



REFERENCE

ERRATA

IC/89/392
 23 March 1990

**PROPAGATION OF ELECTROMAGNETIC RADIATION
 IN A RANDOM FIELD OF GRAVITATIONAL WAVES
 AND SPACE RADIO INTERFEROMETRY**

V.B. Braginsky, N.S. Kardashev, I.D. Novikov and A.G. Polnarev

Page 3 - Line 13

The last five words: ... *for the arbitrary ration of*
 should read: ... *for the large ratio of*

Page 4 - Line 1

The heading of Section 2: *Exact solution of*
 should read: *Solution of*

Page 4 - Line 2

The last five words: ... *with the gauge condition chosen*
 should read: ... *with the gauge condition for electromagnetic field chosen*

Page 6 - (Eq. 2.24)

Instead of: $(b^j e_j) = k^2 \sin^2 \theta (\epsilon_{\alpha\beta} p_{\perp}^{\alpha} p_{\perp}^{\beta}) + k\kappa(1 - \cos \theta) (\epsilon_{\alpha\beta} e^{\alpha} e^{\beta})$
 should read: $(b^j e_j) = k^2 \sin^2 \theta (\epsilon_{\alpha\beta} p_{\perp}^{\alpha} p_{\perp}^{\beta}) + 0(\frac{\kappa}{k})$

Page 6 - Lines 25 to 33

Replace from: *For that purpose ...* to the end of the page.

With:

One can verify that

$$b_i(k^i + \kappa^i) = 0, \quad (2.27)$$

then with (2.27) in view, the gauge condition (2.1) reduces to the following

$$b^n (F e^{-i\varphi_s})_{,n} = 0 \quad (2.28)$$

From (2.14) one can see that

$$b^n = - \epsilon_{im} k^i k^m [e^n + \tilde{e}^n \alpha (\frac{\kappa}{k})], \quad (2.29)$$

Page 7 - Lines 1 to 8

Delete from lines 1 to 8 up to: for brevity).

Replace with:

where \tilde{e}^j is a unit vector collinear to the vector

$$\tilde{b}^n = b^n - e^n (\epsilon_{im} k^i k^m) \quad (2.30)$$

So from (2.29) in the limiting case $\kappa \ll k$ one can reduce (2.28) to

$$e^n (F e^{-i\varphi_s})_{,n} = 0 \quad (2.31)$$

Having in mind that boundary conditions must be put on F at the surface $z = 0$ one may conclude the following. If the function F has the form

$$F = \Phi(z) e^{i\varphi_s}, \quad (2.32)$$

where $\Phi(z)$ is some function of z only, (see also Eq.2.40) then the function F does satisfy the equation (2.31).

Then text continues with:

We would like to find a solution to (2.16) in the form ...

Page 8 - Last line of Table

On the last line of the third column, replace: $\frac{(\tilde{e}\kappa)}{\kappa\sqrt{1-\mu^2}}$

with: $\frac{(\tilde{e}\kappa\Gamma^0)}{\kappa\sqrt{1-\mu^2}}$

Page 22 - Line 7

J.H. Taylor should read as Ref. 30 and not 3.



REFERENCE

IC/89/392

**INTERNATIONAL CENTRE FOR
THEORETICAL PHYSICS**

**PROPAGATION OF ELECTROMAGNETIC RADIATION
IN A RANDOM FIELD OF GRAVITATIONAL WAVES
AND SPACE RADIO INTERFEROMETRY**

V.B. Braginsky

N.S. Kardashev

I.D. Novikov

and

A.G. Polnarev

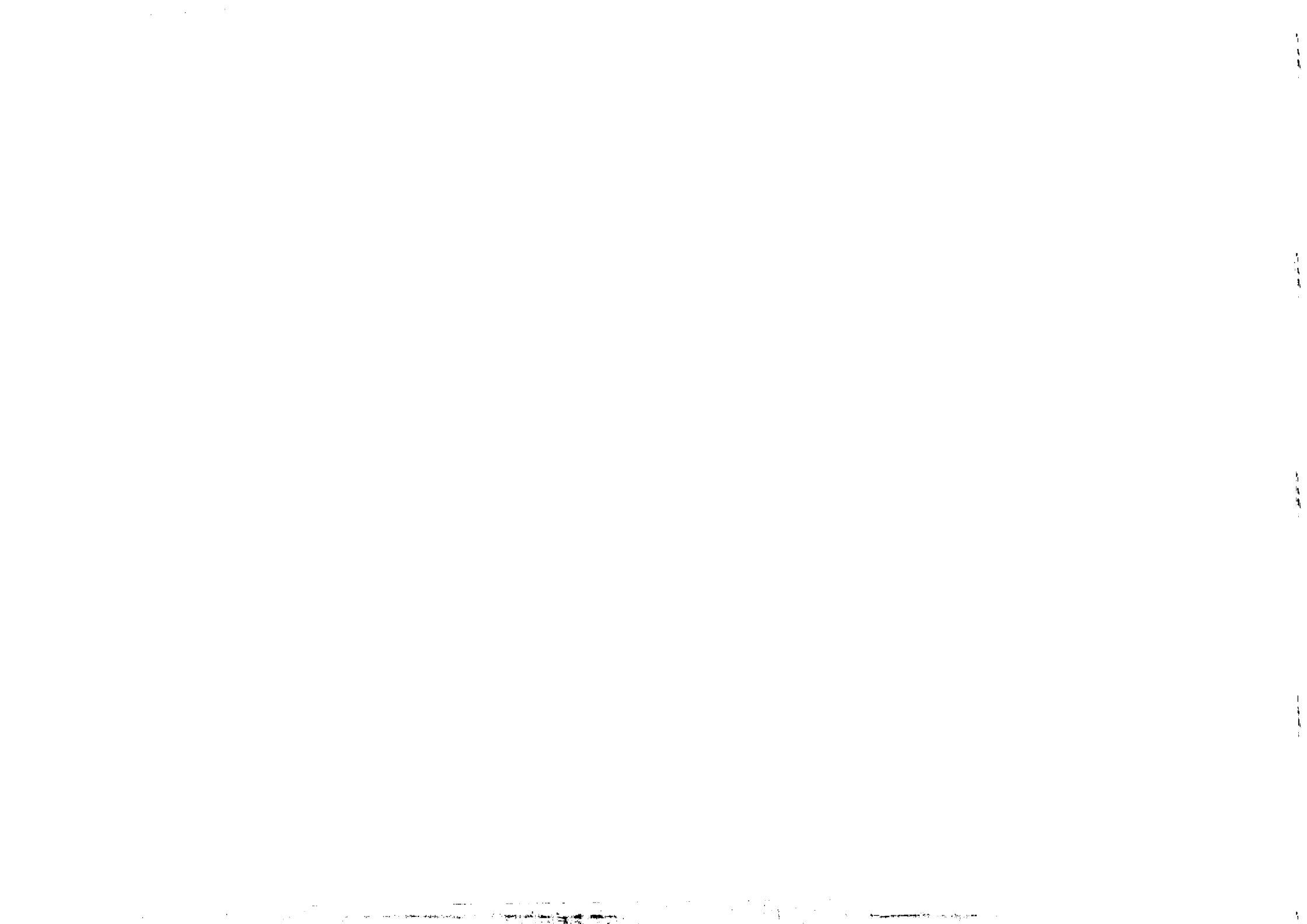


**INTERNATIONAL
ATOMIC ENERGY
AGENCY**



**UNITED NATIONS
EDUCATIONAL,
SCIENTIFIC
AND CULTURAL
ORGANIZATION**

1989 MIRAMARE - TRIESTE



International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

**PROPAGATION OF ELECTROMAGNETIC RADIATION
IN A RANDOM FIELD OF GRAVITATIONAL WAVES
AND SPACE RADIO INTERFEROMETRY ***

V.B. Braginsky

Moscow State University, Moscow, USSR,

N.S. Kardashev

Space Research Institute, Moscow, USSR,

I.D. Novikov **

International Centre for Theoretical Physics, Trieste, Italy

and

A.G. Polnarev

Space Research Institute, Moscow, USSR.

MIRAMARE - TRIESTE

December 1989

Propagation of an electromagnetic wave in the field of gravitational waves is considered. Attention is given to the principal difference between the electromagnetic wave propagation in the field of random gravitational waves and the electromagnetic wave propagation in a medium with a randomly-inhomogeneous refraction index. It is shown that in the case of the gravitation wave field the phase shift of an electromagnetic wave does not increase with distance. The capability of space radio interferometry to detect relic gravitational waves as well as gravitational wave bursts of non cosmological origin are analyzed.

* To be submitted for publication.

** Permanent address: Space Research Institute, Moscow, USSR.

1. INTRODUCTION

One of the main tasks of observational cosmology is to get information about possible existence in space of primordial gravitational waves (PGWs) generated at the very first instants of the Universe. Since PGWs are extremely weakly interacting with the matter they carry direct data about the birth of the Universe.

