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**SOME NOTES ON STATIONARY
VACUUM SPACE-TIMES
WITH SHEARING
NONGEODESIC EIGENRAYS**

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**SOME NOTES ON STATIONARY VACUUM
SPACE-TIMES WITH SHEARING
NONGEODESIC EIGENRAYS**

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I. Horváth, B. Lukács: Some notes on stationary vacuum space-times with shearing nongeodesic eigenrays. KFKI-1989-55/B

ABSTRACT

Here the stationary vacuum problem with eigenray structure $\kappa = \text{const.} \neq 0$, $\sigma = \text{const.} \neq 0$ is considered, as a possible Kerr generalisation. The result is that for the solutions of the Einstein equation this class is empty.

И. Хорват, Б. Лукач: О стационарно-вакуумных решениях с негеодетическими собственными лучами со сдвигами. КФКИ-1989-55/В

АННОТАЦИЯ

Изучен класс стационарно-вакуумных решений уравнения Эйнштейна с собственными лучами, которые имеют постоянные ненулевые сдвиги и отклонения от геодезических линий. Такие решения являются одним из методов обобщения пространства-времени Керра. К этому классу не принадлежит ни одного из решений уравнения Эйнштейна.

Horváth I., Lukács B.: Megjegyzések a nyíró nemgeodétikus sajátugarú stacionárius vákuum téridőkről. KFKI-1989-55/B

KIVONAT

A stacionárius vákuum téridőket vizsgáljuk olyan esetekre, mikor a sajátugarakra κ és σ állandó és nem 0. Ez a Kerr-megoldás általánosításának egy módja. Az eredmény az, hogy ezen osztályba az Einstein egyenlet megoldásai közül egy sem tartozik.

1. INTRODUCTION

An interesting task in General Relativity is to look for external solutions of compact final states of stellar evolution, such as rotating neutron stars and black holes, when Newtonian treatment is already insufficient. Such final states are expected to be axisymmetric in addition to stationarity, but generally not spherical.

There is a solution which can be used as physical starting point for tackling the above problem, the Kerr metric [1]. It is a stationary axisymmetric vacuum solution containing two arbitrary constants, whose canonical notations are m and a . Far-field asymptotic geodesics demonstrate [2] that

$$m = GM/c^2 \quad (1.1a)$$

$$a = J/Mc \quad (1.1b)$$

where M is a mass and J is an angular momentum. Therefore one might regard the solution as the external field of a massive rotating source. Indeed, for far-field approximations the Kerr solution is practically sufficient.

If the above parameters are finite, then the solution approaches the Minkowski space-time asymptotically, therefore the source is localized. (Near the central singularity there are acausal closed timelike orbits [2], [3], but this problem can be eliminated if $a < m$ and, e.g. the matter fills the space up to above the outer horizon.) All the other parameters of the body, as e. g. oblateness, are determined by a and m . Therefore, one cannot expect the Kerr solution to be the general external solution of a rotating stationary body, because even in the Newtonian limit mass and angular momentum are not enough to characterize the gravitational field [4]. Rather the external solution around a fastly rotating neutron star may be a member of a class containing Kerr as special case. Therefore to describe astrophysical situations a generalization of Kerr metric would be needed. For further arguments see Ref. 5.

There is a class containing Kerr as first member, the Tomimatsu-Sato series [6]. However, there the extra third parameter takes only integer values. Interpolation has been formulated by Cosgrove [7] but not in analytic form.

There is a method for solving Einstein equations where the

position of Kerr is very special, so giving ideas to generalize. In the Newman-Penrose spin coefficient method of solving the Einstein equation (8) the Kerr solution belongs to a very special class $\kappa = \sigma = 0$. If we are interested in final states of evolution, stationarity can be assumed. Then the Einstein equations can be reformulated in a 3 dimensional space (9). Again, spin coefficient technique can be used, and the resulted equations are substantially simpler than in 3 dimensions (9). The $\kappa=0, \sigma=0$ class (geodesic shearfree eigenrays (9)) can be fully integrated and contains Kerr-NUT and some plane waves. The only solution in this class with asymptotic flatness is the Kerr. Now it seems that the task is to properly generalize the Ansatz $\kappa=0, \sigma=0$.

Both the $\kappa=0, \sigma \neq 0$, and the $\kappa \neq 0, \sigma=0$ cases have been analytically solved. The first class contains 3 solutions, but neither is the generalisation of Kerr (10). In the second case one obtains two solutions, without free parameters (11), so again they cannot be Kerr generalisations. Then the Kerr generalisations must be somewhere within the generic case $\kappa \neq 0, \sigma \neq 0$. But there they are expected to represent only a cluster of very special metrics. Therefore for them some special relations must hold among the spin coefficients.

The spin coefficients are invariant scalars for coordinate transformations. For axial symmetry in any specific solution there is a relation

$$f(\kappa, \sigma) = 0 \tag{1.1}$$

(For the detailed argumentation see Ref. 5.) A functional dependence of form (1.1) is a reasonable form of Ansatz to formulate the speciality of "generalized Kerr" solutions in the space of spin coefficients. The specific Ansatz would be a specific function f . With such a relation derivatives of κ can be expressed via derivatives of σ , and the number of equations seems to be sufficient to proceed to solve them (5).

However, besides this generic case there is a disjoint special one, when both κ and σ are constant. This case cannot be written into the form (1.1), and therefore must be differently handled. The aim of the present paper is to look for solutions in this special subclass. This does not necessary mean isolation from Kerr, because the constants may be imagined to be arbitrarily close to 0.

2. THE FIELD EQUATIONS

Our starting equations are the triad projections of the vacuum Einstein equations in spin coefficient form using the 3+1 decomposition in the stationary case. The details of the technique can be found in Ref. 8 and will not be repeated here. The unknown quantities are the spin coefficients κ , σ , ρ , τ , ϵ , and the triad ($_{o+}$) components of the complex vector G . However, by appropriate triad choice one can always reach $\epsilon=G_+=0$ [9]. Here this gauge will be used. Then the field equations are as follows:

$$0 = \tau\kappa^* - \kappa^*\kappa^* - \sigma^*(\rho + \rho^*) \quad (2.1a)$$

$$D\tau = \rho\kappa^* - \kappa\sigma^* - \tau^*\sigma^* + \tau\rho + G_0G_+^* \quad (2.1b)$$

$$\delta\rho = -(2\tau\sigma + \kappa(\rho - \rho^*) - G_0G_+^*) \quad (2.1c)$$

$$DG_0 = 2\rho G_0 + G_0^2 - G_0G_0^* - \kappa G_- \quad (2.1d)$$

$$\delta^*G_0^* = -\sigma^*G_+^* - \kappa^*G_0^* - G_0^*G_- \quad (2.1e)$$

$$DG_- = \delta^*G_0 + \rho G_- + \kappa^*G_0 - G_-G_0^* \quad (2.1f)$$

$$\delta G_- = -\rho G_0 + \rho^*G_0 + \tau^*G_- - G_-G_+^* \quad (2.1g)$$

$$D\rho = -\tau\kappa + \kappa\kappa^* + \sigma\sigma^* + \rho\rho + G_0G_0^* \quad (2.1h)$$

where the differential operators D , δ , δ^* have the commutator relations

$$D\delta - \delta D = \rho^*\delta + \sigma\delta^* + \kappa D \quad (2.2)$$

$$\delta\delta^* - \delta^*\delta = \tau^*\delta^* - \tau\delta + (\rho^* - \rho)D$$

The star stands for complex conjugate. The triad is not yet fully fixed: there has remained a restricted rotation of the triad vector $\mathbf{x}_+ = \mathbf{m}$ as follows [9]:

$$\begin{aligned} \mathbf{x}' &= \mathbf{m} e^{iC^0} \\ DC^0 &= 0 \end{aligned} \quad (2.3)$$

By means of this, a phase, constant along the eigenrays, can be conveniently removed from one of the quantities σ , τ or G_- [9]. In the present case, since σ is totally constant by assumption, its phase is constant too, then it can be removed, and the triad becomes completely fixed. Therefore henceforth σ is real and positive.

