4 ⁺	[16,17]	40	5	104(14)	4	119(16)	72(25)	4	119(16)	23(16)						
	20	14	6	5 ⁺	11 ⁺	[18,19]	70	8	42(4)	6	48(5)	44(4)	7	51(7)	2.7(20)	7	48(6)	43(9)	-0.3(30)		
	32	21	9	2 ⁺	11 ⁺	[16-19]	60	12	46(5)	10	54(6)	48(4)	10	59(7)	4.5(22)	10	57(6)	54(12)	7(5)		
¹⁶⁴ Er	9	9	4	2 ⁺	5 ⁺	[20,21]	14	1.6	59(8)	0.8	68(8)	45(9)	0.8	68(8)	11(5)						
	25	14	9	2 ⁺	11 ⁺	[14,22]	80	9	63(5)	9	65(5)	61(5)									
	34	24	10	2 ⁺	11 ⁺	[19-22]	70	7	63(4)	7	65(4)	60(4)	6	67(4)	4.8(21)	6	66(5)	66(6)	5(5)		
¹⁶⁶ Er	11	11	4	2 ⁺	5 ⁺	[24]	500	13	45.7(26)	15	45.6(26)	44(4)	15	45.7(29)	0.1(19)	17	46(3)	43(6)	-0.9(30)		
	21	21	7	2 ⁺	8 ⁺	[25]	400	7	39.1(12)	5	41.9(17)	39.9(11)	6	41.5(19)	0.6(5)	6	41.5(18)	31.2(26)	-0.6(11)		
	16	16	6	3 ⁺	8 ⁺	[26]	4000	8	43.2(5)	4	44.8(5)	43.5(3)	4	44.7(6)	0.60(19)	3	44.7(5)	41.4(16)	-1.1(9)		
	16	16	7	2 ⁺	10 ⁺	[27]	28	14	27(7)	12	42.4(16)	34.5(11)	15	56.2(27)	6(6)	14	44.3(28)	36.5(25)	0(5)		
	64	24	8	2 ⁺	10 ⁺	[24-27]	2100	18	42.8(6)	13	44.8(8)	43.1(5)	14	44.8(8)	0.85(26)	14	44.7(8)	41(3)	-0.9(17)		
¹⁶⁸ Er	10	10	4	2 ⁺	5 ⁺	[28]	500	0.6	37.4(7)	0.4	36.6(9)	37.6(7)	0.4	36.6(9)	-0.5(5)	0.5	36.6(9)	36.8(20)	-0.4(10)		
	23	23	7	2 ⁺	8 ⁺	[29]	19	5	36(5)	5	37(6)	35(6)	5	38(6)	1.8(25)	6	38(6)	37(7)	1(3)		
	21	21	7	2 ⁺	8 ⁺	[29]	17	1.0	37.1(24)	1.0	37.7(25)	36.3(25)	1.1	37.5(26)	0.5(11)	1.1	37.6(27)	36.1(29)	-0.2(13)		
	31	21	7	2 ⁺	8 ⁺	[28,29]	170	0.9	37.4(6)	0.9	37.1(8)	37.5(7)	0.9	36.8(9)	-0.4(4)	0.9	36.8(9)	36.2(16)	-0.7(8)		
	4	4	2	3 ⁺	4 ⁺	[30]	11	5	23(12)	-	20(6)	57(17)	-	20(6)	-19(9)						
¹⁷⁰ Er	4	4	2	3 ⁺	4 ⁺	[31]	0.8	0.4	6(6)	-	6(6)	23(27)	-	6(6)	-9(14)						
	14	14	6	2 ⁺	7 ⁺	[21]	26	15	47(15)												
	11	11	5	2 ⁺	7 ⁺	[22]	28	2.8	65(7)	2.2	51(10)	66(6)	1.3	43(5)	-11(4)						
	19	11	5	2 ⁺	7 ⁺	[30,31,23]	26	9	40(10)	4	25(7)	58(7)	2.5	20(6)	-19(3)						
	16	11	4	2 ⁺	5 ⁺	[33,34]	12	5	27(7)	6	27(8)	28(8)	1.0	20(3)	-12.2(19)	0.9	20(3)	18(3)	-12.8(19)		
¹⁶⁰ Dy	9	9	4	2 ⁺	5 ⁺	[37]	100	2.2	39.9(16)	0.11	43.4(20)	51(3)	0.11	43.4(20)	-3.8(14)						
¹⁶⁴ Yb	6	6	3	2 ⁺	5 ⁺	[39]	230	1.5	43.6(25)	0.04	43.8(20)	51(3)	0.04	43.8(20)	-3.5(14)						

N: number of gamma transitions, N': number of different gamma transitions, n: number of levels
 [29]: the transitions 277.705 and 1075.64 keV are excluded from input data of [29]
 [23]: the transitions 562.3, 1152.5 keV are excluded from input data of [23]
 [37]: a small admixture of M1 multiplicity determined in [38] was subtracted from I_γ[37]

beta decay: [14,15,16,17,20,21,24,25,26,28,30,31,35,37,38,39]
 reaction (N.I., α or β): [18,19,22,27]
 reaction (n, γ) or (n, n' γ): [23,29,34]

Table 4.

Calculated relative intensities of unobserved transitions in beta decay [14] . The nucleus ^{158}Er ($Z_{\beta} = 0.030(16)$, $Z_{\delta} = 0.230(60)$).

$I_i \rightarrow I_f$	E_{β} [keV]	I_{β}	$I_i \rightarrow I_f$	E_{δ} [keV]	I_{δ}
$4_{\beta}^+ \rightarrow 4_{\xi}^+$	729.8	100(80)	$4_{\delta}^+ \rightarrow 2_{\xi}^+$	991.69	$9\left(\begin{smallmatrix} +33 \\ -9 \end{smallmatrix}\right)$
$4_{\beta}^+ \rightarrow 2_{\xi}^+$	1065.07	250(20) ^{a)}	$4_{\delta}^+ \rightarrow 6_{\xi}^+$	213.48	0.69(26)
			$4_{\delta}^+ \rightarrow 3_{\delta}^+$	140.43	0.77(22)
$2_{\delta}^+ \rightarrow 4_{\xi}^+$	292.9	4.1(18)	$4_{\delta}^+ \rightarrow 4_{\xi}^+$	656.57	276(25) ^{a)}

The intensity marked by a) has been measured and it is presented for comparison.

Table 5.

Calculated relative intensities of unobserved transitions in beta decay of the ground [15] and the isomeric [16] state of ^{160}Tm . The nucleus ^{160}Er ($Z_{\beta} = 0.029(5)$, $Z_{\delta} = 0.133(14)$).

$I_i \rightarrow I_f$	E_{β} [keV]	I_{β} [15]	$I_i \rightarrow I_f$	E_{δ} [keV]	I_{δ} [16]
$2_{\beta}^+ \rightarrow 4_{\xi}^+$	617.5	1.5(3)	$4_{\delta}^+ \rightarrow 6_{\xi}^+$	362.9	0.138(26)
$4_{\beta}^+ \rightarrow 6_{\xi}^+$	464.6	0.88(26)	$4_{\delta}^+ \rightarrow 2_{\xi}^+$	1002.8	1.3(7)
$4_{\beta}^+ \rightarrow 4_{\xi}^+$	840.8	3.8(9) ^{a)}	$5_{\delta}^+ \rightarrow 6_{\xi}^+$	550.9	3.4(11)
			$6_{\delta}^+ \rightarrow 6_{\xi}^+$	739.0	$98\left(\begin{smallmatrix} +120 \\ -98 \end{smallmatrix}\right)$
$2_{\delta}^+ \rightarrow 4_{\xi}^+$	464.4	0.187(25)	$6_{\delta}^+ \rightarrow 8_{\xi}^+$	274.3	0.5(5)
$2_{\delta}^+ \rightarrow 0_{\xi}^+$	854.4	8.1(7) ^{a)}	$5_{\delta}^+ \rightarrow 4_{\xi}^+$	926.7	6.7(12) ^{b)}

Intensities marked by a) and b) have been measured [15] and [16] and presented for comparison.

Table 6.

Calculated relative intensities of unobserved transitions in beta decay [15,17] and in reactions (p,4n) [18] and (α ,2n)[19].

The nucleus ^{162}Er ($Z_B = 0,033(7)$, $\beta_B = 0,013(3)$, $Z_F = 0,046(5)$).

$I_i \rightarrow I_f$	E_γ [keV]	I_γ [18]	I_γ [19]	I_γ [15]	I_γ [17]
$2_B^+ \rightarrow 0_B^+$	83.5			0.000029(16)	0.00048(24)
$2_B^+ \rightarrow 0_E^+$	1170.8			0.16(9)	
$2_B^+ \rightarrow 2_E^+$	1068.8			1.1(1) ^{c)}	15.5(8) ^{d)}
$3_F^+ \rightarrow 2_F^+$	101.5	0.0039(13)	0.0037(14)	0.0058(26)	0.059(14)
$3_F^+ \rightarrow 2_E^+$	900.0	6.4(19)	6.0(21)		
$4_F^+ \rightarrow 3_F^+$	126.8	0.066(21)	0.21(7)	0.004(4)	0.021(10)
$4_F^+ \rightarrow 2_F^+$	228.3	0.56(17)	1.8(6)	0.03(3)	0.17(9)
$4_F^+ \rightarrow 6_E^+$	461.67			0.023(29)	0.13(6)
$4_F^+ \rightarrow 4_E^+$	799.0	34(9)	106(29)	1.9(7) ^{c)}	10.8(15) ^{d)}
$4_F^+ \rightarrow 2_E^+$	1027.0	20(6)	63(19)	1.1(⁺¹⁴ ₋₁₁)	
$5_F^+ \rightarrow 4_F^+$	157.0	0.028(7)	0.021(3)		
$5_F^+ \rightarrow 3_F^+$	284.7	0.53(13)	0.41(7)		
$5_F^+ \rightarrow 4_E^+$	956.7	8.55(71) ^{a)}	7.1(2) ^{b)}		
$6_F^+ \rightarrow 5_F^+$	173.0	0.024(4)	0.0152(24)		
$6_F^+ \rightarrow 8_E^+$	362.9	0.027(4)	0.0175(25)		
$7_F^+ \rightarrow 6_F^+$	208.0	0.036(12)	0.027(4)		
$7_F^+ \rightarrow 5_F^+$	382.9	1.8(6)			
$7_F^+ \rightarrow 8_E^+$	573.0		0.56(8)		
$8_F^+ \rightarrow 10_E^+$	269.0	0.0035(12)	0.0025(10)		
$8_F^+ \rightarrow 8_E^+$	776.0		1.0(4)		
$8_F^+ \rightarrow 6_E^+$	1205.7		0.33(15)		
$8_F^+ \rightarrow 7_F^+$	203.6	0.013(14)	0.009(4)		
$8_F^+ \rightarrow 6_F^+$	413.0	1.5(5)			

Table 6 (continue)

$I_i \rightarrow I_f$	E_{γ} [keV]	I_{γ} [18]	I_{γ} [19]	I_{γ} [15]	I_{γ} [17]
$9_f^+ \rightarrow 8_f^+$	261.1	0.025(7)			
$9_f^+ \rightarrow 8_g^+$	1037.1	1.6(5)			
$10_f^+ \rightarrow 9_f^+$	212.6	0.0047(14)			
$10_f^+ \rightarrow 8_f^+$	473.9	1.5(4)			
$10_f^+ \rightarrow 12_g^+$	181.6	0.00027(8)			
$10_f^+ \rightarrow 8_g^+$	1250.2	0.08(6)			
$11_f^+ \rightarrow 10_f^+$	309.6	0.0093(21)			
$11_f^+ \rightarrow 12_g^+$	491.5	0.039(11)			

Intensities marked by a), b), c), d) have been measured and presented for comparison.

Table 7a.

Calculated relative intensities of unobserved transitions in reaction $(\alpha, 2n_g)$ [22] and (n, n'_g) [23].

The rotational β -band of ^{164}Er ($Z_\beta = 0,028(4)$).