Much has been written about the problem of PGW detection (see e.g. [1-16]). Most informative – with respect of PGW detection – are the processes of their interaction with electromagnetic radiation. This paper is devoted to a thorough analysis of these processes and to assessing a possibility to detect PGWs with VLBI space radio interferometry.

The paper discusses how gravitational waves affect the propagation of electromagnetic radiation from remote astrophysical sources and from active generators in the Solar system.

Section 2 presents the exact solution of Maxwell equations in the field of the monochromatic flat gravitational wave for the first order of its amplitude and for the arbitrary ratio of electromagnetic-to-gravitational wave frequencies. To compare this problem with that of electromagnetic wave propagation in a medium whose refraction index differs from unity, an effective refraction index is introduced which corresponds to the gravitational wave.

The results derived in Section 2 are used in Section 3 to calculate the dispersion and the structure function of electromagnetic radiation phase shift in a random field of gravitational waves. The influence of the specific features of gravitational waves (their transversity, their propagation velocity equal to the velocity of light) on the propagation of the electromagnetic waves have been analyzed. The comparison is made with the task of electromagnetic wave propagation in randomly inhomogeneous media [17,18]. It is shown why for stochastic gravitational waves the distance to the source does not enter the solution of the problem.

Section 4 gives restrictions on the PGW spectrum derived from the future observations with the help of space radio interferometry.

Section 5 compares the capabilities of radio interferometry and pulsar timing [10, 19-30] to detect PGWs; PGW wavelength ranges are presented where this or that method appears to be most efficient.

Section 6 briefly discusses some capabilities of space radio interferometry in detecting individual gravitational-wave bursts of astrophysical (rather than cosmological) origin. A new effect called "phase memory" is described.

In closing (Section 7) the results obtained are summarized and discussed from the viewpoint of future space projects.

2. EXACT SOLUTION OF MAXWELL EQUATION IN A GRAVITATIONAL WAVE

For an arbitrary gravitational wave, with no sources and with the gauge condition chosen as

$$A^i_{;i} = 0, \quad (2.1)$$

where A^i is the 4-vector of the electromagnetic field potential, and ";" is the covariant derivative (Latin letter indices run the values 0, 1, 2, 3) Maxwell equations reduce to the following wave equations [31,32]

$$g^{ik} A^j_{;i;k} = 0 \quad i, k = 0, 1, 2, 3. \quad (2.2)$$

The metric tensor for a weak field is written as

$$g_{ik} = \eta_{ik} + h_{ik}, \quad g^{ik} = \eta^{ik} - h^{ik}, \quad (2.3)$$

where $\eta_{ik} = \text{diag}(1, -1, -1, -1)$. Further to the first order in h_{ik} the indices are raised and lowered using a non-perturbed metric tensor η_{ik} . Then it follows from (2.2) that

$$\square A^j + \tilde{L}_m^j A^m = 0, \quad (2.4)$$

where $\square = -\frac{\partial^2}{\partial t^2} + \Delta$, Δ is a conventional Laplacian, the operator \tilde{L}_m^j is determined as

$$\tilde{L}_m^j = -\delta_m^j h^{ik} \frac{\partial^2}{\partial x^i \partial x^k} + (h_m^{jk} + h_m^{kj} - h_m^{kl}) \frac{\partial}{\partial x^k}. \quad (2.5)$$

Eq.(2.5) was derived taking into account the gravitational wave gauge choice:

$$h_{,k}^k = h_k^k = 0 \quad (2.6)$$

and that h_{ik} satisfies the wave equation:

$$\square h_{ik} = 0. \quad (2.7)$$

Eq.(2.4) is easily generalized for the case when electromagnetic and gravitational wave interaction occurs in a homogeneous medium with the refraction index different from 1. To do that, it is sufficient that the light velocity in vacuum, c , in the \square be substituted with the phase velocity of the electromagnetic wave in a homogeneous medium, c_ϕ . Further on, the operator \square_ϕ is always either an operator with $c_\phi = c$ if vacuum is meant, or an operator with $c_\phi \neq c$ if it is the case with a homogeneous medium. Below we assume $c = 1$.

We consider the propagation of a flat monochromatic electromagnetic wave in the field of a flat monochromatic gravitational wave:

$$h_i^k = h \epsilon_i^k \exp(i\varphi_g) \quad (2.8)$$

Here h and φ_g are the amplitude and phase of the gravitational wave; ϵ_i^k is the unit tensor orthogonal to the zero wave vector of the gravitational wave:

$$\kappa_i = \frac{\partial \varphi_g}{\partial x^i}, \quad \kappa_i \kappa^i = 0, \quad \epsilon_i^i \kappa^i = 0. \quad (2.9)$$

The nonperturbed electromagnetic wave is written as:

$$A^{(0)j} = A_0 e^j \exp(i\varphi_e), \quad (2.10)$$

where A_0 and φ_e are the amplitude and phase of the wave, e^j is the unit space-like vector orthogonal to the electromagnetic wave vector k^i

$$k_i = \partial \varphi_e / \partial x^i, \quad e^i k_i = 0. \quad (2.11)$$

The non-perturbed vector-potential $A^{(0)j}$ meets the wave equation $\square_\phi A^{(0)j} = 0$.

In this case the operator \hat{L}_m^j reduces to a matrix. Note that:

$$\hat{L}_m^j A^m = -A_0 h b^j \exp[i(\varphi_g + \varphi_e)] \quad (2.12)$$

and Eq.(2.4) becomes

$$\square_\phi A^j = A_0 h b^j \exp[i(\varphi_e + \varphi_g)] \quad (2.13)$$

where

$$b^j = -(\epsilon_{mn} k^m k^n) e^j + (\epsilon_n^j k^n)(e_m \kappa^m) + (\epsilon_n^j e^n)(k_m \kappa^m) - \kappa^j (\epsilon_{mn} e^m k^n). \quad (2.14)$$

The solution of (2.13) may be presented as:

$$A^j = A_0 e^{i\varphi_e} (e^j + F b^j). \quad (2.15)$$

Here the scalar function F meets the equation

$$-2ik^n \frac{\partial F}{\partial x^n} + \square_\phi F = h e^{i\varphi_e}. \quad (2.16)$$

On the other hand, a perturbed electromagnetic wave may be written as follows:

$$A^j = A_0 (1 + \delta A/A) \exp[i(\varphi_e + \delta\varphi_e)] (e^j + \delta e^j) \approx A_0 e^{i\varphi_e} [(1 + i\delta\varphi_e + \delta A/A) e^j + \delta e^j] \quad (2.17)$$

where $\delta A/A$ is a fractional variation in the amplitude, $\delta\varphi_e$ is a phase shift, while δe^j represents the time delay the deflection and rotation of the polarization vector of the electromagnetic wave. The comparison of (2.17) with (2.15) yields the following expressions for the variation in the amplitude

$\delta A/A$, phase shift $\delta\varphi_e$, polarization vector rotation $\delta\epsilon_e$, the angular deflection $\delta\theta_e$ and delay δt_e of the wave.

$$\delta A/A = -(e_i b^i) \text{Re } F, \quad (2.18)$$

$$\delta\varphi_e = -(e_i b^i) \text{Im } F, \quad (2.19)$$

$$\delta\epsilon_e = -(a_i b^i) \text{Re } F, \quad (2.20)$$

$$\omega_e \delta t_e - \delta\theta_e = \frac{(b^j k_j)}{k} \text{Re } F. \quad (2.21)$$

Here a^j is the unit vector orthogonal to k^j and e^j , k is the 3-vector modulus, i.e. $k = |k^\alpha|$. Greek indices take on the values 1, 2, 3. It should be mentioned that the linear combination of the wave delay and deflection in (2.21) does not depend on how the reference system is chosen whereas each of the quantities δt_e and $\delta\theta_e$ themselves depends on that choice.

