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ABSTRACT

The formulation of a relativistic theory of statevector reduction is proposed and analyzed, and its conceptual consequences are elucidated. In particular, a detailed discussion of stochastic invariance and of local and nonlocal aspects at the level of individual systems is presented.

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1. INTRODUCTION

"One wants to be able to take a realistic view of the world, to talk about the world as if it is really there, even when it is not being observed .... our business is to try to find out about it, and the technique for doing that is indeed to make models and to see how far we can go with them in accounting for the real world" [1]. No one has done more than John Bell to advance and encourage this program of the pursuit of reality. With gratitude we dedicate this paper to him, and go about "our business".

The essential wave aspect of natural phenomena has been made evident by the remarkable success of quantum theory. However the[2] *"indefiniteness, the waviness of the wave function"* is quite difficult to reconcile with the *"definiteness, the particularity of the world of experience"* at the macroscopic level.

The orthodox solution to this so called quantum measurement problem is very well known: instead of statements about properties possessed by physical systems, statements are made about probabilities of getting results if a measurement is performed. However, as repeatedly stressed by Bell[3] there are two fundamental difficulties with this point of view:

i. The need for a classical base which cannot be consistently derived from quantum principles,

ii. The fundamentally shifty character of the level at which one chooses to place the transition from the quantum to the classical, from the small to the large.

The desire to overcome these difficulties motivates the construction of dynamical reduction models, which are based on the following idea. By accepting a "small" modification of the dynamics of quantum theory, one attempts to build up a scheme in which microscopic systems fully exhibit their wave aspects while the macroscopic ones, on the contrary, behave as localized objects in accordance with our perceptions, as a consequence of a single fundamental dynamics governing all phenomena.

This idea has been pursued for some time [4-9] and has recently obtained a particularly simple and effective formulation[10-11] through the introduction of a model (referred to as QMSL-Quantum Mechanics with Spontaneous Localization) in which the wave function is supposed to be subjected to spontaneous collapses corresponding to localizations with an appropriate frequency and scale. It has been shown[11-13] that such a model does not conflict with any known fact about microsystems and leads, in the case of macro-objects, in an extremely short time to states corresponding to definite positions, thus meeting Einstein's requirement (in Pauli's words[14]) that:

"a macro-body must always have a quasi-sharply-defined position in the 'objective description of reality'"

J. Bell[2] in explicating QMSL, immediately identified two aspects of it which required further investigations:

a. The model does not respect the symmetry requirements for systems of identical particles

b. The introduction of the localizations assigns a special role to position and requires a smearing on space, which makes quite problematic to find a relativistic generalization of it.

The first difficulty has been overcome quite recently by the introduction of a dynamical reduction model (referred to as CSL- Continuous Spontaneous Localization), in which the sudden localizations of QMSL have been replaced by a continuous stochastic evolution of the state vector.[15-16]

Steps toward a solution of the second problem, have been made recently with the introduction of a relativistic CSL model[17], in which localizations for fermions are induced by a reducing dynamics for a virtual meson field coupled to the fermions.

In the present paper we consider the problem of describing relativistic dynamical reduction mechanisms. We mainly focus our attention on setting up a general framework for such a theory, by discussing in detail how one has to impose invariance requirements within a stochastic quantum field theoretic scheme.

To summarize, and to underscore the interest of such considerations we can do no better than quote John Bell[18]:

"Now in my opinion the founding fathers were in fact wrong on this point. The quantum phenomena do not exclude a uniform description of micro and macro worlds .... system and apparatus. It is not essential to introduce a vague division of the world of this kind .... But another problem is brought into focus .... I think any sharp formulation of quantum mechanics has a very surprising feature: the consequences of events at one place propagate to other places faster than light .... For me this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory .... It may be that a real synthesis of quantum and relativity theories requires not just technical developements but radical conceptual renewal."

In Section 2 we review the CSL theory, which achieves its results through a nonhermitian randomly fluctuating potential. At the same time we discuss an example of ordinary quantum theory with a hermitian randomly fluctuating potential. Comparison shows that the statistical operator for both theories obeys identically the same evolution equation, resulting in the decay of off-diagonal elements of the statistical operator in the position representation. A number of authors [19-21] have regarded such behaviour of the statistical operator as a satisfactory resolution of the measurement problem. We do not: we regard such behaviour as a necessary, but not sufficient, condition. One must also look at the behaviour at the level of the individual state vector. Unlike the CSL case, in the hermitian case each individual state vector remains forever in a superposition of macroscopically distinguishable states: the off-diagonal elements of the statistical operator disappear just because of the increasing randomization of phase factors mutiplying these states.

In Section 2 we also discuss the nature of Galilean invariance in these theories. A particular sample of the fluctuating field certainly does not look the same in all inertial reference frames. However, the statistical behaviour of the fluctuating field, and the corresponding statistical distribution of the results of identical experiments is the same in all inertial reference frames. We call this stochastic Galilean invariance, and this discussion paves the way for Section 3 in which we consider stochastic Lorentz invariant models.

These models are quantum field theories, with the field locally coupled to a scalar function with a relativistically invariant white noise distribution. We first discuss stochastic Lorentz invariance in the more familiar context of a hermitian coupling term. Upon considering a nonhermitian coupling, which is introduced in order to obtain the desired behaviour at the level of the individual statevector, we are led to employ the Tomonaga-Schwinger formalism; this permits discussion of relativistic transformation properties and of stochastic Lorentz invariance in the context of arbitrary space-like surfaces. The formalism is then specialized to the example already considered in ref.(17). In this case, new divergences emerge besides the ones which are familiar from standard quantum field theory, having their origin in the white noise nature of the stochastic processes which are considered. In a recent paper J.Bell[22] calls attention to Dirac's division of the difficulties of quantum mechanics into first and second class ones (i.e. those connected with wave packet reduction and the infinities of quantum field theories, respectively); we note that this present attempt to generalize models which allow one to overcome first class difficulties seems to lead to an increase of those of the second class.

Section 4 is devoted to a discussion of the local and nonlocal features of the theory. We analyze both the ensemble and the individual levels of description of physical systems. In particular, consideration of the individual level affords us the opportunity to discuss the possibility of attributing

objective properties to individual systems in the micro and the macroscopic case. Such a problem acquires a special interest in a relativistic context, in which one has, at the individual level, nonlocal effects.

The work presented here, although only a step toward a completely satisfactory relativistic theory of state vector reduction, we believe provides justification for repeating the comment of John Bell[2] made after his examination of relativistic aspects of QMSL: *"It takes away the ground of my fear that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance"*.

2. DYNAMICAL REDUCTION MODELS: THE NONRELATIVISTIC CASE.

2.1. *General Considerations*

We are interested in considering stochastic modifications of the standard evolution equation for the state vector describing a physical system. To this purpose we add nonhermitian stochastic terms to the Hamiltonian. These have the property of driving, with the appropriate probabilities, the state vector into one of a "preferred" set of orthogonal linear manifolds whose direct sum spans the whole Hilbert space.

A crucial point, which always has to be kept in mind, is that we are interested in a description at the level of the state vector of any *individual* member of the statistical ensemble. Nevertheless, with the purpose of making some important conceptual distinctions, we start with a description at the level of the *ensemble*. We consider a (nonhamiltonian) evolution equation for the statistical operator of the Quantum Dynamical Semigroup type:

$$d\rho(t)/dt=-i[H,\rho(t)]+\gamma\Sigma_i A_i \rho(t) A_i-(\gamma/2)\{\Sigma_i A_i^2,\rho(t)\} \qquad (2.1)$$

In Eq.(2.1) the operators A_i are assumed to be self-adjoint and commuting.

Let us introduce the projection operators P_σ on the common eigenmanifolds of the operators A_i, with $A_i P_\sigma = a_{i\sigma} P_\sigma$. Then it follows from Eq. (2.1), when one disregards the Hamiltonian term, that

$$d[P_\sigma \rho(t) P_\tau]/dt = -(\gamma/2)\Sigma_i (a_{i\sigma} - a_{i\tau})^2 P_\sigma \rho(t) P_\tau \qquad (2.2)$$

So, the dynamical evolution yields an exponential damping of the terms $P_\sigma \rho P_\tau$ for $\sigma \neq \tau$.

This (block) diagonalization of the statistical operator could mean that there is taking place, as desired, an actual decomposition of the ensemble into subensembles described by state vectors lying in the "preferred eigenmanifolds". We will say in this case, following Stapp [21], that we have Heisenberg reductions. However, one must make the important distinction between this desired behaviour and other different evolutions which only give the same behaviour at the statistical operator level: these will be called von Neumann reductions [1].

2.2 *General Formalism: Ito Approach*

Let us briefly sketch how a stochastic dynamical model for the evolution of the state vector can give rise to von Neumann reductions and generate the dynamical equation (2.1) for the statistical operator.

To this purpose we consider the Ito stochastic equation

$$d|\Psi_B(t)\rangle = [(-iH - (\gamma/2)\Sigma_i A_i^2)dt + i\Sigma_i A_i dB_i]|\Psi_B(t)\rangle \qquad (2.3)$$

where $B(t) = \{B_i(t)\}$ is a real Wiener process characterized by expectation values

$$\langle\langle dB_i(t) \rangle\rangle = 0, \quad \langle\langle dB_i(t) dB_j(t) \rangle\rangle = \gamma \delta_{ij} dt \qquad (2.4)$$

We note that Eq.(2.3) describes a unitary evolution of the state vector for any given realization of the stochastic

process $B_i(t)$ (i.e. $d\langle\Phi_B(t)|\Psi_B(t)\rangle = 0$ for any $|\Phi_B(t)\rangle$, $|\Psi_B(t)\rangle$ as can be immediately seen using Ito calculus).

As one can easily check, if one defines

$$\rho(t) = \langle\langle |\Psi_B(t)\rangle\langle\Psi_B(t)| \rangle\rangle \qquad (2.5)$$

one gets Eq. (2.1) for ρ from Eq.(2.3). It has to be remarked, however, that if consideration is given to the random variables

$$z_\sigma(t)_B = \langle\Psi_B(t)|P_\sigma|\Psi_B(t)\rangle \qquad (2.6)$$

one sees that, when the Hamiltonian term in (2.3) is disregarded, they satisfy the equation:

$$dz_\sigma(t)_B = 0 \qquad (2.7)$$

For a Heisenberg reduction to take place, $z_\sigma(t)_B$ must evolve into zero or one. Equation (2.7) proves that the dynamics is not able to induce such reductions.

We consider now, following refs.(15,16), a different stochastic equation for the state vector, in which the coupling of the stochastic process with the operators A_i is skew-hermitian:

$$d|\Psi_B(t)\rangle = [(-iH - (\gamma/2)\Sigma_i A_i^2)dt + \Sigma_i A_i dB_i]|\Psi_B(t)\rangle \qquad (2.8)$$

This equation does not describe a unitary evolution of the statevector and, in particular, it does not preserve the norm. Therefore it requires a prescription to have a physical meaning. As discussed in refs.(15,16), one gets a consistent theory by considering the normalized state vector $|\Psi_B(t)\rangle/\||\Psi_B(t)\rangle\|$ and assuming that the considered process $B_i(t)$ occurs with a "cooked" probability density obtained by weighting the original Wiener process probability by the factor $\||\Psi_B(t)\rangle\|^2$. This is equivalent to considering the stochastic, norm conserving, nonlinear equation for the state vector (which we will still denote by $|\Psi_B(t)\rangle$)

$$d|\Psi_B(t)\rangle=\{[-iH-(\gamma/2)\Sigma_i(A_i-R_i)^2]dt+\Sigma_i(A_i-R_i)dB_i\}|\Psi_B(t)\rangle \quad (2.9a)$$

where

$$R_i=\langle\Psi_B(t)|A_i|\Psi_B(t)\rangle \qquad (2.9b)$$

Equation (2.9) was also obtained by Gisin[23], following a different argument.

As shown in refs.(15,16), Eq.(2.8) or Eq.(2.9) lead to Eq.(2.1) for the statistical operator. Moreover, when the Hamiltonian term is disregarded, it gives rise to stochastic differential equations for the random variables z_σ defined in Eq.(2.6), which imply that, for $t\to+\infty$, z_σ tends either to zero or to one, the probability of the result +1 being $\langle\Psi(0)|P_\sigma|\Psi(0)\rangle$. Therefore, the dynamics (2.9) is such that, in the long run, any given initial state is driven into one of the eigenmanifolds associated to the operators P_σ. Consequently, the model yields Heisenberg reductions.

