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QUANTUM CHROMODYNAMICS WITH INFINITE NUMBER OF VECTOR MESONS

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We suppose that families of vector mesons ρ, ψ, Υ, contain an infinite number of resonances with gradually increasing widths. The asymptotic freedom requirement involves a relationship between the electronic width of k-th resonance and its mass M_k derivative over the number k, $\frac{dM_k}{dk}$. We use this relationship and show that for the families of ψ and Υ mesons the moment from experimental function $R(s)$ is equal to the sum of the moment from a bare quark loop and the edge term which stems from replacing of summation by integration. These equalities are fulfilled up to 1% for 60 moments in the ψ-meson family and up to 2% for 96 moments in the Υ-meson family. The electronic widths of the resonances at hand and the ρ-meson mass are calculated.

Fig. - , ref. - 7

I. Introduction

The purpose of this paper is to demonstrate some advantages of considering the families of vector mesons ρ, ψ, Υ as consisting of infinite number of narrow resonances with gradually increasing widths. For the families consisting of heavy quarks we use the moment method proposed in ref.[1]. The moment L_n is an infinite sum which can be converted into integral if we use the relationship between electronic width Γ_{ee}^{κ} of the corresponding resonance and its mass derivative $\frac{dM_\kappa}{d\kappa}$ which follows from asymptotic freedom. This integral is exactly equal to the bare loop moment. When replacing summation by integration there arise extra edge terms. The moment from experimental function R(s) appears to be equal to the sum of the bare loop moment and the edge terms. In our paper we take into account the leading edge term which is dependent of a single nonobservable parameter. This parameter is the second derivative from the k-th resonance mass M_k over the number k at k=1. We analytically continue the function $S_\kappa = M_\kappa^2$ from integer nonnegative k to complex k, i.e. determine the derivatives $\frac{\partial^2 S_\kappa}{\partial \kappa^2}$. The n-th moment from experimental function R(s) is equal to the sum of the bare loop moment and the first edge term for the ψ-meson family up to 1% for the first 60 moments.

For the Υ-meson family this equality is fulfilled up to 2% for the first 96 moments.

The paper is organized as follows. In Sec.2 we obtain formulae for heavy quarks. In Sec.3 these formulae are used for the ψ meson family and the electron widths of the known 6 resonances are calculated. In Sec.4 the same formulae are applied to the Υ -meson family. In Sec.5 we consider vector mesons consisting of light quarks. The principal method used in this Section is operator expansion. The electronic widths of ρ (770) and ρ' (1600) mesons and the meson mass are calculated.

2. Formulae for Vector Mesons Consisting of Heavy Quarks

Consider the polarization operator connected with heavy quark vector current $\Pi_a(s)$ and write the dispersion relation for $\Pi_a(s)$ as [1]

$$\Pi_a(s) = \frac{s^2}{\pi} \int_{4m_a^2}^{\infty} \frac{\operatorname{Im} \Pi_a(s')\, ds'}{(s'-s)\, s'^2} \tag{1}$$

where

$$\operatorname{Im} \Pi_a(s) = \frac{1}{3} \alpha\, s\, R_a(s) \tag{2}$$

At $a=c$ we consider the vector current of charmed quarks, i.e. the ψ -meson family and at $a=b$ the vector current of b -quarks, i.e. the Υ -meson family. In approximation of infinite number of narrow resonances with the masses M_K and electronic widths

$$R_a(s) = \frac{9\pi}{\alpha^2} \sum_{K=0}^{\infty} \Gamma_K^{ee} M_K\, \delta(s - M_K^2) \tag{3}$$

determine the moment L_n by the formula [1,2]

$$L_n = \frac{1}{12\pi^2 Q_a^2} \int_{4m_a^2}^{\infty} \frac{R_a(s)\,ds}{s^n} \tag{4}$$