Observe the algebraic equation (2.1a); it is algebraic because in general it contains derivatives of κ and σ , which vanish now. This equation becomes a trivial identity if $\kappa = \sigma = 0$.

Again, observe that the system (2.1-2.2) was closed for integrability for $\kappa = \sigma = 0$. Now the number of unknowns is the same if we regard κ and σ as given (although unspecified) constant

parameters. However, because of the extra equation, now the system is not closed; in the following Section we derive new integrability conditions.

3. INTEGRABILITY CONDITIONS

One may start in two directions: either the algebraic equation can be utilized or some commutator can be applied on an unknown to obtain new equations. We start with the algebraic equation (2.1a). Hence τ can be expressed as

$$\tau = -x^\sigma - (\sigma^\rho(\rho^\sigma \rho^\sigma))/x \quad (3.1)$$

For a while we consider the constants x and σ given. This can be substituted into eqs. (2.1b). The first one gives the rather long explicit expression

$$\begin{aligned} G_- = & (2G_0^\sigma x^\sigma \sigma^\rho G_0^\sigma x + 2x^\sigma \rho^\sigma \sigma^\rho \sigma + 4x^\sigma \sigma^\rho \sigma^\rho x^2 + 2x^\sigma \sigma^\rho \rho^\sigma \sigma + x^\sigma \rho^\sigma \sigma^\rho \sigma^\rho x \\ & - x^\sigma \rho^\sigma \sigma^\rho x \rho + 2x^\sigma \sigma^\rho \sigma^\rho x \sigma + \rho^\sigma \sigma^\rho \sigma^\rho x^2 + \sigma^\rho \sigma^\rho x^2 \rho) / (G_0^\sigma x^\sigma x) \end{aligned} \quad (3.2)$$

if $G_0 \neq 0$. The exceptional case $G_0 = 0$ would lead to $G_- = 0$ via eq. (2.1e); then $G = 0$, therefore the 3-dimensional Ricci tensor vanishes, $f = \text{const.}$, and ω_i can be made 0 [9], so the trivial Minkowski solution is reached. Henceforth $G_0 \neq 0$. There is no need for eq. (2.1d) to be explicitly used in this calculation.

Then G_- can be written into eqs. (2.1f) and (2.1g). Two fairly long equations are obtained which explicitly give the δ^ρ derivatives of ρ and G_0 . (See Appendix.) Observe that the D and δ derivatives of these quantities were already given by the starting equations; the other unknowns G_- and τ have later been expressed via ρ and G_0 .

So now all the first derivatives of ρ and G_0 are known, therefore any new integrability condition for them will already overdetermine the system in some extent. We are going to get these new equations.

Let us start with the commutator (2.2a). In order to evaluate its effect one, of course, needs eqs. (A.1-2). Applying it on ρ an identity is obtained. However, acting on G_0 , the result is not an identity, similarly to e. g. the case $x \neq 0 \neq \sigma$ [10]. However, now no unknown derivative has remained. It gives

$$\begin{aligned}
& (36 G_0^2 x^4 G_0^2 x^3 \sigma - 80 G_0^2 x^3 \sigma^2 G_0^2 x^2 \sigma^2 + G_0^2 x^3 G_0^2 x^4 \rho + \\
& 20 G_0^2 x^2 \rho^2 \sigma^2 G_0^2 x^3 \sigma + 20 G_0^2 x^2 \sigma^2 G_0^2 x^5 + 20 G_0^2 x^2 \sigma^2 G_0^2 x^3 \rho \sigma - \\
& 120 G_0^2 x^2 \sigma^2 G_0^2 x^4 \sigma + 40 G_0^2 x^4 \rho^2 \sigma^2 G_0 x \sigma^3 + 40 G_0^2 x^4 \sigma^2 G_0 x \rho \sigma^3 + \\
& 40 G_0^2 x^3 \rho^2 \sigma^2 G_0 x^2 \sigma^2 + 140 G_0^2 x^3 \rho^2 \sigma^2 G_0 x^4 \sigma + 60 G_0^2 x^3 \rho^2 \sigma^2 G_0 x^2 \rho \sigma^2 + \\
& 40 G_0^2 x^3 \sigma^2 G_0 x^6 + 140 G_0^2 x^3 \sigma^2 G_0 x^4 \rho \sigma + 20 G_0^2 x^3 \sigma^2 G_0 x^2 \rho^2 \sigma^2 + \\
& 6 G_0^2 x^2 \rho^2 \sigma^2 G_0 x^5 - 40 G_0^2 x^2 \rho^2 \sigma^2 G_0 x^3 \sigma^2 - G_0^2 x^2 \rho^2 \sigma^2 G_0 x^5 \rho - \\
& 220 G_0^2 x^2 \sigma^2 G_0 x^5 \sigma - 40 G_0^2 x^2 \sigma^2 G_0 x^3 \rho \sigma^2 + G_0^2 x^2 \rho^2 \sigma^2 G_0 x^6 + 12 \\
& G_0^2 x^2 \rho^2 \sigma^2 G_0 x^4 \rho \sigma - 120 G_0^2 x^2 \sigma^3 G_0 x^4 \sigma^2 + G_0^2 x^2 \sigma^2 G_0 x^6 \rho - 60 G_0^2 \rho^2 \sigma^3 G_0 \\
& x^5 \sigma - 60 G_0^2 \sigma^3 G_0 x^5 \rho \sigma + 4x^5 \rho^2 \sigma^2 \sigma^4 + 24x^5 \rho^2 \sigma^2 x^2 \sigma^3 + 8x^5 \rho^2 \sigma^2 \rho \sigma^4 + \\
& 32x^5 \sigma^2 x^4 \sigma^2 + 24x^5 \sigma^2 x^2 \rho \sigma^3 + 4x^5 \sigma^2 \rho^2 \sigma^4 + 2x^4 \rho^3 \sigma^2 x \sigma^3 + \quad (3.3) \\
& 8x^4 \rho^2 \sigma^2 x^3 \sigma^2 - 4x^4 \rho^2 \sigma^2 x \rho \sigma^3 + 12x^4 \rho^2 \sigma^2 x \sigma^4 - 16x^4 \rho^2 \sigma^2 x^3 \rho \sigma^2 - \\
& 2x^4 \rho^2 \sigma^2 x \rho^2 \sigma^3 + 32x^4 \sigma^2 x^3 \sigma^3 + 12x^4 \sigma^2 x \rho \sigma^4 + 8x^4 \sigma^2 x^3 \rho^2 \sigma^2 + \\
& 4x^4 \sigma^2 x \rho^3 \sigma^3 - 2x^3 \rho^3 \sigma^2 x^2 \rho \sigma^2 + 14x^3 \rho^2 \sigma^2 x^2 \sigma^3 + 4x^3 \rho^2 \sigma^2 x^2 \rho \sigma^2 + \\
& 24x^3 \rho^2 \sigma^2 x^4 \sigma^2 + 12x^3 \rho^2 \sigma^2 x^2 \rho \sigma^3 - 2x^3 \rho^2 \sigma^2 x^2 \rho^3 \sigma^2 + 8x^3 \sigma^2 x^3 \sigma^4 + \\
& 24x^3 \sigma^2 x^4 \rho \sigma^2 + 14x^3 \sigma^2 x^2 \rho^2 \sigma^3 + 4x^2 \rho^3 \sigma^2 x^3 \sigma^2 - 2x^2 \rho^2 \sigma^2 x^3 \rho \\
& \sigma^2 + 12x^2 \rho^2 \sigma^2 x^3 \sigma^3 - 4x^2 \rho^2 \sigma^2 x^3 \rho^2 \sigma^2 + 12x^2 \sigma^2 x^3 \rho \sigma^3 + 2x^2 \sigma^2 x^3 \\
& \rho^3 \sigma^2 + 4x^2 \rho^2 \sigma^2 x^4 \sigma^2 + 8x^2 \rho^2 \sigma^2 x^4 \rho \sigma^2 + 4x^2 \sigma^2 x^4 \rho^2 \sigma^2) = 0
\end{aligned}$$