$I_i \rightarrow I_f$	E_β [keV]	I_β [22]	$I_i \rightarrow I_f$	E_β [keV]	I_β [23]
$6_\beta^+ \rightarrow 2_g^+$	682.0	10.9(25)	$2_\beta^+ \rightarrow 0_\beta^+$	68.5	0.0006(5)
$6_\beta^+ \rightarrow 4_g^+$	1407.0	13(7)	$2_\beta^+ \rightarrow 4_g^+$	1014.4	20(4)
$6_\beta^+ \rightarrow 4_\beta^+$	237.0	1.2(11)	$4_\beta^+ \rightarrow 2_\beta^+$	155.4	0.04(4)
$8_\beta^+ \rightarrow 10_g^+$	551.0	4.3(11)	$4_\beta^+ \rightarrow 6_g^+$	855.5	8(3)
$8_\beta^+ \rightarrow 6_\beta^+$	363.0	10(9)	$6_\beta^+ \rightarrow 4_\beta^+$	237.0	0.20(22)
$8_\beta^+ \rightarrow 8_g^+$	1044.3	15.0(15) ^{a)}	$6_\beta^+ \rightarrow 8_g^+$	682.0	1.8(12)
			$2_\beta^+ \rightarrow 2_g^+$	1223.1	20(3) ^{b)}

Intensities marked by a) and b) have been measured in [22] and [23] and they are presented for comparison.

Table 7b.

Calculated relative intensities of unobserved transitions in beta decay of the ground [20] and the isomeric states of ^{164}Tm [21].

The rotational γ -band of ^{164}Er ($Z_\gamma = 0.063(4)$).

$I_i \rightarrow I_f$	E_γ [keV]	I_γ [20]	$I_i \rightarrow I_f$	E_γ [keV]	I_γ [21]
$3_\gamma^+ \rightarrow 2_\gamma^+$	86.24	0.0018(4)	$5_\gamma^+ \rightarrow 3_\gamma^+$	250.99	2.2(5)
$4_\gamma^+ \rightarrow 6_g^+$	444.17	0.014(4)	$5_\gamma^+ \rightarrow 4_\gamma^+$	138.9	0.113(24)
$4_\gamma^+ \rightarrow 2_\gamma^+$	198.4	0.0096(25)	$5_\gamma^+ \rightarrow 4_g^+$	898.1	42(4) ^{b)}
$4_\gamma^+ \rightarrow 3_\gamma^+$	112.07	0.0012(3)			
$2_\gamma^+ \rightarrow 0_g^+$	860.25	16.6(14) ^{a)}			

Intensities marked by a) and b) have been measured in [20] and [21].

Table 7c.

Calculated relative intensities of unobserved transitions in reactions $(\alpha, 2n\gamma)$ [19,22].
The rotational γ -band ^{164}Er ($Z_\gamma=0.063(4)$).

$I_i \rightarrow I_f$	E_γ [keV]	I_γ [22]	I_γ [19]	$I_i \rightarrow I_f$	E_γ [keV]	I_γ [22]	I_γ [19]
$2_\gamma^+ \rightarrow 4_g^+$	561.46	0.57(9)		$7_\gamma^+ \rightarrow 6_\gamma^+$	185.7	0.45(7)	0.047(6)
$3_\gamma^+ \rightarrow 4_g^+$	647.52		1.08(11)	$7_\gamma^+ \rightarrow 5_\gamma^+$	346.8		2.6(3)
$3_\gamma^+ \rightarrow 2_\gamma^+$	86.24	0.036(6)	0.0020(2)	$7_\gamma^+ \rightarrow 8_\epsilon^+$	519.7		1.24(16)
$4_\gamma^+ \rightarrow 3_\gamma^+$	112.07	0.13(3)	0.0060(6)	$8_\gamma^+ \rightarrow 7_\gamma^+$	200.5	0.31(5)	0.0197(20)
$4_\gamma^+ \rightarrow 2_\gamma^+$	198.4	1.03(24)	0.047(4)	$8_\gamma^+ \rightarrow 10_\epsilon^+$	226.9	0.050(9)	0.0032(4)
$4_\gamma^+ \rightarrow 6_\epsilon^+$	444.17	1.5(3)	0.069(5)	$8_\gamma^+ \rightarrow 6_\epsilon^+$	1130.4	1.0(7)	0.06(4)
$4_\gamma^+ \rightarrow 2_\epsilon^+$	967.81	37(9)	1.71(15)	$9_\gamma^+ \rightarrow 8_\gamma^+$	231.9	0.40(9)	
$5_\gamma^+ \rightarrow 4_\gamma^+$	138.92	0.38(5)	0.0313(25)	$9_\gamma^+ \rightarrow 10_\epsilon^+$	458.8	3.6(9)	
$5_\gamma^+ \rightarrow 3_\gamma^+$	250.99		0.60(5)	$10_\gamma^+ \rightarrow 9^+$	207.4	0.106(18)	
$6_\gamma^+ \rightarrow 5_\gamma^+$	161.1	0.71(14)	0.0199(19)	$10_\gamma^+ \rightarrow 12_\epsilon^+$	101.6	0.0002(5)	
$6_\gamma^+ \rightarrow 4_\gamma^+$	300.3		0.75(7)	$10_\gamma^+ \rightarrow 8_\epsilon^+$	1159.5	0.4(4)	
$6_\gamma^+ \rightarrow 8_\epsilon^+$	334.0	1.00(23)	0.0280(21)	$11_\gamma^+ \rightarrow 10_\gamma^+$	294.9	0.19(3)	
$6_\gamma^+ \rightarrow 4_\epsilon^+$	1059.1		0.81(15)	$11_\gamma^+ \rightarrow 12_\epsilon^+$	396.5	0.44(9)	
$6_\gamma^+ \rightarrow 6_\epsilon^+$	744.2	150(30)		$4_\gamma^+ \rightarrow 4_\epsilon^+$	758.8	94(9) ^{a)}	4.3(1) ^{b)}

Intensities marked by a) and b) have been measured in [22] and [19].

Table 8.

Calculated relative intensities of unobserved transitions in beta decay of ^{166}Tm [24], $^{166\text{m}}\text{Ho}$ [25,26] and in reaction $(\alpha, 2n\gamma)$ [27]. The nucleus ^{166}Er ($Z_\gamma(Z_\delta) = 0,0448(8)$, $Z_\gamma(O_g) = 0,00431(5)$).

$I_i \rightarrow I_f$	E_δ [keV]	I_γ [27]	I_γ [25,26]	I_γ [24]
$2_\delta^+ \rightarrow 4_E^+$	520.9	0.060(15)	0.00048(22)	
$3_\delta^+ \rightarrow 2_\delta^+$	73.5	0.0034(13)	0.00142(3)	0.0345(19)
$3_\delta^+ \rightarrow 4_\delta^+$	594.4	1.8(7)		
$4_\delta^+ \rightarrow 3_\delta^+$	96.8	0.0087(25)	0.00259(8)	0.0536(23)
$4_\delta^+ \rightarrow 6_E^+$	410.8	0.083(24)		
$4_\delta^+ \rightarrow 2_\delta^+$	170.3	0.065(18)		
$5_\delta^+ \rightarrow 4_\delta^+$	119.0	0.047(15)	0.179(3)	0.131(8)
$5_\delta^+ \rightarrow 3_\delta^+$	215.9	0.93(29)		0.258(15)
$5_\delta^+ \rightarrow 4_E^+$	810.3	21(7)		
$6_\delta^+ \rightarrow 5_\delta^+$	140.7	0.052(11)	0.0412(8)	
$6_\delta^+ \rightarrow 8_E^+$	304.8	0.052(11)		
$7_\delta^+ \rightarrow 6_\delta^+$	160.1	0.079(21)	0.0867(14)	
$7_\delta^+ \rightarrow 8_E^+$	464.8	1.5(4)		
$8_\delta^+ \rightarrow 7_\delta^+$	179.7	0.062(13)	0.00292(18)	
$8_\delta^+ \rightarrow 10_E^+$	206.0	0.0061(13)	0.000290(20)	
$9_\delta^+ \rightarrow 8_\delta^+$	195.3	0.053(19)		
$9_\delta^+ \rightarrow 10_E^+$	401.4	0.34(12)		
$9_\delta^+ \rightarrow 8_E^+$	839.8	3.7(14)		
$10_\delta^+ \rightarrow 9_\delta^+$	213.3	0.031(9)		
$10_\delta^+ \rightarrow 12_E^+$	117.1	0.00015(4)		
$10_\delta^+ \rightarrow 10_E^+$	614.7	1.1(3)		
$10_\delta^+ \rightarrow 8_E^+$	1053.1	0.32(10)		
$4_\delta^+ \rightarrow 2_E^+$	875.7	3.8(4) ^{a)}	0.991(11) ^{b)}	21.5(4) ^{c)}

Intensities marked by a), b) and c) have been measured in [27], [26] and [24]

Table 9.

Influence of nonlinear correction upon the values of mixing parameters for the rotational γ -vibrational band of ^{166}Er .

References	$10^3 Z_{\beta}(0_{\beta})$	0	20	40	60
[24, 25, 26, 27]	$\chi^2(3)$	13.8	11.4	10.6	10.3
	$10^3 Z_{\gamma}(2_{\gamma})$	44.7(8)	44.7(8)	44.6(7)	44.5(7)
	$10^3 Z_{\gamma}(0_{\text{g}})$	41(3)	48.5(22)	47.6(16)	47.1(14)
	$10^3 Z_{\beta\gamma}$	-0.9(17)	1.8(7)	1.1(4)	0.78(25)
[24, 25, 26, 27]	$\chi^2(3)$	6.4	5.6	5.1	4.8
	$10^3 Z_{\gamma}(2_{\gamma})$	44.5(6)	44.5(5)	44.4(5)	44.4(5)
	$10^3 Z_{\gamma}(0_{\text{g}})$	41.0(23)	46.7(16)	46.4(12)	46.1(10)
	$10^3 Z_{\beta\gamma}$	-1.0(11)	1.2(5)	0.81(28)	0.59(19)
[24, 25, 26, 27]*	$\chi^2(3)$	4.8	4.4	4.1	3.9
	$10^3 Z_{\gamma}(2_{\gamma})$	44.5(5)	44.5(5)	44.5(5)	44.5(5)
	$10^3 Z_{\gamma}(0_{\text{g}})$	41.0(20)	45.4(15)	45.3(11)	45.2(10)
	$10^3 Z_{\beta\gamma}$	-0.9(10)	0.9(5)	0.62(26)	0.46(18)

- [24] : the intensity of the transition $2_{\gamma}^+ \rightarrow 4_{\text{g}}^+$ was corrected $I_{\gamma} = 0.90(13)$, (the transition 520.945 keV is doublet)
- [25] : the transition 304.81 keV ($6_{\gamma}^+ \rightarrow 8_{\text{g}}^+$) is excluded from input data of [25]
- [27] : the transition 1053.70 keV ($10_{\gamma}^+ \rightarrow 8_{\text{g}}^+$) is excluded from input data of [27]
- [24, 25, 26, 27]* : a small admixture of M1 multipolarity determined in [32] was subtracted from $I_{\gamma}[24, 25, 26, 27]$

Table 10.

Calculated relative intensities of unobserved transitions in beta decay [28] and in reaction (n, γ) [29].
 The nucleus ^{168}Er ($Z_{\beta} = 0.028(4)$, $\{_{\beta\gamma} = -0.0012(4)$, $Q_{\beta\gamma} = 0.112(20)\text{eb}$, $Q_{\beta} = 0.056(9)\text{eb}$, $Z_{\gamma} = 0.0374(6)$).