In approximation of infinite number of narrow resonances eq.(4) takes the form

$$L_n = \frac{1}{12\pi^2 Q_a^2} \frac{9\pi}{\alpha^2} \sum_{k=0}^{\infty} \frac{M_k}{s_k^{n+1}} \Gamma_k^{ee} \tag{5}$$

where $s_k = M_k^2$. It follows from the asymptotic freedom requirement that for large k [3]

$$\Gamma_k^{ee} = \frac{2\alpha^2}{9\pi} R_a^{(0)}(s_k) M_k^{(1)} \tag{6}$$

the function $R_a^{(0)}(s_k)$ is determined by the well-known formula

$$R_a^{(0)}(s_k) = \frac{3}{2} Q_a^2 V_k \left(3 - V_k^2\right) \tag{7}$$

$V_k = \sqrt{1 - 4m_a^2/s_k}$, m_a is the mass of the corresponding quarks, Q_a is its charge. In eq.(6) and in the next formulae we use the notations

$$M_k^{(\ell)} \equiv \frac{d^\ell M_k}{dk^\ell} \quad , \quad s_k^{(\ell)} \equiv \frac{d^\ell s_k}{dk^\ell} \tag{8}$$

It may be proved that function $s_k \equiv s(k)$ given at k=0,1,2,3 ... can be continued to analytical function of the complex derivative k with a cut along the negative axis. To do so, one should conformally map the plane with the cut s inwards the unit circle, i.e. make transformation $W = (\sqrt{s} - \sqrt{s_0})/(\sqrt{s} + \sqrt{s_0})$

and map the plane of the complex variable k inwards the unit circle by transformation $z=(\sqrt{k}-\sqrt{\gamma})/(\sqrt{k}+\sqrt{\gamma})$ (γ is an arbitrary positive number). The the function $W(s)$ at $z=z_k \equiv (\sqrt{k}-\sqrt{\gamma})/(\sqrt{k}+\sqrt{\gamma})$ k=0,1,2, ... takes the values $W_k \equiv (\sqrt{s_k}-\sqrt{s_c})/(\sqrt{s_k}+\sqrt{s_c})$. Furthermore we can apply the theorems presented in the Walsh's book[4] (Chapter 10). According to these theorems one may construct analytical in the unit circle function $W(z)$ which takes the values $W(z_k)=W_k$ at the points z_k (k=0,1, ...). If the series $\sum_{k=0}^{\infty}(1-|z_k|)$ converges, such function is unique. Making inverse transformation from the unit circle to the cut plane, let us prove the above formulated statement. To avoid imsunderstanding note, that we have not proved that $s(k)$ is analytical function of k but have proved that function $s(k)$ can be continued from integer nonnegative values of k to analytical function with the cut along the negative axis. In this sense analytical properties of the function $s(k)$ have not been proved but only postulated. We may suppose another analytical properties of the function $s(k)$ map it inwards the unit circle and repeat the proofs. Then we shall get another analytical function. The theorem that two analytical functions which coincide in arbitrarily small region, coincide everywhere in the analytical region, does not work because these two functions coincide only at integer nonnegative values of k.

Let us make use of eqs.(6-8) and write eq.(5) in the form

$$L_n = \frac{1}{8\pi^2}\,\frac{1}{(4m_a^2)^{n+1}}\,\tilde{L}_n \tag{9}$$

$$\tilde{L}_n = \left(\frac{4m_a^2}{s_0}\right)^{n+1}\sum_{k=0}^{\infty}\frac{s_k^{(1)}}{(s_k/s_0)^{n+1}}\left(1+\frac{4m_a^2}{s_0}\,\frac{s_0}{s_k}\sqrt{1-\frac{4m_a^2}{s_0}\,\frac{s_0}{s_k}}\right) \tag{10}$$

Replace summation by integration in eq.(10) employing the well-known Euler-Maclaurin formula (EMF)[5] which we write as

$$\sum_{k=p}^{\infty}F(k)=\int_p^{\infty}F(t)\,dt+\frac{1}{2}F(p)-\sum_{k=1}^{n-1}\frac{B_{2k}}{(2k)!}F^{(2k-1)}(p)+R \tag{11}$$