The theory introduced in refs.(15,16), i.e. CSL, is obtained when consideration is given to a system of identical particles. The set of operators A_i of Eq.(2.1) is identified with the set N(x), $x\in\mathbb{R}^3$, where

$$N(x)=\left[\frac{\alpha}{2\pi}\right]^{3/2}\int dq\ e^{-\frac{\alpha}{2}(q-x)^2}a^{\dagger}(q)\ a(q) \qquad (2.10)$$

In Eq. (2.10) the operators $a(q),a^{\dagger}(q)$ are the annihilation and creation operators of a particle at q, so that N(x) has the meaning of an average (over a volume of the order of $\alpha^{-3/2}$) particle number density. In the case of a single particle, choosing $\lambda=\gamma(\alpha/4\pi)^{3/2}$ and disregarding the Hamiltonian term, Eq.(2.1) becomes

$$\frac{d\rho(t)}{dt}=\lambda\left[\frac{\alpha}{\pi}\right]^{3/2}\int dx\ e^{-\frac{\alpha}{2}(q-x)^2}\rho(t)\ e^{-\frac{\alpha}{2}(q-x)^2}-\lambda\rho(t) \qquad (2.11)$$

2.3 *Stochastic Potential: Stratonovich Approach*

We continue to examine the specialization of the operators A_i to N(x). Also, for a better understanding of the line we will follow to get a relativistic generalization of dynamical reduction models, we turn to the Stratonovich version of the stochastic dynamics we have considered in the previous subsection[2]. We will do this with reference only to the case of a single particle, disregarding the standard Hamiltonian term, i.e. to the dynamics described by Eq.(2.11)

In the case of von Neumann reductions, the Stratonovich equation corresponding to Eq.(2.3) is, in the coordinate representation:

$$i\frac{\partial\Psi_V(q,t)}{\partial t}=V(q,t)\Psi_V(q,t) \qquad (2.12)$$

where

$$V(q,t)=\left[\frac{\alpha}{\pi}\right]^{3/4}\int dx\ e^{-\frac{\alpha}{2}(q-x)^2}\dot{B}(x,t) \qquad (2.13)$$

Since $\dot{B}$ is a white noise, the stochastic potential V(q,t) satisfies

$$\langle\langle V(q,t)\rangle\rangle=0,\quad \langle\langle V(q,t)V(q',t')\rangle\rangle=\lambda e^{-\frac{\alpha}{4}(q-q')^2}\delta(t-t') \qquad (2.14)$$

Let us denote by T the time interval $[t_0,t]$. For a given stochastic potential $V(q,\tau)$, $\tau\in T$, and a given initial state $|\Psi(t_0)\rangle$, the solution of Eq.(2.12) at time t is

$$\Psi_V(q,t)= e^{-i\int_{t_0}^{t} d\tau\, V(q,\tau)}\,\Psi(q,t_0) \qquad (2.15)$$

The presence of the stochastic potential implies that the evolution generates a statistical ensemble. This ensemble is the union of the pure states $|\Psi_V(t)\rangle$ with appropriate weights $P(V)$. This ensemble is associated to the statistical operator

$$\rho(t)=\langle\langle|\Psi_V(t)\rangle\langle\Psi_V(t)|\rangle\rangle=\int D[V]\,P[V]\,|\Psi_V(t)\rangle\langle\Psi_V(t)| \qquad (2.16)$$

with obvious meaning of the symbols.

The coordinate representation of Eq.(2.16) reads

$$\rho(q,q',t)=\langle\langle e^{i\int_{t_0}^{t} d\tau[V(q',\tau)-V(q,\tau)]}\rangle\rangle\rho(q,q',t_0) \qquad (2.17)$$

The expression at the r.h.s. is easily evaluated by using the characteristic functional associated with the mean value and covariance defined by (2.14). We then get

$$\rho(q,q',t)=e^{-\lambda(t-t_0)[1-e^{-\frac{\alpha}{4}(q-q')^2}]}\rho(q,q',t_0) \qquad (2.18)$$

which is the solution of Eq.(2.11) in the coordinate representation.

Eq.(2.16) defines a map $\Sigma^P(t,t_0)$ from pure states to statistical operators which has to be extended linearly to the set of trace class operators. This extension will be denoted by $\Sigma(t,t_0)$. Note that, since the covariance in (2.14) depends only on the difference $t-t'$, $\Sigma(t,t_0)$ is actually a function of the difference $t-t_0$.

Our description of the physics of the process tells us that, given a pure state or an inhomogeneous ensemble at time t_0, it evolves into a definite ensemble at the time t'. We can now consider the statistical operator $\rho(t')$ as describing an initial situation, and we can follow its evolution from t' to $t>t'$. The final situation will be described by

$$\rho(t)= \Sigma(t,t')[\Sigma(t',t_0)[\rho(t_0)]] \qquad (2.19)$$

On the other hand, one can consider the preparation of $\rho(t_0)$ at time t_0 and its evolution up to t getting

$$\rho^{\sim}(t)=\Sigma(t,t_0)\rho(t_0) \qquad (2.20)$$

We shall call the condition $\rho^{\sim}(t)=\rho(t)$, i.e.

$$\Sigma(t,t')\Sigma(t',t_0)=\Sigma(t,t_0) \qquad (2.21)$$

the "consistency requirement" for the evolution. In our case since, as already remarked $\Sigma(t,t_0)=\Sigma(t-t_0)$, condition (2.21) amounts to

$$\Sigma(\Delta_1)\Sigma(\Delta_2)=\Sigma(\Delta_1+\Delta_2), \quad \forall\ \Delta_1,\Delta_2\geq 0 \qquad (2.22)$$

which is automatically satisfied [25] since eq.(2.1) is of the Quantum Dynamical Semigroup type.

It is useful to remark that, if one considers stochastic evolution equations for the state vector of type (2.12) and assumes that $V(q,t)$ is a Gaussian noise with zero mean and covariance $A(q-q',t-t')$, then it can be easily proved (see Appendix A) that the necessary and sufficient condition in order that (2.21) be satisfied is that $V(q,t)$ is white in time.

The fact that the stochastic equation (2.12) does not describe Heisenberg reduction processes for the state vector follows trivially by observing that, from Eq.(2.15), for any given realization of the potential

$$|\Psi_V(q,t)|^2=|\Psi(q,t_0)|^2, \quad \forall\ t. \qquad (2.23)$$

The suppression of the off-diagonal elements of the statistical operator in the coordinate representation does not correspond therefore to a localization of the particle but is simply due to a randomization of the relative phases at different space points. This is a typical mechanism for von Neumann reductions.

To describe Heisenberg reductions we then consider the Stratonovich analogue of Eq.(2.8), i.e.

$$\frac{\partial\Psi_V(q,t)}{\partial t} = [V(q,t)-\lambda]\Psi_V(q,t) \tag{2.24}$$

where $V(q,t)$ is given by (2.13). The counterterm $-\lambda$ in Eq.(2.24) guarantees that the square norm average is conserved. For a given stochastic potential $V(q,t)$, the solution of Eq.(2.24) is:

$$\Psi_V(q,t) = e^{-\lambda(t-t_0)+\int_{t_0}^{t} d\tau V(q,\tau)}\,\Psi(q,t_0) \tag{2.25}$$

from which it is evident that the norm is not conserved in a specific process. However, if we define, in complete analogy to the previous case, $\rho(t)$ by means of Eq.(2.16), with $|\Psi_V(t)\rangle$ satisfying (2.24), we get the same equation (2.11) for the statistical operator. The solution (2.18) of this equation, as already remarked, displays the suppression of the off-diagonal elements in the coordinate representation.

Note that since the vectors $|\Psi_V(t)\rangle$ given by (2.25) are not of norm one, to give a physical meaning to the definition (2.16), we are led to read it as

$$\rho(t)=\int D[V]\,P[V]\,\| |\Psi_V(t)\rangle \|^2 \frac{|\Psi_V(t)\rangle}{\| |\Psi_V(t)\rangle \|}\,\frac{\langle\Psi_V(t)|}{\| |\Psi_V(t)\rangle \|} \tag{2.26}$$

This amounts to assuming that the physically meaningful probability distribution for the stochastic potential is not $P(V)$ but

$$P_C[V]=P[V]\,\| |\Psi_V(t)\rangle \|^2 \tag{2.27}$$

where $V=V(q,t)$ is a sample function with support in the time interval $[t_0,t]$. Note that the passage from $P[V]$ to $P_C[V]$ corresponds to the cooking of the stochastic process referred to earlier. Through the appearance of $\| |\Psi_V(t)\rangle \|^2$, this makes the evolution equation for the normalized state vector nonlinear (see Eq.(2.9)).

2.4 *The Cooked Probability Density*

It is appropriate to discuss here in more detail, the cooked probability density (2.27). We stress that, for a real and positive functional $\tilde{P}[V]$, defined over the sample functions $V(q,t)$ for an arbitrary time interval, to be interpreted as a probability density, it has to satisfy two fundamental requirements, firstly the completeness relation

$$\int D[V]\,\tilde{P}[V]=1 \tag{2.28}$$

and, secondly a compatibility property we are now going to discuss. Let us consider two contiguous time intervals $T_1=[t_0,t']$ and $T_2=[t',t]$ and the sample functions $V_1(q,\tau)$ with $\tau\in T_1$, $V_2(q,\tau)$ with $\tau\in T_2$ and $V(q,\tau)$ with $\tau\in T_1 \cup T_2$, together with their associated probability density functionals $\tilde{P}[V_1]$, $\tilde{P}[V_2]$ and $\tilde{P}[V]$. An arbitrary sample function $V(q,\tau)$ with $\tau\in T_1\cup T_2$, can be identified with the two sample functions $V_1(q,\tau)$ and $V_2(q,\tau)$ obtained by restricting the time support of $V(q,\tau)$ to T_1 and T_2, respectively. The probability density $\tilde{P}[V]$ can then be written as $\tilde{P}[V_1,V_2]$. We can now express the compatibility property through the equation

$$\tilde{P}[V_1]=\int D[V_2]\,\tilde{P}[V_1,V_2] \tag{2.29}$$

Of course, the probability density functional $P[V]$ induced by white noise obeys Eqs.(2.28), (2.29). We have now to check that the cooked probability density $P_C[V]$ defined by (2.27) actually satisfies (2.28) and (2.29) too. Concerning (2.28) we remark that the fact that it is satisfied

$$\int D[V]P_C[V]=\int D[V]P[V]\|\,|\Psi_V(t)>\|^2=1 \qquad (2.30)$$

follows from the property of Eq.(2.24) of conserving the average of the square norm. Concerning (2.29) we consider a given initial state $|\Psi(t_0)>$. Then, by definition

$$P_C[V_1]=P[V_1]\|\,|\Psi_{V1}(t')>\|^2 \qquad (2.31a)$$

and

$$P_C[V]=P[V]\|\,|\Psi_V(t)>\|^2 \qquad (2.31b)$$

Now, $P[V]=P[V_1]\cdot P[V_2]$ due to the noise being white in time. Moreover, from the linearity of the evolution equation we see that

$$|\Psi_V(t)>= |\Psi_{V2}(t)>\ \|\,|\Psi_{V1}(t')>\| \qquad (2.32)$$

where $|\Psi_{V2}(t)>$ is the state that evolves, according to V_2, from the normalized state $|\Psi_{V1}(t')>/\|\,|\Psi_{V1}(t')>\|$. We then have

$$\int D[V_2]\ P_C[V_1,V_2]=\int D[V_2]P[V_1]P[V_2]\|\,|\Psi_{V2}(t)>\|^2\|\Psi_{V1}(t')>\|^2= P[V_1]\|\,|\Psi_{V1}(t')>\|^2\int D[V_2]P[V_2]\|\,|\Psi_{V2}(t)>\|^2 \qquad (2.33)$$

Since, according to Eq.(2.30), the integral in the last term of (2.33) is equal to one, we have that the last term in Eq.(2.33) is, according to (2.31a), equal to $P_c[V_1]$, so that property (2.29) for P_c is proved.

This result follows from a combined use of the white noise property of $V(q,t)$ in time and the specific cooking

prescription. In Appendix B we show that the choice of a particular noise which is not white in time would, together with our standard cooking prescription, lead to a P_c which does not satisfy (2.29).

Since, as already stated, Eq.(2.24) together with the cooking prescription is the Stratonovich analogue of Eq.(2.8), it leads to Heisenberg reductions.

2.5. *Stochastic Galileian Invariance*

To bring out some concepts which will be useful in Section 3, it is appropriate to consider the transformation and the invariance properties of the dynamical reduction models considered above.