$I_i \rightarrow I_f$	E_{γ} [keV]	I_{γ} [29]	I_{γ} [28]	$I_i \rightarrow I_f$	E_{γ} [keV]	I_{γ} [29]	I_{γ} [28]
$0_{\beta}^+ \rightarrow 2_{\gamma}^+$	396.0	0.31(18)		$8_{\beta}^+ \rightarrow 10_{\gamma}^+$	494.3	0.031(25)	
$2_{\beta}^+ \rightarrow 0_{\beta}^+$	59.1	0.015(6)		$8_{\beta}^+ \rightarrow 8_{\gamma}^+$	962.1	0.029(28)	
$2_{\beta}^+ \rightarrow 4_{\gamma}^+$	281.5	0.08(4)	0.000017(10)	$8_{\beta}^+ \rightarrow 6_{\gamma}^+$	1341.58	0.10(10)	
$2_{\beta}^+ \rightarrow 3_{\gamma}^+$	380.6		0.00035(13)	$2_{\beta}^+ \rightarrow 4_{\gamma}^+$	1012.28	99(9) ^{a)}	0.016(2) ^{b)}
$2_{\beta}^+ \rightarrow 2_{\gamma}^+$	455.2		0.00072(22)				
$2_{\beta}^+ \rightarrow 0_{\gamma}^+$	1276.4		0.013(3)	$3_{\gamma}^+ \rightarrow 2_{\gamma}^+$	74.6	0.88(6)	0.0273(10)
$4_{\beta}^+ \rightarrow 6_{\gamma}^+$	147.2	0.003(3)	0.00000005($\frac{+7}{-5}$)	$4_{\gamma}^+ \rightarrow 3_{\gamma}^+$	99.0	1.01(6)	0.0122(5)
$4_{\beta}^+ \rightarrow 5_{\gamma}^+$	293.6		0.0000018(11)	$4_{\gamma}^+ \rightarrow 6_{\gamma}^+$	446.0		0.1345(20)
$4_{\beta}^+ \rightarrow 4_{\gamma}^+$	416.4		0.00019(10)	$5_{\gamma}^+ \rightarrow 4_{\gamma}^+$	122.8	1.05(7)	0.000131(8)
$4_{\beta}^+ \rightarrow 3_{\gamma}^+$	515.3		0.00026(13)	$5_{\gamma}^+ \rightarrow 3_{\gamma}^+$	221.8		0.00252(15)
$4_{\beta}^+ \rightarrow 2_{\gamma}^+$	589.9		0.000047(25)	$6_{\gamma}^+ \rightarrow 5_{\gamma}^+$	146.3	0.67(4)	
$4_{\beta}^+ \rightarrow 2_{\beta}^+$	134.8		0.00010(6)	$7_{\gamma}^+ \rightarrow 6_{\gamma}^+$	169.0	0.182(14)	
$6_{\beta}^+ \rightarrow 7_{\gamma}^+$	183.9	0.015(10)		$8_{\gamma}^+ \rightarrow 7_{\gamma}^+$	191.6	0.019(3)	
$8_{\beta}^+ \rightarrow 8_{\gamma}^+$	265.8	0.0011(9)		$8_{\gamma}^+ \rightarrow 10_{\gamma}^+$	227.7	0.0018(3)	
$8_{\beta}^+ \rightarrow 7_{\gamma}^+$	457.3	0.017(14)		$8_{\gamma}^+ \rightarrow 6_{\gamma}^+$	1075.64	0.70(13)	
$8_{\beta}^+ \rightarrow 6_{\gamma}^+$	626.4	0.022(18)		$3_{\beta}^+ \rightarrow 2_{\gamma}^+$	815.984	3000(240) ^{a)}	93.4(9) ^{b)}

Intensities marked by a) and b) have been measured in [29] and [28].

Table 11.

Calculated relative intensities of unobserved transitions in the decay of the ground [30] and isomeric [31] states of ^{170}Ho and in reaction (n, n'_{γ}) [23].

The nucleus ^{170}Er ($Z_{\beta} = 0.0096(17)$, $\beta_{\beta\gamma} = 0.0033(5)$, $Z_{\gamma} = 0.040(10)$).

$I_i \rightarrow I_f$	E_{γ} [keV]	I_{γ} [30]	I_{γ} [31]	I_{γ} [23]
$2_{\beta}^+ \rightarrow 0_{\beta}^+$	68.8	0.0185(27)		0.023(4)
$4_{\beta}^+ \rightarrow 2_{\beta}^+$	168.1			1.14(17)
$6_{\beta}^+ \rightarrow 4_{\beta}^+$	273.4			2.1(8)
$6_{\beta}^+ \rightarrow 8_{\beta}^+$	486.4			0.11(5)
$2_{\gamma}^+ \rightarrow 4_{\beta}^+$	674.4		5(3)	
$2_{\gamma}^+ \rightarrow 2_{\beta}^+$	855.9		200(140)	
$3_{\gamma}^+ \rightarrow 2_{\beta}^+$	76.1	0.033(6)	0.27(5)	0.020(5)
$4_{\gamma}^+ \rightarrow 3_{\beta}^+$	92.9	0.009(5)	0.07(5)	0.0128(24)
$4_{\gamma}^+ \rightarrow 2_{\beta}^+$	169.0	0.08(4)	0.6(4)	0.114(21)
$4_{\gamma}^+ \rightarrow 6_{\beta}^+$	562.9	0.36(19)	2.6(19)	0.50(14)
$5_{\gamma}^+ \rightarrow 3_{\beta}^+$	226.0		3.6(8)	0.39(11)
$5_{\gamma}^+ \rightarrow 4_{\beta}^+$	133.1		0.26(6)	0.028(8)
$5_{\gamma}^+ \rightarrow 6_{\beta}^+$	696.0		31(8)	
$6_{\gamma}^+ \rightarrow 5_{\beta}^+$	176.5		0.10(7)	0.010(8)
$6_{\gamma}^+ \rightarrow 4_{\beta}^+$	309.6		2.9(21)	0.29(21)
$6_{\gamma}^+ \rightarrow 8_{\beta}^+$	498.8		0.26(19)	0.027(20)
$6_{\gamma}^+ \rightarrow 4_{\beta}^+$	1153.0		7(5)	0.7(5)
$7_{\gamma}^+ \rightarrow 6_{\beta}^+$	145.3			0.005(4)
$7_{\gamma}^+ \rightarrow 8_{\beta}^+$	642.6			0.5(5)
$3_{\beta}^+ \rightarrow 2_{\beta}^+$	931.8	179(10) ^{a)}	1640(90) ^{b)}	94(10) ^{c)}

Intensities marked by a), b) and c) have been measured in [30], [31] and [23].

Table 12.

Calculated relative intensities of unobserved transitions in beta decay [33] and in reaction (n, γ) [34].
 The nucleus ^{158}Gd ($Z_{\beta} = 0.013(3)$, $\{_{\beta\gamma} = -0.0035(9)$, $Q_{\beta\gamma} = 0.80(22)\text{eb}$, $Z_{\gamma} = 0.020(3)$, $Z_{\beta\gamma} = -0.0122(19)$).

$I_i \rightarrow I_f$	E_{γ} [keV]	I_{γ} [34]	I_{γ} [33]	$I_i \rightarrow I_f$	E_{γ} [keV]	I_{γ} [34]	I_{γ} [33]
$0_{\beta}^+ \rightarrow 2_{\gamma}^+$	9.0	0.00000022(10)	0.000000008(5)	$3_{\gamma}^+ \rightarrow 2_{\gamma}^+$	78.4	0.038(3)	0.00058(20)
$2_{\beta}^+ \rightarrow 0_{\gamma}^+$	63.7	0.019(3)	0.00021(5)	$4_{\gamma}^+ \rightarrow 3_{\gamma}^+$	92.9	0.0294(20)	
$2_{\beta}^+ \rightarrow 2_{\gamma}^+$	72.7	0.008(4)	0.00009(5)	$5_{\gamma}^+ \rightarrow 4_{\gamma}^+$	122.9	0.0270(15)	
$4_{\beta}^+ \rightarrow 4_{\gamma}^+$	48.2	0.00047(21)		$2_{\gamma}^+ \rightarrow 0_{\gamma}^+$	1187.13	401(24) ^{a)}	14.7(10) ^{b)}
$2_{\beta}^+ \rightarrow 4_{\gamma}^+$	998.36	158(9) ^{a)}	1.27(14) ^{b)}				

Intensities marked by a) and b) have been measured in [34, and [33].

Table 13.

Calculated relative intensities of unobserved transitions in beta decay of ^{160}Ho [35,36] and ^{160}Tb [37].

The nucleus is ^{160}Dy ($Z_{\beta} = -0,014(9)$, $Z_{\beta}(2_{\beta}^+) = 0,0434(20)$, $Z_{\beta}(0_{\beta}^+) = 0,051(3)$).

$I_i \rightarrow I_f$	E_{β} [keV]	I_{β} [35]	$I_i \rightarrow I_f$	E_{β} [keV]	I_{β} [37]
$2_{\beta}^+ \rightarrow 0_{\beta}^+$	69.1	0.0021(5)	$3_{\beta}^+ \rightarrow 2_{\beta}^+$	82.9	0.0062(5)
$2_{\beta}^+ \rightarrow 0_{\beta}^+$	1349.6	19.0(20) ^{a)}	$4_{\beta}^+ \rightarrow 3_{\beta}^+$	106.7	0.00055(5)
			$4_{\beta}^+ \rightarrow 2_{\beta}^+$	189.7	0.0043(4)
			$4_{\beta}^+ \rightarrow 6_{\beta}^+$	574.8	0.0148(10)
			$5_{\beta}^+ \rightarrow 4_{\beta}^+$	132.7	0.000140(17)
			$5_{\beta}^+ \rightarrow 3_{\beta}^+$	239.4	0.0027(3)
			$2_{\beta}^+ \rightarrow 0_{\beta}^+$	966.160	83.3(24) ^{b)}

Intensities marked by a) and b) have been measured in [35] and [37].

Table 14.

Calculated relative intensities of unobserved transitions in beta decay [39]. The nucleus ^{164}Yb ($Z_{\beta} = 0.023(6)$, $Z_{\beta} = 0.122(15)$).

$I_i \rightarrow I_f$	E_{β} [keV]	I_{β} [38]	$I_i \rightarrow I_f$	E_{β} [keV]	I_{β} [39]
$4_{\beta}^+ \rightarrow 6_{\beta}^+$	563.0	3.3(10)	$4_{\beta}^+ \rightarrow 6_{\beta}^+$	384.6	0.098(24)
			$4_{\beta}^+ \rightarrow 2_{\beta}^+$	1021.4	1.1(6)
$2_{\beta}^+ \rightarrow 4_{\beta}^+$	478.0	1.64(26)	$2_{\beta}^+ \rightarrow 0_{\beta}^+$	863.89	71.8(35) ^{a)}

The intensity marked by a) has been measured in [39] and is presented for comparison.

Table 15

Resulting values of the physical constants of the ground states and quadrupole vibrational states

Nucleus	Refs.	X^2	C_{GG} [eb]	Q_T [eb]	Q_S [eb]	$10^3 \epsilon_T$	$10^3 \epsilon_S$	$10^3 \epsilon_{\mathcal{A}f}$	$10^3 \alpha_f$	$10^3 \alpha_S$	$10^3 z_f$	$10^3 z_{\mathcal{A}f}$	$10^3 z_S$	$10^3 \lambda_{\mathcal{A}f}$
^{158}Er	[14]	a)	1.66(15)	0.29(5)	0.25(12)	-6.2(23)	-5(5)	=0	1.2(4)	0.7(6)	230(60)	=0	30(16)	=0
		5	1.66(10)	0.27(3)	0.17(3)	-7.1(16)	-5.8(15)	17(4)	0.95(28)	0.59(24)	210(40)	-26(10)	57(9)	-11.2(24)
^{162}Er	[16-19]	a)	2.24(4)	0.287(18)	0.21(7)	-1.20(16)	-0.7(11)	=0	0.125(22)	0.07(10)	46(5)	=0	7(11)	=0
		9	2.24(4)	0.296(17)	0.12(4)	-1.61(24)	-0.7(6)	-4.0(19)	0.17(3)	0.04(4)	59(7)	4.7(21)	13(9)	4.6(26)
^{164}Er	[19-23]	a)	2.38(4)	0.285(11)	0.23(10)	-1.54(12)	-2.6(14)	=0	0.150(16)	0.27(13)	63(4)	=0	20(4)	=0
		b)												
^{166}Er	[24-27]	a)	2.39(6)	0.260(7)	0.19(9)	-0.950(17)	-4(3)	=0	0.0644(21)	0.3(3)	42.6(6)	=0	50(40)	=0
		14	2.39(6)	0.261(6)	0.134(13)	-0.988(22)	-	-0.67(26)	0.0890(25)	-	44.6(6)	0.55(28)	-	0.55(28)
^{168}Er	[28,29]	a)	2.390(21)	0.260(5)	0.048(8)	-0.828(21)	-0.50(13)	=0	0.073(7)	0.0101(25)	37.4(5)	=0	25(5)	=0
		7	2.39(4)	0.260(9)	0.048(5)	-0.83(4)	-0.51(9)	0.10(25)	0.075(6)	0.0303(25)	37.3(12)	-0.04(11)	25(3)	-0.2(5)
		8	2.39(6)	0.258(13)	0.056(9)	-0.81(6)	-0.65(14)	0.60(20)	0.072(8)	0.015(5)	36.5(10)	-0.33(13)	28(4)	-1.2(4)
^{170}Er	[30,31,23 ^a]	a)	2.37(5)	0.229(12)	0.097(15)	-0.78(20)	-0.54(24)	=0	0.061(17)	0.022(10)	40(10)	=0	13(5)	=0
		2.0	2.372(22)	0.230(6)	0.093(9)	-0.85(9)	-0.41(5)	-2.4(6)	0.068(8)	0.016(5)	44(4)	2.4(6)	10.5(21)	2.5(6)
^{158}Ga	[33,34 ^a]	a)	2.231(21)	0.209(10)	0.103(10)	-0.52(15)	-0.41(18)	=0	0.040(12)	0.019(8)	27(7)	=0	9(4)	=0
		3	2.228(16)	0.209(7)	0.100(5)	-0.48(11)	-0.52(10)	2.6(7)	0.036(9)	0.023(5)	25(5)	-3.3(9)	11.6(21)	-2.4(6)
		5	2.227(20)	0.209(10)	0.097(7)	-0.46(14)	-0.58(13)	4.0(5)	0.035(11)	0.025(7)	24(7)	-4.6(11)	13(3)	-3.5(8)
^{160}Dy	[35,36,37 ^a]	a)	2.27(4)	0.250(11)	0.125(12)	-0.98(8)	0.8(4)	=0	0.067(10)	-0.043(24)	43.6(25)	=0	-14(9)	=0
		0.04	2.27(3)	0.250(9)	0.115(8)	-0.98(6)	0.72(27)	2.2(12)	0.088(5)	-0.038(13)	43.6(20)	-2.6(13)	-14(6)	-1.9(11)