where

$$F^{(e)}(k)=\frac{\partial^e}{\partial k^e}F(k) \tag{12}$$

B_{2k} are Bernulli numbers, R is the EMF residual term which can be written as

$$R=\frac{B_{2n}}{(2n)!}\sum_{k=p}^{\infty}F^{(2n)}(k+\theta) \tag{13}$$

$0 \le \theta \le 1$. The first five Bernulli numbers are equal to $B_0 = 1$, $B_1 = -1/2$, $B_2 = 1/6$, $B_4 = -1/30$, $B_6 = 1/42$.
All the Bernulli numbers from odd numbers are zero, $B_{2k+1} = 0$, except for B_1 . To derive EFM it is necessary for the function F(k) to have 2n continuous derivatives over k. EFM will be effective if these derivatives are not too large. Emphasize that EFM contains finite number of terms and is exact. When applying EFM to eq.(10) one should select separately the term

with k=0, i.e. replace the sum by the integral, starting from k=1. This is done for two reasons: (1) eq.(6) is derived for large k and one should better not apply it to the ground state. Note that in nonrelativistic quantum mechanics the analogous formula[3]

$$|\psi_K(0)|^2 = \frac{\mu^2}{\sqrt{2}\pi^2} E_K^{1/2} \frac{dE_K}{dK} \tag{14}$$

for the considered in[3] potentials has an accuracy ~ 2% for k=0 and a few parts of a per cent for k > 0. (2) The main reason for which EFM should be applied starting from k=1 is that, as will be shown below, the values $\mathfrak{z}_\circ^{(\ell)}$ radpidly increase with ℓ increasing and thus EMF becomes noneffective. Let us write

$$L_n = L_{n,0} + \Delta L_{n,1} + \Delta_{n,3} + \Delta L_{n,5} + \dots \tag{15}$$

where

$$L_{n,0} = \frac{1}{8\pi^2} \int_{4m_a^2}^{\infty} \frac{v(3-v^2)}{s^{n+1}} ds = \frac{1}{4\pi^2 (4m_a^2)^n} \tilde{L}_{n,0} \tag{16}$$

$$\tilde{L}_{n,0} = \frac{3\cdot 2^n\cdot(n+1)(n-1)!}{(2n+3)!!} \tag{17}$$

$L_{n,o}$ is the n-th moment from the bare quark loop

$$\Delta L_{n,1} = -f_n + \frac{1}{8\pi^2}\left[u_{0n} + \frac{1}{2}u_{1n} - \frac{1}{12}u_{1n}^{(1)}\right] \tag{18}$$

is the first correction arising from replacing summation by integration in eq.(10). The values entering eq.(18) are the following

$$f_n = \frac{1}{8\pi^2} \int_{4m_a^2}^{s_1} \frac{v(3-v^2)}{s^{n+1}} ds \tag{19}$$

$$u_{on} = \frac{1}{(4m_a^2)^n} V_o (3-V_o^2) \frac{s_o^{(1)}}{s_o} \left(\frac{4m_a^2}{s_o}\right)^n \tag{20}$$

$$u_{1n} = \frac{1}{(4m_a^2)^n} V_1 (3-V_1^2) \frac{s_1^{(1)}}{s_1} \left(\frac{4m_a^2}{s_1}\right)^n \tag{21}$$

$\Delta L_{n,3}$, $\Delta L_{n,5}$ are the next corrections arising from replacing summation by integration

$$\Delta L_{n,3} = \frac{1}{8\pi^2} \frac{1}{720} u_{1n}^{(3)} \tag{22}$$

$$\Delta L_{n,5} = -\frac{1}{8\pi^2} \frac{1}{30240} u_{1n}^{(5)} \tag{23}$$

In what follows we will restrict ourselves to the first correction. First we will obtain a convenient formula to calculate f_n . Replace in eq.(19) the integration variable $s = 4m_a^2/(1-v^2)$. Then we get for f_n

$$f_n = \frac{1}{4\pi^2} \frac{1}{(4m_a^2)^n} \tilde{f}_n \tag{24}$$

$$f_n = 2\chi_{n-1} - \chi_n - \chi_{n+1} \tag{25}$$

where

$$\chi_n = \int_0^{v_1} (1 - v^2)^n dv \tag{26}$$

For χ_n we may readily get the recurrent formula

$$\chi_n = \left[v_1 (1 - v_1^2)^n + 2n \chi_{n-1} \right] / (2n+1) \tag{27}$$

Since $\chi_0 = v_1$, then using eq.(27) one can easily calculate all values of χ_n and, consequently, find the values of

$$\tilde{f}_n = \left(\frac{1}{n} + \frac{1}{2n+3} \right) \chi_n - v_1 (1 - v_1^2)^n / n - v_1 (1 - v_1^2)^{n+1} / (2n+3) \tag{28}$$