To calculate scalar products that enter (2.16) to (2.21) we present 4-vectors k^j and κ^j in the following form:

$$k^j = \delta_0^j k^0 + \delta_a^j (\cos\theta \cdot p_{||}^\alpha + \sin\theta \cdot p_\perp^\alpha) k, \quad (2.22)$$

$$\kappa^j = \delta_0^j \kappa^0 + \delta_a^j (\cos\theta \cdot \eta_{||}^\alpha + \sin\theta \cdot \eta_\perp^\alpha) \kappa. \quad (2.23)$$

Here θ is the angle the 3-vectors \vec{k} and $\vec{\kappa}$ form, κ is the modulus of the 3-vector $\vec{\kappa}$, $\kappa = |\kappa^\alpha|$; $p_{||}^\alpha$ is the unit vector aligned along κ^α , p_\perp^α is the unit vector perpendicular to κ^α and respectively $-\eta_{||}^\alpha$ is the unit vector aligned along k^α and η_\perp^α is the unit vector perpendicular to k^α . The above presentation with (2.22) and (2.23) permits the angular θ -dependence of (2.18) to (2.21) to be straightforwardly identified. As follows from (2.14)

$$(b^j e_j) = k^2 \sin^2\theta (\epsilon_{\alpha\beta} p_{||}^\alpha p_\perp^\beta) + k\kappa(1 - \cos\theta) (\epsilon_{\alpha\beta} e^\alpha e^\beta) \quad (2.24)$$

$$(b^j a_j) = -k\kappa \sin^2\theta (\epsilon_{\alpha\beta} p_\perp^\alpha a^\beta) (e_\alpha \eta_\perp^\alpha) + k\kappa(1 - \cos\theta) (\epsilon_{\alpha\beta} e^\alpha e^\beta) - k\kappa \sin^2\theta (\epsilon_{\alpha\beta} p_\perp^\alpha e^\beta) (a_\alpha \eta_\perp^\alpha) \quad (2.25)$$

$$(b^j k_j)/k = k\kappa \sin^3\theta (\epsilon_{\alpha\beta} p_\perp^\alpha p_\perp^\beta) (e_\alpha \eta_\perp^\alpha). \quad (2.26)$$

The Maxwell equations (2.2) are derived for the case when the gauge condition (2.1) is met, thus the solution (2.15) should satisfy (2.1). For that purpose the coordinate transformation is required:

$$x'^n = x^n + h A_k^n x^k, \quad (2.27)$$

here, with (2.15) in view, the gauge condition (2.1) reduces to the following condition on the transformation matrix A_k^n :

$$h A_k^n k_n e^k = i b^n F_{,n} - (k^n b_n) F, \quad (2.28)$$

the second condition on A_k^n results from the requirement that Eq.(2.13) be invariant against (2.27) transformation, since

$$\square_\phi = \square'_\phi - 2h \left[\left(\frac{1}{c^2} - 1 \right) A^{nl} \frac{\partial^2}{\partial x^{nl} \partial x^m} + A^{nm} \frac{\partial^2}{\partial x^n \partial x^m} \right] \quad (2.29)$$

the additional condition on A_m^n is as follows:

$$A_m^n k^n k_m + \left(\frac{1}{c_\phi^2} - 1 \right) A_0^m k^0 k_m = 0 \quad (2.30)$$

choosing the OZ along the 3-vector \vec{k} and the OX along the \vec{e} vector permits (2.29) and (2.30) to be rewritten as:

$$h(k_0 A_1^0 + k_3 A_1^3) = i b^n F_{,n} - (k_n b^n) F, \quad (2.31)$$

$$A_0^0 - \left(\frac{1}{c_\phi} + c_\phi \right) A_0^3 - A_3^3 = 0. \quad (2.32)$$

We now employ a new coordinate system (writing new variables however without primes for brevity). We would like to find a solution to (2.16) in the form which would make it possible to present the solution of the problem of electromagnetic wave propagation in the random field of gravitational waves in the form identical to the solution of the problem of electromagnetic wave propagation in the medium with random perturbations of the refraction index.

It will help us to reveal a cardinal difference between the solution patterns of the first and second problems and make important conclusions.

We first consider the problem of electromagnetic wave propagation in a medium with the perturbed refraction index. We assume that the refraction index n_0^* of the medium is homogeneous with minor perturbations added as a flat monochromatic wave

$$\delta n^* = |\delta n^*| e^{i\varphi^*}, \quad (2.33)$$

where $\varphi^* = v_\phi \omega - \vec{\ell} \vec{x}$, $|\delta n^*|$ is the perturbation amplitude, v_ϕ – is its phase velocity, ω – its frequency and $\vec{\ell}$ – is the wave vector of the perturbation.

The wave equation for an EM wave in the medium is written as [17, 18]

$$-n^{*2} \frac{\partial^2 A^j}{\partial t^2} + \Delta A^j = 0 \quad (2.34)$$

For the first order of $|\delta n^*|$ we get [17, 18]

$$\square_\phi A^j = 2 \delta n^* \frac{\partial^2 A^j}{\partial t^2}. \quad (2.35)$$

The solution of (2.35) may be written as:

$$A^j = A_{0j} e^{i\varphi^*} (e^j + F^* e^j) \quad (2.36)$$

and F^* in this case meets the following equation:

$$-2 i k^n \frac{\partial F^*}{\partial x^n} + \square_\phi F^* = 2 |\delta n^*| e^{i\varphi^*}, \quad (2.37)$$

which reduces to (2.16) when $2 |\delta n^*|$ is substituted with h . This permits the effective refraction index $n_{GW} = 1 + \delta n_{GW}$ to be introduced which corresponds to a gravitational wave where

$$\delta n_{GW} = \frac{h}{2 k^2} (b^j e_j) e^{i\varphi^*}. \quad (2.38)$$

In the case $\kappa \ll k$ and it is this case that is most interesting, it follows from (2.38)

$$\delta n_{GW} \simeq \frac{1}{2} h e^{i\varphi^*} \sin^2 \theta \cos 2\tilde{\varphi}, \quad (2.39)$$

where $\tilde{\varphi}$ is the polarization angle between the principal axis of the polarization tensor $\epsilon_{\alpha\beta}$ and the \vec{k} vector projection onto the plane perpendicular to the $\vec{\kappa}$ vector. That gravitational waves are transverse manifests itself in the fact that the refraction index depends on θ and becomes zero at $\theta = 0$ when \vec{k} and $\vec{\kappa}$ are parallel.

We will now try and get a solution for with Eqs.(2.16) and (2.37), each of which can be reduced to the other with the simple substitution of variables as has been shown above.

The solution is written as:

$$\vec{F} = \vec{h} \exp \left\{ i \vec{\kappa} \left[\vec{v}_\phi t - \sqrt{1 - \mu^2} (\cos \psi x + \sin \psi y) \right] + i k z \right\} f(z). \quad (2.40)$$

Here we use notations in the following sense:

Symbol or Designation	In the medium	In the gravitational wave
\vec{F}	\vec{F}^*	F
\vec{h}	$2 \delta n^* $	h
$\vec{\kappa}$	$\ell = \vec{\ell} $	$\kappa \equiv \vec{\kappa} = \kappa^0$
\vec{v}_ϕ	$v_\phi / c_\phi = v_\phi h_0^*$	$1 / c_\phi = h_0^*$
μ	$(\vec{k} \vec{\ell}) / k \ell$	$(\vec{k} \vec{\kappa}) / k \kappa = \cos \theta$
$\cos \psi$	$(\vec{e} \vec{\ell}) / e \sqrt{1 - \mu^2}$	$\frac{(\vec{e} \vec{\kappa})}{\kappa \sqrt{1 - \mu^2}}$

Substituting (2.40) into (2.16) or (2.37) yields the following condition for the function $f(z)$ to be found:

$$\frac{d^2 f}{dz^2} + \Omega_0^2 f = e^{-i\Omega z} \quad (2.41)$$

where

$$\Omega_0 = \sqrt{(k + \bar{\kappa} \bar{v}_\phi)^2 - \bar{\kappa}^2 (1 - \mu^2)} \quad (2.42)$$

and

$$\bar{\Omega} = k + \bar{\kappa} \mu. \quad (2.43)$$

The solution of Eq.(2.16) or Eq.(2.37), taking account of (2.40) and (2.41) and meeting the boundary conditions

$$\bar{F}|_{z=0} = \frac{\partial \bar{F}}{\partial z}|_{z=0} = 0, \quad (2.44)$$

becomes

$$\bar{F} = \frac{\bar{h} e^{i\psi}}{2\Omega_0} \left[\frac{1 - e^{i(\bar{\Omega} - \Omega_0)z}}{\Omega_0 - \bar{\Omega}} + \frac{1 - e^{i(\bar{\Omega} + \Omega_0)z}}{\Omega_0 + \bar{\Omega}} \right], \quad (2.45)$$

where $\bar{\varphi} = \varphi_g$ for gravitational waves or $\bar{\varphi} = \varphi^*$ for the inhomogeneous medium.