While this equation is considerably longer than its counterpart was for $x=0$, now all the derivatives, together with τ and G_0 , have been substituted, so eq. (3.3) is algebraic. Henceforth we concentrate on this equation and the equations containing $D\rho$ and DG_0 . These 3 equations will be sufficient to prove that no solution exists in the subclass investigated here.

4. THE RADIAL EQUATIONS

Our equations are: eq. (3.3) (not repeated here), and the radial equations for ρ and G_0 . The latter ones come from eqs. (2.1), τ and G_0 substituted:

$$DG_0 = 2\rho G_0 + G_0^2 - G_0 G_0' - xG_0 \quad (4.1)$$

$$D\rho = (x' + (\sigma'(\rho + \rho'))/x')x + xx' + \sigma\sigma' + \rho\rho' + G_0 G_0'$$

(4.2)

Let us see the steps in details first for the special case

when x is real. For complex x the derivation is similar. For a real x the imaginary part of eq. (3.3) gives a simple equation, linear in $\theta_0 \theta_0^*$:

$$\theta_0 \theta_0^* (x^2 + 2\rho_1 \sigma) - 4\sigma^3 \rho_1 = 0 \quad (4.3)$$

where

$$\rho = \rho_1 + i\rho_2 \quad (4.4)$$

Observe that eq. (4.3) overdetermines the system (4.1-2). Therefore acting on it repeatedly by D , all the D derivatives can be substituted from the system, thus obtaining a sequence of new algebraic equations, if not identities. Doing so, the first differentiation gives:

$$(2\sigma(3\rho_1^2 x^4 + 6\rho_1^2 x^2 \sigma^2 + 6\rho_1 x^4 \sigma - 4\rho_1 x^2 \sigma^3 - 8\rho_1 \sigma^5 - \rho_2^2 x^4 + 2\rho_2^2 x^2 \sigma^2 + 2x^6 - 3x^4 \sigma^2 - 2x^2 \sigma^4)) / x^2 = 0 \quad (4.5)$$

where eq. (4.3) has been used. Acting again with D , one gets:

$$3\rho_1^3 x^6 + 6\rho_1^3 x^4 \sigma^2 + 15\rho_1^2 x^6 \sigma + 34\rho_1^2 x^4 \sigma^3 + 20\rho_1^2 x^2 \sigma^5 - 5\rho_1 \rho_2^2 x^6 - 2\rho_1 \rho_2^2 x^4 \sigma^2 + 6\rho_1 x^8 + 27\rho_1 x^6 \sigma^2 + 10\rho_1 x^4 \sigma^4 - 24\rho_1 x^2 \sigma^6 - 16\rho_1 \sigma^8 - 3\rho_2^2 x^6 \sigma + 2\rho_2^2 x^4 \sigma^3 + 4\rho_2^2 x^2 \sigma^5 + 6x^8 \sigma - x^6 \sigma^3 - 10x^4 \sigma^5 - 4x^2 \sigma^7 = 0 \quad (4.6)$$

At this point it is worthwhile to separate eq. (4.1) into real and imaginary parts as

$$D\rho_1 = 2x^2 + \sigma^2 + \theta_0 \theta_0^* + \rho_1^2 - \rho_2^2 + 2\rho_1 \quad (4.7)$$

$$D\rho_2 = 2\rho_1 \rho_2 \quad (4.8)$$

From eq. (4.6) one can eliminate ρ via eq. (4.5), unless $x^2 - 2\sigma^2 = 0$. Assuming that it is not 0, one obtains

$$(4\rho_1 x^4 (-3\rho_1^2 x^4 - 9\rho_1^2 x^2 \sigma^2 - 6\rho_1^2 \sigma^4 - 6\rho_1 x^4 \sigma + 6\rho_1 \sigma^5 - x^6 + 2x^4 \sigma^2 - x^2 \sigma^4)) / (x^2 - 2\sigma^2) = 0 \quad (4.9)$$

Hence $\rho_1 = \text{const.}$, being eq. (4.9) an algebraic equation of constant coefficients for ρ_1 . After that, in the same way, eq. (4.6) gives $\rho_2 = \text{constant}$ as well, which, via eq. (4.8) gives that either ρ_1 or ρ_2 is 0. By straightforward but tedious calculations one can show that these cases do not give solutions.

Still we have the subcase $x^2 - 2\sigma^2 = 0$. Then eq. (4.5) is a second order algebraic equation for ρ_1 , with two solutions:

$$\rho_1 = 0, \text{ or}$$

$$\rho_1 = \frac{6\sigma x^2 - 4\sigma^3 - 8\sigma^5/x^2}{6\sigma^2 + 3x^2} \quad (4.10)$$

(constant in both cases). In the first case eq. (4.3) would give $G_0 G_0^* = 0$ excluded earlier; in the second again ρ_1 is constant, and we can repeat the previous arguments. So for real x there is no solution. For complex x again we can start with the imaginary part of eq. (3.3). With the definitions

$$x = x_1 + ix_2 \quad (4.11)$$

$$\Omega = x_1^2 - x_2^2 \quad (4.12)$$

one gets

$$\begin{aligned} & G_0 G_0^* ((xx^*)^2 \rho_2 \Omega / \sigma + (xx^*)^2 \rho_{11} 2x_1 x_2 / \sigma + 8xx^* \rho_1 \sigma x_1 x_2 + 8xx^* \Omega x_1^* \\ & x_2 - 124\sigma^2 x_1 x_2 \Omega) - 4(xx^*)^2 \sigma \rho_1 \rho_2 + xx^* (\rho_1^2 - \rho_2^2) 4x_1 x_2 \Omega - xx^* \sigma \\ & 2\rho_1 \rho_2 (x_1^4 - 6x_1^2 x_2^2 + x_2^4) + 2\rho_1 (28(xx^*)^2 \sigma x_1 x_2 + (\sigma - \\ & 6\sigma^3 (1/\Omega)) (6x_1^5 x_2 - 14x_1^3 x_2^3 + 6x_1 x_2^5)) + (\rho_1^2 + \rho_2^2) 4x_1 x_2 \Omega (12\sigma\sigma \\ & - xx^*) + (4(xx^*)^2 - 22xx^* \sigma^2 - 12\sigma^4) 4(x_1^3 x_2 - x_1 x_2^3) = 0 \end{aligned} \quad (4.13)$$

In the generic case hence $G_0 G_0^*$ can be expressed. Acting on eq. (4.13) by D and eliminating $G_0 G_0^*$, we get eq. (A.3) (because of its extreme length relegated to the Appendix) Now, substituting $G_0 G_0^*$ into the real part of eq. (3.3) one obtains a rather long equation, not displayed here. Apart from constants, both that one and eq. (4.11) contains only ρ_1 and ρ_2 . Acting again on them by D and substituting the D derivatives from eqs. (4.2), one finally gets 6 equations for the variables ρ_1 and ρ_2 and for the constants σ , x_1 and x_2 . Such a system is either overdetermined or the equations are algebraically dependent. Checking the coefficient matrix by REDUCE code it turned out to be the first case. Therefore no solution exists in this subclass.

