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**PHENOMENOLOGICAL STUDY OF**  
**THE POLARIZED LEPTO PRODUCTION**

**P. Chiappetta<sup>\*)</sup> and G. Girardi**  
LAPP, BP. 110, 74941 Annecy-le-Vieux Cedex

**ABSTRACT**

In this note we make a phenomenological study of the recent EMC data on polarized structure function  $g_1^P(x)$ . Using general principles like Regge behaviour and positivity we derive simple parametrizations for quark and gluon polarized distribution function and obtain a consistent description of experimental data.

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<sup>\*)</sup> Permanent address Centre de Physique Théorique, CNRS Luminy, Case 907, F-13288 Marseille Cedex 9.

The measurement of the polarized proton structure function in deep inelastic lepton scattering by the EMC group<sup>1)</sup> has triggered a renewal of interest on an old subject, the spin of the proton and how it is shared between its constituents. The interest of this measurement is that it covers very low values of  $x$ , allowing a reliable calculation of the first moment,  $M_1^P$ , of the spin dependent structure function  $g_1^P(x)$ . At moderate  $x$  values, the measured values of  $g_1^P(x)$  and of  $M_1^P$  coincide with previous SLAC results<sup>2)</sup>, giving confidence on the reality of the effect. One has:

$$M_1^P(Q^2) = \int_0^1 dx g_1^P(x, Q^2) = 0.126 \pm 0.010 \pm 0.015 \quad (1)$$

in violent disagreement with the Ellis-Jaffe<sup>3)</sup> sum rule which states that  $M_1^P \sim 0.19$ , based on SU(3) flavor and the assumption that  $s$  quarks do not carry net spin.

$M_1^P$  is related through operator product expansion to the forward matrix element of the axial vector current between proton states<sup>4)</sup>, more precisely

$$M_1^P = \frac{1}{2} \left( \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right) \quad (2)$$

where

$$\Delta q = \int_0^1 dx [q_+(x) - q_-(x) + \bar{q}_+(x) - \bar{q}_-(x)] \sim \langle P \text{ pol} | \bar{q} \gamma_0 \gamma_5 q | P \text{ pol} \rangle \quad (3)$$

The indices  $\pm$  refer to situations where the quark spin is parallel or antiparallel to the proton spin. Isospin invariance and SU(3) flavor symmetry give us simple relations for the non-singlet components of  $M_1^P$ <sup>5)</sup>:

$$\Delta u - \Delta d = g_A = F + D \quad (4)$$

$$\Delta u + \Delta d - 2\Delta s = 3F - D \quad (5)$$

These relations together with the values  $F = 0.477 \pm 0.011$  and  $D = 0.755 \pm 0.011$ , obtained from hyperon decays<sup>6)</sup>, give

$$\Delta u + \Delta d + \Delta s \approx 0.00 \pm 0.24 \quad (6)$$

that is quarks carry none (or nearly so) of the proton spin, a rather surprising result in view of other successes of the naive quark model. In particular the measured value of  $M_1^P$  implies that s quarks give a rather large negative contribution<sup>7)</sup>

$$\Delta s = -0.23 \pm 0.08 \quad (7)$$

These observations have motivated many speculations<sup>8,9)</sup>, an interesting one being the description of the nucleon as a skyrmion, most of its spin being due to gluons and angular momentum<sup>9)</sup>.

However this is not all the story because it was soon realised<sup>10)</sup> that because of the axial U(1) anomaly, the matrix elements of the axial current receive a gluon contribution, such that what one calls  $\Delta q$  is in fact

$$\Delta \tilde{q} = \Delta q - \frac{\alpha_s}{2\pi} \Delta g \quad (8)$$

$\Delta q$  is the true quark contribution to the proton spin and

$$\Delta g = \int_0^1 dx [g_{+1}(x, Q^2) - g_{-1}(x, Q^2)] \quad (9)$$

is the contribution of helicity  $\pm 1$  gluons to the proton spin. It is clear that (4) and (5) are insensitive to this contribution, which modifies drastically eq.(2); indeed, including 2-loop QCD corrections<sup>11)</sup>, it reads now

$$M_1^P = \frac{1}{18} \left(1 - \frac{\alpha_s}{\pi}\right) \left(4\Delta u + \Delta d + \Delta s - 6 \frac{\alpha_s}{2\pi} \Delta g\right) \quad (10)$$

Using (4) and (5) one obtains

$$M_1^P = \frac{1}{18} \left(1 - \frac{\alpha_s}{\pi}\right) (9F - D + 6\Delta s - 6\Delta \Gamma) \quad (11)$$

with  $\Delta \Gamma = \frac{\alpha_s}{2\pi} \Delta g$ , which in view of the measured values entails

$$\Delta \tilde{s} = \Delta s - \Delta \Gamma \sim -0.20 \pm 0.08 \quad (12)$$

which agrees with the results of  $\bar{\nu}_p$  scattering at low energy indicating<sup>12)</sup>

$$\Delta \tilde{s} = -g_A \eta, \quad \eta = 0.12 \pm 0.07. \quad (13)$$

With these informations on the integrals of polarisation distributions we can look at  $g_1^P(x)$  itself and also at  $\int_x^1 dx g_1^P(x)$ , which were presented in ref.1).