Let us start by limiting our considerations to the evolution equation for the statistical operator and let us consider two observers O and O' related by a transformation of the Galilei group. We take the so called passive point of view according to which the two observers look at the same physical situation. For simplicity, let us suppose that the transformation connecting O and O' is a translation in space of an amount **a** and a translation in time of an amount τ, so that

$$\mathbf{r}'=\mathbf{r}-\mathbf{a},\quad t'=t-\tau \qquad (2.34)$$

Let the observer O describe the physical situation at his subjective time t by the statistical operator $\rho(t)$. Observer O', at the same objective time, i.e. at his subjective time $t'=t-\tau$, will describe the physical situation by the statistical operator

$$\rho'(t')=U(\mathbf{a})\rho(t)U^\dagger(\mathbf{a}) \qquad (2.35)$$

where $U(\mathbf{a})=\exp[i\mathbf{P}\cdot\mathbf{a}]$ is the usual unitary operator inducing the space translation. The dynamical equation for the statistical operator for observer O' is then

$$\frac{d\rho'(t')}{dt'} = U(a)\frac{d\rho(t)}{dt}U^{\dagger}(a) \qquad (2.36)$$

Substituting Eq.(2.1), describing the evolution of the statistical operator for the observer O, into the r.h.s. of Eq.(2.36) one gets:

$$\frac{d\rho'(t')}{dt'} = -i\ U(a)[H(t),\rho(t)]U^{\dagger}(a)$$
$$+\gamma\Sigma_i U(a)A_i\rho(t)A_i U^{\dagger}(a)-\frac{\gamma}{2}\ U(a)\{\Sigma_i A_i^{\ 2},\rho(t)\}U^{\dagger}(a). \qquad (2.37)$$

If H is invariant under space and time translations

$$H'(t')=U(a)H(t)U^{\dagger}(a)=H(t') \qquad (2.38)$$

and if, moreover

$$\Sigma_i U(a)A_i U^{\dagger}(a)XU(a)A_i U^{\dagger}(a)=\Sigma_i A_i X A_i \qquad (2.39)$$

for any bounded operator X, then Eq.(2.37) implies

$$\frac{d\rho'(t')}{dt'} = -i[H,\rho'(t')]+\gamma\ \Sigma_i A_i\rho'(t')A_i\ -\frac{\gamma}{2}\ \{\Sigma_i A_i^{\ 2},\rho'(t')\} \qquad (2.40)$$

i.e. the theory is invariant for space and time translations. If the same holds for all transformations of the restricted Galilei group we have invariance for the transformations of this group. QMSL and CSL actually possess this invariance property.

Nonetheless, it is important to stress that there is a difference between equations of the type we are considering and the usual case in which one has a purely Hamiltonian evolution, with respect to the connection between invariance and representations of the symmetry group. This key difference arises from the fact that while in the standard case one can always relate the statistical operators used by O and O' to

describe the physical situation at the same subjective time t, in the present case this cannot be done in general, when one considers negative values of τ in Eq.(2.34). In fact, let us suppose that O, at his own time t=0 is dealing with a physical system described by a pure state $\rho(0)=|\Psi\rangle\langle\Psi|$. Since the dynamical evolution transforms pure states into statistical mixtures, there is no way for O to prepare a physical situation at his own time $\tau<0$ described by a statistical operator such that it evolves into the pure state $\rho(0)$ at t=0. Correspondingly, there is no way for O' to prepare at his own time t'=0 a statistical operator such that its evolved state at his time $-\tau>0$ is $|\Psi\rangle\langle\Psi|$.

However, if the active point of view is taken and O', at his time t'=0, prepares the same state $\rho(0)$, and the above stated invariance requirements are satisfied, then O and O' will observe the same dynamical evolution for the same (subjective) initial situation.

Coming now to the group theoretic point of view, since for the above reasons the map Σ_t from a pure state is not defined for negative t, one has to consider the proper Galilei semigroup G_+, with only forward time translations[28]. Any transformation $g\in G_+$ can be expressed as a transformation of the subgroup G_0 of G_+ which does not contain time translations, times a forward time translation

$$g\in G_+:\ g=g_\tau\ g_0 \qquad (2.41)$$

The map on the Banach space of the trace class operators

$$g:\rho\to\rho_g,\ \ \rho_g=\Sigma_\tau[U(g_0)\rho U^{\dagger}(g_0)] \qquad (2.42)$$

where $U(g_0)$ is the usual unitary representation of G_0 and Σ_τ is such that, for $\tau>0$, $\Sigma_\tau\rho(t)=\rho(t+\tau)$ is the solution of Eq.(2.1), is then easily checked to yield a representation of G_+.

Up to now we have discussed the invariance properties of dynamical reduction models from the point of view of the statistical operator. However, since we are interested here in the evolution equation for the state vector, it is appropriate to discuss the problem of the invariance also at this level. For simplicity, we will limit ourselves here to the discussion of space translations.

Let us then start by considering the Stratonovich equation (2.12). If we denote by O' an observer whose reference frame is translated by an amount a with respect to the frame of O, he will experience the potential

$$V'(q',t)=V(q'+a,t) \tag{2.43}$$

so that, for a particular realization of V, there is no invariance.

However, since we are dealing with a fundamentally stochastic theory, the invariance requirement has to be formulated in an appropriate way. We will say that the theory is stochastically invariant under space translations if, for all observers O', translated by any a with respect to O, the stochastic ensemble of potentials is the same. This is equivalent to requiring that, if V(q,t) is a possible sample function for O, then V(q-a,t), for any a, is also a possible sample function for him, having the same probability of occurrence of V(q,t), i.e.

$$\mathcal{P}[V(q,t)]=\mathcal{P}[V(q-a,t)] \tag{2.44}$$

Note that this is automatically guaranteed by the form (2.14) for the mean value and covariance function of the gaussian noise.

In the case of the model based on Eq.(2.24) describing Heisenberg reduction processes, a separate discussion is needed, since the stochastic invariance requirement has to be referred to the cooked probabilities which depend on the initial state vector. Let us consider two observers O and O' and suppose they prepare the same (subjective) state $|\Psi(0)\rangle$ at time t=0. The probability density of occurrence of the same

(subjective) potential V(q,t) is, for the two observers,

$$\begin{aligned}\mathcal{P}_C^{O'}[V(q,t)]&=\mathcal{P}^{O'}[V(q,t)]\,\| |\Psi_V^{O'}(t)\rangle\|^2\\ \mathcal{P}_C^{O}[V(q,t)]&=\mathcal{P}^{O}[V(q,t)]\,\| |\Psi_V^{O}(t)\rangle\|^2\end{aligned} \tag{2.45}$$

Since $|\Psi_V^{O'}\rangle$ and $|\Psi_V^{O}\rangle$ are the solutions of Eq.(2.21) with the same (subjective) potential and satisfy the same initial conditions, they coincide. Moreover, due to Eq.(2.44) $\mathcal{P}^{O'}[V(q,t)]=\mathcal{P}^{O}[V(q,t)]$, implying

$$\mathcal{P}_C^{O'}[V(q,t)]=\mathcal{P}_C^{O}[V(q,t)] \tag{2.46}$$

This guarantees the invariance from the active point of view, i.e. the observers cannot, by making physical experiments in their own frames, discover that they are displaced. They agree on the statistical distributions of the future outcomes.

3. RELATIVISTIC, STOCHASTICALLY INVARIANT REDUCTION MODELS.

In trying to set up the framework for a relativistic generalization of reduction models we adopt the quantum field theoretic point of view. We remark that the analogue of the idea of considering, within a nonrelativistic framework, a stochastic potential V(q,t) consists in assuming that the Lagrangian density for fields contains a stochastic interaction term. In the two following subsections we will consider model theories which are analogues of the nonrelativistic ones based on Eqs.(2.12) and (2.24), respectively.

3.1 *Quantum Field Theory with a Hermitian Stochastic Coupling*

Let us consider, in the context of quantum field theory, the Lagrangian density

$$L(x)=L_0(x)+L_I(x)V(x) \tag{3.1}$$

where L_0 and L_I are Lorentz scalar functions of the fields (for the moment we do not need to specify the fields we deal with). We assume that L_I does not depend on the derivatives of the fields, and that V(x) is a c-number stochastic process which is a scalar with respect to transformations of the restricted Poincare' group, i.e. that under the change of variables $x'=\Lambda x+b$, it transforms according to

$$V'(x')=V[\Lambda^{-1}(x'-b)] \tag{3.2}$$

We will also assume that V(x) is a Gaussian noise with mean zero and, to get a relativistic stochastically invariant theory, that its covariance is an invariant function

$$\langle\langle V(x)V(x')\rangle\rangle = A(x-x') \tag{3.3}$$

with $A(\Lambda^{-1}x)=A(x)$.

As discussed in the previous section stochastic invariance requires different observers to agree on the unfolding of physical processes. This, in turn, is guaranteed by the condition that the family of all sample functions V(x) and the probability density of occurrence of the same (subjective) sample function be the same for all observers. This is achieved by requiring that, for a single observer

$$P[V(x)]=P[V(\Lambda(x+b))] \tag{3.4}$$

We stress that property (3.4) holds automatically if the covariance is a relativistically invariant function. In fact, from

$$P[V(x)]=\frac{1}{N}\, e^{-\frac{1}{2}\int\int dx\, dx'\, V(x)\tilde{A}(x-x')V(x')} \tag{3.5}$$

(where we have denoted by $\tilde{A}(x-x')$ the function satisfying $\int dx''A(x-x'')\tilde{A}(x''-x')=\delta(x-x')$) one gets immediately, using

the scalar nature of A and consequently of $\tilde{A}$, that

$$P[V(\Lambda(x+b))]=P[V(x)] \tag{3.6}$$

The most natural generalization of the case discussed in the previous section is obtained by assuming that V(x) is a white noise in all variables, i.e.

$$\langle\langle V(x)V(x')\rangle\rangle=A(x-x')=\lambda\delta(x-x') \tag{3.7}$$

It is appropriate to make a brief digression on this specific choice. Obviously one could consider a more general Gaussian process satisfying (3.6). For example any function

$$\tilde{V}(x)=\int dz\,\omega(z-x)V(z) \tag{3.8}$$

where V(z) satisfies (3.7) and $\omega(\Lambda^{-1}x)=\omega(x)$, would be equally acceptable from the point of view of stochastic invariance. However, if such a choice is made, one can prove by arguments similar to those of section 2, that one cannot define in a consistent way the statistical operator for the considered process. Moreover, when one tries to pass to a formalism yielding Heisenberg reductions one would meet difficulties with the compatibility requirement for the cooked probability density (see the discussion in Appendices A and B). Let us come back now to our general problem.

We study, first of all, the physical consequences of the stochastic coupling $L_I(x)V(x)$. In Schrödinger's picture we have, for a given V(x), the evolution equation

$$i\frac{d|\Psi_V(t)\rangle}{dt}=\left[H_0-\int dx\; L_I(x,0)V(x,t)\right]|\Psi_V(t)\rangle \tag{3.9}$$

where H_0 is the Hamiltonian corresponding to L_0. Eq.(3.9) implies

$$|\Psi_V(t)\rangle = T\, e^{-iH_0 t + i\int_0^t d\tau \int dx\, L_I(x,0)V(x,\tau)} |\Psi(0)\rangle \tag{3.10}$$

This equation shows how, for a given initial state $|\Psi(0)\rangle$ one gets an ensemble of states $|\Psi_V(t)\rangle$ at time t, according to the particular realization of the stochastic process. The statistical ensemble can then be described by the statistical operator obtained by averaging over the sample functions (see Eq.(2.5)). In the case under consideration one gets a closed evolution equation for the statistical operator. In fact, we observe that, due to the fact that $L_I(x)$ does not depend on the derivatives of the fields

$$[L_I(x,0), L_I(x',0)]=0 \quad \forall x,x' \tag{3.11}$$

and due to the presence of the time ordering in (3.10), we have

$$\begin{aligned}\rho(t+\epsilon) = \langle\langle &\Big[1 - iH_0\epsilon + i\int_t^{t+\epsilon} d\tau \int dx\, L_I(x,0)V(x,\tau) - \\ &-\frac{1}{2}\int_t^{t+\epsilon} d\tau \int_t^{t+\epsilon} d\tau' \int dx \int dx' L_I(x,0)L_I(x',0)V(x,\tau)V(x',\tau')\Big]. \\ &|\Psi_V(t)\rangle\langle\Psi_V(t)|.\Big[1 + iH_0\epsilon - i\int_t^{t+\epsilon} d\tau \int dx\, L_I(x,0)V(x,\tau) - \\ &\frac{1}{2}\int_t^{t+\epsilon} d\tau \int_t^{t+\epsilon} d\tau' \int dx \int dx'\, L_I(x,0)L_I(x',0)V(x,\tau)V(x',\tau')\Big]\rangle\rangle\end{aligned} \tag{3.12}$$

We recall now the properties associated with a zero mean gaussian probability distribution

$$\begin{aligned}&\langle\langle V(x_1,t_1)\ldots.V(x_n,t_n)\rangle\rangle = 0 \qquad \text{for n odd} \\ &\langle\langle V(x_1,t_1)\ldots..V(x_n,t_n)\rangle\rangle = \Sigma_{\text{all pairs}} \langle\langle V(x_i,t_i)V(x_j,t_j)\rangle\rangle \\ &\qquad \ldots\ldots\ldots\ldots \langle\langle V(x_k,t_k)V(x_l,t_l)\rangle\rangle \qquad \text{for n even}\end{aligned} \tag{3.13}$$

From (3.12) we then have

$$\begin{aligned}\frac{d\rho(t)}{dt} = &-i[H_0,\rho(t)] + \lambda\int dx\, L_I(x,0)\rho(t)L_I(x,0) \\ &-\frac{\lambda}{2}\left\{\int dx L_I^2(x,0),\ \rho(t)\right\}\end{aligned} \tag{3.14}$$

Note that the obtained equation is of the Lindblad type and this fact by itself guarantees that the map Σ_t defined by $\rho(t)=\Sigma_t\rho(0)$ satisfies Eq. (2.22).