Generalizing the solution (2.45) for the case when \bar{h} is a slowly varying function of z gives

$$F = \frac{e^{i\psi}}{2\Omega_0} \left[\frac{\bar{h}(z) - \bar{h}(0)e^{i(\bar{\Omega} - \Omega_0)z}}{\Omega_0 - \bar{\Omega}} + \frac{\bar{h}(z) - \bar{h}(0)e^{i(\bar{\Omega} + \Omega_0)z}}{\Omega_0 + \bar{\Omega}} \right]. \quad (2.46)$$

The condition of slow \bar{h} variation is the following inequality

$$\left| \frac{\bar{h}'}{(\bar{\Omega} - \Omega_0)\bar{h}} \right| \ll 1. \quad (2.47)$$

In another limiting case when

$$\left| \frac{\bar{h}'}{(\bar{\Omega} - \Omega_0)\bar{h}} \right| \gg 1 \quad (2.48)$$

there is a resonance in the solutions of Eqs.(2.16) and (2.37). We denote the characteristic distance by z_* , which is

$$z_* = \min \{ z, |\bar{h}'/\bar{h}|_{z=0} \}. \quad (2.49)$$

Then the resonance condition can be rewritten as:

$$|\bar{\Omega} - \Omega_0| z_* \ll 1 \quad (2.50)$$

or, as follows from (2.42) and (2.43), as

$$\left| \mu - \bar{v}_\phi - \frac{\bar{\kappa}}{2k} (1 - \bar{v}_\phi^2) \right| \ll \frac{\bar{\Omega} + \Omega_0}{2k\bar{\kappa}z_*}. \quad (2.51)$$

In the limiting case $\bar{\kappa} \ll k$ we have $\bar{\Omega} \simeq k + \bar{\kappa} \bar{v}_\phi$, so that $\bar{\Omega} - \Omega_0 \simeq \bar{\kappa}(\mu - \bar{v}_\phi)$, $\bar{\Omega} + \Omega_0 \simeq 2k$, here the solution of (2.46) far from resonance and the resonance condition (2.50) are simplified and become, respectively:

$$\bar{F} \simeq \frac{e^{i\psi}}{2k\bar{\kappa}} \frac{\bar{h}(z) - \bar{h}(0)e^{i\bar{\kappa}(\mu - \bar{v}_\phi)z}}{\bar{v}_\phi - \mu} \quad (2.52)$$

$$|\mu - \bar{v}_\phi| \ll (\kappa z_*)^{-1} \quad (2.53)$$

When the resonance condition (2.50) is met function \bar{F} grows linearly with the distance the electromagnetic wave passes. This turns out to be extremely important when the problem of electromagnetic wave passage in a random field of perturbations (see Section 3) is solved

$$\bar{F} \simeq -\frac{ie^{i\psi}}{2k} \bar{h}(0) z_* \quad \text{at} \quad \bar{v}_\phi^2 - (\bar{\kappa} z_*)^{-1} < \mu < \bar{v}_\phi + (\bar{\kappa} z_*)^{-1}. \quad (2.54)$$

For further considerations it is convenient to introduce the following function:

$$\psi(x, z) = \begin{cases} \frac{e^{i\psi} - \bar{h}(z)/\bar{h}(0)}{iz_*} & \text{at } xz_* \gg 1 \\ z & \text{at } xz_* \ll 1 \end{cases} \quad (2.55)$$

Then for both limiting cases (2.47) and (2.48) the function can be written as:

$$\bar{F} = \frac{e^{i\psi}}{2k} \bar{h}(0) \psi[\bar{\kappa}(\mu - \bar{v}_\phi), z]. \quad (2.56)$$

Further let us consider the effect of relic gravitational waves on the propagation of electromagnetic radiation from distant sources with large red shifts Z_* . With due account taken of the adiabatic damping of gravitational waves caused by Universe expansion, the amplitude of the gravitational wave is a slowly varying function of the distance z the electromagnetic wave passes.

When the effect of the primordial gravitational waves on an electromagnetic wave is analyzed it is convenient to turn to conformal time η , $d\eta = dt/a$ where a is the scale factor and to employ a wave equation in the curved space time [32] which is the generalization of Eqs.(2.2):

$$g^{ik} A_{;i;k}^j + R_m^j A^m = 0, \quad (2.57)$$

where R_m^j is the Ricci tensor. The second term in (2.57) can be neglected if $k/H \ll 1$ and $\kappa/H \ll 1$, where H is the Hubble constant; (here and on the point means η -derivative). In fact for a zero approximation gravitational wave amplitude this term is of the order of $H^2 A_0$, whereas the term responsible for the adiabatic damping of electromagnetic waves for the zero-approximation gravitational wave amplitude is of the order of $H\kappa A_0 \gg H^2 A_0$. The first-order correction, in terms of the gravitational wave amplitude, to the Ricci tensor is zero

$$\delta R_m^j = 0. \quad (2.58)$$

It follows from (2.58) that

$$h_k^i = h_0 \frac{a_0}{a} \epsilon_k^i \quad (2.59)$$

Here h_0 and a_0 is the wave amplitude and the scale factor for today. As $\kappa/H \lesssim 1$ the behaviour of gravitational waves differs from the adiabatic law, see [15, 33]. Substituting h_k^i from (2.59) into (2.57) and

$$A^j = \nu^j / a^2 \quad (2.60)$$

and using instead of (2.3) the metric

$$g_{ik} = a^2 (\eta_{ik} + a_0/a \cdot h_0 \epsilon_{ik}) \quad (2.61)$$

$$g^{ik} = 1/a^2 (\eta^{ik} - \frac{a_0}{a} h_0 \epsilon^{ik}) \quad (2.62)$$

(indices in the case of the vector ν^j and the tensor ϵ_k^j are raised and lowered with the help of tensor η_k^j), we get instead of (2.4)

$$\square \nu^j + \frac{a_0}{a} \hat{L}_m^j \nu^m = 0 \quad (2.63)$$

where $\square = -\frac{\partial^2}{\partial \tau^2} + \Delta$ and \hat{L}_m^j is given by (2.5) with the quantity $h_0 \epsilon^{ik}$ instead of h^{ik} .

The solution of (2.63) is given by (2.56) if $h(0)$ is substituted with $h_0(1 + Z_s)$ and $h(z)/h(0)$ with $(1 + Z_s)^{-1}$

$$i\delta\hat{\varphi}_e \equiv i\delta\varphi_e + \delta A/A = (1 + Z_s)\delta n_{GW} k\psi[\kappa(\mu - \bar{v}_\phi), z] \quad (2.64)$$

3. PROPAGATION OF AN ELECTROMAGNETIC WAVE IN A RANDOM FIELD OF GRAVITATIONAL WAVES AND COMPARISON WITH THE CASE OF SPATIAL AND TEMPORAL FLUCTUATIONS OF THE MEDIUM REFRACTION INDEX

In a random field of gravitational waves $\langle \delta\hat{\varphi}_e \rangle = 0$. Here and below $\langle \rangle$ denotes the averaging over the random ensemble of gravitational waves. The correlation function for the non-polarized PGW noise is written as

$$\begin{aligned} \Gamma(t, \bar{\tau}, t', \bar{\tau}') &\equiv \langle \delta\hat{\varphi}_e(t, \bar{\tau}) \delta\hat{\varphi}_e^*(t', \bar{\tau}') \rangle = \frac{3}{4\pi} (1 + Z_s)^2 H^2 k^2 \int_{a_s}^{\infty} \frac{d\kappa}{\kappa^3} \varphi(\kappa) \cdot \\ &\cdot \exp[i\kappa(\eta - \eta' - z - z')] \int_{-1}^1 d\mu (1 - \mu^2)^2 \Psi[\kappa(\mu - \bar{v}_\phi), z] \cdot \\ &\cdot \Psi^*[\kappa(\mu - \bar{v}_\phi), z'] \int_0^{2\pi} d\psi \exp[-i\kappa\sqrt{1 - \mu^2} \Delta \rho \cos \psi] \end{aligned} \quad (3.1)$$

Here the asterisk stands for complex conjugacy.

$$\Delta \rho = \sqrt{(x - x')^2 + (y - y')^2}, \quad \kappa_* = 2\pi/\tau$$

where τ is the duration of observation; as the literature on pulsar timing caused by cosmological gravitational waves often mentions, (see [10]), this limitation on low frequencies is associated with the fact that the observation time is non-sufficient for a wave with $\kappa < \kappa_*$ to undergo at least one oscillation, thus their effect cannot be distinguished from the systematic variation of any task parameters. As a result, the low frequency contribution to mean-square quantities is zero. The quantity $\phi(\kappa)$ that enters (3.1) is the spectrum of gravitational wave background at the present moment, determined as:

$$\phi(\kappa) \frac{d\kappa}{\kappa} \approx \frac{1}{32\pi G} \left(\frac{3H^2}{8\pi G} \right)^{-1} 4\pi \kappa^4 \langle |h_0|^2 \rangle d\kappa \quad (3.2)$$

The density of gravitational wave background is expressed in terms of $\phi(x)$ as

$$\Omega_{GW} \equiv \epsilon_{GW}/\epsilon_{cr} = \int \phi(\kappa) \frac{d\kappa}{\kappa}$$

where $\epsilon_{cr} = 3H^2/8\pi G$ is the critical density at which the dimensionless parameter of the mean density $\Omega = 1$. It follows from (3.2) that

$$\langle |h_0|^2 \rangle = \frac{3}{\pi} \phi(\kappa)/\kappa^3 \quad (3.3)$$

The relation (3.3) is taken into account in (3.1). From (3.1) for the correlation function we get the squared dispersion of the phase fluctuation putting $\eta' = \eta$; $x = x'$; $y = y'$ and $z = z'$:

$$\sigma^2 = \Gamma(t, \bar{\tau}, t, \bar{\tau}) = \frac{3}{2} (1 + Z_s)^2 H^2 k^2 \int_{\kappa_*}^{\infty} \frac{d\kappa}{\kappa^3} \phi(\kappa) I_\mu \quad (3.4)$$

where

$$I_\mu = \int_{-1}^1 d\mu |\psi[\kappa(\mu - \bar{v}_\phi), z]|^2 q(\mu) \quad (3.5)$$

Here $q(\mu) = (1 - \mu^2)^2$.