Now we turn to the special case when the coefficient of $G_0 G_0^*$ vanishes. Then eq. (3.3) reduces to

$$\begin{aligned}
& - 4 (xx^*)^2 \sigma \sigma \rho_1 \rho_2 + xx^* (\rho_1^2 - \rho_2^2) 4x_1 x_2 \Omega - xx^* 2 \rho_1 \rho_2 \\
& (x_1^4 - 6x_1^2 x_2^2 + x_2^4) + 2\rho_1 (14 (xx^*)^2 \sigma 2 x_1 x_2 + (\sigma - 6\sigma^3) \quad (4.14) \\
& (1/\Omega) (6 x_1^5 x_2 - 14x_1^3 x_2^3 + 6x_1 x_2^5)) + (\rho_1^2 + \rho_2^2) 4x_1 x_2 \Omega \\
& (12\sigma\sigma - xx^*) + (4(xx^*)^2 - 22xx^*\sigma\sigma - 12\sigma^4) 4(x_1^3 x_2 - x_1 x_2^3) = 0
\end{aligned}$$

Now, from the vanishing bracket one can either express ρ_2 as

$$\rho_2 = a\rho_1 + b \quad (4.15)$$

with constant a and b, known functions of x and σ (cf. (4.13)), or

$$\Omega = 0 \quad (4.16)$$

In the first case ρ_2 can be written into eq. (4.14). What remains, is an algebraic equation of second order for ρ_1 with constant coefficients, therefore

$$D\rho_1 = D\rho_2 = 0. \quad (4.17)$$

(In the special case when coefficients of all the 3 terms in the quadratic equation vanish, we have 3 homogeneous equations for 3 variables, without nontrivial solution.)

Then from eq. (4.2):

$$\begin{aligned}
& 2(x_1^2 + x_2^2) + \sigma^2 + G_0 G_0^* + \rho_1^2 - \rho_2^2 + 2\sigma\rho_1(x_1^2 - x_2^2) + \\
& (1/\Omega) = 0 \quad (4.18)
\end{aligned}$$

$$2\rho_1\rho_2 + 4x_1x_2\sigma\rho_1(1/\Omega) = 0 \quad (4.19)$$

Now, from eq. (4.18) $G_0 G_0^*$ is const. Comparing this with eq. (4.1) and (3.2), one gets

$$\begin{aligned}
& 2\rho G_0 G_0^* x^{*2} + G_0 G_0^* x^{*2} (G_0 - G_0^*) - (2 G_0^* x^* \sigma^* G_0 x + 2 x^{*2} \rho^* \sigma^* \sigma \\
& + 4 x^{*2} \sigma^* x^2 + 2 x^{*2} \sigma^* \rho \sigma + x^* \rho^{*2} \sigma^* x - x^* \rho^* \sigma^* x \rho + 2 x^* \\
& \sigma^{*2} x \sigma + \rho^* \sigma^{*2} x^2 + \sigma^{*2} x^2 \rho) \\
& (4.20)
\end{aligned}$$

Finally, the real part of eq. (3.3) is utilized. Apart from x_1, x_2 and σ , it contains $G_0 G_0^*$, substituted from eq. (4.18), ρ_1 , substituted by solving (4.14), and ρ_2 , substituted from eq. (4.15). Then we have 5 equations for 5 unknown x_1, x_2, G_{01}, G_{02} and σ . ($G_0 = G_{01} + iG_{02}$)

However, each equation is homogeneous in them. Therefore one can introduce the new pentad of unknowns ($\sigma, x_1/\sigma, x_2/\sigma, G_{01}/\sigma, G_{02}/\sigma$). Then σ occurs in the equations as a (nonzero) trivial

multiplicator and can be removed. There remains 5 equations for 4 quantities. Checking algebraic dependence, again they are overdetermined and no solution exists.

Finally, if $\Omega=0$, the vanishing bracket in eq. (4.13) reduces to

$$\rho_1 x_1 = 0 \quad (4.21)$$

(all the other common factors being positive). But from the definition of Ω , if it is 0, $x_1=0$ leads to $x=0$, excluded in this paper. For $\rho_1=0$ we can compare eqs. (2.1c) and (4.18). The first, having substituted τ and G_- from eqs. (3.1-2), in our special case gives

$$2xx'' + \sigma^2 - \rho_2^2 + G_0 G_0'' = 0 \quad (4.22)$$

while from the second:

$$xx'' + \sigma^2 - \rho_2^2 + G_0 G_0'' = 0 \quad (4.23)$$

Hence $x=0$, excluded in the present study.

Now we are ready and the whole class has turned out to be empty. It is interesting to note that such solutions, where one of σ and x is 0, and the other is a nonzero constant, do not exist either as can be seen by inspecting the solutions in Refs. 10, 11.

5. CONCLUSIONS

We have found that stationary vacuum solutions with the eigenray structure $x=\text{const.}$, $\sigma=\text{const.}$ do not exist if the constants are not zero. The physical meaning of this negative result is hard to be quite explicitly seen; it is true that the above 2 spin coefficients characterize the deviation from geodesics and the shear of the eigenray congruence, but constant deviation does not mean too much. Again we have arrived at an isolation of the Kerr solution. The mathematical reason of the negative result is clear enough: comparing cases $x=0$, $\sigma=0$ and $x=\text{const.}\neq 0$, $\sigma=\text{const.}\neq 0$, in the first case eq. (2.1a) is an identity, while in the latter this equation exists but, of course is a differential equation not for the constant quantities but for the other ones, governed also by the further equations. So in the latter case there is greater chance for the system to be overdetermined, and this is definitely which has happened.

Therefore all asymptotically flat and axisymmetric Kerr generalizations must be in the subclass characterized by the existence of a relation $f(x,\sigma)=0$. This subclass needs a completely different approach and will be object of forthcoming papers.

APPENDIX

Here three rather long equations are collected which would have disturbed the reader in the main text. First we are obtaining the δ^* derivatives of the quantities ρ and G_0 . For this, we start from eqs. (2.1f-g). Substituting thither τ and G_- from eqs.(3.1-2) and known derivatives from the system (2.1) there remain only $\delta^*\rho$ and δ^*G_0 , and one gets eqs. (A.1-2) (see below).

