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CHIRAL SYMMETRY BREAKING IN QED_3: BIFURCATION OF THE FERMIONIC SELF-ENERGY

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ABSTRACT

We study the existence of a bifurcation point in the Schwinger-Dyson equation of 2+1 dimensional quantum electrodynamics with N fermions. We find evidence for the existence of a critical behavior, such that chiral symmetry breaking may occur only for a small number of flavors.

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The importance of chiral symmetry breaking (χSB) for the strong interaction is largely known, and it is through this mechanism that the quarks obtain their effective mass. It is also expected that this phenomenon occurs only when the coupling constant of the theory is larger than a certain critical value, above which the fermionic self-energy bifurcates into a non-trivial solution.

A series of studies of the bifurcation of the quark self-energy has been done by Atkinson et al[1-3], and here we will apply some of their ideas to 2+1 dimensional quantum electrodynamics (QED_3) with N fermions.

QED_3 is a super-renormalizable gauge theory, which mimics some of the main features of QCD;it has a coupling constant (e^2) with dimension of mass, offering a natural scale to which most of the dynamics can be related; and working in the large N limit we can sum sets of diagrams under controlable approximations, going beyond perturbative results[4]. The techniques of refs. (1-3) will allow us to determine the existence, or not, of a bifurcation point in the Schwinger-Dyson equation of the fermionic self-energy, in particular we may verify if chiral symmetry breaking occurs for any value of N. If this is not the case (as will be shown) the use of the large N expansion to study χSB in this theory may not be consistent.

The massless QED_3 lagrangian density is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \sum_{j=1}^{N} \bar{\psi}_j \gamma^{\nu} (i\, \partial_{\nu} - e A_{\nu}) \psi_j \quad , \qquad (1)$$

which has a global chiral symmetry U(2N), and a mass term $m\bar{\psi}\psi$ would break this symmetry to U(N)xU(N). The gauge boson propagator in Landau gauge is

$$D_{\mu\nu}(k) = \frac{g_{\mu\nu} - k_{\mu}k_{\nu}/k^2}{k^2\, [1 + \Pi(k)]} \quad , \qquad (2)$$

where, to leading order in 1/N expansion, $\Pi(k)$ is given by

$$\Pi(k) = \frac{\alpha}{k} \quad , \qquad (3)$$

with $\alpha = e^2N/8$ fixed. The inverse fermionic propagator is

$$S^{-1}(p) = -\gamma_{\mu}\, p^{\mu}\, [1 + A(p)] + \tilde{\Sigma}(p) \quad , \qquad (4)$$

where A(p) is the wave function renormalization, which is perturbatively generated, and can be neglected in leading order of 1/N. Using the lowest order vertex $\Gamma^{\mu} \approx \gamma^{\mu}$, the Schwinger-Dyson gap equation of the fermionic propagator (after angular integration) is[4]

$$\Sigma(x) = \frac{4}{\pi^2 N x} \int_0^{\infty} dy\, \frac{y\, \Sigma(y)}{y^2 + \Sigma(y)^2}\, \mathrm{Ln}\left[\frac{x+y+1}{|x-y|+1}\right] \quad , \qquad (5)$$

where the variables $x=p/\alpha$, $y=k/\alpha$ and $\Sigma(p)=\tilde{\Sigma}(p)/\alpha$ were used.

If equation (5) has a bifurcation point it means that there is a critical value N_c, such that a non-trivial solution $\Sigma(x)$ exists only when $N \le N_c$. According to ref.(1), to find non-trivial small solutions of the non-linear equation (5), we study the linearized equation, i.e., the functional derivative of (5) evaluated at $\Sigma=0$[1]. Writing $\delta\Sigma(x)=f(x)$, we obtain

$$f(x) = \frac{4}{\pi^2 N} \int_0^\infty dy \, \frac{f(y)}{xy} \, \mathrm{Ln} \left[\frac{x+y+1}{|x-y|+1}\right] \qquad (6)$$

This is a Fredholm equation and if its kernel is L^2 (square integrable) there is a discrete spectrum solution. The smallest eigenvalue for which (6) has a non-trivial solution is the first bifurcation point of the non-linear problem. Moreover, if the kernel is symmetric we have the condition

$$1 \le \frac{4}{\pi^2 N_c} \, \| K \| \qquad , \qquad (7)$$

with

$$\| K \|^2 = \int_0^\infty dx \int_0^\infty dy \; K(x,y)^2 \qquad , \qquad (8)$$

where $K(x,y)$ is the kernel of the integral equation. The kernel of equation (6) is symmetric but it is not L^2 due to the infrared divergences.