a) X^2 are different for the \mathcal{A} - and f - band and they are in the tabs 2 and 3.b) the mixing parameters $\epsilon_{\mathcal{A}f}$ and $\lambda_{\mathcal{A}f}$ are not related for this nucleus by relation $\epsilon_{\mathcal{A}f} \lambda_{\mathcal{A}f} = \epsilon_{\mathcal{A}f}^2$, because this version of calculations is not displayedc) the \mathcal{A} - f transitions are included in the analysis and the values of $Q_{\mathcal{A}f}$ and $\alpha_{\mathcal{A}f}$ are determined too:

$$^{168}\text{Er} (Q_{\mathcal{A}f} = 0.112(20) \cdot 10^3 \text{ eb}, \alpha_{\mathcal{A}f} = -0.80(30)), \quad ^{158}\text{Ga} (Q_{\mathcal{A}f} = 0.80(22) \cdot 10^3 \text{ eb}, \alpha_{\mathcal{A}f} = -5.7(22))$$

Table 16.

Determination of mixing parameters for the ground band.

Nucleus	N	I _{min}	I _{max}	Refs.	Alaga		Mixing of wave functions			
					$\chi^2(A)$	Q _{gg} [eb]	$\chi^2(I)$	Q _{gg} [eb]	$10^3(\alpha_\lambda + \alpha_\beta)_{\text{ground}}$	$10^3(\alpha_\lambda + \alpha_\beta)_{\text{vibr.}}^a)$
¹⁵⁸ Er	7	2 ⁺	16 ⁺	[45]	1.4	1.72(3)	1.6	1.72(4)	-0.02(30)	1.6(4)
¹⁶⁰ Er	8	2 ⁺	18 ⁺	[46]	0.2	2.067(20)	0.2	2.068(22)	-0.02(16)	
¹⁶² Er										0.21(6)
¹⁶⁴ Er	5	2 ⁺	12 ⁺	[41]	1.0	2.333(13)	1.4	2.333(16)	0.01(15)	0.65(17)
¹⁶⁶ Er	5	2 ⁺	12 ⁺	[42,43]	2.2	2.346(17)	1.6	2.336(17)	0.21(14)	0.096(8) ^{b)}
¹⁶⁸ Er	5	2 ⁺	12 ⁺	[42,43]	2.8	2.365(28)	1.5	2.396(26)	-0.22(10)	0.087(10)
¹⁷⁰ Er	6	2 ⁺	12 ⁺	[42-44]	2.2	2.371(18)	2.3	2.362(21)	0.13(14)	0.085(9)
¹⁵⁸ Gd	5	2 ⁺	12 ⁺	[45]	0.5	2.228(9)	0.4	2.231(9)	-0.10(12)	0.059(8)
¹⁶⁰ Dy	7	2 ⁺	14 ⁺	[46]	4.5	2.22(5)	5.0	2.24(6)	-0.15(25)	0.050(15)
¹⁶⁴ Yb	9	2 ⁺	18 ⁺	[41]	0.8	2.162(23)	0.4	2.191(28)	-0.27(14)	

a) Determination of mixing parameters from the physical constants of the quadrupole vibrational states

b) Only α_β

FIGURE CAPTIONS

FIG. 1 Levels of rotational bands of the β -, γ -vibrational states and the ground state in even-even Er isotopes ($A = 158-170$) excited in the beta decay. The values of I^π and the experimental (asterisk) or calculated mean lives (τ) of the individual levels are shown. Levels introduced in beta decay preliminarily are marked by a dashed line.

FIG. 2 Levels of rotational bands of the β -, γ -vibrational states and the ground state in even-even $N = 94$ isotones ($A = 158-164$) excited in beta decay. The values of I^π and the experimental (asterisk) or calculated mean lives (τ) of the individual levels are shown. Levels introduced in beta decay preliminarily are marked in a dashed line.

FIG. 3 The mixing parameters of the rotational β -band wave functions of $N = 94$ nuclei ($A = 158-164$) and $Z = 68$ nuclei ($A = 158-170$). The values of Z_β determined using microscopic model with both pair and quadrupole-quadrupole interaction in Woods-Saxon mean field [51] are marked by empty circles.

FIG. 4 The mixing parameters of the rotational γ -band wave functions of $N = 94$ nuclei ($A = 158-164$) and $Z = 68$ nuclei ($A = 158-170$). The values of Z_γ determined using microscopic model with both pair and quadrupole-quadrupole interaction in Woods-Saxon mean field [51] are marked by empty circles.

The values of $(Q_{\gamma\gamma} - Q_{gg}) / Q_{\gamma\gamma}$ determined using re-orientation effect in Coulomb excitation of 2^+_g levels [13] are marked by a cross.

FIG. 5 Comparison of experimental and calculated values of $B(E2)$ probabilities for the interband gamma transitions in ^{158}Er . Ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ were calculated with values $Z_{\beta} = 0.030(16)$ ($\chi^2(1) = 30$) - empty circles, or $Z_{\beta} = 0.056(4)$, $\xi_{\beta\gamma} = -0.011(11)$ ($\chi^2(2') = 0.2$) - full circles and $Z_{\gamma} = 0.23(6)$ ($\chi^2(1) = 12$) - empty circles, or $Z_{\gamma} = 0.16(12)$, $Z_{\beta\gamma} = -0.07(9)$ ($\chi^2(2') = 14$) - full circles.

FIG. 6 Comparison of experimental and calculated values of $B(E2)$ probabilities for the interband gamma transitions in ^{160}Er . Ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ were calculated for transitions related with the gamma-band using the Alaga rules ($\chi^2(A) = 27$) - empty circles, or with value $Z_{\gamma} = 0.133(1)$ ($\chi^2(1) = 0.9$) - full circles. $B(E2)_{\text{calc}}$ for transitions de-exciting β -band were calculated using the Alaga rules ($\chi^2(A) = 7$) - empty circles, or with $Z_{\beta} = 0.029(5)$ ($\chi^2(1) = 0.5$) - full circles.

FIG. 7 Comparison of experimental and calculated values of $B(E2)$ probabilities for the interband gamma transitions in ^{162}Er . Ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ were calculated with parameters $Z_{\gamma} = 0.059(7)$, $Z_{\beta\gamma} = 0.0045(22)$ ($\chi^2(2') = 10$) and $Z_{\beta} = 0.007(11)$ ($\chi^2(1) = 16$).

Experimental values of I_{γ} were obtained in beta decay (empty circles) and in reactions $(p,4n\gamma)$, $(\alpha,2n\gamma)$ (full circles).

a) Determined value 2.5(7) for this transition is not shown in fig. 7.

FIG. 8 Comparison of experimental and calculated values of B(E2) probabilities for the transitions in ^{164}Er . Ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ were calculated with parameters: $Z_{\gamma}(2_{\gamma}) = 0.068(5)$, $Z_{\gamma}(0_g) = 0.068(8)$, $Z_{3\gamma} = 0.005(5)$ ($\chi^2(3) = 6$) and $Z_3 = 0.028(4)$ ($\chi^2(1) = 3$).

Experimental values of I_{γ} were obtained in beta decay (empty circles) in reactions $(\alpha,2n\gamma)$ or $(p,2n\gamma)$ (full circles) and in reactions $(n,n'\gamma)$ (cross).

FIG. 9 Comparison of experimental and calculated values of B(E2) probabilities for the transitions de-exciting levels of γ -band in ^{166}Er . Ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ were calculated with parameters $Z_{\gamma}(2_{\gamma}) = 0.0448(8)$, $Z_{\gamma}(0_g) = 0.0431(5)$ ($\chi^2(2) = 13$).

Experimental values of I_{γ} were obtained in beta decay (empty circles) in reactions $(\alpha,2n\gamma)$ (full circles).

a) Determined value 0.412(22) for this transition is not shown in fig. 9.

FIG. 10 Comparison of experimental and calculated values of B(E2) for transitions in ^{166}Er for the different number of parameters: $Z_{\gamma} = 0.0427(5)$, ($\chi^2(1) = 9.2$) -

full circles, $Z_{\gamma}(2_{\gamma}) = 0.0445(5)$, $Z_{\gamma}(0_{\text{g}}) = 0.0410$
 (20), $Z_{\beta\gamma} = -0.0009(10)$, ($\chi^2(3) = 4.8$) - cross,
 $Z_{\gamma}(2_{\gamma}) = 0.0445(5)$, $Z_{\gamma}(0_{\text{g}}) = 0.0452(10)$, $Z_{\beta\gamma} =$
 $= 0.00046(18)$, $Z_{\beta} = 0.06$, ($\chi^2(3) = 3.9$) - empty cir-
 cles. Experimental data used are described in tab.
 9 [24, 25, 26, 27]*.

FIG. 11 Comparison of experimental and calculated values of
 B(E2) for the transitions de-exciting levels of γ -
 and β -band in ^{168}Er . Values of $B(E2)_{\text{calc}}$ were calcu-
 lated with parameter $Z_{\gamma} = 0.0374(6)$ ($\chi^2(1) = 0.9$)
 and $Z_{\beta} = 0.025(5)$ ($\chi^2(1) = 9$).
 Experimental values of I_{γ} were obtained in beta de-
 cay (empty circles) and in reactions (n, γ) (full
 circles).

FIG. 12 Comparison of experimental and calculated values of
 B(E2) for the transition excited in beta decay in
 ^{170}Er . Values of $B(E2)_{\text{calc}}$ were calculated using
 the Alaga rules ($\chi^2(A) = 6$ for γ -band and $\chi^2(A) =$
 $= 22$ for β -band) (empty circles) and with parameters
 $Z_{\gamma} = 0.013(5)$, $Z_{\beta\gamma} = -0.017(7)$ ($\chi^2(2') = 1.3$) and
 $Z_{\beta} = 0.011(9)$ ($\chi^2(1) = 16$) (full circles).

FIG. 13 Comparison of experimental and calculated values of
 B(E2) for the transitions in ^{158}Gd . Values of $B(E2)_{\text{calc}}$
 were calculated with parameters: $Z_{\beta}(0_{\beta}) = 0.0094(21)$,
 $Z_{\beta}(0_{\text{g}}) = 0.013(3)$, $Z_{\beta\gamma} = -0.0018(7)$, ($\chi^2(3) = 4$)
 or $Z_{\gamma}(2_{\gamma}) = 0.020(3)$, $Z_{\gamma}(0_{\text{g}}) = 0.019(3)$, $Z_{\beta\gamma} =$
 $= -0.0115(17)$, $Z_{\beta} = 0.01$, ($\chi^2(3) = 0.8$).

Experimental values were obtained in beta decay (empty circles) and in reactions (n, γ) (full circles).

FIG. 14 Comparison of experimental and calculated values of $B(E2)$ for the transitions in ^{160}Dy . Values of $B(E2)_{\text{calc}}$ were calculated with parameters: $Z_{\beta} = -0.014(9)$ ($\chi^2(1) = 5$) or $Z_{\gamma} = 0.0438(20)$, $Z_{\beta\gamma} = -0.0035(14)$, ($\chi^2(2) = 0.04$).

FIG. 15 Comparison of experimental and calculated values of $B(E2)$ for the transitions in ^{164}Yb . Ratios $[B(E2)_{\text{calc}}/B(E2)_{\text{exp}}]^{1/2}$ were calculated with parameters: $Z_{\beta} = 0.023(6)$, ($\chi^2(1) = 17$) or $Z_{\gamma} = 0.122(15)$, ($\chi^2(1) = 13$).

FIG. 16 The physical constants of transitions ($Q_{gg}, Q_{\beta}, Q_{\gamma}$) and levels (matrix elements of the Coriolis interaction $h_{\beta g}, h_{rg}, h_{\beta\gamma}$). The values of Q_{gg} determined by I.A. Mitropolskij [40] using the superconducting model are marked by trianagl.