$$\begin{aligned}
 & (2G_0^* x^3 G_0 x^2 \delta^* \rho - G_0^{*2} x^3 G_0^2 x^3 + 8 G_0^{*2} x^2 \delta^* G_0^2 x^2 \rho \\
 & x^2 \sigma + 2 G_0^* x^4 G_0 x^4 + 4 G_0^* x^4 G_0 x^2 \rho \sigma + 14 G_0^* x^3 \rho^* \sigma^* \rho \\
 & G_0 x \sigma^2 + G_0^* x^3 \rho^* G_0 x^3 \rho + 27 G_0^* x^3 \delta^* G_0 x^3 \sigma + 14 G_0^* x^3 \sigma^* G_0 x \rho \sigma^2 + 6 G_0^* x^2 \rho^* \sigma^* G_0 x^2 \sigma + 2 G_0^* x^2 \rho^* \sigma^* G_0 x^4 \\
 & x^4 - 2 G_0^* x^2 \rho^* \sigma^* G_0 x^2 \rho \sigma + 12 G_0^* x^2 \sigma^* G_0 x^2 \sigma^2 + 2 G_0^* x^2 \sigma^* G_0 x^4 \rho + 4 G_0^* x^2 \sigma^* G_0 x^2 \rho^2 \sigma + 8 G_0^* x^* \rho^* \sigma^* \rho^2 \sigma \\
 & G_0 x^3 \sigma + 8 G_0^* x^* \sigma^* G_0 x^3 \rho \sigma + 2 x^4 \rho^* \sigma^* \sigma^3 + 12 x^4 \rho^* \sigma^* \sigma^2 \sigma^2 + 4 x^4 \rho^* \sigma^* \rho \sigma^3 + 16 x^4 \sigma^* x^4 \sigma + 12 x^4 \sigma^* x^2 \rho \sigma^2 \\
 & \sigma^2 + 2 x^4 \sigma^* \rho^2 \sigma^3 + x^3 \rho^* \sigma^3 \sigma^* x \sigma^2 + 4 x^3 \rho^* \sigma^2 \sigma^* x^3 \sigma - 2 x^3 \rho^* \sigma^* x \rho \sigma^2 + 6 x^3 \rho^* \sigma^* x \sigma^3 - 8 x^3 \rho^* \sigma^* x^3 \rho \sigma - \\
 & x^3 \rho^* \sigma^* x \rho^2 \sigma^2 + 16 x^3 \sigma^* x^3 \sigma^2 + 6 x^3 \sigma^* x \rho \sigma^3 + 4 x^3 \sigma^* \rho^* \sigma^2 \sigma + \dots^3 \sigma^* x \rho^3 \sigma^2 - x^2 \rho^* \sigma^* \rho^2 \rho \sigma + 7 x^2 \rho^* \sigma^* \rho^2 \sigma^2 \\
 & \sigma^2 x^2 \sigma^2 + 2 x^2 \rho^* \sigma^* x^2 \rho^2 \sigma + 12 x^2 \rho^* \sigma^* x^4 \sigma + 6 x^2 \rho^* \sigma^* x^2 \rho \sigma^2 - x^2 \rho^* \sigma^* x^2 \rho^3 \sigma + 4 x^2 \sigma^* x^2 \sigma^3 + 12 x^2 \sigma^* x^4 \rho \sigma \\
 & + 7 x^2 \sigma^* x^2 \rho^2 \sigma^2 + 2 x^* \rho^* \sigma^* x^3 \sigma - x^* \rho^* \sigma^* \sigma^2 x^3 \rho \sigma + 6 x^* \rho^* \sigma^* x^3 \sigma^2 - 2 x^* \rho^* \sigma^* x^3 \rho^2 \sigma + 6 x^* \sigma^* x^3 \rho \sigma^2 \\
 & + x^* \sigma^* x^3 \rho^3 \sigma + 2 \rho^* \sigma^* x^4 \sigma + 4 \rho^* \sigma^* x^4 \rho \sigma \\
 & \sigma + 2 \sigma^* x^4 \rho^2 \sigma) / (2 G_0^* x^3 G_0 x^3) = 0
 \end{aligned}$$

(A.1)

$$\begin{aligned}
& (2\delta(\mathbb{G}_0^*) x^3 \rho^* \sigma^* \mathbb{G}_0 x^2 \sigma + 4 \delta(\mathbb{G}_0^*) x^3 \sigma^* \mathbb{G}_0 x^4 + 2 \delta(\mathbb{G}_0^*) x^3 \sigma^* \mathbb{G}_0 x^2 \rho \sigma + \delta(\mathbb{G}_0^*) x^2 \rho^* \sigma^* \mathbb{G}_0 x^3 - \delta(\mathbb{G}_0^*) x^2 \rho^* \sigma^* \mathbb{G}_0 x^4 \\
& + 2 \delta(\mathbb{G}_0^*) x^2 \sigma^* \sigma^* \mathbb{G}_0 x^3 \sigma + \delta(\mathbb{G}_0^*) x^* \rho^* \sigma^* \sigma^* \mathbb{G}_0 x^4 + \delta(\mathbb{G}_0^*) x^* \sigma^* \sigma^* \mathbb{G}_0 x^4 \rho - 2 \delta(\rho^*) \mathbb{G}_0^* x^3 \sigma^* \mathbb{G}_0 x^2 \sigma - 2 \delta(\rho^*) \mathbb{G}_0^* x^2 \rho^* \sigma^* \mathbb{G}_0 x^3 \\
& + \delta(\rho^*) \mathbb{G}_0^* x^2 \sigma^* \mathbb{G}_0 x^3 \rho - \delta(\rho^*) \mathbb{G}_0^* x^* \sigma^* \sigma^* \mathbb{G}_0 x^4 - 4 \mathbb{G}_0^* x^2 x^3 \rho^* \sigma^* \mathbb{G}_0 x \sigma^2 + \mathbb{G}_0^* x^2 x^3 \rho^* \mathbb{G}_0^2 x^3 - 4 \mathbb{G}_0^* x^2 x^3 \sigma^* \mathbb{G}_0^2 x \sigma^2 \\
& - 8 \mathbb{G}_0^* x^2 x^3 \sigma^* \mathbb{G}_0 x^3 \sigma - 4 \mathbb{G}_0^* x^2 x^3 \sigma^* \mathbb{G}_0 x \rho \sigma^2 - \mathbb{G}_0^* x^2 x^3 \mathbb{G}_0^2 x^3 \rho - 2 \mathbb{G}_0^* x^2 x^2 \rho^* \sigma^* \mathbb{G}_0 x^2 \sigma + 2 \mathbb{G}_0^* x^2 x^2 \rho^* \sigma^* \mathbb{G}_0 x^2 \rho \sigma \\
& - 4 \mathbb{G}_0^* x^2 x^2 \sigma^* \mathbb{G}_0 x^2 \sigma^2 - 2 \mathbb{G}_0^* x^2 x^2 \sigma^* \mathbb{G}_0^2 x^2 \rho \sigma - 2 \mathbb{G}_0^* x^2 x^2 \rho^* \sigma^* \mathbb{G}_0 x^3 \sigma + 2 \mathbb{G}_0^* x^2 x^2 \sigma^* \mathbb{G}_0^2 x^3 \sigma \\
& - 2 \mathbb{G}_0^* x^2 x^2 \sigma^* \mathbb{G}_0 x^3 \rho \sigma - 2 \mathbb{G}_0^* x^4 \rho^* \sigma^* \sigma^3 - 2 \mathbb{G}_0^* x^4 \rho^* \sigma^* \mathbb{G}_0 \sigma^3 - 12 \mathbb{G}_0^* x^4 \rho^* \sigma^* x^2 \sigma^2 - 4 \mathbb{G}_0^* x^4 \rho^* \sigma^* \rho \sigma^3 - 12 \mathbb{G}_0^* x^4 \sigma^* \mathbb{G}_0 x^2 \sigma^2 \\
& - 2 \mathbb{G}_0^* x^4 \sigma^* \mathbb{G}_0 \rho \sigma^3 - 16 \mathbb{G}_0^* x^4 \sigma^* x^4 \sigma - 12 \mathbb{G}_0^* x^4 \sigma^* x^2 \rho \sigma^2 - 2 \mathbb{G}_0^* x^4 \sigma^* \rho^2 \sigma^3 - \mathbb{G}_0^* x^3 \rho^* \sigma^3 \sigma^* x \sigma^2 - \mathbb{G}_0^* x^3 \rho^* \sigma^2 \sigma^* \mathbb{G}_0 x \sigma^2 \\
& - 4 \mathbb{G}_0^* x^3 \rho^* \sigma^2 \sigma^* x^3 \sigma + 2 \mathbb{G}_0^* x^3 \rho^* \sigma^2 \sigma^* x \rho \sigma^2 - 6 \mathbb{G}_0^* x^3 \rho^* \sigma^2 x \sigma^3 - 2 \mathbb{G}_0^* x^3 \rho^* \sigma^* \mathbb{G}_0 x^3 \sigma - \mathbb{G}_0^* x^3 \rho^* \sigma^* \mathbb{G}_0 x \rho \sigma^2 + 8 \mathbb{G}_0^* x^3 \rho^* \sigma^* x^3 \rho \sigma + \mathbb{G}_0^* x^3 \rho^* \sigma^* x \rho^2 \sigma^2 \\
& - 4 \mathbb{G}_0^* x^3 \sigma^* \mathbb{G}_0 x \sigma^3 - 16 \mathbb{G}_0^* x^3 \sigma^* x^3 \sigma^2 - 6 \mathbb{G}_0^* x^3 \sigma^* x \rho \sigma^3 - 4 \mathbb{G}_0^* x^3 \sigma^* \mathbb{G}_0 x^5 - 4 \mathbb{G}_0^* x^3 \sigma^* \mathbb{G}_0 x^3 \rho \sigma - 4 \mathbb{G}_0^* x^3 \sigma^* \mathbb{G}_0 x \rho^2 \sigma^2 - 4 \mathbb{G}_0^* x^3 \sigma^* x^3 \rho^2 \sigma - 2 \mathbb{G}_0^* x^3 \sigma^* x \rho^3 \sigma^2 \\
& - \mathbb{G}_0^* x^2 \rho^* \sigma^3 \sigma^* \mathbb{G}_0 x^2 \sigma + \mathbb{G}_0^* x^2 \rho^* \sigma^3 \sigma^* x^2 \rho \sigma - 7 \mathbb{G}_0^* x^2 \rho^* \sigma^2 \sigma^* x^2 \sigma^2 - \mathbb{G}_0^* x^2 \rho^* \sigma^2 \sigma^* \mathbb{G}_0 x^2 \rho \sigma - 2 \mathbb{G}_0^* x^2 \rho^* \sigma^2 \sigma^* x^2 \rho^2 \sigma - 5 \mathbb{G}_0^* x^2 \rho^* \sigma^2 \mathbb{G}_0 x^2 \sigma^2 - 12 \mathbb{G}_0^* x^2 \rho^* \sigma^*
\end{aligned}$$