The nonhamiltonian terms in Eq.(3.14) imply a suppression of the off-diagonal elements of the statistical operator in the basis of the common eigenstates of the commuting operators $L_I(x,0)$. Putting

$$L_I(x,0)|\ldots\nu\ldots\rangle = \nu(x)|\ldots\nu\ldots\rangle \tag{3.15}$$

one gets, when the Hamiltonian term in (3.14) is disregarded

$$\langle..\nu..|\rho(t)|..\nu'..\rangle = e^{-\frac{\lambda t}{2}\int dx[\nu(x)-\nu'(x)]^2} \langle..\nu..|\rho(0)|..\nu'..\rangle \tag{3.16}$$

As in the nonrelativistic case, however, for a single realization of the stochastic potential V(x,t), the state vector is not driven into one of the eigenmanifolds characterized by a given $\nu(x)$, since $|\langle\ldots\nu\ldots|\Psi_V(t)\rangle|^2$ does not change with time. These considerations point out that, in order to have Heisenberg reductions, one has to resort to a skew-hermitian coupling with the noise.

Equation (3.14) for the statistical operator is not manifestly covariant, even though, from the procedure which has been followed to derive it, we know that the theory is stochastically invariant. To obtain a manifestly covariant description of the statistical operator evolution, we note that the model presented above is obviously equivalent to the following scheme:

i.- Assume that the fields are solutions of the Heisenberg equations obtained in the standard way from the Lagrangian density $L_0(x)$ (note that we do not require $L_0(x)$ to describe free fields)

ii.- Assume that the evolution of the state vector is governed by the Tomonaga-Schwinger equation

$$i\frac{\delta|\Psi_V(\sigma)\rangle}{\delta\sigma(x)} = - L_I(x)V(x)\ |\Psi_V(\sigma)\rangle \qquad (3.17)$$

$L_I(x)$ being a function of the fields considered in i) which does not involve their derivatives. As a consequence of the assumptions about $L_I(x)$, for any two points $x,x'\in\sigma$, σ being a space-like surface, $[L_I(x),L_I(x')]=0$, and consequently Eq. (3.17) is integrable.

Let us consider the formal solution of Eq. (3.17):

$$|\Psi_V(\sigma)\rangle = T\ e^{i\int_{\sigma_0}^{\sigma}dx\ L_I(x)V(x)}\ |\Psi(\sigma_0)\rangle \qquad (3.18)$$

Defining

$$\rho(\sigma)=\langle\langle|\Psi_V(\sigma)\rangle\langle\Psi_V(\sigma)|\rangle\rangle \qquad (3.19)$$

using (3.18), and following the procedure outlined in Eqs.(3.12) to (3.14) we get the Tomonaga-Schwinger equation for the statistical operator

$$\frac{\delta\rho(\sigma)}{\delta\sigma(x)} = \lambda\ L_I(x)\rho(\sigma)L_I(x)-\frac{\lambda}{2}\ \{\ L_I^2(x),\rho(\sigma)\} \qquad (3.20)$$

which is manifestly covariant.

3.2 *Quantum Field Theory with Heisenberg Reductions.*

In this subsection we present a stochastically invariant theory yielding Heisenberg reductions. To this purpose we keep assumption i) of the previous subsection and we replace ii) by the requirement that $|\Psi(\sigma)\rangle$ instead of being governed by Eq.(3.17), obeys the following equation of the Tomonaga-Schwinger type

$$\frac{\delta|\Psi_V(\sigma)\rangle}{\delta\sigma(x)} = [L_I(x)V(x)-\lambda L_I^2(x)]|\Psi_V(\sigma)\rangle \qquad (3.21)$$

The main difference between the two equations (3.17) and (3.21) derives from the skew-hermitian character of the coupling to the stochastic c-number field. At the r.h.s. of (3.21) a term guaranteeing the conservation of the average value of the square norm of the state appears. It is important to remark that Eq.(3.21), for a given sample potential, does not conserve the norm of the state vector.

Let $|\Psi_V(\sigma)\rangle$ be the solution of Eq.(3.21) for a given realization of the stochastic potential

$$|\Psi_V(\sigma)\rangle = T e^{\int_{\sigma_0}^{\sigma}dx\ [L_I(x)V(x)-\lambda L_I^2(x)]}\ |\Psi(\sigma_0)\rangle \qquad (3.22)$$

and let us define the stochastic average

$$\rho(\sigma)= \langle\langle|\Psi_V(\sigma)\rangle\langle\Psi_V(\sigma)|\rangle\rangle \qquad (3.23)$$

Following the same procedure of the previous subsection one sees that $\rho(\sigma)$ still satisfies Eq.(3.20) derived in the Hermitian case.

As in the nonrelativistic case we have then two conceptually different dynamical evolutions for the state vector, i.e. (3.17) and (3.21), which give rise to the same dynamics for the statistical operator and therefore to the same physical predictions at the ensemble level. The very definition (3.23) of the statistical operator, when confronted with the fact that the equation for the state vector does not preserve the norm, implies the adoption of the point of view that a cooking procedure, analogous to the one discussed in section 2.4, is necessary. This means that one has to consider normalized vectors $|\Psi_V(\sigma)\rangle/\||\Psi_V(\sigma)\rangle\|$ and has to attribute to the considered realization $V(x)$ of the stochastic potential, having support in the space-time region lying between the two space-like hypersurfaces σ_0 and σ, not the probability density $P[V(x)]$ given by (3.5), but a cooked probability density $P_C[V(x)]$ given by

$$P_C[V(x)]=P[V(x)]\||\Psi_V(\sigma)\rangle\|^2 \tag{3.24}$$

In the above equation $|\Psi_V(\sigma)\rangle$ is the solution of Eq.(3.21) satisfying

$$|\Psi_V(\sigma_0)\rangle=|\Psi_0\rangle. \tag{3.25}$$

Before discussing the cooking procedure, the role of the counterterm and the relativistic invariance of the theory, an important remark is necessary. As we have discussed in section 2.5, at the level of the statistical operator the map Σ_t does not exist when $t<0$. For this reason, even at the state vector level, we will only consider Eq.(3.21) as yielding the evolution from the state vector associated to a given space-like surface σ_0 to space-like surfaces lying entirely in the future of σ_0.

For what concerns the properties of the cooking procedure one can immediately see that Eq.(3.20) preserves the trace of ρ which amounts to the statement that Eq.(3.21) preserves the average of the square norm of the state vector. In particular this implies

$$\int D[V]P_C[V]=\int D[V]P[V]\||\Psi_V(\sigma)\rangle\|^2=1 \tag{3.26}$$

which shows that the requirement (2.28) on the cooked

probability density is satisfied. One can also easily prove, by the same procedure we have followed in the nonrelativistic case, that the cooked probability density satisfies the compatibility condition.

3.2.1 *Transformation Properties and Invariance of the Theory*

We discuss now the transformation properties of the theory for a given realization of the stochastic potential, in going from a given reference frame O to another one O' related to it by a transformation of the restricted Poincare' group

$$(\Lambda,b):\ x\to x'=\Lambda x+b \tag{3.27}$$

We remind the reader that in the Tomonaga—Schwinger formalism of conventional quantum field theory each reference frame O is able to assign a statevector to each space-like hypersurface. Our first concern is to demonstrate that the consistency of the composition law for Lorentz transformations remains intact in the present use of the Tomonaga—Schwinger formalism.

Suppose that the transformation (3.27) involves a boost and consider a given space-like surface σ for O. The surface which is subjectively the same for O' involves points which lie in the past of the surface σ for O. Our previous discussion has pointed out that we will only use the Tomonaga-Schwinger equation to go from a given space-like surface σ to surfaces lying entirely in the future of σ. Therefore, contrary to the standard case we are not allowed to raise here the following question: which state vector $|\Psi'(\sigma)\rangle$ would O' associate to his subjective surface σ to describe the same physical situation described by O who assigns the state vector $|\Psi(\sigma)\rangle$ to his subjective surface σ ?

We can, however, legitimately consider subjective surfaces $\tilde{\sigma}'$ for O', such that they lie in the future of the surface σ for O. Suppose the observer O associates the state vector $|\Psi_O(\sigma)\rangle$ to his subjective surface σ to describe the physical situation. Let us denote by $\sigma^{\sim}$ the surface of O which is objectively the same as the above-mentioned surface $\sigma^{\sim\prime}$ for O'. Then O associates to $\sigma^{\sim}$ the state $|\Psi_O(\sigma^{\sim})\rangle$ obtained by solving Eq.(3.21) with the initial condition that it reduces to $|\Psi_O(\sigma)\rangle$ on σ. We have

$$|\Psi_O(\sigma^{\sim})\rangle = S_V(\sigma^{\sim},\sigma)|\Psi_O(\sigma)\rangle / \| S_V(\sigma^{\sim},\sigma)|\Psi_O(\sigma)\rangle \| \tag{3.28}$$

with

$$S_V(\sigma^{\sim},\sigma) = T\, e^{\int_{\sigma}^{\sigma^{\sim}} dx\, [L_I(x)V(x) - \lambda L_I^{2}(x)]} \tag{3.29}$$

Then the observer O' will associate to his surface $\sigma^{\sim\prime}$ the state vector

$$|\Psi_{O'}(\sigma^{\sim\prime})\rangle = U(\Lambda,b)|\Psi_O(\sigma^{\sim})\rangle = U(\Lambda,b)S_V(\sigma^{\sim},\sigma)|\Psi_O(\sigma)\rangle / \| S_V(\sigma^{\sim},\sigma)|\Psi_O(\sigma)\rangle \| \tag{3.30}$$

In Eq.(3.30), $U(\Lambda,b)$ is the unitary operator whose infinitesimal generators P^{μ} and $J^{\mu\nu}$ are obtained in the standard way from the Lagrangian density $L_0(x)$. Let now σ, $\sigma^{\sim}$, $\sigma^{\sim\sim}$ be three space-like surfaces for O each lying entirely in the future of the previous ones. Let us consider two other observers O' and O'' related by two successive Lorentz transformations (the generalization to Poincare' transformations is straighforward): $O'=\Lambda_1 O$, $O''=\Lambda_2 O'$, and let us denote by σ', $\sigma^{\sim\prime}$, $\sigma^{\sim\sim\prime}$ and σ'', $\sigma^{\sim\prime\prime}$, $\sigma^{\sim\sim\prime\prime}$ the above surfaces as seen by O' and O'', respectively.

The map (3.29), for a given realization of the stochastic potential, has the following property. Suppose O assignes the state $|\Psi_O(\sigma)\rangle$ to the surface σ. Then O' assigns the state (3.30) to the surface $\sigma^{\sim\prime}$. For O' this state evolves according to the Tomonaga-Schwinger equation (3.21) with $V'(x')=V(\Lambda_1^{-1}x')$ from $\sigma^{\sim\prime}$ to $\sigma^{\sim\sim\prime}$

$$|\Psi_{O'}(\sigma^{\sim\sim\prime})\rangle = S'_{V'}(\sigma^{\sim\sim\prime},\sigma^{\sim\prime})|\Psi_{O'}(\sigma^{\sim\prime})\rangle / \| S'_{V'}(\sigma^{\sim\sim\prime},\sigma^{\sim\prime})|\Psi_{O'}(\sigma^{\sim\prime})\rangle \| \tag{3.31}$$

The observer O'' will describe the final situation by assigning the state vector

$$|\Psi_{O''}(\sigma^{\sim\sim\prime\prime})\rangle = U(\Lambda_2)|\Psi_{O'}(\sigma^{\sim\sim\prime})\rangle \tag{3.32}$$

to the surface $\sigma^{\sim\sim\prime\prime}$. On the other hand, one can consider the evolution from σ to $\sigma^{\sim\sim}$ as seen from O

$$|\Psi_O(\sigma^{\sim\sim})\rangle = S_V(\sigma^{\sim\sim},\sigma)|\Psi_O(\sigma)\rangle / \| S_V(\sigma^{\sim\sim},\sigma)|\Psi_O(\sigma)\rangle \| \tag{3.33}$$

and then look at it from $O''=\Lambda_2\Lambda_1 O$, getting the state

$$|\Psi_{O''}^{*}(\sigma^{\sim\sim\prime\prime})\rangle = U(\Lambda_2\Lambda_1)|\Psi_O(\sigma^{\sim\sim})\rangle \tag{3.34}$$

For consistency $|\Psi_{O''}^{*}(\sigma^{\sim\sim\prime\prime})\rangle$ must coincide with $|\Psi_{O''}(\sigma^{\sim\sim\prime\prime})\rangle$. This can be easily proved to hold.

Although we have just seen that the theory implies an assignment of a statevector to a hypersurface by any observer that fulfills the Lorentz (also Poincare') group requirements, this does not mean that the description is Lorentz invariant. In fact, because a particular realization of the stochastic potential V looks different from two different reference frames, the map $S_V(\sigma^{\sim},\sigma)$ obviously depends upon the reference frame O. This shows that, at the individual level the theory does not posess the property of standard (i.e. nonstochastic) Lorentz invariance. However, for stochastic Lorentz invariance one must consider the ensemble of possible sample potentials. When one takes into account the Lorence invariance of the requirement (3.3) for the correlation function $\langle\langle V(x)V(x')\rangle\rangle$, and the invariance of the cooking procedure that must be performed to get the physics of the problem, one can easily prove, along the same lines as in the nonrelativistic case, that there is stochastic invariance in the state vector language, i.e. the stochastic ensemble of evolution operators $S_V(\sigma^{\sim},\sigma)$ is the same in each reference frame.