Let us find $u_{1n}^{(1)}$

$$\begin{aligned} u_{1n}^{(1)} = & \left[v_1 (3 - v_1^2) \right]^{(1)} \cdot s_1^{(1)} / s_1^{n+1} + v_1 (3 - v_1^2) s_1^{(2)} / s_1^{n+1} \\ & - (n+1) v_1 (3 - v_1^2) {s_1^{(1)}}^2 / s_1^{n+2} \end{aligned} \tag{29}$$

In further transformations we used

$$\left[v_1 (3 - v_1^2) \right]^{(1)} = 3 (1 - v_1^2) v_1^{(1)} = 3 \cdot \frac{4m_e^2}{s_1} \cdot v_1^{(1)} \tag{30}$$

$$v_1^{(1)} = \frac{1}{2v} \frac{4m_e^2}{s_1^2} s_1^{(1)} \tag{31}$$

Finally we get

$$u_{1n}^{(1)} = \frac{1}{(4m_a^2)^n}\left[\frac{1.5}{V_1}\left(\frac{4m_a^2}{s_1}\right)^2\left(\frac{s_1^{(1)}}{s_1}\right)^2 + V_1\left(3-V_1^2\right)\left(\frac{s_1^{(2)}}{s_1}\right) - (n+1)\cdot V_1\cdot\left(3-V_1^2\right)\left(\frac{s_1^{(1)}}{s_1}\right)^2\right]\left(\frac{4m_a^2}{s_1}\right)^n \tag{32}$$

and

$$\Delta L_{n,1} = \frac{1}{4\pi^2(4m_a^2)^n}\Delta\tilde{L}_{n,1}$$

$$\Delta\tilde{L}_{n,1} = -\tilde{f}_n + \frac{1}{2}V_0\left(3-V_0^2\right)\left(1-V_0^2\right)^n s_0^{(1)}/s_0 + \left[\frac{1}{4}V_1\left(3-V_1^2\right)s_1^{(1)}/s_1\right. \tag{33}$$

$$-\frac{1}{16}\frac{1}{V_1}\left(1-V_1^2\right)^2\left(s_1^{(1)}/s_1\right)^2 - \frac{1}{24}V_1\left(3-V_1^2\right)s_1^{(2)}/s_1 + \left.+\frac{1}{24}V_1\left(3-V_1^2\right)\left(s_1^{(1)}/s_1\right)^2(n+1)\right]\left(1-V_1^2\right)^n \tag{34}$$

Write

$$\tilde{L}_n = \tilde{L}_{n,0} + \Delta\tilde{L}_{n,1} \tag{35}$$

$\tilde{L}_n$ should be considered as the n-th moment (up to the factor $4\pi^2(4m_a^2)^n$) from experimental function $R_a(s)$. Then $\tilde{L}_{n,0}$ is the n-th moment from the bare quark loop. In order to obtain $\tilde{L}_n$ from $\tilde{L}_{n,0}$ one should add to the latter the term $\Delta\tilde{L}_{n,1}$ connected with the edge terms which arise when replacing summation by integration with EMF. It is essential that $\tilde{L}_{n,1}$ is determined by single nonobservable parameter $s_1^{(2)}$.

3. Υ -Meson Family

Nowadays 6 Υ -mesons have been found (see the Table). The simplest mass formula which describes these 6 mesons is

$$s_k = s_0 + a\left(\sqrt{k+b} - \sqrt{b}\right) \tag{36}$$

where a and b are the parameters which can be found from the requirement $s_1 = M^2_{1Exp}$, $s_2 = M^2_{2Exp}$

$$a = 17.6973 \text{ GeV}^2, \qquad b = 0.246 \tag{37}$$

At these values of a and b eq.(36) gives the values

$$\begin{aligned} M_3 &= \sqrt{s_3} = 10.611 \text{ GeV} \\ M_4 &= \sqrt{s_4} = 10.825 \text{ GeV} \\ M_5 &= \sqrt{s_5} = 11.011 \text{ GeV} \end{aligned} \tag{38}$$