$$\begin{aligned}
& \sigma^2 x^4 \sigma - 6 G_0^* x^2 \rho^* \sigma^2 x^2 \rho \sigma^2 + 2 G_0^* x^2 \rho^* \sigma^* G_0 x^2 \rho^2 * \\
& \sigma + G_0^* x^2 \rho^* \sigma^* x^2 \rho^3 \sigma - 4 G_0^* x^2 \sigma^3 x^2 \sigma^3 - 7 G_0^* x^2 * \\
& \sigma^2 G_0 x^2 \rho \sigma^2 - 12 G_0^* x^2 \sigma^2 x^4 \rho \sigma - 7 G_0^* x^2 \sigma^2 x^2 \rho^2 \sigma^2 - \\
& - 2 G_0^* x^* \rho^3 \sigma^2 x^3 \sigma + 5 G_0^* x^* \rho^2 \sigma^2 G_0 x^3 \sigma + G_0^* x^* * \\
& \rho^2 \sigma^2 x^3 \rho \sigma - 6 G_0^* x^* \rho^* \sigma^3 x^3 \sigma^2 - 2 G_0^* x^* \rho^* \sigma^2 G_0 x^5 + \\
& G_0^* x^* \rho^* \sigma^2 G_0 x^3 \rho \sigma + 2 G_0^* x^* \rho^* \sigma^2 x^3 \rho^2 \sigma + 2 G_0^* x^* * \\
& \sigma^3 G_0 x^3 \sigma^2 - 6 G_0^* x^* \sigma^3 x^3 \rho \sigma^2 - 2 G_0^* x^* \sigma^2 G_0 x^3 \rho^2 \sigma - \\
& G_0^* x^* \sigma^2 x^3 \rho^3 \sigma - 2 G_0^* \rho^2 \sigma^3 x^4 \sigma - 2 G_0^* \rho^* \sigma^3 G_0 x^4 \sigma * \\
& - 4 G_0^* \rho^* \sigma^3 x^4 \rho \sigma - 2 G_0^* \sigma^3 G_0 x^4 \rho \sigma - 2 G_0^* \sigma^3 x^4 \rho^2 * \\
& \sigma) / (G_0^* x^3 G_0 x^3) = 0
\end{aligned}$$

(A.2)

Hence one can directly get $\delta^* \rho$ and $\delta^* G_0$ by trivial algebra. For the shorthand notation Ω see eq. (4.12).

Our last equation here is the result when acting by D on the algebraic expression (4.13). One gets:

$$\begin{aligned}
 & (48 \rho_1^3 \sigma^3 x_1^3 x_2 - 48 \rho_1^3 \sigma^3 x_1 x_2^3 + 32 \rho_1^2 \rho_2 (1/\Omega) \sigma^3 x_1^4 \cdot \\
 & x_2^2 + 32 \rho_1^2 \rho_2 (1/\Omega) \sigma^3 x_1^2 x_2^4 + 8 \rho_1^2 \rho_2 (1/\Omega) \sigma x_1^6 x_2^2 + \\
 & 16 \rho_1^2 \rho_2 (1/\Omega) \sigma x_1^4 x_2^4 + 8 \rho_1^2 \rho_2 (1/\Omega) \sigma x_1^2 x_2^6 - 6 \rho_1^2 \rho_2 \cdot \\
 & \sigma^3 x_1^4 - 12 \rho_1^2 \rho_2 \sigma^3 x_1^2 x_2^2 - 6 \rho_1^2 \rho_2 \sigma^3 x_2^4 - 3 \rho_1^2 \rho_2 \sigma \cdot \\
 & x_1^6 + 15 \rho_1^2 \rho_2 \sigma x_1^4 x_2^2 + 15 \rho_1^2 \rho_2 \sigma x_1^2 x_2^4 - 3 \rho_1^2 \rho_2 \sigma \cdot \\
 & x_2^6 + 16 \rho_1^2 (1/\Omega)^2 \sigma^4 x_1^7 x_2 - 80 \rho_1^2 (1/\Omega)^2 \sigma^4 x_1^5 x_2^3 - 80 \cdot \\
 & \rho_1^2 (1/\Omega)^2 \sigma^4 x_1^3 x_2^5 + 16 \rho_1^2 (1/\Omega)^2 \sigma^4 x_1 x_2^7 + 4 \rho_1^2 (1/\Omega)^2 \cdot \\
 & \sigma^2 x_1^9 x_2 - 16 \rho_1^2 (1/\Omega)^2 \sigma^2 x_1^7 x_2^3 - 40 \rho_1^2 (1/\Omega)^2 \sigma^2 x_1^5 x_2^5 - \\
 & - 16 \rho_1^2 (1/\Omega)^2 \sigma^2 x_1^3 x_2^7 + 4 \rho_1^2 (1/\Omega)^2 \sigma^2 x_1 x_2^9 + 52 \rho_1^2 \cdot \\
 & (1/\Omega) \sigma^4 x_1^5 x_2 - 124 \rho_1^2 (1/\Omega) \sigma^4 x_1^3 x_2^3 + 52 \rho_1^2 (1/\Omega) \sigma^4 x_1 \cdot \\
 & x_2^5 - 4 \rho_1^2 (1/\Omega) \sigma^2 x_1^7 x_2 + 20 \rho_1^2 (1/\Omega) \sigma^2 x_1^5 x_2^3 + 20 \rho_1^2 \cdot \\
 & (1/\Omega) \sigma^2 x_1^3 x_2^5 - 4 \rho_1^2 (1/\Omega) \sigma^2 x_1 x_2^7 + 20 \rho_1^2 \sigma \sigma \sigma \sigma \sigma^2 x_1^3 \cdot \\
 & x_2 + 20 \rho_1^2 \sigma \sigma \sigma \sigma \sigma^2 x_1 x_2^3 + 5 \rho_1^2 \sigma \sigma \sigma \sigma \sigma^2 x_1^5 x_2 + 10 \rho_1^2 \cdot \\
 & \sigma \sigma \sigma \sigma \sigma^2 x_1^3 x_2^3 + 5 \rho_1^2 \sigma \sigma \sigma \sigma \sigma^2 x_1 x_2^5 + 32 \rho_1^2 \sigma^4 x_1^3 x_2^3 + 32 \rho_1^2 \cdot \\
 & \sigma^4 x_1 x_2^5 + 42 \rho_1^2 \sigma^2 x_1^5 x_2 + 58 \rho_1^2 \sigma^2 x_1^3 x_2^3 + 42 \rho_1^2 \sigma^2 \cdot \\
 & x_1 x_2^5 - 16 \rho_1 \rho_2^2 (1/\Omega) \sigma^3 x_1^5 x_2 + 16 \rho_1 \rho_2^2 (1/\Omega) \sigma^3 x_1^3 x_2^5 + \\
 & + 48 \rho_1 \rho_2^2 \sigma^3 x_1^3 x_2 - 48 \rho_1 \rho_2^2 \sigma^3 x_1 x_2^3 - 16 \rho_1 \rho_2^2 \sigma x_1^5 \cdot \\
 & x_2 + 16 \rho_1 \rho_2^2 \sigma x_1 x_2^5 + 2 \rho_1 \rho_2 (1/\Omega)^2 \sigma^2 x_1^{10} - 10 \rho_1 \rho_2 \cdot \\
 & (1/\Omega)^2 \sigma^2 x_1^8 x_2^2 - 12 \rho_1 \rho_2 (1/\Omega)^2 \sigma^2 x_1^6 x_2^4 + 12 \rho_1 \rho_2 \cdot \\
 & (1/\Omega)^2 \sigma^2 x_1^4 x_2^6 + 10 \rho_1 \rho_2 (1/\Omega)^2 \sigma^2 x_1^2 x_2^8 - 2 \rho_1 \rho_2 (1/\Omega)^2 \cdot \\
 & \sigma^2 x_2^{10} - 4 \rho_1 \rho_2 (1/\Omega) \sigma^4 x_1^6 - 4 \rho_1 \rho_2 (1/\Omega) \sigma^4 x_1^4 x_2^2 + 4 \cdot
 \end{aligned}$$