In the language of the statistical operator, invariance is evident from the manifestly covariant Tomonaga-Schwinger form (3.20) of the evolution equation.

3.2.2. *Reduction Properties*

Once we have guaranteed the invariance of the formalism by using its Tomonaga-Schwinger formulation, in order to discuss specific features of the process, we can consider t= const hyperplanes in the Schrödinger picture. In so doing, the equation corresponding to (3.21) is

$$\frac{d|\Psi_V(t)\rangle}{dt} = \left[-iH_0+\int dx[L_I(x,0)V(x,t)-\lambda L_I^2(x,0)]\right]|\Psi_V(t)\rangle \quad (3.35)$$

This is a Stratonovich equation for the state vector. By standard procedures one can consider the corresponding Ito stochastic dynamical equation

$$d|\Psi_V(t)\rangle=\{[-iH_0-(\lambda/2)\int dx L_I^2(x,0)]dt+\int dx L_I(x,0)dV(x)\}|\Psi_V(t)\rangle \quad (3.36)$$

where dV(x) is a real Wiener process satisfying

$$\langle\langle dV(x)\rangle\rangle=0,\quad \langle\langle dV(x)dV(y)\rangle\rangle=\lambda\delta(x-y)dt \quad (3.37)$$

Note that both Eq.(3.35) and (3.36) do not conserve the norm of the state vector but they conserve the average of its squared norm.

As discussed in Subsection 2.2 one can take two equivalent attitudes to describe the physics of the process. One can solve Eq.(3.35) or (3.36) for a given initial condition, and then one can consider the normalized vectors $|\Psi_V(t)\rangle/\||\Psi_V(t)\rangle\|$ at time t and assume that the probability of their occurrence is obtained by cooking the probability density of occurrence of V(x), i.e. by mutiplying it times $\||\Psi_V(t)\rangle\|^2$. Alternatively, one can consider the nonlinear stochastic dynamical equation

$$d|\Psi_V(t)\rangle=\{[-iH_0-(\lambda/2)\int dx\,(L_I(x,0)-\langle L_I(x,0)\rangle)^2]dt+ \int dx(L_I(x,0)-\langle L_I(x,0)\rangle)dV(x)\}|\Psi_V(t)\rangle \quad (3.38)$$

(where $\langle L_I(x,0)\rangle=\langle\Psi_V(t)|L_I(x,0)|\Psi_V(t)\rangle$),without cooking, i.e. using just the probability weighting of V(x).

As shown in refs.(15-16), when one disregards the Hamiltonian term in (3.38), the evolution leads the state vector to enter one of the common eigenmanifolds of the commuting operators $L_I(x,0)$. The theory induces therefore Heisenberg reductions, as required.

3.3. *The Model*

In this Subsection we will consider some specific choices for the Lagrangian densities L_0 and L_I which, when used in connection with the formalism presented in Subsection 3.2, yield stochastically invariant relativistic reduction models. The goal is to build up a framework leading to localization in position of the basic constituents of all matter.

The simplest and most immediate idea would be to introduce a fermion field for particles of mass M and to choose for the lagrangian density the expressions

$$L_0(x)=\bar{\Psi}(x)(i\gamma^\mu\partial_\mu-M)\Psi(x),\qquad L_I(x)=\bar{\Psi}(x)\Psi(x) \quad (3.39)$$

However, in the nonrelativistic limit, the dynamics induced by the above choice, would lead to infinitely sharp position localizations for the fermions, and this, as well known[13], is unacceptable.

We have then to enrich the formalism. This can be done by following the proposal put forward in ref.(17). One considers a fermion field coupled to a real scalar meson field and chooses

$$L_0(x)=\frac{1}{2}\left[\partial_\mu\Phi(x)\partial^\mu\Phi(x)-m^2\Phi^2(x)\right]+\bar{\Psi}(x)\left[i\gamma^\mu\partial_\mu-M\right]\Psi(x)+\eta\bar{\Psi}(x)\Psi(x)\Phi(x) \tag{3.40}$$

$$L_I(x)=\Phi(x)$$

The introduction of the meson field coupled to the fermion field allows one to overcome the difficulty of the infinitely sharp localization for fermions met by the previous model. In Schrödinger representation the evolution equation for the state vector corresponding to the choice (3.40) is

$$\frac{d|\Psi_V(t)\rangle}{dt} = \left[-iH_0+\int dz[\Phi(z,0)V(z,t)-\lambda\Phi^2(z,0)]\right]|\Psi_V(t)\rangle \tag{3.41}$$

Let us consider now the nonrelativistic infinite mass limit for fermions, and let us confine our discussion to the sector containing one fermion (note that in the limit the fermion number is a conserved quantity). The state of a fermion at position q is the "dressed" state

$$|1q\rangle=a^\dagger(q)A(q)|0\rangle \tag{3.42}$$

where $a^\dagger(q)$ is the creation operator for a fermion at q and $A(q)|0\rangle=|m_q\rangle$ is a coherent state which can be characterized as either the common eigenstate of the annihilation operators of physical mesons with eigenvalue zero or as the common eigenstate of the annihilation operators b(k) of bare mesons with momentum k, with eigenvalues $(\eta/\sqrt{2})\exp[-ikq]/(2\pi k_0)^{3/2}$.

To be rigorous, in the three dimensional case, one should introduce an ultraviolet cut-off on the momentum of mesons in the interaction term to avoid ultraviolet singularities. In the limit in which the cut-off is removed, the meson states $|m_q\rangle$, $|m_{q'}\rangle$ tend to become orthogonal for $q \neq q'$. In this way, due to the coupling of the fermion field to the meson field, the "position" of one fermion turns out to be strictly correlated to states of the meson field which are approximately orthogonal.

We note that the mean value of $\Phi(z,0)$ in the state $|m_q\rangle$

turns out to be

$$\langle m_q|\Phi(z,0)|m_q\rangle=f(z-q)=\frac{\eta}{4\pi}\frac{e^{-m|z-q|}}{|z-q|} \tag{3.43}$$

In what follows, in order to illustrate the localization properties of the model for physical fermions, we make a gross simplification, i.e. we treat the states $|m_q\rangle$ as eigenstates of $\Phi(z,0)$ pertaining to the eigenvalue $f(z-q)$. Let us then consider the physical state for one fermion

$$|\Psi(t)\rangle=\int dq\Psi(q,t)|1q\rangle \tag{3.44}$$

By substituting (3.44) into Eq.(3.41) and disregarding the standard Hamiltonian H_0, we get the equation for $\Psi(q,t)$:

$$\frac{\partial\Psi_V(q,t)}{\partial t} =\int dz f(z-q)V(z,t)\Psi_V(q,t)-\frac{\lambda\eta^2}{8\pi m}\Psi_V(q,t) \tag{3.45}$$

i.e.

$$\frac{\partial\Psi_V(q,t)}{\partial t} = \tilde{V}(q,t)\Psi_V(q,t) - \frac{\lambda\eta^2}{8\pi m}\Psi_V(q,t) \tag{3.46}$$

with $\tilde{V}(q,t)$ a Gaussian noise with zero mean and covariance

$$\langle\langle\tilde{V}(q,t)\tilde{V}(q',t')\rangle\rangle=\frac{\lambda\eta^2}{8\pi m}e^{-m|q-q'|}\delta(t-t') \tag{3.47}$$

Equation (3.46) is essentially the same as Eq.(2.24) of CSL for the case of a single particle. If one considers the sector with N fermions, in the above approximations, one gets an equation of the CSL type (see Eqs.(2.8) and (2.10)) with the operator

$$D(x)=\frac{m^2}{4\pi}\int dz\,\frac{e^{-m|z-x|}}{|z-x|}\,a^\dagger(z)a(z) \tag{3.48}$$

taking the place of N(x) and $(\lambda\eta^2)/m^4$ taking the place of γ.

Thus it appears reasonable that the model (3.40) posesses the desired localizing features. However it also presents a serious difficulty. The evolution equation (3.14) for the statistical operator, specialized to the Lagrangian (3.40), is

$$\frac{d\rho(t)}{dt} = -i[H_0,\rho(t)]+\lambda\int dz\Phi(z,0)\rho(t)\Phi(z,0)-\frac{\lambda}{2}\left\{\int dz\Phi^2(z,0),\rho(t)\right\} \quad (3.49)$$

Let us consider the Hamiltonian H for the free meson field; by using (3.49) one can evaluate the increase per unit time of the mean value of H, getting

$$\frac{d\langle H\rangle}{dt} = -\frac{\lambda}{2}\int dz \, \langle[\Phi(z,0),[\Phi(z,0),H]]\rangle \quad (3.50)$$

i.e.

$$\frac{d\langle H\rangle}{dt} = \frac{\lambda}{2}\int dz \, \delta(0) \quad (3.51)$$

Therefore, the increase per unit time and per unit volume of the mean value of the energy of the meson field, turns out to be infinite. So, in addition to the desidered reduction behaviour, the model displays an undesidered additional behaviour: because the white noise source is locally coupled to the meson field, it copiously produces mesons out of the vacuum. We note that the now outlined difficulty does not show up in the nonrelativistic approximation of the model discussed above (Eqs. (3.45) to (3.48)) due to the gross simplification of treating the states $|m_q\rangle$ as eigenstates of $\Phi(z,0)$.

As stated in the introduction the attempt we have made here to get rid, at the relativistic level, of first class difficulties of quantum mechanics, has increased the second class difficulties (i.e. the divergences) which affect quantum field theories. We note, if one considers the lattice version of the model we have presented here, that the increase per unit time of the mean value of the energy per unit volume then

turns out to be finite and obviously depends on the lattice spacing and on the parameters of the model.

4. LOCAL AND NONLOCAL FEATURES.

As is well known, the quantum theory of measurement, in addition to the difficulties discussed in the introduction which constitute the main motivation for the consideration of dynamical reduction models, presents some further difficulties arising specifically from the assumed instantaneous nature of the collapse of the wave function.

In particular, at the individual level of description, nonlocal features as well as odd aspects (from the relativistic point of view) emerge. Such problems have already been extensively discussed in the literature[27-30], in the case of standard quantum mechanics. It is interesting to look at them from the perspective of relativistic dynamical reduction models.

4.1. *Quantum Theory with the Reduction Postulate.*

4.1.1 *Objective Properties of Individual Systems.*

Suppose one accepts it as meaningful, within standard quantum theory, to consider an individual level of description with the possibility of attributing objective properties to a quantum system. Then a natural attitude corresponding to the one first introduced in the celebrated EPR paper[35] is to assume the following. If an individual physical system S is associated to a definite state vector $|\Psi\rangle$ which is an eigenstate of an observable A pertaining to the eigenvalue a, then one can state that *"S has the property a"* or that *"there exists an element of physical reality"* . We remark that if we denote by P_a the projection operator on the closed linear manifold of the eigenstates of A belonging to the eigenvalue a, then

$$\langle\Psi|P_a|\Psi\rangle=1 \qquad (4.1)$$

We want to stress, however, that even within non relativistic standard quantum mechanics, one is compelled to take the attitude of attributing objective properties to a system even when condition (4.1) is valid only to an extremely high degree of accuracy. To clarify this statement, we can think e.g. of the spin measurement of a spin 1/2 particle by a Stern-Gerlach apparatus. In such a case, the two spin values are strictly correlated to two states Ψ_1 and Ψ_2 describing the spatial degrees of freedom. Even though these wave functions are appreciably different from zero in two distant regions, their supports cannot have a void intersection. As a consequence even an absolutely precise measurement of the position cannot reduce the state vector exactly to an eigenstate of the spin component. The final state unavoidably exhibits an (extremely slight) entanglement of position with spin variables and as such cannot be an eigenstate of a spin operator.

Incidentally we remark that the above considerations are even more appropriate in the case of dynamical reduction models. In fact, on the one hand, such models, with the requirement that they induce Heisenberg reductions, are introduced just with the purpose of implying, at the individual level, the emergence of objective properties for macroscopic objects (in particular the property of being in one place rather than in another). Correspondingly, they imply the emergence of objective properties also for microscopic systems, at least when they interact with macroscopic measuring like devices. On the other hand, as is well known and has been repeatedly stressed in refs.(16,17), within dynamical reduction models, the unavoidable persistence of the tails, the tiny but non zero terms corresponding to the terms of a linear superposition which have been suppressed by the spontaneous localization process, prevents us from asserting with absolute certainity that the "macroscopic pointers" are in a definite space region.

The conclusion is that if one whishes to attribute objective properties to individual systems one has to accept that such an attribution is legitimate even when the mean value of the projection operator on the eigenmanifold associated to the eigenvalue corresponding to the attributed property is not exactly equal to 1, but is extremely close to it.