FIG. 17 Comparison of experimental and calculated values of $B(E2)$ probabilities for the intraband transitions de-exciting the rotational levels of the ground state. The values of $B(E2)_{\text{calc}}$ were calculated using the Alaga rules (empty circles) or determined assuming the mixing of wave functions of β -g and γ -g states (full circles).

FIG. 18 Values of parameter $(\alpha_A + \alpha_f)$ determined from analysis of B(E2) probabilities of intraband transitions in the ground band

FIG. 1

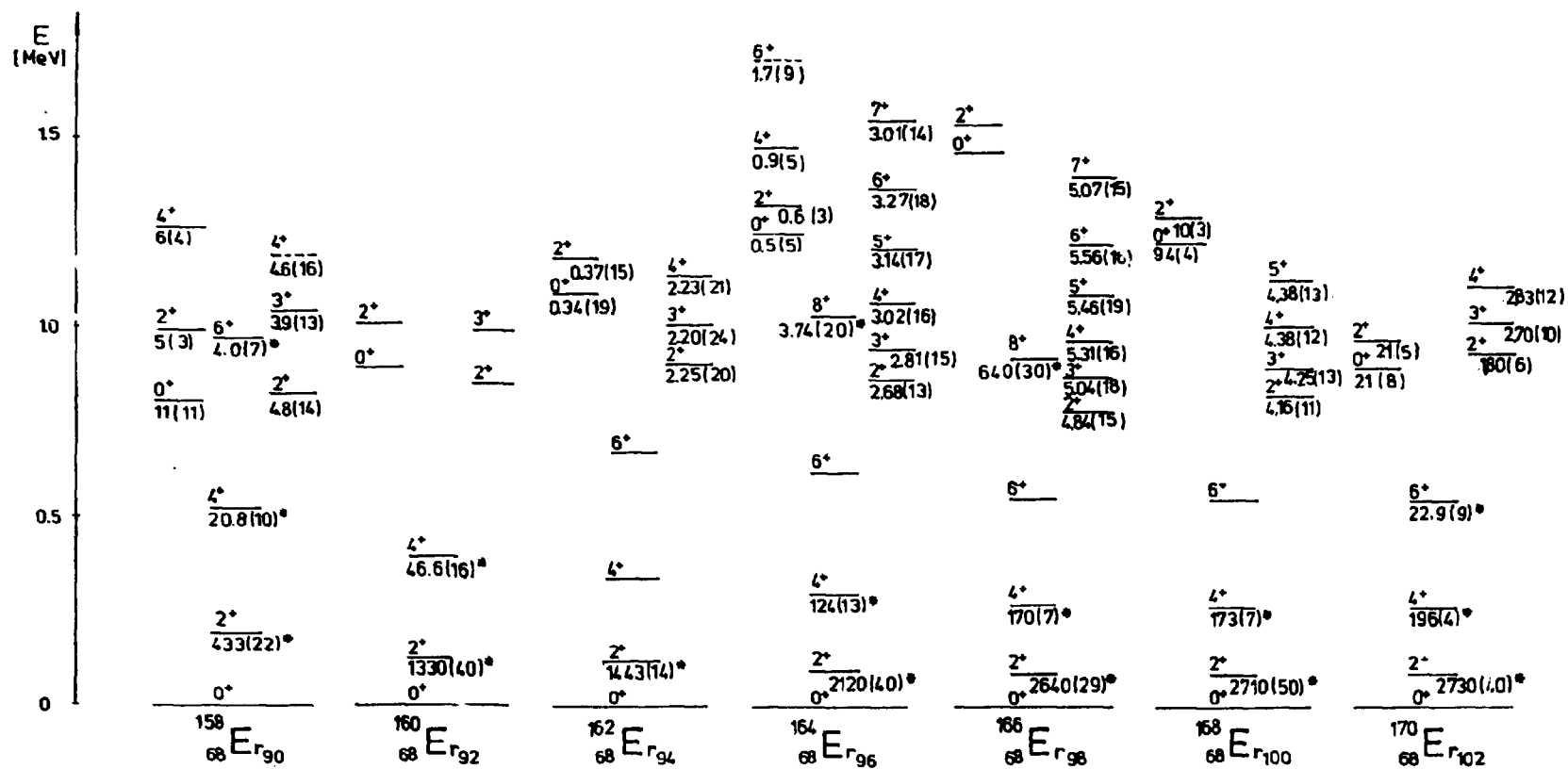


FIG. 2

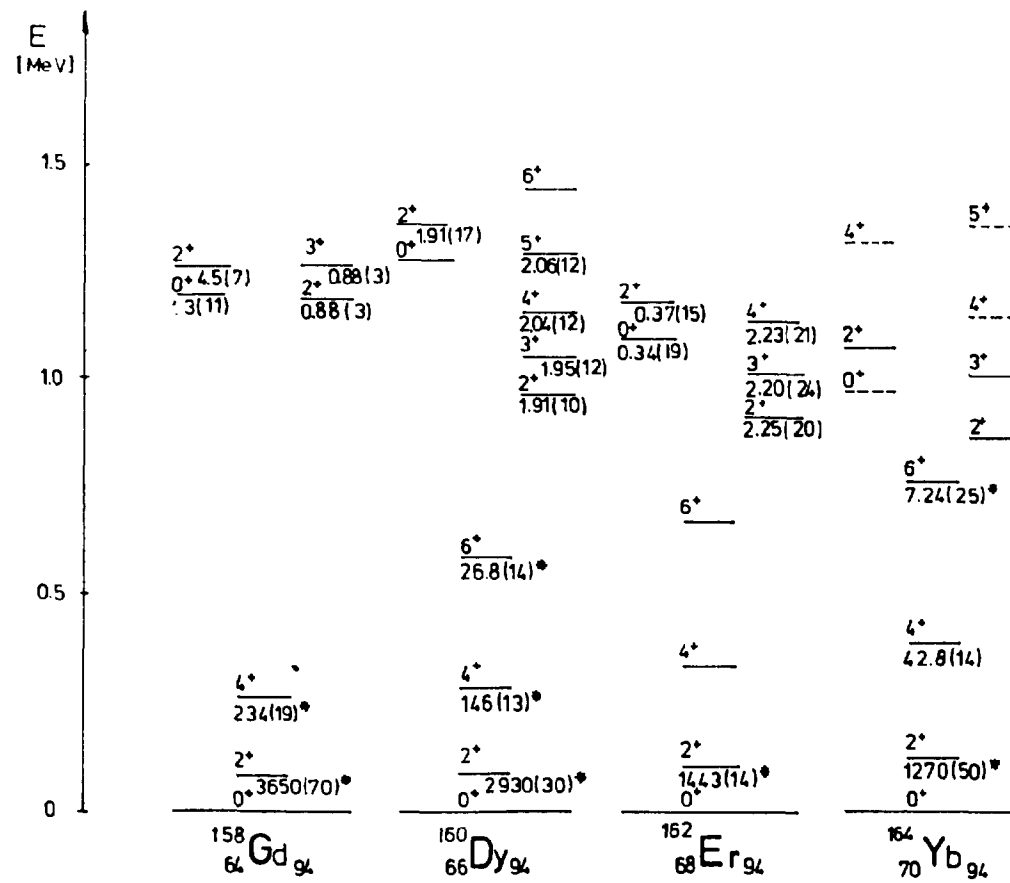


FIG. 3

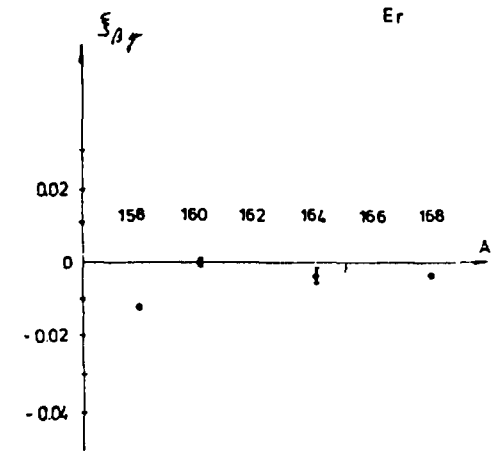
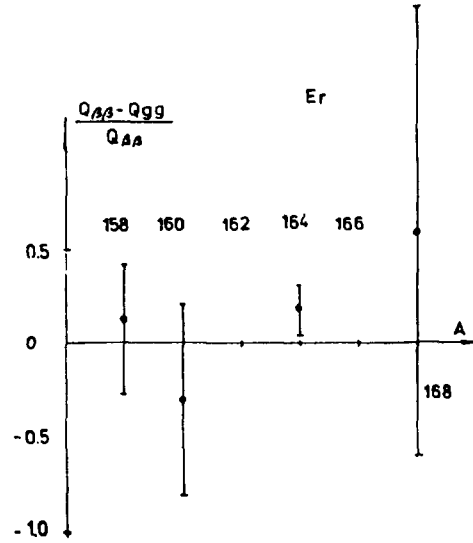
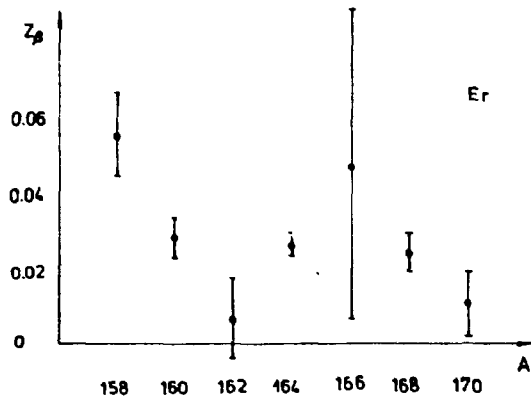
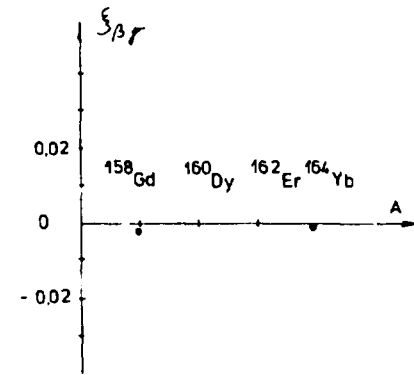
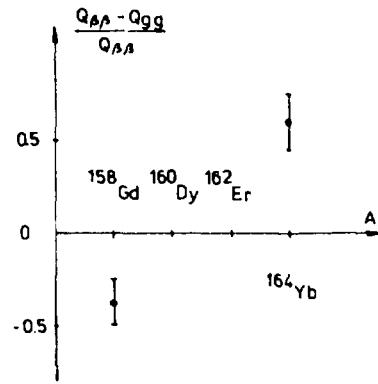
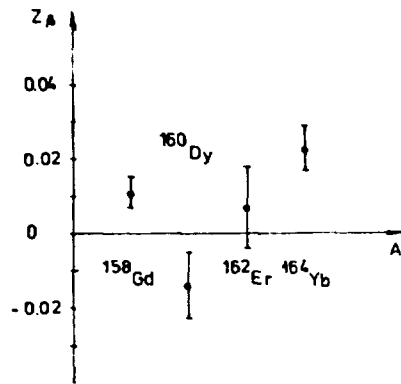