$$\begin{aligned}
& \rho_1 \rho_2 (1/\Omega) \sigma^4 x_1^2 x_2^4 + 4 \rho_1 \rho_2 (1/\Omega) \sigma^4 x_2^6 - 2 \rho_1 \rho_2 (1/\Omega) \sigma^2 x_1^8 + 12 \rho_1 \rho_2 (1/\Omega) \sigma^2 x_1^6 x_2^2 - 12 \rho_1 \rho_2 (1/\Omega) \sigma^2 x_1^2 x_2^6 + \\
& 2 \rho_1 \rho_2 (1/\Omega) \sigma^2 x_2^8 + 3 \rho_1 \rho_2 \sigma_0 \sigma_0^* x_1^6 + 3 \rho_1 \rho_2 \sigma_0 \sigma_0^* x_1^4 x_2^2 - \\
& - 3 \rho_1 \rho_2 \sigma_0 \sigma_0^* x_1^2 x_2^4 - 3 \rho_1 \rho_2 \sigma_0 \sigma_0^* x_2^6 + 4 \rho_1 \rho_2 \sigma^2 x_1^6 + \\
& + 4 \rho_1 \rho_2 \sigma^2 x_1^4 x_2^2 - 4 \rho_1 \rho_2 \sigma^2 x_1^2 x_2^4 - 4 \rho_1 \rho_2 \sigma^2 x_2^6 - \\
& 168 \rho_1 (1/\Omega)^2 \sigma^5 x_1^7 x_2 + 912 \rho_1 (1/\Omega)^2 \sigma^5 x_1^5 x_2^3 - 912 \rho_1 (1/\Omega)^2 \sigma^5 x_1^3 x_2^5 + 168 \rho_1 (1/\Omega)^2 \sigma^5 x_1 x_2^7 + 16 \rho_1 (1/\Omega)^2 \sigma^3 x_1^9 x_2 - \\
& - 96 \rho_1 (1/\Omega)^2 \sigma^3 x_1^7 x_2^3 + 96 \rho_1 (1/\Omega)^2 \sigma^3 x_1^3 x_2^7 - \\
& 16 \rho_1 (1/\Omega)^2 \sigma^3 x_1 x_2^9 + 24 \rho_1 (1/\Omega) \sigma_0 \sigma_0^* \sigma^3 x_1^5 x_2 - 24 \rho_1 (1/\Omega) \sigma_0 \sigma_0^* \sigma^3 x_1 x_2^5 + 3 \rho_1 (1/\Omega) \sigma_0 \sigma_0^* \sigma x_1^7 x_2 + 8 \rho_1 (1/\Omega) \sigma_0 \sigma_0^* \sigma x_1^5 x_2^3 - \\
& - 8 \rho_1 (1/\Omega) \sigma_0 \sigma_0^* \sigma x_1^3 x_2^5 - 8 \rho_1 (1/\Omega) \sigma_0 \sigma_0^* \sigma x_1 x_2^7 + 16 \rho_1 (1/\Omega) \sigma^5 x_1^5 x_2 - 16 \rho_1 (1/\Omega) \sigma^5 x_1 x_2^5 + \\
& 104 \rho_1 (1/\Omega) \sigma^3 x_1^7 x_2 + 52 \rho_1 (1/\Omega) \sigma^3 x_1^5 x_2^3 - 52 \rho_1 (1/\Omega) \sigma^3 x_1^3 x_2^5 - 104 \rho_1 (1/\Omega) \sigma^3 x_1 x_2^7 + 8 \rho_1 (1/\Omega) \sigma x_1^9 x_2 + 16 \rho_1 (1/\Omega) \sigma x_1^7 x_2^3 - \\
& - 16 \rho_1 (1/\Omega) \sigma x_1^3 x_2^7 - 8 \rho_1 (1/\Omega) \sigma x_1 x_2^9 - 48 \rho_1 \sigma_0 \sigma_0^* \sigma^3 x_1^3 x_2 + 48 \rho_1 \sigma_0 \sigma_0^* \sigma^3 x_1 x_2^3 + 16 \rho_1 \sigma_0 \sigma_0^* \sigma x_1^5 x_2 - 16 \rho_1 \sigma_0 \sigma_0^* \sigma x_1 x_2^5 - 144 \rho_1 \sigma^5 x_1^3 x_2 + \\
& 144 \rho_1 \sigma^5 x_1 x_2^3 + 123 \rho_1 \sigma^3 x_1^5 x_2 - 128 \rho_1 \sigma^3 x_1 x_2^5 - 2 \rho_2^3 (1/\Omega) \sigma x_1^8 + 4 \rho_2^3 (1/\Omega) \sigma x_1^4 x_2^4 - 2 \rho_2^3 (1/\Omega) \sigma x_2^8 + 2 \rho_2^3 \sigma^3 x_1^4 + 4 \rho_2^3 \sigma^3 x_1^2 x_2^2 + 2 \rho_2^3 \sigma^3 x_2^4 + \rho_2^3 \sigma x_1^6 - \\
& 5 \rho_2^3 \sigma x_1^4 x_2^2 - 5 \rho_2^3 \sigma x_1^2 x_2^4 + \rho_2^3 \sigma x_2^6 + 132 \rho_2^2 (1/\Omega) \sigma^4 x_1^5 x_2 - 276 \rho_2^2 (1/\Omega) \sigma^4 x_1^3 x_2^3 + 132 \rho_2^2 (1/\Omega) \sigma^4 x_1 x_2^5 - 16 \rho_2^2 (1/\Omega) \sigma^2 x_1^7 x_2 + 16 \rho_2^2 (1/\Omega) \sigma^2 x_1^5 x_2^3 + 16 \rho_2^2 (1/\Omega) \sigma^2 x_1^3 x_2^5 - 16 \rho_2^2 (1/\Omega) \sigma^2 x_1 x_2^7 -
\end{aligned}$$