Defining

$$\delta Q(x) = q_+(x) - q_-(x) + \bar{q}_+(x) - \bar{q}_-(x) \quad (14)$$

the function  $g_1^P(x)$  can be cast under the following form:

$$g_1^P(x) = \frac{1}{18} \left( 1 - \frac{\alpha_s}{\pi} \right) [4\delta U(x) + \delta D(x) + \delta S(x) - 6 \delta \Gamma(x)] \quad (15)$$

where  $\delta U(x)$  and  $\delta D(x)$  receive contributions from the valence and sea quarks, for these latters we assume

$$\delta(u_s + \bar{u})(x) = \delta(d_s + \bar{d})(x) = A\delta S(x) \quad (16)$$

$\delta S(x)$  contains both quarks and antiquarks and  $A$  is a constant which takes values in the range 1 (SU(3) symmetric case) to 2.5<sup>13)</sup>, this yields

$$g_1^P(x) = \frac{1}{18} \left( 1 - \frac{\alpha_s}{\pi} \right) [4\delta u_v(x) + \delta d_v(x) + (5A+1)\delta S(x) - 6 \delta \Gamma(x)] \quad (17)$$

The explicit forms of  $\delta q(x)$  and  $\delta \Gamma(x)$  are determined in the following way :  $q_{\pm}(x)$  and  $g_{\pm}(x)$  which represent the number distribution of partons with helicity aligned or opposite to the nucleon spin are positive quantities. They are related to the unpolarized and polarized distribution by

$$\begin{aligned} q(x) &= q_+(x) + q_-(x) \\ \delta q(x) &= q_+(x) - q_-(x) \end{aligned} \quad (18)$$

The functions  $q(x)$  being measured, a number of parametrizations exist which reproduce fairly well experimental results. For definiteness we take the forms proposed by M. Diemoz et al<sup>13)</sup>, namely for valence quarks

$$x u_v(x) = 0.237 x^{0.38} (1-x)^{2.11} [1 - 17.724(1-x) - 9.993(1-x)^2 - 4.439(1-x)^3] \quad (19)$$

$$d_v(x) = 0.574(1-x) u_v(x) \quad (20)$$

and for the sea and the gluons:

$$xs(x) = x\bar{s}(x) = 0.4x\bar{u}(x) = 0.1(1-x)^{8.5} (1 - 4.18x + 20.3x^2 - 15.3x^3) \quad (21)$$

$$xg(x) = 3.34 (1-x)^{5.06} (1 - 0.177x) \quad (22)$$

which corresponds to the choice  $A = 2.5$ . We have checked that our results are unaffected if we use different parametrizations of the unpolarized structure functions that exist in the literature.

As far as the polarized distributions are concerned we propose to cast them under the simple form

$$\delta q(x) = C_q \frac{x^{\alpha_q} (1-x)^{\beta_q}}{B(\alpha_q+1, \beta_q+1)} \quad (23)$$

where the normalizing factor  $B(\alpha_q+1, \beta_q+1)$ , the Euler function, takes into account the  $x$  dependence whilst  $C_q$  is determined using (4) and (5), namely

$$C_{u_v} = 2F - (A-1) \Delta s \quad (24)$$

$$C_{d_v} = F - D - (A-1) \Delta s \quad (25)$$

The exponent  $\alpha_q$  is the same for all flavors including gluons and estimated from the Regge behaviour at low  $x$  values which is, here, governed by the non natural parity trajectory<sup>14</sup>) of the  $a_1(1270)$ . One expects

$$\delta q(x) \underset{x \rightarrow 0}{\sim} x^{-\alpha_R(0)} \quad (26)$$

where  $\alpha_R(0)$  is the intercept of the trajectory which is estimated to be<sup>15</sup>)

$$\alpha_R(0) = -0.14 \pm 0.20 \quad (27)$$

There are a priori many parameters at hand and one surely could find a set of values such that we could fit very accurately the data on  $g_1^P(x)$  and  $\int_x^1 g_1^P(x) dx$ . However our aim is