Fig. 4

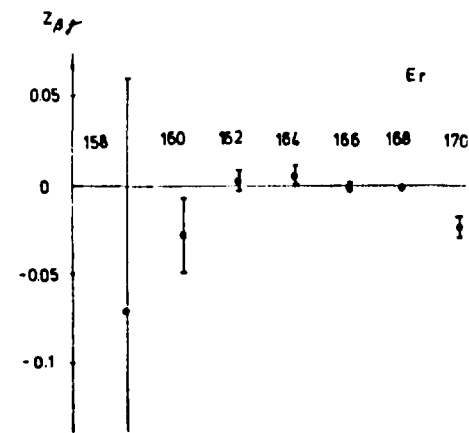
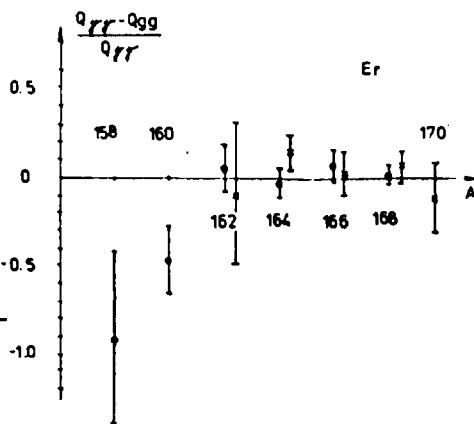
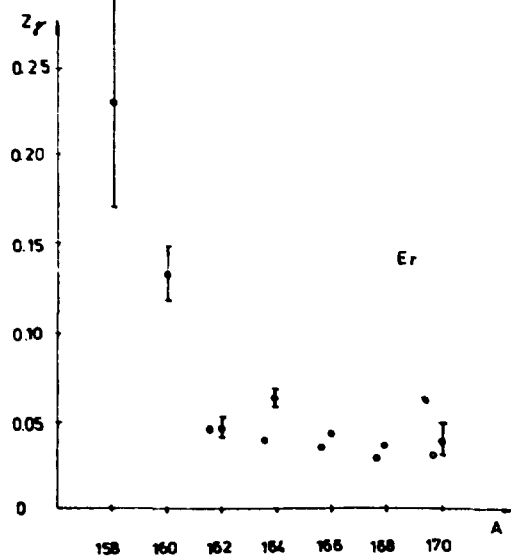
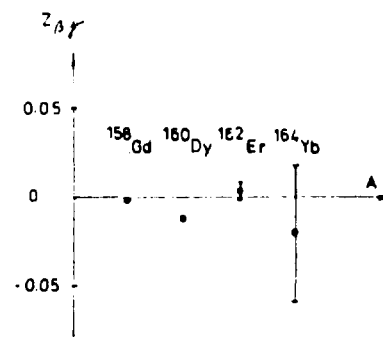
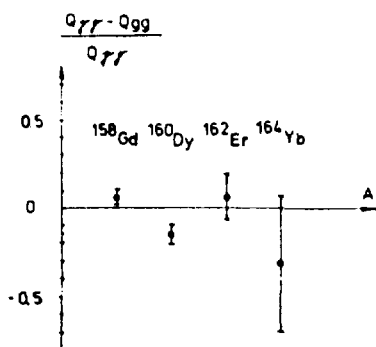
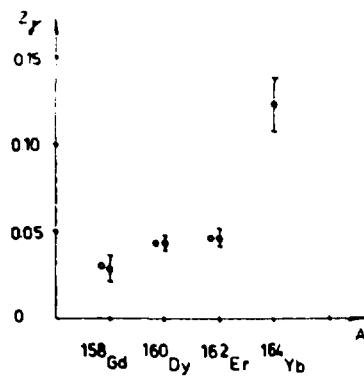


Fig. 5

$^{158}_{68}\text{Er}_{90}$

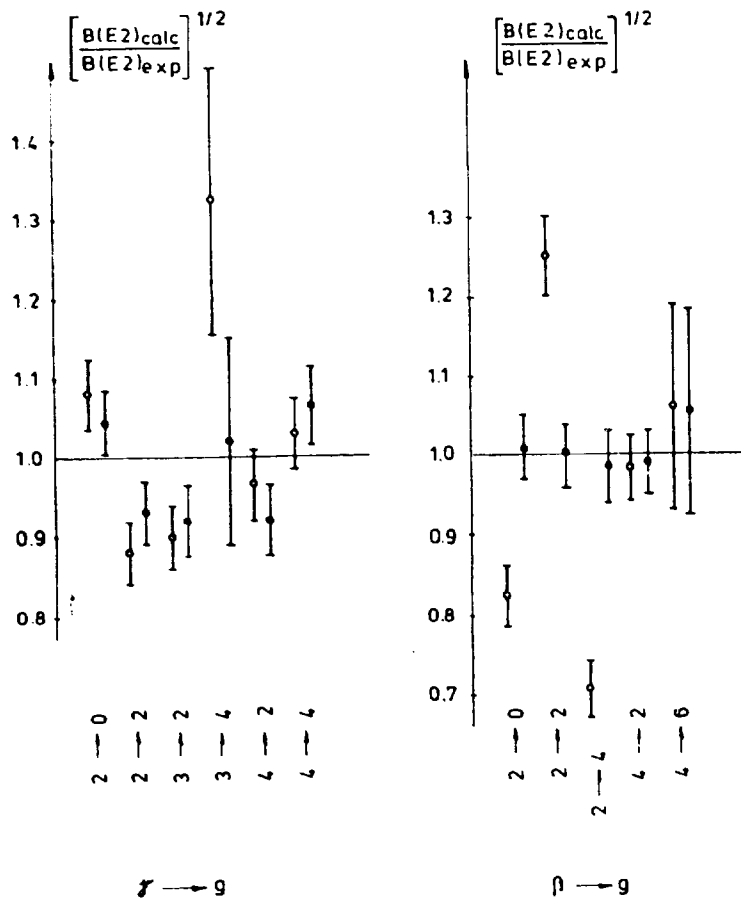


Fig. 6

$^{160}_{68}\text{Er}_{92}$

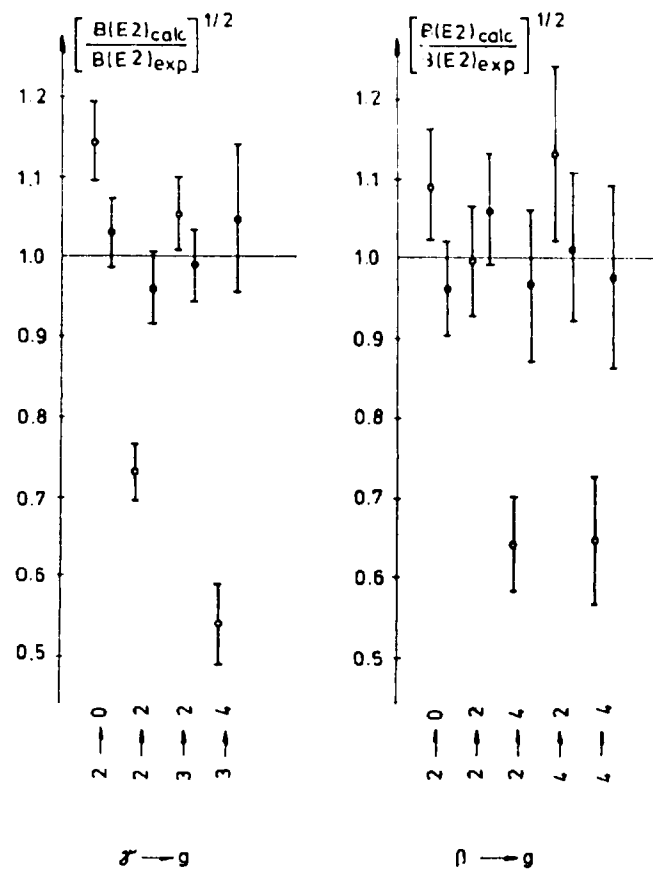
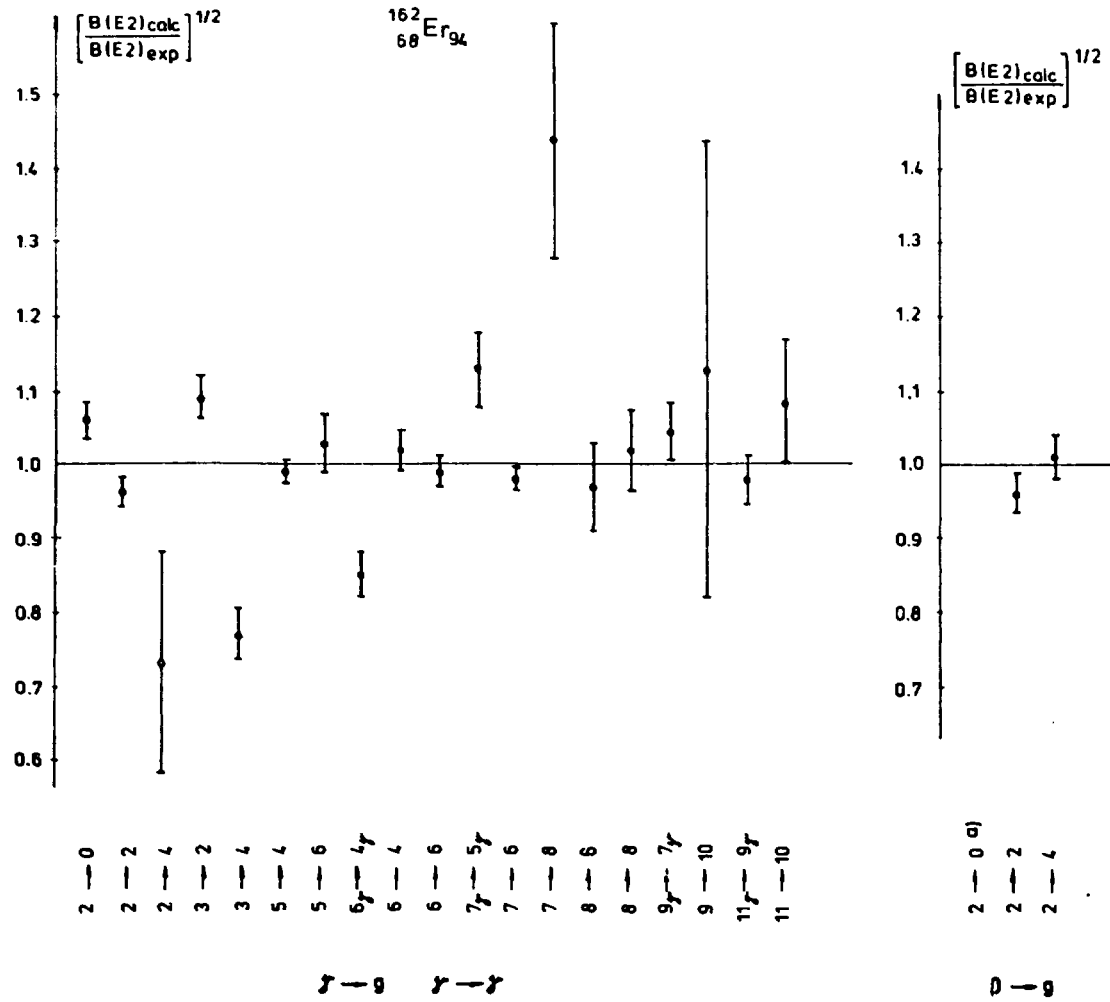
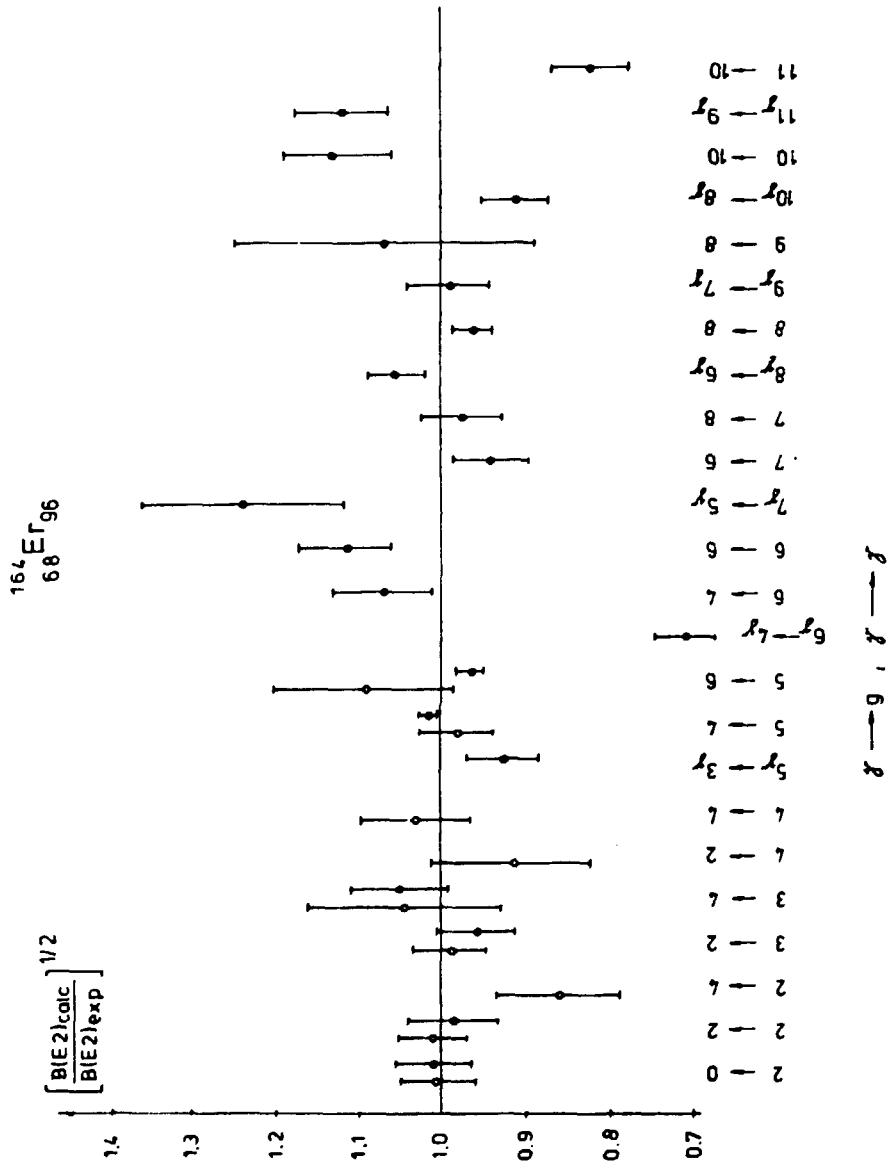
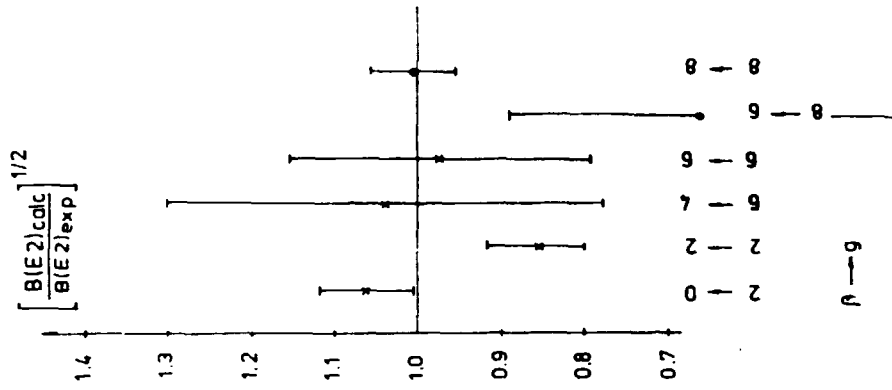