$$\begin{aligned}
& 4\rho_2^2 \theta_{00} \theta_{00}^* \sigma^2 x_1^3 x_2 - 4\rho_2^2 \theta_{00} \theta_{00}^* \sigma^2 x_1 x_2^3 - \rho_2^2 \theta_{00} \theta_{00}^* \sigma^5 x_1 x_2 - 2\rho_2^2 \theta_{00} \theta_{00}^* \sigma^3 x_1^3 x_2^3 - \rho_2^2 \theta_{00} \theta_{00}^* \sigma^5 x_1 x_2^5 - 34\rho_2^2 \sigma^2 x_1^5 x_2 - 42\rho_2^2 \sigma^2 x_1^3 x_2^3 - \\
& 34\rho_2^2 \sigma^2 x_1 x_2^5 + 2\rho_2 (1/\Omega) \theta_{00} \theta_{00}^* \sigma x_1^8 - 4\rho_2 (1/\Omega) \theta_{00} \theta_{00}^* \sigma x_1^4 x_2^4 + \\
& 2\rho_2 (1/\Omega) \theta_{00} \theta_{00}^* \sigma x_2^8 + 2\rho_2 (1/\Omega) \sigma^3 x_1^8 - 4\rho_2 (1/\Omega) \sigma^3 x_1^4 x_2^4 + \\
& 2\rho_2 (1/\Omega) \sigma^3 x_2^8 + 4\rho_2 (1/\Omega) \sigma x_1^{10} + 4\rho_2 (1/\Omega) \sigma x_1^8 x_2^2 - 8\rho_2 (1/\Omega) \sigma x_1^6 x_2^4 - \\
& - 8\rho_2 (1/\Omega) \sigma x_1^4 x_2^6 + 4\rho_2 (1/\Omega) \sigma x_1^2 x_2^8 + 4\rho_2 (1/\Omega) \sigma x_2^{10} - \\
& 2\rho_2 \theta_{00} \theta_{00}^* \sigma^3 x_1^4 - 4\rho_2 \theta_{00} \theta_{00}^* \sigma^3 x_1^2 x_2^2 - 2\rho_2 \theta_{00} \theta_{00}^* \sigma^3 x_2^4 - \rho_2 \theta_{00} \theta_{00}^* \sigma x_1^6 + \\
& 5\rho_2 \theta_{00} \theta_{00}^* \sigma x_1^4 x_2^2 + 5\rho_2 \theta_{00} \theta_{00}^* \sigma x_1^2 x_2^4 - \rho_2 \theta_{00} \theta_{00}^* \sigma x_2^6 - 2\rho_2 \sigma^5 x_1^4 - \\
& 4\rho_2 \sigma^5 x_1^2 x_2^2 - 2\rho_2 \sigma^5 x_2^4 - 5\rho_2 \sigma^3 x_1^6 - 7\rho_2 \sigma^3 x_1^4 x_2^2 - 7\rho_2 \sigma^3 x_1^2 x_2^4 - \\
& 5\rho_2 \sigma^3 x_2^6 - 2\rho_2 \sigma x_1^8 + 8\rho_2 \sigma x_1^6 x_2^2 + 20\rho_2 \sigma x_1^4 x_2^4 + 8\rho_2 \sigma x_1^2 x_2^6 - \\
& 2\rho_2 \sigma x_2^8 - 132 (1/\Omega) \theta_{00} \theta_{00}^* \sigma^4 x_1^5 x_2 + 276(1/\Omega) \theta_{00} \theta_{00}^* \sigma^4 x_1^3 x_2^3 - \\
& 132(1/\Omega) \theta_{00} \theta_{00}^* \sigma^4 x_1 x_2^5 + 16(1/\Omega) \theta_{00} \theta_{00}^* \sigma^2 x_1^7 x_2 - 16(1/\Omega) \theta_{00} \theta_{00}^* \sigma^2 x_1^5 x_2^3 - \\
& - 16(1/\Omega) \theta_{00} \theta_{00}^* \sigma^2 x_1^3 x_2^5 + 16(1/\Omega) \theta_{00} \theta_{00}^* \sigma^2 x_1 x_2^7 - 132(1/\Omega) \sigma^6 x_1^5 x_2^3 + \\
& 276(1/\Omega) \sigma^6 x_1^3 x_2^5 - 132(1/\Omega) \sigma^6 x_1 x_2^7 - 248(1/\Omega) \sigma^4 x_1^7 x_2 + \\
& 272(1/\Omega) \sigma^4 x_1^5 x_2^3 + 272(1/\Omega) \sigma^4 x_1^3 x_2^5 - 248(1/\Omega) \sigma^4 x_1 x_2^7 + \\
& 32(1/\Omega) \sigma^2 x_1^9 x_2 - 64(1/\Omega) \sigma^2 x_1^5 x_2^5 + 32(1/\Omega) \sigma^2 x_1 x_2^9 + 4\theta_{00} \theta_{00}^* \sigma^2 x_1^3 x_2^3 + \\
& + 4\theta_{00} \theta_{00}^* \sigma^2 x_1 x_2^3 + \theta_{00} \theta_{00}^* \sigma^2 x_1^5 x_2 + 2\theta_{00} \theta_{00}^* \sigma^2 x_1^3 x_2^3 + \theta_{00} \theta_{00}^* \sigma^2 x_1 x_2^5 + \\
& 4\theta_{00} \theta_{00}^* \sigma^4 x_1^3 x_2 + 4\theta_{00} \theta_{00}^* \sigma^4 x_1 x_2^3 + 43\theta_{00} \theta_{00}^* \sigma^2 x_1^5 x_2 + 60\theta_{00} \theta_{00}^* \sigma^2 x_1^3 x_2^3 + \\
& 43\theta_{00} \theta_{00}^* \sigma^2 x_1 x_2^5 + 2\theta_{00} \theta_{00}^* \sigma^7 x_1 x_2 + 6\theta_{00} \theta_{00}^* \sigma^5 x_1^3 x_2^3 + 6\theta_{00} \theta_{00}^* \sigma^5 x_1 x_2^5 + \\
& 2\theta_{00} \theta_{00}^* \sigma^7 x_1 x_2 + 34\sigma^4 x_1^5 x_2 + 42\sigma^4 x_1^3 x_2^3 + 34\sigma^4 x_1 x_2^5 + 68\sigma^2 x_1^7 x_2 + \\
& 152\sigma^2 x_1^5 x_2^3 + 152\sigma^2 x_1^3 x_2^5 + 68\sigma^2 x_1 x_2^7) = 0
\end{aligned}$$

(A.3)

In this form of the equation $\theta_{00} \theta_{00}^*$ is not completely eliminated. It could be done by using again eq. (4.13), but it would be of little help to get a handy expression.

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