4.1.2. *Nonlocality.*

Nonlocal features[3] of quantum mechanics arise from the fact that, due to the instantaneous nature of the collapse of the wave function, possible actions performed in a certain space region can, under specific circumstances, induce immediate changes in distant regions. In this connection two important questions arise: first, do these changes correspond to some modifications of the physical situation in the distant region ? Secondly, are these modifications detectable, so that one can take advantage of them to send faster-than-light signals ?

In order for the above questions to have an unambiguous meaning, it is necessary to specify at which level of description of physical processes one is raising them. In particular it is important to make a clear disctinction between the ensemble and the individual levels of description.

To understand the above situation one can make reference either to the well known EPR-Bohm type set-up for an "entangled" state of a composite system $S=S_1+S_2$, the components being far apart and non-interacting, or to the position measurement of a particle whose state is the linear superposition of two distant packets. In the first case, as is well known, at the level of the individual members of the ensemble, the far away system (let us say S_2) is (32) "*steered or piloted into one or the other type of state*" according to the measurement which is performed on S_1 and the specific result which is obtained. In the second case, let us write

$\Psi(x,t)=\Psi_1(x,t)+\Psi_2(x,t)$, $\Psi_1(x,t)$ and $\Psi_2(x,t)$ being appreciably different from zero only in two far apart regions α_1 and α_2, respectively. Then, a measurement aimed to test whether the particle is in α_1 and yielding, e.g., the answer "no" ("yes"), instantaneously collapses $\Psi(x,t)$ to $\Psi_2(x,t)$ ($\Psi_1(x,t)$). Correspondingly the quantity $\int_{\alpha_2} dx|\Psi(x,t)|^2$ (i.e. " the mean value"[4] of the projection operator on region α_2), changes from 1/2 to either 1 or 0 according to the outcome of the position measurement at α_1. This puts into evidence how, if interpreted as a theory describing individual systems, quantum mechanics exhibits nonlocal features.

The situation is quite different when looked at from the ensemble point of view. In fact, as is well known [33] no measurement procedure in a given region can change the statistical distribution of prospective measurement results in a distant region.

These remarks, although made in the context of ordinary quantum theory with a reduction postulate, are not essentially modified (i.e. the word "instantaneously" must be changed to "rapidly") in the case of the CSL theory with its reduction dynamics.

4.1.3. *Relativistic Oddities with Observations.*

In the above analysis we have discussed a measurement process in a given reference frame O. The consideration of the instantaneous change of the state vector induced by a measurement raises interesting questions when looked at by different observers. Since the distance between the two space regions α_1 and α_2 mentioned above can be arbitrarily large, even the passage to a reference frame which is moving with respect to O with an arbitrarily small velocity can change the time order of simultaneous (for O) events occurring in the two regions.

To illustrate briefly the main points of the problem we consider the observer O looking at a system of one particle in the state $\Psi(x,t)=\Psi_1(x,t)+\Psi_2(x,t)$ which is a superposition of

two well localized wave packets propagating in opposite directions with respect to the origin x=0. Disregarding the extension and the spreading of the wave packets we can represent the situation by the space-time diagram of Fig.1, in which the two world lines 1 and 2 are associated to Ψ_1 and Ψ_2, respectively. Suppose that, at the space time point $C=(x_1,t_1)$ there is a device designed to test whether the particle is there or not, and let us suppose that, in the specific individual case we are considering, the result of the test is "yes". This is a covariant statement on which all observers

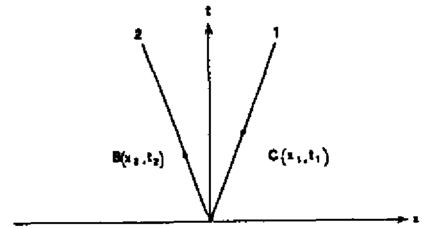

Fig.1 : World lines of two well-localized wave packets 1 and 2, belonging to a single particle which is detected at event C.

must agree. If one adopts the wave packet reduction postulate of standard quantum theory and one assumes that the collapse occurs for each reference frame along the hyperplane t'=const, where t' is the subjective time of the event C for such a frame, one meets a puzzling situation. Let us in fact consider an objective point B on world line 2, which is space-like separated from C and which is labelled by (x_2,t_2) (see Fig.1). According to O, $t_2<t_1$ and, by the above assumption, no reduction has occurred at time t_2 and the state vector is $\Psi(x,t_2)$. If one considers the projection operator P_2 on the space region around x_2, one has $\langle\Psi|P_2|\Psi\rangle = 1/2$. Accordingly we could say that the situation is such that, at time t_2, O cannot attribute to the particle the "property" of being or not being in the region around x_2.

However, there exists an observer O' such that $t_2'>t_1'$, where t_2' and t_1' are the time labels attributed by O' to the events B and C, respectively. For O' the particle has triggered the detector in C at t_1'. Therefore at t_2' the state of the system is Ψ_1. Then, for O', the mean value of the projection operator P_2 at t_2' is zero [5]. Observer O' can then state that the particle has the property of "not being around B". Thus, O and O' do not agree on a statement referring to a local property at an objective space-time point.

It is useful to note that this ambiguity occurs only for the points of the world line 2 which are spacelike with respect to C; for a point B in the past of C all observers agree in stating that the particle has no definite local property while for a point B in the future of C all observers agree in saying that the particle "is not around B".

The above discussion follows essentially the one given in ref.(28). The consideration of these kinds of difficulties have led various authors to take different attitudes. Bloch[29] and Aharonov and Albert[30] derive from this the conclusion that one cannot attach an objective meaning to wave functions for individual systems. Hellwig and Kraus[29] have tried to solve the ambiguity about the wave function at a given objective space time point by requiring that the collapse of the state vector due to the measurement at C takes place on the surface which delimits the past light cone originating from C. Thus, at points outside the past light cone the statevector is reduced, while at points inside the past light cone the statevector is unreduced. This is a covariant statement and leads the authors to the identification of a unique state vector to be associated to any given space-time point. However, such a prescription implies that there are space–like surfaces (those crossing the past cone of C) to which it is not possible to associate a definite state vector. This, as nicely illustrated by Aharonov and Albert[30], forbids the consideration of nonlocal observables on these hypersurfaces; for example it does not allow one to speak consistently of the total charge of the system. Moreover, the assumption that the reduction occurs on

the hypersurface delimiting the past light cone raises conceptual difficulties with the cause-effect relation: in certain reference frames the cause would seem to occur later than the effect

4.2 *Relativistic Reduction Models*

We discuss here the local and nonlocal features of reduction models in the relativistic case. In order to investigate whether the dynamics presented in Section 3.2 induces nonlocal effects we make reference to the procedure outlined in ref.(34), i.e. we consider whether a modification of the Lagrangian density in a space-time region C, can have effects in a region B which is space-like separated from it (this will be discussed in Subsections 4.2.1. and 4.2.2). In particular, since we want to study the possibility of nonlocal effects due to the reducing character of the dynamics, we will take into account modifications of the Lagrangian density $\mathcal{L}_I$ coupled to the noise.

The problems which we want to discuss require the consideration of "local observables". By this expression we mean the integral of a function of the interaction picture fields and their derivatives:

$$A_I(\sigma)=\int dx' f_\alpha(x') F[\Phi_I(x'),\partial_\mu\Phi_I(x')] \qquad (4.2)$$

with $f_\alpha(x)$ a function of class C^∞ with a compact support α on the space-like surface σ. The physically interesting quantities, for our analysis, are the mean values[4] of such local observables. As usual it is necessary to make precise the level at which the nonlocality problem is discussed. We will consider it, as before, both at the ensemble and at the individual level.

At this last level, we will discuss also questions analogous to those considered in Sect. 4.1.3 which arose when one looked from a relativistic point of view at the wave packet reduction postulate. In the present context, they emerge naturally from the relativistic dynamics described by the Tomonaga-Schwinger equation. In particular, it turns out that, for all Tomonaga-Schwinger surfaces coinciding on α, the mean value of the local observable depends upon the specific Tomonaga-Schwinger surface on which it is evaluated (see Subsection 4.2.3). This is not the case with the Tomonaga-Schwinger description of an ordinary relativistic quantum field theory, and it gives rise to interesting questions about the possibility of attributing objective properties to the systems which we will discuss in Subsection 4.2.4.

4.2.1. *Ensemble Level*

As already emphasized, at the ensemble level, the statistical operator and therefore the physics of the two models considered in Sect.3 coincide. Thus, to investigate properties referring to the statistical ensemble, one can make reference to the stochastic dynamics with hermitian coupling, which can be easily handled by familiar methods.

With reference to the model of Sect. 3.1, we consider the mean value of a local observable $A_I(\sigma)$

$$\langle A_I(\sigma)\rangle = Tr\{A_I(\sigma)\rho_I(\sigma)\} \qquad (4.3)$$

Let us denote by $U_V(\sigma,\sigma_0)$ the evolution operator

$$U_V(\sigma,\sigma_0) = T\, e^{i\int_{\sigma_0}^{\sigma} dx\, L_I(x)V(x)} \qquad (4.4)$$

and by $A_{HV}(\sigma)=U^{\dagger}_V(\sigma,\sigma_0)A_I(\sigma)U_V(\sigma,\sigma_0)$ the observable in the Heisenberg picture which corresponds to $A_I(\sigma)$ for the realization V of the stochastic potential. Let $A_H(\sigma)$ be the stochastic average over V of $A_{HV}(\sigma)$:

$$A_H(\sigma) = \int D[V]\,P[V]\,A_{HV}(\sigma) \qquad (4.5)$$

We then have

$$\langle A_I(\sigma)\rangle = Tr\,\{A_H(\sigma)\rho(\sigma_0)\} \qquad (4.6)$$

The support of $A_I(\sigma)$ defines a partition of space-time into three regions: the future, the past, and the set of points which are space-like separated from all points belonging to this support. (see Fig.2).

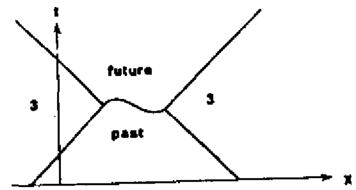

Fig.2 : The support of the local observable $A_I(\sigma)$, and the set of points 3 bearing a space-like relation to this support.

We choose now a space-time region C entirely contained in region 3 and we consider a modification of the Lagrangian density $L_I(x)$ coupled to the noise. We replace $L_I(x)$ with a new density $L_I^{\sim}(x)=L_I(x)+\Delta L_I(x)$, with $\Delta L_I(x)$ different from zero only for $x\in C$.. If $A_{HV}^{\sim}(\sigma)$ denotes the local observable in the Heisenberg picture, when we replace $L_I(x)$ with $L_I^{\sim}(x)$, we have

$$A_{HV}^{\sim}(\sigma) = \left[T e^{i\int_{\sigma_0}^{\sigma} dx\,\Delta L_I(\Phi_{HV}(x))V(x)}\right]^{\dagger} A_{HV}(\sigma) \left[T e^{i\int_{\sigma_0}^{\sigma} dx\,\Delta L_I(\Phi_{HV}(x))V(x)}\right] \qquad (4.7)$$

The fields $\Phi_{HV}(x)$ which appear in $\Delta L_I(x)$ are the fields in Heisenberg picture for the original Lagrangian density $L_0(x)+L_I(x)V(x)$. The appearence of $\Delta L_I(x)$ actually restricts the integration in the exponential to the space-like region C, which is space-like separated with respect to the support of

$A_{HV}(\sigma)$. It follows that the exponential commutes with $A_{HV}(\sigma)$, and therefore

$$A_{HV}^{\sim}(\sigma)=A_{HV}(\sigma) \tag{4.8}$$

for any given realization of the stochastic potential. One then has

$$A_{H}^{\sim}(\sigma)=A_{H}(\sigma) \tag{4.9}$$

i.e., due to Eq.(4.6), at the level of the statistical ensemble any modification of $\mathcal{L}_I(x)$ in a space-time region C cannot cause physical changes in regions which are space-like separated from it. We stress that this conclusion is true for the case of nonhermitian coupling as well as for the case of hermitian coupling, even though the argument was carried out in terms of the hermitian coupling alone, as it depends solely upon the statistical operator which is identical for both couplings.

4.2.2. *Individual Level*

From the result (4.8) of the previous Section it is also evident that, in the case of an hermitian coupling,(i.e. for (3.17)) a variation of the Lagrangian density $\mathcal{L}_I(x)$ in a region C has no effect on the mean value of any local observable with support spacelike separated from C, even at the level of an individual system(i.e., for any realization of the stochastic potential). This property is related to the fact that, in this case, no Heisenberg reduction takes place.