FIG. 7





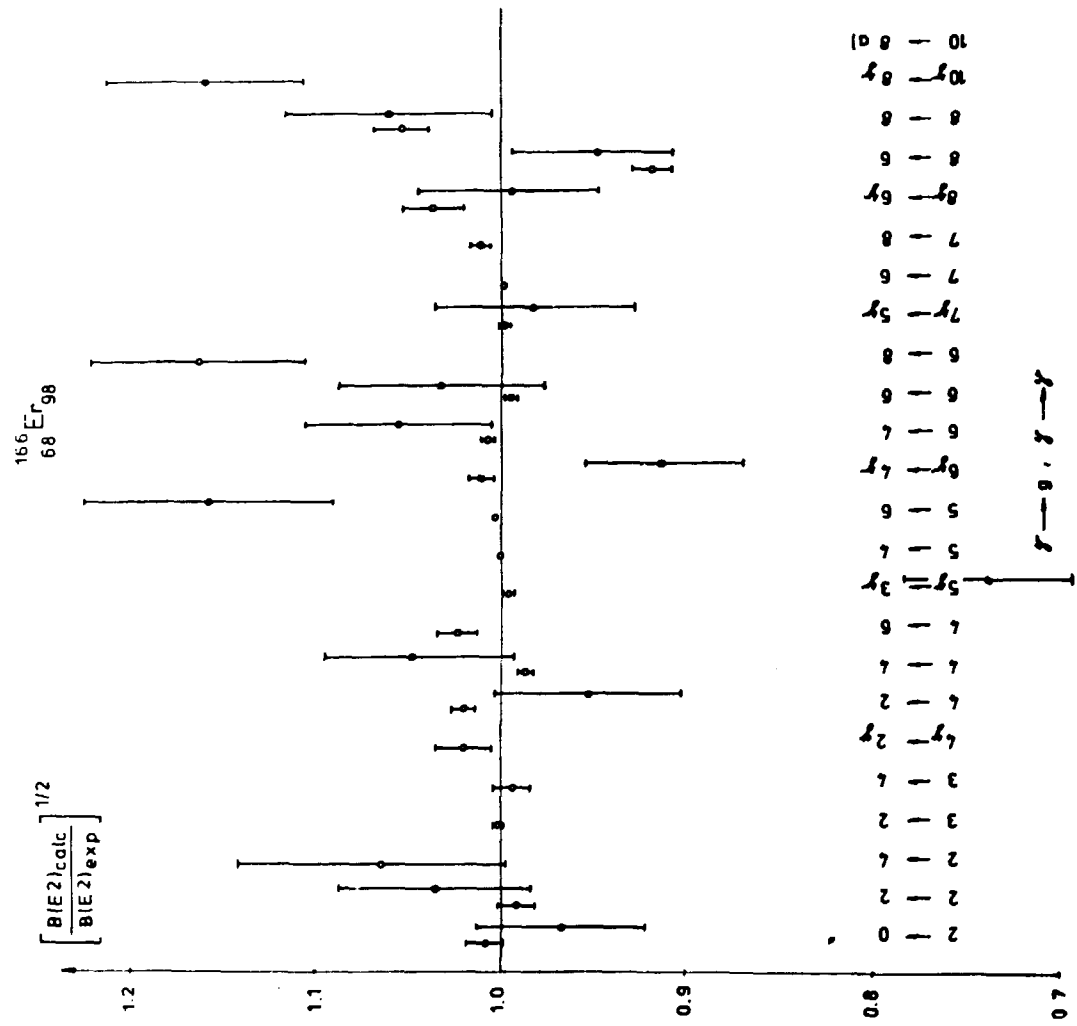


Fig. 4

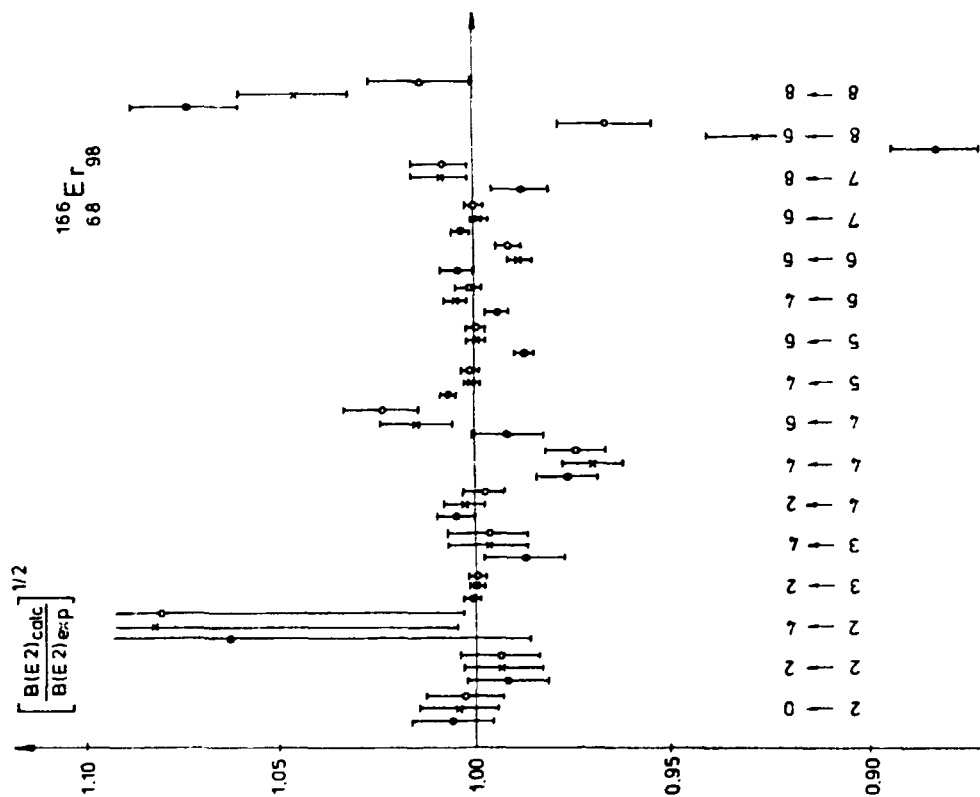
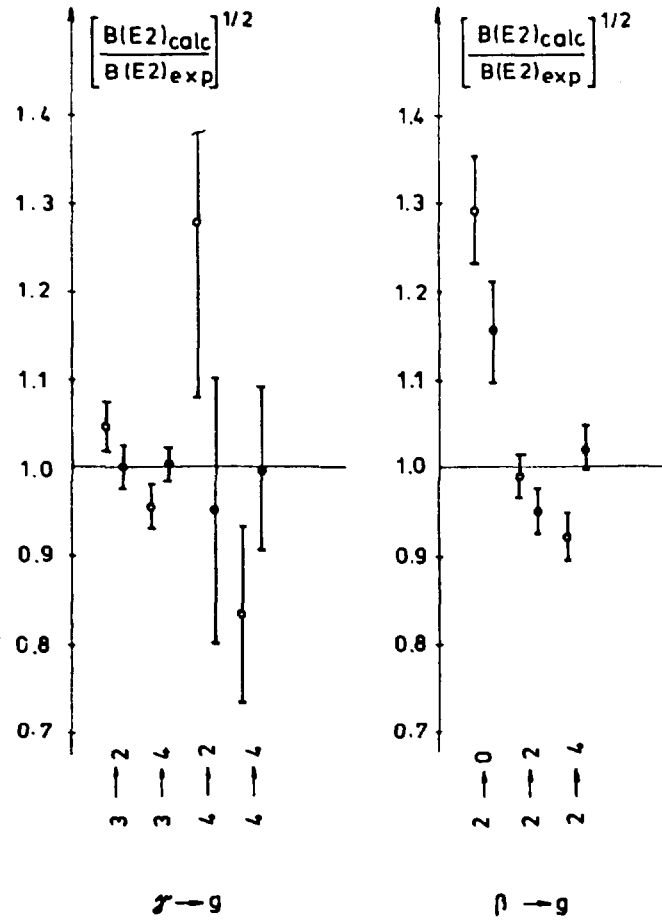