Before the integral (3.5) is taken let us estimate the contribution from the angular range near the resonance (see (2.50)). As follows from (2.54) and (2.55)

$$\begin{aligned} dI_\mu^{(M)} &\simeq z^2 \int_{\bar{v}_\phi - (\kappa z)^{-1}}^{\bar{v}_\phi + (\kappa z)^{-1}} q(\mu) d\mu \simeq 2z^2 \left[q \Big|_{\mu=\bar{v}_\phi} (\kappa z_*)^{-1} + 1/6 \frac{d^2 q}{d\mu^2} \Big|_{\mu=\bar{v}_\phi} (\kappa z_*)^{-3} \right] = \\ &= 2q \Big|_{\mu=\bar{v}_\phi} (z_*/\kappa) + 1/3 \frac{d^2 q}{d\mu^2} \Big|_{\mu=\bar{v}_\phi} (\kappa^3 z_*)^{-1} \end{aligned} \quad (3.6)$$

It is evident from (3.6) why the factor of distance lacks in the dispersion relation for the case of random gravitational waves. As $\bar{v}_\phi = 1$ for the latter then the coefficient of z_* in (3.6) is proportional to $q|_{\mu=1}$, and $q|_{\mu=1} \simeq (1 - \mu^2)^2|_{\mu=1} = 0$ because of the gravitational wave transversity. Therefore, in this case, the contribution of resonance waves to the squared dispersion

is inversely rather than directly proportional to the distance the EM wave passes. This is contrary to the situation with randomly inhomogeneous media for which $\bar{v}_\phi \simeq 0$ and $q|_{\mu=0} \neq 0$.

The same conclusion can be made if a random field of gravitational waves is considered as a superposition of wave packets with finite cross-sections rather than flat waves. Presented in this way the above mentioned specific features of gravitational waves manifest themselves as inevitable spread of wave packets in the direction perpendicular to that of their propagation.

It is interesting to note that in the case when the phase velocity of the electromagnetic wave in plasma exceeds the light velocity ($\bar{v}_\phi \equiv c/c_\phi < 1$), the distance factor does exist, but the effect in this case is very weak. Indeed,

$$|\bar{v}_\phi - 1| \sim \omega_{pe}^2/k^2 \quad (3.7)$$

where $\omega_{pe}^2 = \frac{4\pi n e^2}{m_e}$, ω_{pe} is the plasma frequency, $n = n_0(1+Z_s)^3$ is the number density of electrons. Assuming that the matter density in the Universe is critical ($\Omega = 1$) we have

$$\omega_{pe}^2 \simeq 5 \cdot 10^3 (1+Z_s)^3 H_s^2$$

and

$$|\bar{v}_\phi - 1| \sim 10^{-10} \left(\frac{\lambda_e}{1 \text{ cm}} \right)^2 (1+Z_s)^2 \quad (3.8)$$

Obviously, $(q|_{\mu=\bar{v}_\phi})^{1/2} \simeq |\bar{v}_\phi - 1|$. Thus to achieve the input from the distance factor (we assume that the distance is equal to Hubble's distance) the length of the gravitational wave should satisfy the inequality

$$\lambda_g < R_H (\bar{v}_\phi - 1)^2 \approx 10^{-8} \left(\frac{\lambda_e}{1 \text{ cm}} \right)^4 (1+Z_s)^6 \text{ cm}$$

which shows that the effect is extremely weak.

In principle, random squared phase shift could be proportional to the distance from the source in the quadratic-in-amplitude approximation.

This effect is also extremely weak because of the smallness of gravitational wave amplitude.

Indeed, for the quadratic-in-amplitude phase increase that depends on the distance to exceed the phase increase linear in amplitude the condition $(1+Z_s) h\sqrt{R/\lambda_g} > 1$ should be fulfilled; in this case the dimensionless density of gravitational waves $\Omega_{GW}(\kappa)$ should meet the inequality (even for $R \simeq R_H$):

$$\Omega_{GW}(\kappa) > (T_H \kappa) (1+Z_s)^{-2}$$

Since for the wavelength range we are interested in the factor $(T_H \kappa) \gtrsim 10^{10}$, whereas the factor $(1+Z_s)^{-2} \gtrsim 4 \cdot 10^{-2}$, we then see that the quadratic effect is important only for inadmissibly high Ω_{GW} .

Thus below we assume $\bar{v}_\phi = 1$ and neglect the contribution from resonant flat waves. In that case the integral (3.5) is easily taken and the dispersion (3.4) is

$$\sigma^2 = 4 \left[(1+Z_s)^2 + 1 \right] H^2 k^2 \int_{\kappa_0}^{\infty} \frac{d\kappa}{\kappa^5} \phi(\kappa) \quad (3.9)$$

In practice, the value to be measured is the structure function determined as [17, 18]

$$\begin{aligned} D(t, \vec{r}, t', \vec{r}') &\equiv \langle |\delta\hat{\varphi}_e(t, \vec{r}) - \delta\hat{\varphi}_e(t', \vec{r}')|^2 \rangle = \\ &= \sigma^2(t, \vec{r}) + \sigma^2(t', \vec{r}') - 2 \text{Re} \Gamma(t, \vec{r}, t', \vec{r}') \end{aligned} \quad (3.10)$$

We now consider a space radio interferometer where the base is perpendicular to the direction toward the source while phase measurements are strictly synchronized, that is, $t = t'$, ($\eta = \eta'$).

With the help of (3.10) we calculate the transverse structure function.

$$D(\Delta \vec{\rho}) \equiv D(t, \vec{r}, t, \vec{r} + \Delta \vec{\rho}), \quad (3.11)$$

where $(\Delta \vec{\rho} \cdot \vec{r}) = 0$. We get from (3.3), (3.6), (3.10) and (3.11):

$$\begin{aligned} D(\Delta \vec{\rho}) &= \frac{3}{2\pi} (1+Z_s)^2 H^2 \kappa^2 \int_{\kappa_0}^{\infty} \frac{d\kappa}{\kappa^5} \phi(\kappa) \int_{-1}^1 d\mu (1+\mu)^2 \left[1 - \frac{2 \cos(\kappa z(1-\mu))}{1+Z_s} \right. \\ &\quad \left. + \frac{1}{(1+Z_s)^2} \right] I_\psi, \end{aligned} \quad (3.12)$$

where

$$I_\psi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[1 - \cos(\kappa \Delta \rho \sqrt{1-\mu^2} \cos \psi) \right] d\psi \quad (3.13)$$

The integral ψ reduces to the zero Bessel function $\mathcal{I}_0(\alpha)$ [34]

$$I_\psi \simeq 2\pi \mathcal{I}_0(\alpha) \quad (3.14)$$

where $\alpha = \kappa \Delta \rho \sqrt{1-\mu^2} = 2\pi(L/\lambda_g) \sqrt{1-\mu^2}$ here L - the base length of the interferometer. In asymptotical relations for long waves when $\alpha \ll 1$

$$I_\psi \simeq \frac{\pi}{2} \kappa^2 L^2 (1-\mu^2), \quad (3.15)$$

then in the short-wave limit when $\alpha \gg 1$ we get

$$I_\psi \simeq 2\pi \quad (3.16)$$

In the asymptotical relations (3.15) and (3.16) the integral over μ is easily taken, the result being:

$$D(\Delta \vec{\rho}) \equiv D(L) \simeq \frac{6}{5} k^2 L^2 H^2 \left[(1+Z_s)^2 + 1 \right] \int_{\kappa_0}^{\infty} d\kappa \frac{\phi(\kappa)}{\kappa^3} G(\kappa), \quad (3.17)$$

where

$$G(\kappa) \approx \begin{cases} 1, & \kappa L \ll 1 \\ \frac{2\pi}{3\kappa^2 L^2}, & \kappa L \gg 1 \end{cases} \quad (3.18)$$

The next section will deal with the numerical estimates of the structure function under various assumptions about the spectrum of cosmological gravitational waves.