to show that general principles such as Regge behaviour at low  $x$  and positivity constraints allow for a good description of the data with reasonable values of the parameters in the game. For instance we kept  $\alpha_R(0) = -\alpha_q = -0.14$  fixed for all flavors, though a better fit could be obtained for larger values as allowed by (27) -  $\alpha_R(0) \sim +0.07$ . In the first place, we studied the valence quark distributions (polarized and unpolarized) for four different values of  $\Delta s (= 0.02, -0.05, -0.12, -0.20)$ , which is the relevant parameter to fix the normalizations according to eq.(24-25). Then we seek for the lowest values of  $\beta_{u_v}$  ( $\beta_{d_v} = \beta_{u_v} + 1$ ) which

make  $u_{v\pm}(x)$  and  $d_{v\pm}(x)$  positive everywhere. We obtain  $\beta_{uv} \geq 3.05$  which is close to our expectations based on naive constituent counting rules, except for  $\Delta_s = -0.20$  which requires  $\beta \geq 10$ , which is unnatural.

The same method using positivity constraints can be applied for determining  $\beta_s$  and  $\beta_g$ . However, to achieve this goal, we have to scan over the values of  $\Delta_s$  and  $\Delta_g$  which are tied together through the values of  $M_1^P$ . The outcome of this study is shown in the tables below, where it appears that  $\beta_s$  and  $\beta_g$  assume reasonable values except when  $M_1^P$  reaches its lower bound and/or  $\Delta_s = -0.20$ . These circumstances are also illustrated in the predicted behaviour of  $g_1^P(x)$  and  $\int_x^1 g_1^P(x) dx$  which is shown in the figures (The labels A,B,C refer to the different values of  $M_1^P$  respectively : .151, .126, .101 ). In particular for  $M_1^P = 0.101$ ,  $g_1^P(x)$  becomes sizeably negative at low  $x$  values in disagreement with the data, also for  $M_1^P = 0.151$  one notices that  $\int_x^1 g_1^P(x) dx$  is everywhere above measured values. Nevertheless we

think that our results are satisfactory since we have not attempted to fit the data but to show that it is possible to describe the data starting from simple guiding principles.

It is clear from this study that  $\Delta_s$  cannot be too negative ( $< -0.20$ ) and that the gluon contribution although positive in most cases could be marginally negative. However if  $\Delta\Gamma = -0.16 \pm 0.08$  as calculated recently<sup>16)</sup>, one would obtain  $\Delta_s$  in the range  $-0.26, -0.42$ , values for which no realistic parameters can be obtained. Although it is obvious that a confirmation of the EMC measurement with a more precise determination of  $g_1^P(x)$  at low  $x$  is needed, other experiments are necessary to obtain full information on the nucleon spin. The measurement of  $g_1^n(x)$  is crucial for testing the Bjorken's sum rule, lepton pair production from polarized proton proton collisions offers an opportunity to measure the size and the sign of  $\Delta_s$ <sup>17)</sup>. As far as  $\Delta_g$  is concerned, photoproduction of heavy quark pairs via polarized photon-gluon fusion should provide useful information<sup>18)</sup>. If  $\Delta_g$  is large, its effect is expected to enhance the high  $p_\perp$  jet production in polarized deep inelastic scattering<sup>19)</sup>.

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$M_1^P$ ↓	$\Delta s = +0.02$ $\beta_s = 8.7$	$\Delta s = - 0.05$ $\beta_s = 8.7$	$\Delta s = - 0.12$ $\beta_s = 10.5$
0.151	$\Delta g = 2.96$ $\beta_g = 8.5$	$\Delta g = 1.2$ $\beta_g = 6.0$	$\Delta g = - 0.56$ $\beta_g = 5.5$
0.126	$\Delta g = 5.0$ $\beta_g = 15.$	$\Delta g = 3.24$ $\beta_g = 9.5$	$\Delta g = 1.48$ $\beta_g = 6.5$
0.101	values undetermined	$\Delta g = 5.3$ $\beta_g = 17.$	$\Delta g = 3.5$ $\beta_g = 10.$

Table : Summary of the lowest values of  $\beta_s$  and  $\beta_g$  satisfying positivity constraints



### Figure captions

Figs. 1 a – b – c :  $g_1^P(x)$  versus  $x$ .

The three figures correspond to different values of  $\Delta s = + 0.02, - 0.05, - 0.12$   
Curves A, B, C correspond to different values of  $M_1^P = 0.151, 0.126, 0.101$ . Data points  
are from ref.1 (squares) and from ref.2 (diamonds and stars )

Figs. 2 a – b – c :  $\int_x^1 g_1^P(x) dx$  versus  $x$ .

The presentation is the same as in Fig. 1