We can study the possible solution of equation (6) even in the case when the kernel is not L^2. If we remenber that x and y are defined as p/α and k/α, and the integral is rapidly damped for momenta larger than α, we can make the following approximation in equation (6):

$$f(x) = \frac{4}{\pi^2 N} \int_0^\infty dy \, \frac{f(y)}{xy} \left[(x+y) - |x-y| \right] \quad . \tag{9}$$

Defining a new function $\sigma(x)=f(x)/\sqrt{x}$ eq. (9) is reduced to

$$\sigma(x) = \frac{8}{\pi^2 N} \left\{ \int_0^\infty dy \, \frac{\sigma(y)}{\sqrt{xy}} + \int_0^\infty dy \, \sqrt{xy} \, \sigma(y) \right\} \quad , \tag{10}$$

and introducing new variables $u=\ln\sqrt{x}$ and $v=\ln\sqrt{y}$, we finally arrive to

$$\sigma(u) = 2\lambda \int_0^\infty dv \, e^{-|u-v|} \, \sigma(v) \quad , \tag{11}$$

which is knwon as the Lalesco-Picard equation[5] ,and where $\lambda=8/\pi^2 N$. This equation has the general solution

$$\sigma(u) = A \, e^{\sqrt{1-4\lambda}\, u} + B \, e^{-\sqrt{1-4\lambda}\, u} \tag{12}$$

and it exists for any value of λ real and positive! The proof that (11) has no bifurcation point can be seen in Tricomi's book (ref. 5). If a cutoff is introduced into eq.(6) it is possible that our conclusion does not hold (see, for instance, the second work of ref. 4).

The linearization we have performed is not reliable for the infrared limit of eq. (5). As pointed out by Atkinson and Johnson[6], a much better approach would be the substitution of

$y^2+\Sigma(y)^2$ in the denominator of Eq. (5) by $y^2+\Sigma(0)^2$. Expanding the logarithm in eq. (5) and retaining the leading terms we obtain

$$\Sigma(x) = \frac{6}{\pi^2 N x}\left\{\int_0^p dy\,\frac{y\,\Sigma(y)}{y^2+\Sigma(y)^2}\,\frac{x}{y+1} + \int_p^\infty dy\,\frac{y\,\Sigma(y)}{y^2+\Sigma(y)^2}\,\frac{y}{x+1}\right\} . \qquad (13)$$

This integral equation is equivalent to the following differential equation

$$\Sigma(x)'' + \frac{2}{x}\,\Sigma(x)' + \lambda\,\frac{\Sigma(x)}{x^2+\Sigma(0)^2} = 0 \quad . \qquad (14)$$

where

$$\lambda = \frac{8}{\pi^2 N} \quad , \qquad (15)$$

and whose solution is

$$\Sigma(x) = \Sigma(0)\;{}_2F_1\left(\frac{1}{4}+\frac{1}{4}\gamma,\frac{1}{4}-\frac{1}{4}\gamma;\frac{3}{2};-\frac{x^2}{\Sigma(0)^2}\right) \quad , \qquad (16)$$

where $\gamma=\sqrt{1-4\lambda}$.

Notice that eq. (14) is identical to eq. (3.3) of ref. (6), and solution (16) exists only for $\lambda > 1/4$ [F1] ! Therefore we see that there is a critical value N_c above which there is not chiral symmetry breaking

$$\lambda > \frac{1}{4} \quad \Rightarrow \quad N < \frac{32}{\pi^2} \quad . \qquad (17)$$

Evidence for the existence of such critical value of N was also found in the analysis of an effective potential for composite operators[7], and such small number cast some doubts in the consistency of the 1/N expansion for the study of chiral symmetry breaking through the use of the Schwinger-Dyson equation. It must also be noticed that the linearization around $\Sigma(0)$ was necessary to obtain this result, and this is a much better approximation to the actual non-linear solution[8].

Finally, it is interesting to investigate if with an improved ansatz for the vertex function the above result will be modified. Such approach has been pursued by Atkinson in the case of QCD[9], and here we recall that the behavior of the theory for all momenta is best described in terms of dimensionless running coupling constant

$$\bar{\alpha}(p) = \frac{\alpha}{p\,[1+\Pi(p)]} \qquad . \qquad (18)$$

Hence, we adopt the ansatz

$$\Gamma^{\mu}(k,p) = \gamma^{\mu}\,\frac{1}{1+\alpha/\beta(k,p)} \qquad , \qquad (19)$$

where $\beta(k,p) = \max(k,p)$. Introducing (19) into equation (6), this last equation will be reduced to

$$f(x) = \frac{4}{\pi^2 N}\int_0^{\infty} dy\,\frac{f(y)}{xy[1+1/\beta(x,y)]}\,\mathrm{Ln}\left[\frac{x+y+1}{|x-y|+1}\right] \quad . \qquad (20)$$

Expanding the logarithmic function as we did to arrive at eq.(13), we obtain an integral with a L^2 symmetric kernel, well behaved in the infrared and ultraviolet regions, for which we can compute the expression (8). Consequently we verify that condition (7) leads to the constraint

$$N \leq \frac{8}{\sqrt{3}\,\pi^2} \tag{21}$$

which, again, is a quite small number and even more restrictive than condition (17).

In conclusion, we have seen, using very simple methods, that the Schwinger-Dyson equation for the fermionic self-energy of QED_3 with N fermions shows a critical behavior , with non-trivial solutions for the dynamical mass appearing only for small values of fermion flavors. This result persisted even when we used an improved vertex function in the gap equation. Such result cast doubts in the consistency of the 1/N expansion as a tool to study chiral symmetry breaking in QED_3, and a deeper study of this behavior, where the full non-linearity of the gap equation is attacked remains yet to be done.

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