4. STRUCTURE FUNCTION OF PHASE FLUCTUATIONS; LIMITATIONS ON THE DENSITY OF COSMOLOGICAL GRAVITATIONAL WAVES

Comparison of the structure function (3.17) with the future radiointerferometry data makes it possible to derive limitations on the energy density of primordial gravitational waves at an arbitrary frequency κ over a special range $\Delta\kappa \simeq \kappa$ and to do it independently of the assumptions about the spectrum of cosmological gravitational waves. And indeed it is evident from (3.5) and (3.17) that the contribution to the structure function from waves with $\Delta\kappa \simeq \kappa$ is equal to

$$D(\kappa, L) \simeq \frac{k^2 H^2}{\kappa^2} [(1 + Z_s)^2 + 1] \Omega_{GW}(\kappa) \begin{cases} L^2, & 2\pi/\tau < \kappa \ll L^{-1} \\ 8\kappa^{-2}, & \kappa \gg L^{-1} \end{cases} \quad (4.1)$$

If the experiment provides a certain sensitivity in determining the structure function for phase shift (we denote it as δ) then – even if we have not got positive results in primordial gravitational waves measurements – we can, with the help of (4.1), obtain the following limitations on $\Omega_{GW}(\kappa)$

$$\Omega_{GW}(\kappa) < \frac{\delta^2 \kappa^2}{k^2 H^2 [(1 + Z_s)^2 + 1]} \begin{cases} L^2, & 2\pi/\tau < \kappa \ll L^{-1} \\ \kappa^2/8, & \kappa \gg L^{-1} \end{cases} \quad (4.2)$$

A maximally severe limitation on $\Omega_{GW}(\kappa)$ is reached for wavelengths with the period of the order of observation time and is written as:

$$\begin{aligned} \Omega_{GW} &\simeq 2\delta^2 \left(\frac{T_H}{\tau}\right)^2 \left(\frac{\lambda_e}{L}\right)^2 [(1 - Z_s)^2 + 1]^{-1} \simeq \\ &\simeq 2 \cdot 10^{-6} \delta^2 \left(\frac{\tau}{1 \text{ year}}\right)^{-2} \left(\frac{\lambda_e}{1 \text{ cm}}\right)^2 \left(\frac{L}{1 \text{ A.U.}}\right)^{-2} \left[\frac{(1 + Z_s)^2 + 1}{2}\right]^{-1} \end{aligned} \quad (4.3)$$

where $T_H \cdot \frac{2}{3} H^{-1} \approx 20$ billion years – is the Hubble time.

For an arbitrarily-long gravitational wave, electromagnetic wave phase radiointerferometry yields the following limitations on $\Omega_{GW}(\kappa)$

$$\begin{aligned} \Omega_{GW}(\kappa) &< \left(\frac{\delta}{3 \cdot 10^{-2}}\right)^2 \left(\frac{\lambda_e}{1 \text{ cm}}\right)^2 \left[\frac{(1 + Z - s)^2 + 1}{2}\right]^{-1} \left(\frac{\lambda_g}{1 \text{ A.U.}}\right)^{-2} \\ &\begin{cases} \left(\frac{L}{1 \text{ A.U.}}\right)^2, & \lambda_g \gtrsim 3L \\ \left(\frac{\lambda_g}{1 \text{ A.U.}}\right)^2, & \lambda_g \lesssim 3L \end{cases} \end{aligned} \quad (4.4)$$

(see Fig.1 where limitations on Ω_{GW} are given as a function of wavelength, and Fig.2 with the same as a function of baselines).

The above described limitations on Ω_{GW} may be valid for some cosmological models predicting a specific type of gravitational wave spectrum. Thus models based on inflation and

phase transitions in the early Universe, within the wavelength range we are interested in, predict a flat spectrum of PGWs. Here [35–40]

$$\phi(\kappa) \simeq \Omega_\gamma (t_{pe}/t_{inf})^2 \quad (4.5)$$

where Ω_γ is the dimensionless density of relic radiation t_{inf} is the time from the beginning of Universe expansion till the beginning of its inflation. This time is $t_{inf} \simeq t_{pe}$ in quantum-gravitational inflation models and $t_{inf} \simeq (10^3 \div 10^4)t_{pe}$ in the models of inflation due to phase transitions at the Grand-Unification energy. To measure such a RGW spectrum with a space radio interferometer the accuracy of phase determination should be of the order of

$$\delta \leq 5 \frac{t_{pe}}{t_{inf}} \left[\frac{(1 + Z_s)^2 + 1}{2}\right]^{1/2} \left(\frac{\tau}{1 \text{ year}}\right) \left(\frac{\lambda_e}{1 \text{ cm}}\right)^{-1} \left(\frac{L}{1 \text{ A.U.}}\right) \quad (4.6)$$

when $Z_s \simeq 3$ and $t_{pe}/t_{inf} \simeq 2 \cdot 10^{-4}$ the required δ is

$$\delta \simeq 3 \cdot 10^{-3} \left(\frac{\tau}{1 \text{ year}}\right) \left(\frac{\lambda_e}{1 \text{ cm}}\right)^{-1} \left(\frac{L}{1 \text{ A.U.}}\right) \quad (4.7)$$

which seems quite realistic.

Models with ring cosmological strings predict the following for the range of interest: $\Omega_{GW} \simeq 10^{-6}$, [41–47] therefore in this case the accuracy with which the phase should be determined is written as:

$$\delta \simeq 3 \cdot 10^{-2} \left(\frac{\tau}{1 \text{ year}}\right) \left(\frac{\lambda_e}{1 \text{ cm}}\right)^{-1} \left(\frac{L}{1 \text{ A.U.}}\right) \quad (4.8)$$

In fact real-world requirements on δ should be an order of magnitude more stringent for the measurements to ensure the observed effect.

5. SPACE RADIOINTERFEROMETRY AND PULSAR TIMING: COMPARISON OF THEIR CAPABILITIES FOR ESTIMATING LIMITATIONS ON RELIC GRAVITATIONAL WAVES

This section had to be included in the paper since the data on pulsar timing may currently yield the most stringent limitations on relic gravitational waves in the wavelength range of the order 1 to 10 light years (as to the 10 to 10³ Mpc range the most stringent limitations can be obtained from the isotropy of relic (background) radiation, see e.g. [36, 48–54]).

The aim of the comparison made below is to determine the range of wave lengths within which space radiointerferometry may compete with, or even be better, as to its capabilities, than pulsar timing. Fairly rough estimates will do for such an analysis.

In pulsar timing the accuracy with which wave amplitudes h_0 are measured depends on the following quantities: Δt is the inevitable systems error in determining the time of arrival of radio pulses and τ_{PT} is the observation time. The quantity Δt is now determined by the accuracy of the Solar system [28] and for RSR 1937 + 21, is of the order of tenths of a microsecond. The quantity τ_{PT} is of the order of 10 years. Therefore, the value measured is of the order of:

$$h_0 \sim \Delta t / \tau. \quad (5.1)$$

For the length of the gravitational wave λ_g this imposes the following limitation on Ω_{GW}

$$\Omega_{GW} \tau \lesssim \left(\frac{R_H}{\lambda_g} \right)^2 \left(\frac{\Delta t}{\tau} \right)^2 \quad \text{for } \lambda_g \ll \tau_{PT}. \quad (5.2)$$

The accuracy of h_0 measurements in terms of phase shift in the radiointerferometer (see Sections 2 and 3) is:

$$h_0 \simeq \frac{\delta}{1 + Z_s} \frac{\lambda_e}{L} \quad \text{for } L < \lambda_g \ll \tau_{SI} \quad (5.3)$$

the respective limitations on Ω_{GW} are

$$\Omega_{GW} \tau \lesssim \left(\frac{R_H}{\lambda_g} \right)^2 \left(\frac{\delta}{1 + Z_s} \right)^2 \left(\frac{\lambda_e}{L} \right)^2. \quad (5.4)$$

Comparison of (5.1) with (5.3) or (5.2) with (5.4) shows that $\Omega_{GW} \tau \lesssim \Omega_{GW} \tau < 1$ when

$$\delta < (1 + Z_s) \frac{\Delta t \cdot L}{\tau \lambda_e} \simeq 2 \cdot 10^{-2} \left(\frac{1 + Z_s}{4} \right) \left(\frac{\Delta t}{0.1 \mu s} \right) \left(\frac{\tau_{PT}}{10 \text{ years}} \right)^{-1} \left(\frac{\lambda_e}{1 \text{ cm}} \right)^{-1} \left(\frac{L}{1 \text{ A.U.}} \right). \quad (5.5)$$

Thus, at fairly realistic δ the space radiointerferometry may compete with pulsar timing. It is essential to emphasize here that if 1 light year to 10 light years is the wavelength range within which the most stringent limitations can be derived for pulsar timing, the respective wavelength range for space radiointerferometer is $\lambda_g \lesssim 1$ light year. In other words, not only could space interferometry be competitive with pulsar timing, it can also be complementary to the latter for the other wavelength range.

Another factor in favour of radiointerferometry is the one associated with adiabatic damping of relic gravitational waves. For distant sources with large red-shifts the phase shift is determined by more intense relic gravitational waves near the source.