FIG. 10

FIG. 13

$^{170}_{68}\text{Er}_{102}$



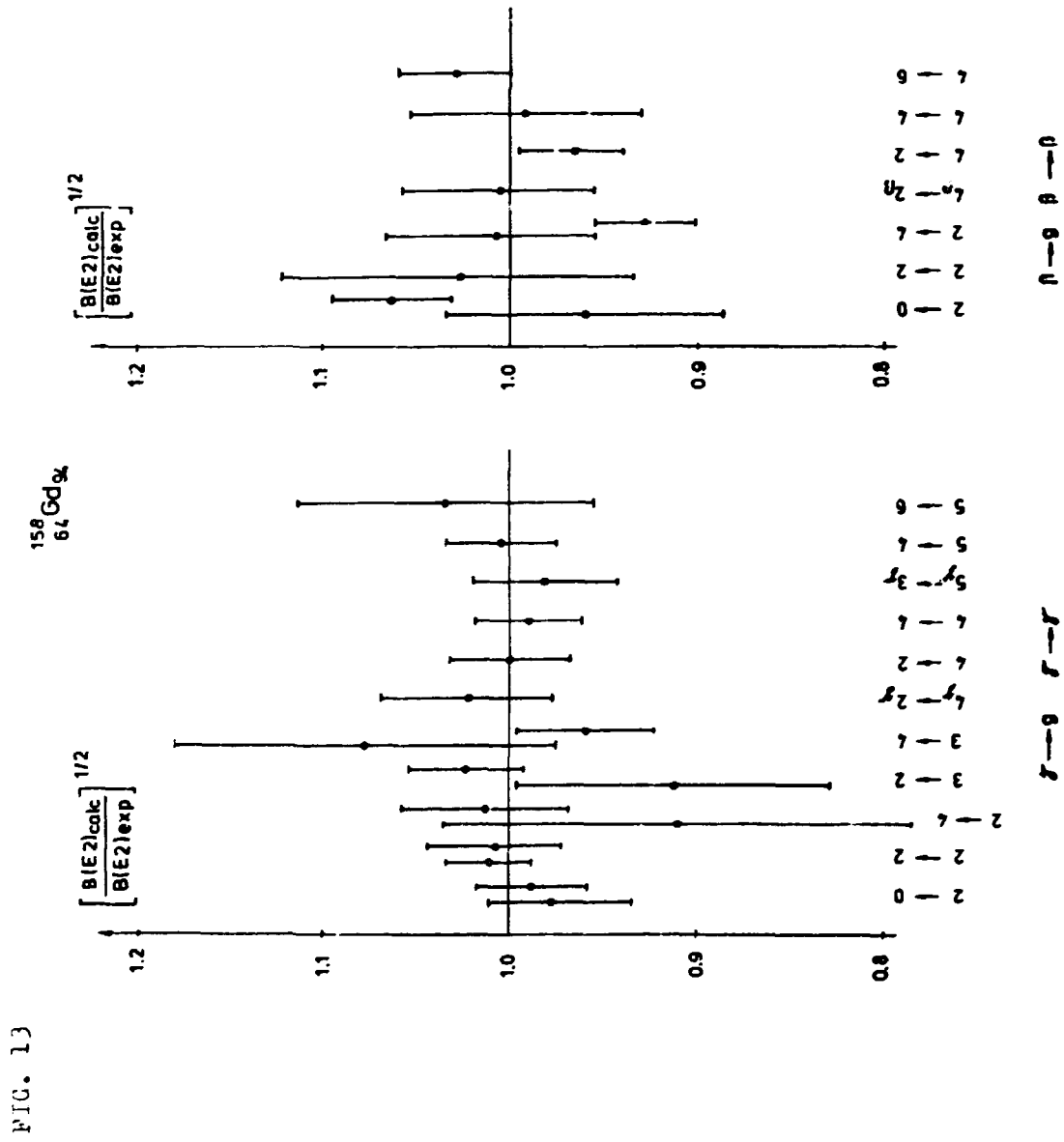


FIG. 14

¹⁶⁰Dy₉₄
56

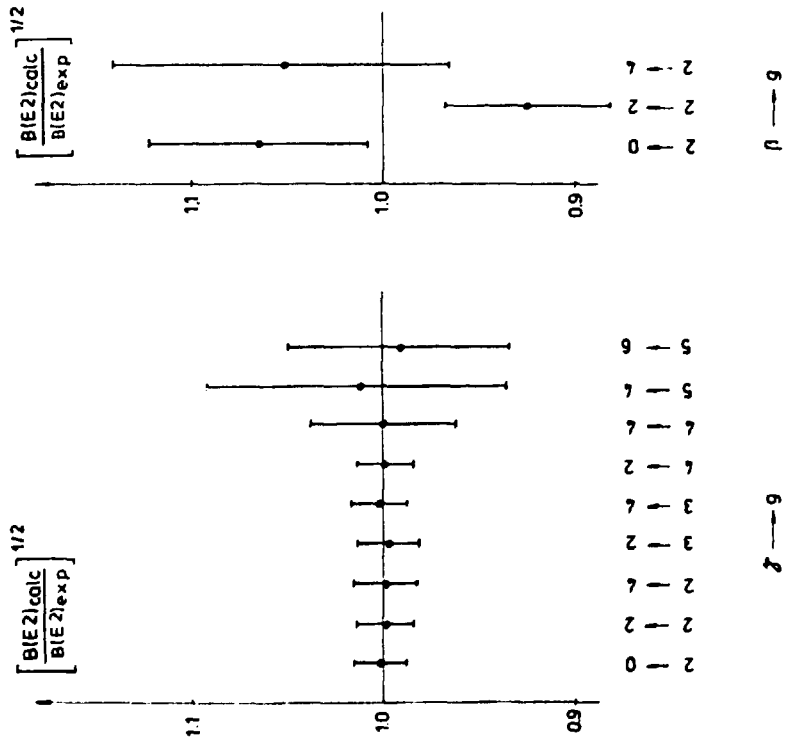


FIG. 15

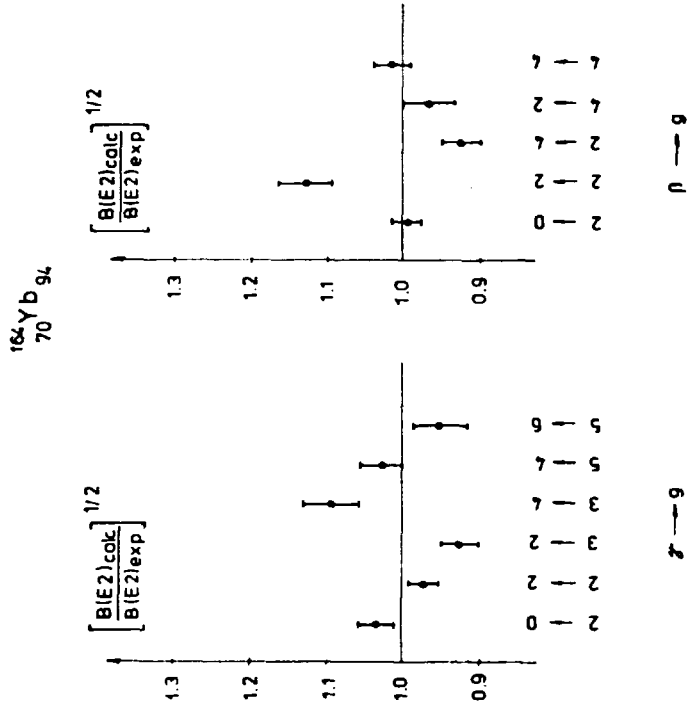
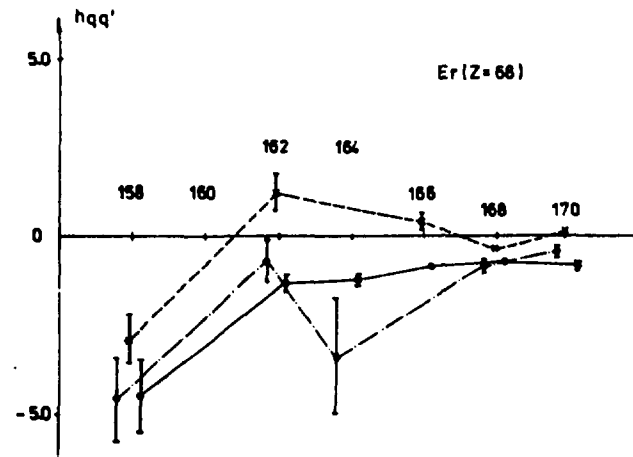
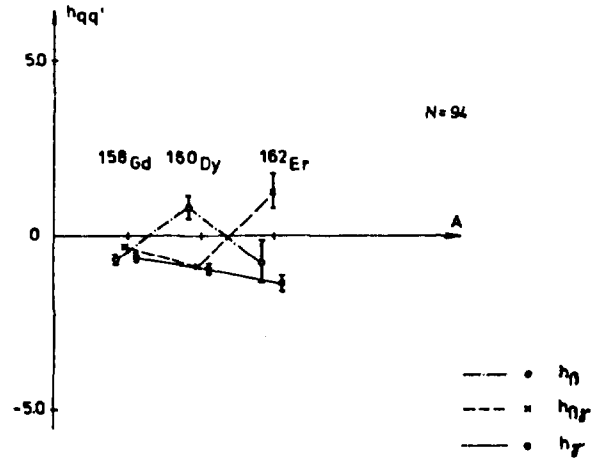
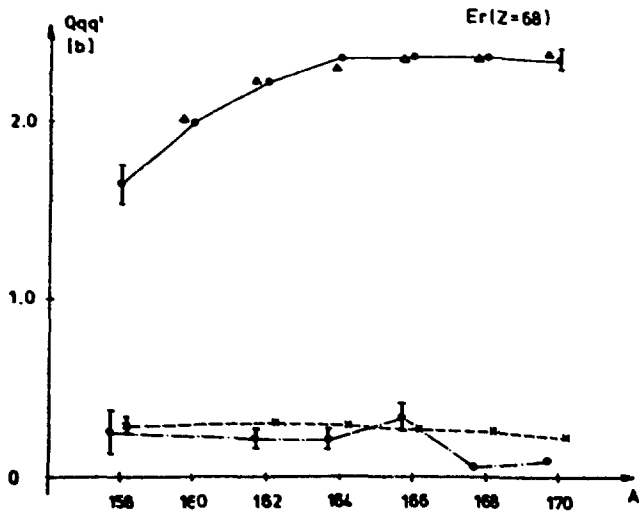
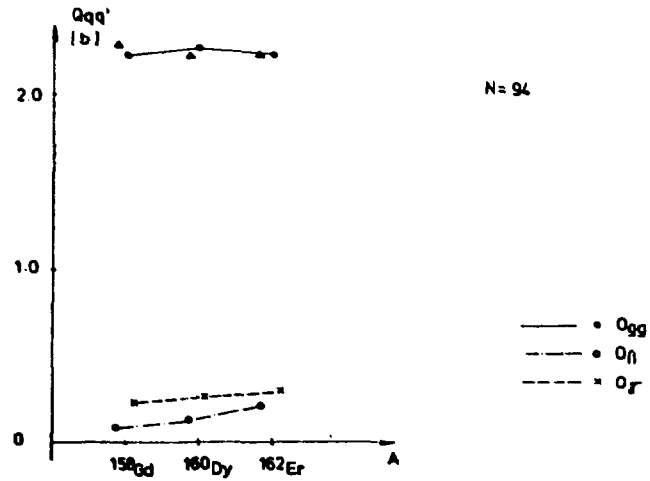
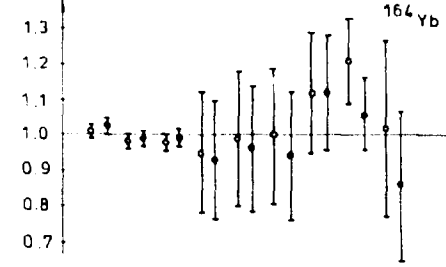
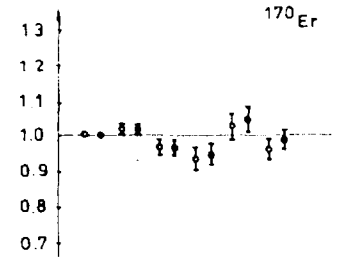
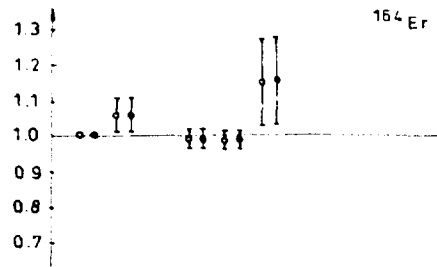
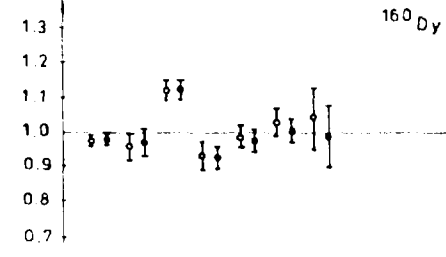
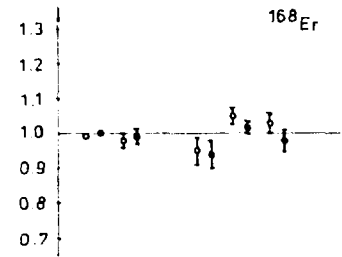
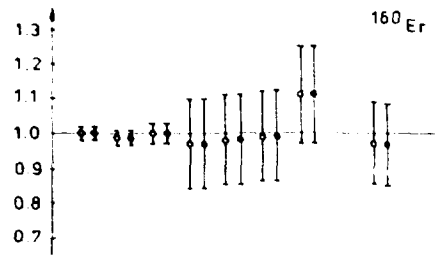
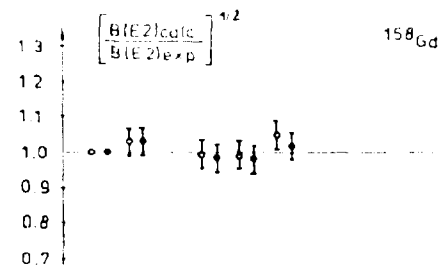
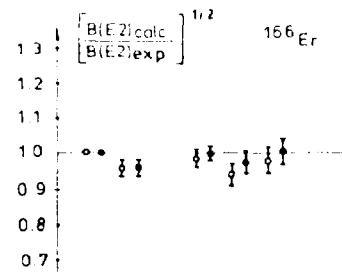
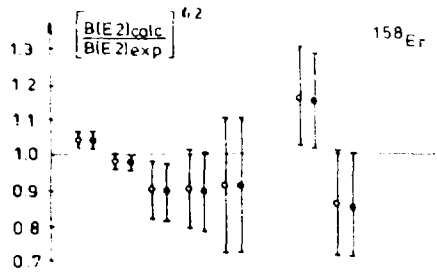


FIG. 16





2 → 0
4 → 2
6 → 4
8 → 6
10 → 8
12 → 10
14 → 12
16 → 14
18 → 16

J → 9

2 → 0
4 → 2
6 → 4
8 → 6
10 → 8
12 → 10

J → 9

2 → 0
4 → 2
6 → 4
8 → 6
10 → 8
12 → 10
14 → 12
16 → 14
18 → 16

J → 9

Fig. 10