The situation is quite different in the case of a non-hermitian coupling. In fact, let us consider Eq.(3.21) and the operator $S_V(\sigma,\sigma_0)$ given by (3.29). The mean value of a local observable $A_I(\sigma)$ is then

$$\langle A_I(\sigma)\rangle=\frac{\langle\Psi_V(\sigma)|A_I(\sigma)|\Psi_V(\sigma)\rangle}{\||\Psi_V(\sigma)\rangle\|^2}=\frac{\langle\Psi(\sigma_0)|S_V^{\dagger}(\sigma,\sigma_0)A_I(\sigma)S_V(\sigma,\sigma_0)|\Psi(\sigma_0)\rangle}{\|S_V(\sigma,\sigma_0)|\Psi(\sigma_0)\rangle\|^2} \tag{4.10}$$

We now replace in (3.21) $\mathcal{L}_I(x)$ by $\mathcal{L}_I(x)+\Delta\mathcal{L}_I(x)$, $\Delta\mathcal{L}_I(x)$ being different from zero only for $x\in C$, and we denote by $S_V^{\Delta}(\sigma,\sigma_0)$ the corresponding evolution operator. The mean value $\langle A_I^{\Delta}(\sigma)\rangle$ of the same local observable, for the same initial condition, is now

$$\langle A_I^{\Delta}(\sigma)\rangle=\frac{\langle\Psi(\sigma_0)|S_V^{\Delta\dagger}(\sigma,\sigma_0)A_I(\sigma)S_V^{\Delta}(\sigma,\sigma_0)|\Psi(\sigma_0)\rangle}{\|S_V^{\Delta}(\sigma,\sigma_0)|\Psi(\sigma_0)\rangle\|^2} \tag{4.11}$$

Note that in general

$$\langle A_I^{\Delta}(\sigma)\rangle\neq\langle A_I(\sigma)\rangle \tag{4.12}$$

in spite of the fact that $[\Delta\mathcal{L}_I(x), A_I(\sigma)]=0$, $\forall x$.

4.2.3 ***Mean Values of Local Observables and Oddities in Relativistic Reduction Models.***

Let us consider a physical system satisfying the initial condition $|\Psi(\sigma_0)\rangle=|\Psi_0\rangle$ on the space-like surface σ_0, the local observable A and two arbitrary space-like surfaces σ_1 and σ_2 coinciding on the support α of A (see Fig.3).

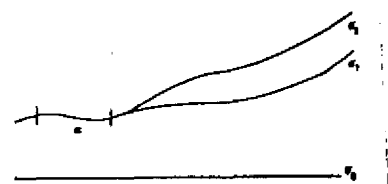

Fig.3 : The space-like surfaces σ_1 and σ_2 coinciding on the support α of local observable A.

When the dynamics (3.17) due to a hermitian interaction is considered, for any given realization of the stochastic potential, as is well known, the mean value of A in the state $|\Psi_V(\sigma_1)\rangle$ coincides with the one in the state $|\Psi_V(\sigma_2)\rangle$. There follows, at the individual level for the case of the hermitian coupling and, as a consequence, at the ensemble level for both cases of hermitian and skew-hermitian coupling,that the mean value of a local observable does not depend on the particular space-like surface which one chooses among all those coinciding on its support. This shows that also in the case of dynamical reduction models, at least at the ensemble level, one can consistently define, as in standard quantum field theory, local observables.

Again, the situation at the individual level is quite different in the skew-hermitian case. In fact, for a given realization of the stochastic potential one has

$$\frac{\langle\Psi_V(\sigma_2)|A|\Psi_V(\sigma_2)\rangle}{\||\Psi_V(\sigma_2)\rangle\|^2} = \frac{\langle\Psi_V(\sigma_1)|S_V^\dagger(\sigma_2,\sigma_1)AS_V(\sigma_2,\sigma_1)|\Psi_V(\sigma_1)\rangle}{\|S_V(\sigma_2,\sigma_1)|\Psi_V(\sigma_1)\rangle\|^2} \quad (4.13)$$

which, in general, is different from $\langle\Psi_V(\sigma_1)|A|\Psi_V(\sigma_1)\rangle/\||\Psi_V(\sigma_1)\rangle\|^2$ even though the space-time region spanned in tilting σ_1 into σ_2 is space-like separated from the support α of A, and, consequently

$$[A,S_V(\sigma_2,\sigma_1)]=0 \quad (4.14)$$

This dependence, at the individual level, of the mean value of a local observable upon the space-like surface (among those coinciding on the support) over which it is evaluated, is not *per se* a difficulty of the theory. It becomes however a difficulty if one wishes to claim that such a mean value corresponds to an objective property of an individual system.

Before facing this problem (see next Subsection), a deeper analysis of the implications of relativistic reduction models for microscopic (case a) below) and macroscopic (case b)) systems is necessary.

case a).

Let us start by reconsidering the case in Subsect. 4.1.3, of a microscopic system coupled to a macroscopic one which acts as a "measuring apparatus" in the sense of dynamical reduction models. Let A_1 and A_2 be two local observables of the microsystem whose supports α_1 and α_2 are space-like separated , and suppose the macroscopic system is devised to measure A_1. For our purposes we can ignore the hamiltonian evolution for the operators and we consider the Tomonaga-Schwinger evolution equation of the state vector, for a specific realization of the stochastic potential

$$\frac{\delta|\Psi(\sigma)\rangle}{\delta\sigma(x)} = [iL_{1-S}(x)+L_I(x)V(x)-\lambda L_I^2(x)]|\Psi(\sigma)\rangle \quad (4.15)$$

Here $L_{1-S}(x)$ (describing the local system-apparatus interaction) and $L_I(x)$ may be taken as different from zero only in a space-time region C which is space-like with respect to α_2 (see Fig.4).

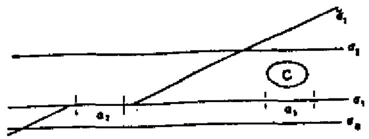

Fig.4 : A macroscopic apparatus measures local observable A_1 in space-time region C. A_1's support α_1 is space-like separated with respect to α_2, the support of another local observable A_2.

Let us assume that the local observables A_1 and A_2 have a purely point spectrum with eigenvalues 0 and 1, and let us consider the initial state

$$|\Psi(\sigma_0)\rangle=(1/\sqrt{2})[|\Psi_1\rangle+|\Psi_2\rangle]|\chi_1\rangle \qquad (4.16)$$

with

$$A_i|\Psi_j\rangle=\delta_{ij}|\Psi_j\rangle;\ i,j=1,2 \qquad (4.17)$$

$|\chi_1\rangle$ being the untriggered apparatus state. Let us furthermore assume that the particular realization of the stochastic potential V(x) is one of those "yielding the result 1 for the measurement of A_1". The situation is then the following:

i. The state associated to σ_0 and σ_1 is $|\Psi(\sigma_0)\rangle$

ii. The state associated to σ_2 is (N being a normalization factor)

$$|\Psi(\sigma_2)\rangle=\frac{1}{N}\, e^{\int_{\sigma_1}^{\sigma_2} dx\, [iL_{1-S}(x)+L_I(x)V(x)-\lambda L_I^2(x)]}\, |\Psi(\sigma_0)\rangle \qquad (4.18)$$

which, under the assumptions which have been made, is approximately an eigenstate of A_2 pertaining to the eigenvalue zero.

iii. The state associated to $\tilde{\sigma}_1$ is also $|\Psi(\sigma_2)\rangle$.

Indeed, the relativistic CSL dynamics considered in Sect.3.3 is such that, when a space-like hypersurface crosses the region C towards the future, no matter what is the behaviour in regions far apart from C, the state vector associated to this hypersurface collapses to the eigenstate of A_1 corresponding to the eigenvalue which has been found.

Looking at the problem from the point of view of the evolution from σ_1 to σ_2 one could be tempted to say that, since the mean value of A_2 has become practically zero as a consequence of the "measurement" in the space time region C, an element of physical reality associated with A_2 has emerged. This is a nonlocal effect of the type of those occurring in an EPR set up.

However, one must realize that the same change of the mean value of A_2 occurs when one considers the Tomonaga-Schwinger evolution from σ_1 to $\tilde{\sigma}_1$, in accordance with iii. This gives rise to an ambiguity in the mean value of A_2, i.e. in a quantity that, when the support α_2 shrinks to zero, refers to a unique objective space-time point. This is not surprising; it corresponds simply to the emergence, within the relativistic reducing dynamics, of the aspects discussed in Subsect.4.1.3 for the standard quantum theory with a reduction postulate. In fact, one can remark that $\tilde{\sigma}_1$ can be approximately identified with a t'=const hyperplane for a boosted observer for which the interaction with the macro-object has already taken place[6].

case b).

Let us discuss now the same problem for macroscopic systems. We consider a situation analogous to the previous one but in which there are two macroscopic systems performing measurements of the observables A_1 and A_2. The initial condition is given by assigning to the surface σ_0 the state

$$|\Psi(\sigma_0)\rangle=(1/\sqrt{2})[|\Psi_1\rangle+|\Psi_2\rangle]|\chi_1\rangle|\chi_2\rangle \qquad (4.19)$$

where $|\chi_1\rangle$ and $|\chi_2\rangle$ refer to the untriggered apparatuses. The evolution equation, with the usual approximation, is now

$$\frac{\delta|\Psi(\sigma)\rangle}{\delta\sigma(x)}=[iL_{1-S}(x)+iL_{2-S}(x)+L_{I1}(x)V(x)+L_{I2}(x)V(x)-\lambda L_{I1}^2(x)-\lambda L_{I2}^2(x)]|\Psi(\sigma)\rangle \qquad (4.20)$$

with obvious meaning of the symbols. To clearly define the situation from the physical point of view we assume that the time which is necessary in order that the microsystem triggers the apparatus is sensibly shorter than the typical reduction time for the apparatus. This means that in the above equation we can consider $L_{1-S}(x)$ and $L_{2-S}(x)$ to be different from zero only in the regions C_1 and B_1, respectively, and $L_{I1}(x)$ and $L_{I2}(x)$ in the regions C_2 and B_2, respectively, as shown in Fig.5.

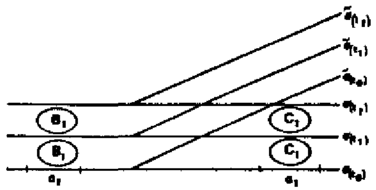

Fig.5 : Measurements take place in C_1 and B_1, followed by reduction dynamics in C_2 and B_2, of local observables A_1 and A_2, respectively.

Let us also assume that the specific realization of the stochastic potential is one leading to the value 1 for A_1. We are interested in discussing the states of the macrosystem used to measure A_2 and the mean values of its observables on various hypersurfaces. In particular, let $\tilde{A}_2$ be the observable of the apparatus corresponding to the yes-no experiment asking whether the result 0 has been found in a measurement of A_2. We consider a t=const hypersurface $\sigma(t)$ and also the bent hypersurfaces $\tilde{\sigma}(t)$ containing the spatial support of $\tilde{A}_2$ at time t (see Fig.5). The situation can now be summarized as follows:

i. For $t<t_0$ the state associated to any surface $\sigma(t)$ or $\tilde{\sigma}(t)$ has always the form of a factorized state; one of the factors

refers to the apparatus 2 and is $|\chi_2\rangle$. Note that what changes in going from $\sigma(t)$ to $\tilde{\sigma}(t)$ is the state of the system+apparatus 1.

ii. For $t=t_1$ the state associated to $\sigma(t_1)$ is

$$|\Psi(\sigma(t_1))\rangle=(1/\sqrt{2})[|\Psi_1\rangle|\chi_1^1\rangle|\chi_2^0\rangle+|\Psi_2\rangle|\chi_1^0\rangle|\chi_2^1\rangle] \qquad (4.21)$$

with the obvious meaning that the superscripts identify the states of the macroscopic apparatuses which have been triggered by the interaction with the microsystem, these states being labelled by the eigenvalues which have been found.

From (4.21) one sees that the state $|\Psi(\sigma(t_1))\rangle$ is not a factorized state and as a consequence it cannot be an eigenstate of any observable of apparatus 2. In particular the mean value of $\tilde{A}_2$ in the state (4.21) is 1/2.

However, it is important to remark that the state to be associated to the surface $\tilde{\sigma}(t_1)$ drawn in Fig.5 is, for the particular realization of the stochastic potential

$$|\Psi(\tilde{\sigma}(t_1))\rangle=|\Psi_1\rangle|\chi_1^1\rangle|\chi_2^0\rangle \qquad (4.22)$$

This state is factorized and it is an eigenstate of $\tilde{A}_2$.

iii. The state to be associated with any surface $\sigma(t)$ and $\tilde{\sigma}(t)$ when $t>t_2$, is once more a factorized state with the factor $|\chi_2^0\rangle$ for the apparatus 2.

The conclusion is that, even though the dependence of the mean value of a local observable upon the space-like surface in which it is evaluated is present also in the case of macro-objects, this dependence occurs only for a time interval of the order of the one which is necessary for the reduction to take place.

4.2.4. *Objective Properties of Micro and Macroscopic Systems.*

We started this section by relating the possibility of attributing objective properties to individual systems to requirement (4.1) being satisfied to an extremely high degree of accuracy. In the relativistic case, however, as shown with great detail in the previous Subsection, the mean value of a projection operator associated to a local observable is affected by an ambiguity depending on the space-like surface used to evaluate it, and, under specific circumstances, by changing the surface its value can vary from e.g. 1/2 to almost exactly 1. This shows that the above definition of objective properties for individual systems is inadequate, and must be made more precise.

We think that the appropriate attitude is the following: when considering a local observable A on its associated support we say that an individual system has the objective property a,(a being an eigenvalue of A), only when the mean value of P_a is extremely close to one, when evaluated on all space-like hypersurfaces containing the support of A.