6. ON A POSSIBILITY TO DETECT GRAVITATIONAL WAVE BACKGROUND AND BURSTS WITH A SPACE RADIOINTERFEROMETER. "MEMORY OF PHASE" EFFECT

If satellites of RADIOASTRO type are used to record low-frequency gravitational radiation, with microwave interferometers to measure small variations in distance [55], it is possible

to achieve a sensitivity similar to the optimistic estimates for relic gravitational wave background. We now estimate the requirements which must be imposed on such satellites and interferometers. Let the expected value of metric amplitude fluctuations be $\langle h_0^2 \rangle^{1/2} = 1 \cdot 10^{-18}$ in the frequency range $\Delta f_{GW} \simeq f_{GW} \simeq 10^{-1} \text{ Hz}^*$. Then the requirement to the compensation level for non-gravitational accelerations of satellites should be relatively milder. These accelerations should not be higher than

$$2 \pi^2 f_{GW}^2 L \langle h_0^2 \rangle^{1/2} \simeq 3 \cdot 10^{-10} \frac{\text{cm}}{\text{c}^2} \left(\frac{f_{GW}}{10^{-3} \text{ Hz}} \right)^2 \cdot \left(\frac{L}{1 \text{ A.U.}} \right) \cdot \left(\langle h_0^2 \rangle^{1/2} / 10^{-18} \right). \quad (6.1)$$

If three satellites are used and two radiointerferometers between two pairs with a common mcw-self-oscillator then the requirement to the relative stability of the self-oscillator frequency $\Delta \omega_0 / \omega_0$ would not be too stringent either:

$$\frac{\Delta \omega_0}{\omega_0} \lesssim \frac{1}{2} \beta^{-1} \langle h_0^2 \rangle^{1/2} \simeq 10^{-14} \left(\frac{\beta}{10^{-4}} \right)^{-1} \left(\frac{\langle h_0^2 \rangle^{1/2}}{10^{-18}} \right) \quad (6.2)$$

where β is the fractional difference of two distances between two pairs of satellites.

The recorded phase-shift value is relatively large

$$\Delta \varphi_e \simeq \frac{\pi}{\lambda_e} L \langle h_0^2 \rangle^{1/2} \simeq 1 \cdot 10^{-4} \text{ rad} \left(\frac{\lambda_e}{1 \text{ cm}} \right)^{-1} \left(\frac{L}{1 \text{ A.U.}} \right) \left(\frac{\langle h_0^2 \rangle^{1/2}}{10^{-18}} \right). \quad (6.3)$$

To compensate for phase fluctuations caused by interplanetary plasma it is necessary that each radiointerferometer should have two microwave frequencies. Requirements to the dynamic range of the phase measuring device are not too severe: about 5 orders of magnitude since for $\lambda_e \simeq 1 \text{ cm}$ and $L = 1.5 \cdot 10^{13} \text{ cm}$ the additional phase shift caused by plasma is of the order of 10 rad [56].

The signal-to-noise ratio is the most serious obstacle if we are to achieve a sensitivity at the level $\Delta \varphi_e \simeq 1 \cdot 10^{-4} \text{ rad}$. If W is the microwave power of the wave that returned to the emitting antenna after its retranslation by one of the satellites, then the standard quantum limit of fluctuations, $\Delta \varphi_{SQL}$, is as known, equal to [57]:

$$\Delta \varphi_{SQL} \simeq \sqrt{\frac{\hbar \omega_0 f_{GW}}{W}} \quad (6.4)$$

For the condition $\Delta \varphi_e \simeq \Delta \varphi_{SQL} \simeq 10^{-4} \text{ rad}$ to be fulfilled at $L \simeq 1.5 \cdot 10^{13} \text{ cm}$ and the transmitter power 10^8 erg/s , antennas aboard the satellites should be of the order of 10^3 cm in diameter and the power gain should be 200 dB. The latter requirement can be appreciably reduced, though in this

* According to [15], $\langle h_0^2 \rangle^{1/2}$ is $\frac{10^{-21}}{f_{GW}} \div \frac{10^{-20}}{f_{GW}}$ if $\Delta f_{GW} \sim f_{GW}$, while $\langle h_0^2 \rangle^{1/2} \simeq (3 \div 10) \cdot 10^{-20}$, according to [38] at $f_{GW} \simeq 10^{-4} \text{ Hz}$.

case two high-stability self-oscillators would be needed for two satellites with $\frac{\Delta\omega}{\omega_0} \approx (h_0^2)^{1/2} \approx 1 \cdot 10^{-10}$. There are indications that this level of frequency stability is possible [58]. Note that the accumulation of data about phase fluctuation correlation during a long period of time in two branches of the radio interferometer will reduce the detection threshold.

Possible use of space radiointerferometry for gravitational wave detection is not restricted only to cosmological background. Space interferometers can also be used to detect individual bursts generated by such astrophysical processes as supernova explosions, by two gravitating bodies passing each other, etc. It is essential that when a single burst, without memory ($h(-\infty) = h(+\infty) = 0$) (see [59-62] for bursts with memory), is passing through an electromagnetic wave moving towards us from the source, the wave gains an additional phase shift:

$$\Delta\varphi_e \approx \frac{\omega_e}{2} \int h_{zz} dz \approx \frac{1}{2} h_{zz} \left(\frac{\lambda_g}{\lambda_e} \right) \quad (6.5)$$

If the emission and retranslation of an electromagnetic wave can be provided for a long time, the above mentioned phase shift will be memorized forever, with which the quantity $\Delta\varphi_e$ could be measured very accurately, since that accuracy is determined by $N^{-1/2}$, where N is the number of photons used.

A similar system also operates in a laser-interferometer with mirrors multiply reflecting a laser beam [13, 63, 64].

In the case of a space radiointerferometer the range of gravitational waves detected with its help shifts to low frequencies. This property makes space radiointerferometry a unique way to detect gravitational wave bursts whose duration varies from minutes to days.

7. CONCLUSION

We have shown that the propagation of electromagnetic waves in a random gravitational wave field considerably differs from electromagnetic wave propagation in randomly inhomogeneous media. Their behaviour is specific since gravitational waves are first - transverse, second - tensorial, thus they are "twice transverse" (the solution includes the factor $\sin^2 \theta$, rather than merely $\sin \theta$, as would be the case for certain vector transverse fields) and also since the propagation velocity of gravitational waves is exactly equal to the light velocity. Due to the cumulative effect of the above factors the squared phase dispersion does not grow with distance to the source of electromagnetic waves as is the case for randomly inhomogeneous media. As was already mentioned above, however, despite even the above said, space radiointerferometry will provide non-trivial limitations on Ω_{GW} (or it may even lead to relic gravitational wave detection).

Principally new possibilities should also be mentioned of optical interferometry which, reducing the length of the wave used, results in greater sensitivity of phase measurements in λ_{radio}/λ by a factor of 10^5 .

Acknowledgments

The authors express their most sincere gratitude to A. Illarionov, D. Kompaneetz, L. Gritschuk and M. Sazhin for useful discussions during the preparation of this paper. One of the authors (I.D.N.) would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste.

REFERENCES

- [1] M.J. Rees, MNRAS 154 (1971) 187.
- [2] G. Dautcourt, Proc. IAV Symp. "Confrontation of cosmological theories with observational data" ed. M.S. Longair, Dordrecht. Reidel, (1974) 299.
- [3] F.B. Estabrook and H.D. Wahlquest, Gen. Rel. Grav. 6 (1975) 439.
- [4] M.B. Sazhin, Astron. J. 55 (1978) 65.
- [5] J.W. Armstrong, R. Woo and F.B. Estabrook, Astrophys. J. 230 (1979) 570.
- [6] B. Bertotti and B.J. Carr, Astrophys. J. 236, No.3 (1980) 1000.
- [7] B. Mashhoon and L.P. Grishchuck, Astrophys. J. 236 (1980) 990.
- [8] B.J. Carr, Astron. Astrophys. 89 (1980) 6.
- [9] R.W. Hellings, Phys. Rev. D23 (1981) 832.
- [10] B. Bertotti, B.J. Carr and M.J. Rees, MNRAS 203 (1983) 945.
- [11] V.B. Braginskii, V.P. Mitrofanov and V.N. Yakimov, Moscow State University preprint No.1 (1985) 4.
- [12] J.D. Anderson and B. Mashhoon, Astrophys. J. 290, No.2 (1985) 445.
- [13] K.S. Thorne, in *Three Hundred Years of Gravitation* (New York, 1987) p.330.
- [14] L.P. Grishchuck, Proceed.GR.11, Stockholm (1987) 86.
- [15] L.P. Grishchuck, UFM 156, No.2 (1988) 297.
- [16] J.L. Boulanger and G. Le Denmat and Tourenc Ph., Phys. Lett. A126, No.4 (1988) 213.
- [17] S.M. Rytov, U.A. Kravzov and V.I. Tatarskiy, Vvedenic v statisticheskju radiofiziku, Chart II, U. Nauka (1978).
- [18] K.S. Gogelashvili and V.I. Shishov, Itogi nauki i tehniki, Serija Radiofiziki, Fizicheskie osnovi elektroniki, Acustika, Tom I VINITI, M (1981).
- [19] M.B. Sazhin, Astron. Circular No.1002 (1978) 1.
- [20] S. Detweiler, Astrophys. J. 234 (1979) 1100.
- [21] B. Mashhoon, MNRAS 199 (1982) 659.
- [22] R.W. Romani and J.H. Taylor, Astrophys. J. 265, No.2 (1983) L35.
- [23] R.W. Hellings and G.S. Downs, Astrophys. J. 265, No.2 (1983) L39.
- [24] R. Blandford, R. Narayan and R.W. Romani, J. Astrophys. Astron. (India) 5 (1984) 369.
- [25] M.M. Davis, J.H. Taylor, J.M. Weisberg and D.C. Backer, Nature 315 (1985) 547.
- [26] L.M. Krauss, Nature 313 (1985) 32.
- [27] B. Carr, Nature 315 (1985) 540.
- [28] D.C. Backer and R.W. Hellings, Ann. Rev. Astron. Astrophys. 24 (1986) 557.
- [29] A.G. Polnarev, Space Research Institute, preprint No.1355 (1988).
- [3] J.H. Taylor, Trudi soveshanja v Tbilisi i Moskve (1989).
- [31] L.D. Landau and E.M. Lifshic, Teoria polja, M, Nauka (1973).
- [32] G.W. Misner, K.S. Thorne and J.A. Wheeler, *Gravitation* (Freeman and Company, San Francisco, 1973).
- [33] L.P. Grishchuck, ZhETF (Russian) 67 (1974) 825.
- [34] I.S. Gradshtein and I.M. Rizhik, Tablici Integralov, summ, riadov i proizredenii, M, Nauka (1977) p.1108.
- [35] A.A. Starobinsky, Pisma v Astro. Zh. 3 (1979) 719.
- [36] A.A. Starobinsky, Pisma v Astro. Zh. 9 (1983) 579.
- [37] C.J. Hogan, MNRAS 218 (1986) 629.
- [38] D.V. Deryagin, D.Yu. Grigoriev, V.A. Rubakov and M.V. Sazhin, Mod. Phys. Lett. A1 (1986) 593.
- [39] V.A. Rubakov, M.V. Sazhin and A.V. Veryashin, Phys. Lett. 115B (1987) 189.
- [40] R. Matzner, "The pregalactic cosmic gravitational wave background", preprint The University of Texas (1988).
- [41] C.J. Hogan and M.J. Rees, Nature 311 (1984) 109.
- [42] T. Vachaspati and A. Vilenkin, Phys. Rev. D31 (1985) 3052.
- [43] A. Albrecht and N. Turok, Phys. Rev. Lett. 54 (1985) 1868.
- [44] R.H. Brandenberger, A. Albrecht and N. Turok, preprint No.SF-ITP-15 (1986).
- [45] A. Vilenkin, in *Three Hundred Years of Gravitation* (1987) p.499.
- [46] D.P. Bennett and F. Bouchet, Phys. Rev. Lett. 60 (1988) 257.
- [47] R.W. Romani, Phys. Lett. B215 (1988) 477.

- [48] G. Dautcourt, MNRAS **144** (1969) 255.
- [49] A.G. Doroshkevich, I.D. Novikov and A.G. Polnarev, in *Experimental Gravitation* (Accad. Nazionale dei Lincei, Roma, 1977) p.91.
- [50] R. Fabbri and M.D. Pollack, Phys. Lett. **125B** (1983) 445.
- [51] P.J. Adams, R.W. Hellings and R.L. Zimmerman, Astrophys J. **280** (1984) L39.
- [52] A.G. Polnarev, Astron. Zh. **62** (1985) 1041.
- [53] A.A. Starobinsky, Pisma v Astron. Zh. **11** (1985) 323.
- [54] A.G. Polnarev, Proceedings GRG-11, Sweden (1986) 491.
- [55] Radiastron, preprint Space Research Institute (1986).
- [56] O.I. Yakovlev, *Rasprostranenie radiovoln v cosmoze* (Nauka, 1985).
- [57] V.B. Braginsky, ZhETF **53** (1967) 1434.
- [58] R.F.C. Vessot, Proceedings of the Workshop on Relativistic Gravitational Experiments in Space, Annapolis, Maryland (1988).
- [59] J.B. Zeldovich and A.G. Polnarev, Astron. Zh. **51** (1974) 30.
- [60] V.B. Braginsky and L.P. Grishuck, ZhETF **89** (1985) 744.
- [61] V.B. Braginsky and K.S. Thorne, Nature **327** (1987) 123.
- [62] L.P. Grishuck and A.G. Polnarev, ZhETF (1989) in press.
- [63] V.N. Rudenki and M.B. Sazhin, Kvantovaja Electrodinamika **7** (1980) 2344.
- [64] V.V. Kulagin, A.G. Polnarev and V.N. Rudenko, ZhETF **91** (1986) 1553.

FIGURE CAPTIONS

Fig.1 Schematical picture of the dependence of the upper limit for the RGW energy density as a function of the wave length which could be obtained from the space radio interferometry (see Eq.(4.4)). Here

$$\dot{\Omega} = \left(\frac{\delta}{3 \cdot 10^{-2}} \right)^2 \left(\frac{\lambda_e}{1 \text{ cm}} \right)^2 \left[\frac{(1 + Z_s)^2 + 1}{2} \right]^{-1} \left(\frac{L}{1 \text{ A U}} \right)^{-4}$$

Fig.2 The same as Fig.1, but as a function of space radio interferometry baseline (see Eq.(4.4)). Here

$$\Omega_{..} = \left(\frac{\delta}{3 \cdot 10^{-2}} \right)^2 \left(\frac{\lambda_e}{1 \text{ cm}} \right)^2 \left[\frac{(1 + Z_s)^2 + 1}{2} \right]^{-1} \left(\frac{\lambda_B}{1 \text{ A U}} \right)^{-4}$$

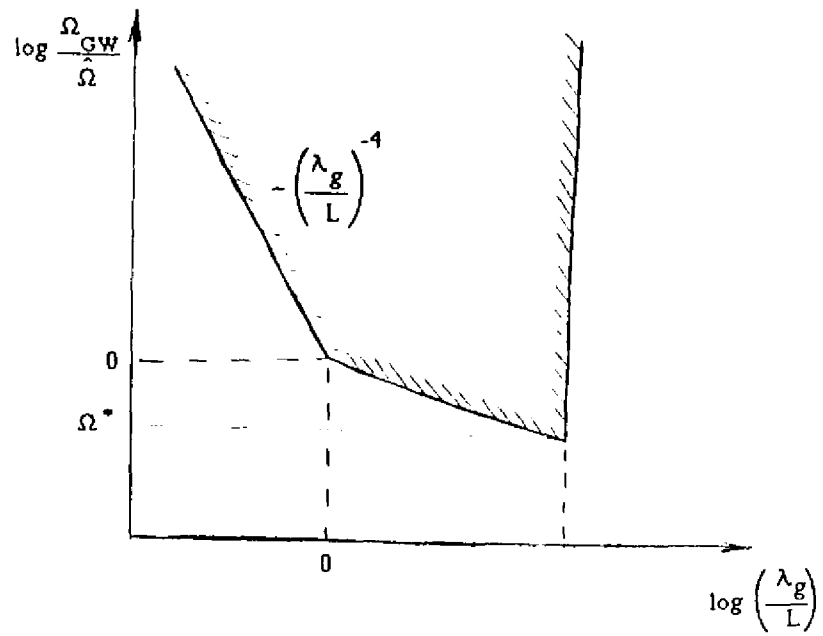


Fig. 1

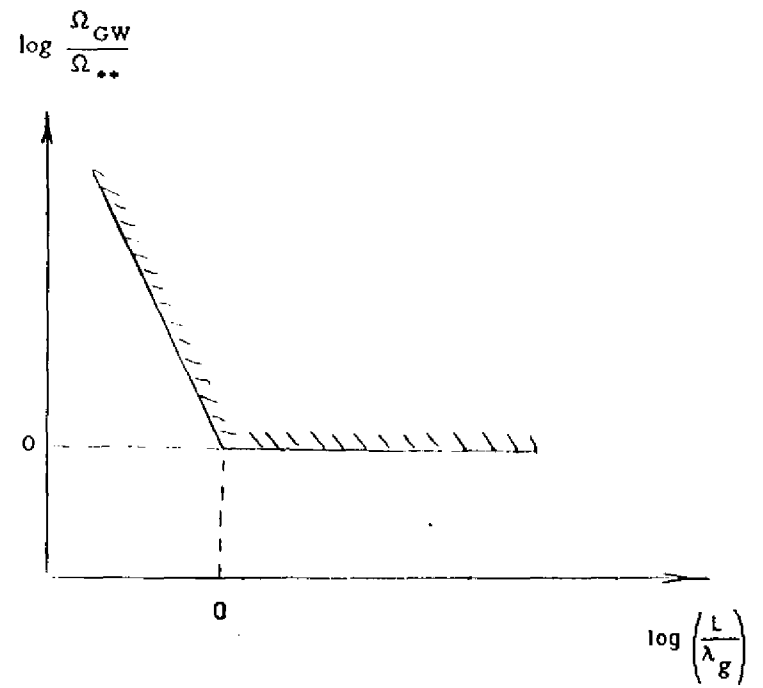


Fig. 2

Stampato in proprio nella tipografia
del Centro Internazionale di Fisica Teorica