Thus, according to this prescription, one cannot attribute an objective property to an individual system when there is an appreciable dependence of the mean value of the local observable upon the surface used to evaluate it.

Let us analyze the implications of this attitude in the cases of microscopic and macroscopic systems. For a microsystem, with reference to case a) of the previous subsection, we observe that no objective property corresponding to a local observable can emerge as a consequence of a "measurement process" performed in a region which is space-like separated from the support of the considered observable. This does not mean that microsystems cannot acquire objective local properties as a consequence of a measurement performed in another space-time region; in fact, with reference to the discussion in a) and to an EPR-Bohm-like set-up one can remark that if one considers the spin component of particle 2, when the particle is in the future of the

region in which the spin of particle 1 has been measured, then one can attribute to particle 2 the objective local property of having its spin "up" or "down".

We wish to emphasize again that the discussion under b) has shown that the impossibility of associating local properties to macrosystems lasts only for a time interval of the order of that which is necessary for the "spontaneous dynamical reduction" to take place. In fact, before the macroapparatus 2 interacts with the microsystem the state of the apparatus is obviously well defined and corresponds to the untriggered state, independently of the considered surface. After the reduction ensuing from the interaction of the microsystem with it, apparatus 2 is again in a well defined state, corresponding to the result which it has registered. Moreover this result is "correctly" correlated to the result registered by apparatus 1 [7].

In conclusion, the dynamical reduction model presented in this work, together with the prescription for the attribution of objective properties to physical systems proposed in this Subsection, allows one to overcome the difficulties discussed in Subsection 4.1.3. The theory assigns a state vector to any space-like hypersurface and the dependence, at the individual level, of the mean value of a local observable upon the specific space-like surface used to evaluate it, does not constitute a difficulty. It simply requires a precise and appropriate criterion for relating the objective properties of a physical system to the mean values of local observables: in particular, this criterion permits the attribution of objective local properties to macro-objects, at the individual level. In a sense, the above analysis should have proven once more that dynamical reduction models meet the requirement put forward by J.S.Bell[3] for an exact and serious formulation of quantum mechanics, i.e. that it should "*allow electrons to enjoy the cloudiness of waves, while allowing tables and chairs, and ourselves, and black marks on photographs, to be rather definetely in one place rather than another, and to be described in* classical terms."

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APPENDIX A.

In this Appendix we show that the consistency condition (2.21) for the statistical operator evolution will be satisfied if, and only if, the random fluctuations have a white noise time behaviour.

Let us denote by $A(q,t;q',t')=A(q-q',t-t')$ the covariance function which is a positive definite and symmetric ($A(q,t;q',t')=A(q',t';q,t)$) function. Eq. (2.17) defines, when the covariance function $A(q,t;q',t')$ is used to evaluate the stochastic average appearing in it, the coordinate representation of the operator $\Sigma(t,t_0)\rho(t_0)$. One then sees that, for the consistency requirement (2.21) to be satisfied, the following equality must hold:

$$\int_{t_0}^{t}d\tau\int_{t_0}^{t}d\tau'[2A(0,\tau-\tau')-A(q-q',\tau-\tau')-A(q'-q,\tau-\tau')]=$$

$$\int_{t_0}^{t'}d\tau\int_{t_0}^{t'}d\tau'[2A(0,\tau-\tau')-A(q-q',\tau-\tau')-A(q'-q,\tau-\tau')]+ \qquad (A.1)$$

$$\int_{t'}^{t}d\tau\int_{t'}^{t}d\tau'[2A(0,\tau-\tau')-A(q-q',\tau-\tau')-A(q'-q,\tau-\tau')]$$

for any $t_0<t'<t$ and arbitrary fixed q and q'.

If the noise is white in time, i.e.

$$A(q-q',t-t')=B(q-q')\delta(t-t') \qquad (A.2)$$

one immediately checks that (A.1) is satisfied.

We prove now the necessity. To this purpose we assume that $A(q,t;q',t')$ instead of being of form (A.2) is a continuous function of its arguments. We consider $t=t'+\epsilon$ with ϵ

arbitrarily small. Condition (A.1) implies:

$$\int_{t_0}^{t'}d\tau\int_{t'}^{t'+\epsilon}d\tau'\, f(\tau-\tau')+\int_{t'}^{t'+\epsilon}d\tau\int_{t_0}^{t'}d\tau' f(\tau-\tau')=0 \qquad (A.3)$$

for any t', where

$$f(\tau-\tau')=2A(0,\tau-\tau')-A(q-q',\tau-\tau')-A(q'-q,\tau-\tau'). \qquad (A.4)$$

Due to the fact that $f(\tau)$ is a continuous and even function, (A.3) implies, to the first order in ϵ:

$$\int_{t_0}^{t'}d\tau\, f(\tau-t')=0, \quad \forall t' \qquad (A.5)$$

With the change of variables $z=\tau-t'$, (A.5) becomes

$$\int_{t_0-t'}^{0}dz\, f(z)=0 \;,\; \forall t'>t_0 \qquad (A.6)$$

Since $f(z)$ is a continuous function, (A.6) implies $f(z)=0$.

Suppose one evalutes the integral $\int dxdx'A(x,x')F(x)F(x')$, where x is a shorthand for (q,t), choosing for $F(x)$ a factorized function $F(x)=h(q)g(t)$ with $\int dqh(q)=0$. Then the vanishing of $f(z)$ implies, by (A.4)

$$\int dxdx'A(x,x')F(x)F(x')=\int dqh(q)\int dq'h(q')\int d\tau\int d\sigma A(0,\tau-\sigma)g(\tau)g(\sigma)=0 \qquad (A.7)$$

We see that the l.h.s. of (A.7) can vanish for functions $F(x)$ which do not vanish almost everywhere. This is absurd since it contradicts the hypothesis that $A(x,x')$ be positive definite.

APPENDIX B.

In this Appendix we show that the compatibility property (2.29) for the cooked probability density can only be satisfied if the random fluctuations have a white noise time behaviour.

Let us consider the stochastic equation for the state vector analogous to (2.24)

$$\frac{\partial \tilde{\Psi}_V(q,t)}{\partial t} = V(q,t)\tilde{\Psi}_V(q,t) \tag{B.1}$$

In Eq.(B.1), $V(q,t)$ is a Gaussian noise with zero mean and covariance function $A(q-q',t-t')$ which, instead of being of form (A.2), is supposed to be a continuous function of its arguments.

For a given initial condition $\Psi_V^{\sim}(q,t_0)=\Psi(q,t_0)$, let us consider the state vectors

$$\Psi_V(q,t)=\tilde{\Psi}_V(q,t)e^{-g(t-t_0)} \tag{B.2}$$

In (B.2) $g(t-t_0)$ is a "counterterm" which has to be chosen in such a way that the average of the square norm of the state vectors $\Psi_V(q,t)$ be conserved:

$$g(t-t_0)= \int_{t_0}^{t} d\tau \int_{t_0}^{t} d\tau' A(0,\tau-\tau') \tag{B.3}$$

Note that when $A(q-q',t-t')$ has the form (A.2), then $g(t-t_0)$ reduces to $(t-t_0)B(0)$, as it must.

As usual, we consider the normalized state vectors $|\Psi_V(t)\rangle/\||\Psi_V(t)\rangle\|$ and the cooked probability density $P_C[V]=P[V]\||\Psi_V(t)\rangle\|^2$, where $P[V]$ is the probability density associated to the Gaussian noise with zero mean and covariance $A(q-q',t-t')$. As a consequence, the equation for the statistical operator turns out to coincide with the one which one would obtain in the hermitian case with the same expectation values for the gaussian noise.

According to (B.1) and (B.2), the square modulus of $\Psi_V(q,t)$ is

$$|\Psi_V(q,t)|^2 = e^{2\int_{t_0}^{t} d\tau V(q,\tau)-2g(t-t_0)} |\Psi(q,0)|^2 \tag{B.4}$$

With reference to the discussion following Eq.(2.28) we have, for the cooked probability density $P_C[V] = P_C[V_1,V_2]$:

$$\begin{aligned} P_C[V]=&\frac{1}{N}\int d\tilde{q}\,|\Psi(\tilde{q},0)|^2 e^{-\frac{1}{2}\int_{t_0}^{t} d\tau\int_{t_0}^{t} d\sigma\iint dq\,d\xi\, V(q,\tau)\tilde{A}(q-\xi,\tau-\sigma)V(\xi,\sigma)} \\ &\cdot e^{2\int_{t_0}^{t} d\tau V(\tilde{q},\tau)-2g(t-t_0)} = \\ &\frac{1}{N}\int d\tilde{q}\,|\Psi(\tilde{q},0)|^2 e^{-\frac{1}{2}\int_{t_0}^{t} d\tau\int_{t_0}^{t} d\sigma\iint dq\,d\xi\, W(q,\tau,\tilde{q},t)\tilde{A}(q-\xi,\tau-\sigma)W(\xi,\sigma,\tilde{q},t)} \end{aligned} \tag{B.5}$$

In Eq. (B.5), we have denoted by $A^{\sim}$ the inverse of the covariance function A and by $W(q,\tau,q^{\sim},t)$ the function:

$$W(q,\tau,\tilde{q},t)=V(q,\tau)-2\int_{t_0}^{t} d\eta\; A(q-\tilde{q},\tau-\eta) \tag{B.6}$$

Using (B.5) one can easily evaluate $P_C^{\sim}[V_1]=\int\mathcal{D}[V_2]P_C[V_1,V_2]$, getting:

$$\begin{aligned} \tilde{P}_C[V_1]= &\frac{1}{N'}\int d\tilde{q}\,|\Psi(\tilde{q},0)|^2 \\ &e^{-\frac{1}{2}\int_{t_0}^{t'} d\tau\int_{t_0}^{t'} d\sigma\iint dq\,d\xi\, W(q,\tau,\tilde{q},t)\tilde{A}(q-\xi,\tau-\sigma)W(\xi,\sigma,\tilde{q},t)} \end{aligned} \tag{B.7}$$

For the compatibility requirement (2.29) to be satisfied, $P_c^{\sim}[V_1]=P_c[V_1]$ must hold for any $\Psi(q,0)$, where $P_c[V_1]$ is given by (B.7) with the variable t' replacing t in the arguments of the functions W appearing there. This implies

$$\int_{t_o}^{t'} d\sigma A(q-\tilde{q},\tau-\sigma)=\int_{t_o}^{t} d\sigma A(q-\tilde{q},\tau-\sigma) \quad \forall\ q,\tilde{q};\ \forall\ t_o<\tau<t'<t \qquad (B.8)$$

Equation (B.8), together with the assumption that $A(q-q',t-t')$ is a continuous function, implies $A(q-q',t-t')\equiv 0$, which is absurd.

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FOOTNOTES

[1] We note that H.P. Stapp adds to the terms "Heisenberg reductions" and "von Neumann reductions", when he is using them in our sense, the specification "objective". Since in this paper we limit our considerations to models which

represent an actual modification of quantum mechanics, we have suppressed, for brevity this specification.

[2] See also the treatment of ref.(24).

[3] For an exhaustive discussion the reader is referred to the excellent book by M. Redhead (31).

[4] We are using the common phrase "mean value" to represent diagonal matrix elements like (4.1), even though the statistical connotation of this phrase has no meaning in our discussion.

[5] Obviously, to be rigorous, both the statement that the state is Ψ_1 or Ψ_2, as well the consideration of the projection operators $P_1$and P_2 are not correct because one should consider a relativistic description of the system and of the observables. However, since O' is moving with a very small velocity v<<c with respect to O, the above approximations are appropriate.

[6] The bending of the surface at the left of α_2 shown in Fig.4 is allowed since, under the assumptions we have made, $L_1(x)=0$ in that region

[7] Perhaps it is worth noticing that it would be possible to give another covariant prescription for the attribution of objective local properties to physical systems. More precisely one could, for any local observable A, consider the mean value of the projection operator P_a on one of A's eigenmanifolds evaluated for the state vector associated to the surface which delimits the future light cone of the support of A. Then, if this mean value is extremely close to 1, one asserts that the system has the objective property a. This is quite different from the previously considered criterion (i.e. that the mean value be extremely close to one on all hypersurfaces containing the support of A) and would, in case a) of the previous subsection, lead to the assignment of the objective property corresponding to the value zero for the observable A_2 to the microsystem, contrary to what would occur by the adoption of the previous criterion.

This attitude would correspond to the following particular interpretation, at the relativistic level, of the EPR criterion for elements of physical reality: "if there exists *at least one* observer who can *predict*, almost (in the above specified sense) with certainty and without disturbing a system in any way, the value of a physical quantity, then there exists an element of physical reality corresponding to that quantity".

We do not want to enter here into a detailed discussion of the conceptual implications involved in adopting the above prescription. We will analyze them in a forthcoming paper. We believe that they lead to some conceptual difficulties in connection with the cause-effect relation. This is not surprising since the considered prescription is analogous, in the present context, to the Hellwig-Kraus[29] postulate about wave packet reduction. For these reasons we drop the criterion considered in this footnote.

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