Comparing the mass values M_3, M_4, M_5 with the table ones we see that the mass formula (32) describes the - meson masses up to ~ 30 MeV. That is why we will not write more complicated formulae with great number of parameters. It follows from the mass formula (36) that

$$s_k^{(1)} = \frac{a}{2\sqrt{k+b}}, \quad s_k^{(2)} = -\frac{a}{4(k+b)^{3/2}}, \quad s_k^{(3)} = \frac{3a}{8(k+b)^{5/2}}, \quad s_k^{(4)} = -\frac{15a}{16(k+b)^{7/2}} \tag{39}$$

If we apply EMF starting from k=0, we get

$s_0^{(1)} = 17.84 \text{ GeV}^2$, $s_0^{(2)} = -36.26 \text{ GeV}^2$, $s_0^{(3)} =$
$= 221.11 \text{ GeV}^2$, $s_0^{(4)} = -2247.01 \text{ GeV}^2$.

The fast growth of derivatives $s_0^{(i)}$ with the number does not allow us to apply EMF starting from k=0.

At k=1

$s_1^{(1)} = 7.93\ \mathrm{GeV}^2$, $\quad s_1^{(2)} = -3.18\ \mathrm{GeV}^2$, $\quad s_1^{(3)} = 3{,}83\ \mathrm{GeV}^2$, $s_1^{(4)} = -7.68\ \mathrm{GeV}^2$, i.e. the derivatives over number at k=1 are small and EMF must work well.

Let us choose for definiteness the mass value of b-quark $m_b = 4.1$ GeV[3]. Then the right- and left-handed parts of eq.(35) can be easily calculated. The right- and left-handed parts of eq.(35) for the first 96 parameters coincide with an accuracy better than 2%. At n=100 the difference between the right- and left handed parts of eq.(35) is 18% . At n=104 it is 1250%.

Using eqs.(6),(39) we calculate at $m_b = 4.1$ GeV the electronic widths of all 6 Υ -mesons in the zero in α_s approximation. The account of the first α_s correction reduces to replacing[1]

$$R_b^{(0)}(s_k) \to R_b^{(1)}(s_k) = R_b^{(0)}(s_k)\left(1 + \delta_k\right) \tag{40}$$

where

$$\delta_k = \frac{2}{3}\alpha_s\left[\frac{\pi}{v_k} - \frac{1}{2}\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)(3+v_k)\right] \tag{41}$$

At $\alpha_s = 0.15$ for δ_k we obtain

$\delta_0 = 0.397$, $\quad \delta_1 = 0.308$, $\quad \delta_2 = 0.274$, $\quad \delta_3 = 0.256$, $\delta_4 = 0.235$, $\quad \delta_5 = 0.226$.

The values for the calculated electronic widths with the account of the first α_s correction are given in the Table.

4. ψ -Meson Family

At present 6 ψ -mesons have been found (see Table). To describe them we will use the mass formula (36). However, unlike Υ meson family, the K-dependence of K for the ψ -meson family is not so regular as in the Υ -meson family, ψ_2 and ψ_4 fall out from the smooth curve. It is impossible to find coefficients from first three resonances because corresponding equations have no solutions. Of course, one can write a more complicated formula with a greater number of parameters and describe all 6 ψ -mesons with it. But if one uses such a formula, then because of urregular K-dependence of s_K the $s_K^{(\ell)}$ derivatives will be large and EFM will not effectively work. On the other hand, according to potential model ψ_2 and ψ_4 mesons are D-states and they must be described by another mass formula. We will describe by eq. (36) four ψ -mesons with the mass squares

$$\tilde{s}_0 = s_0 \, , \quad \tilde{s}_1 = s_1 \, , \quad \tilde{s}_2 = s_3 \, , \quad \tilde{s}_3 = s_5 \tag{42}$$

The a and b values for the terms of the ψ -meson family in s -state are

$$a = 7.2517 \text{ GeV} \, , \qquad b = 0.3993 \tag{43}$$

For the mass $\tilde{M}_3 = \sqrt{\tilde{s}_3}$ calculated by eq. (36) with the parameters (43) we obtain $\tilde{M}_3 = 4.287 \text{ GeV}$ instead of $\tilde{M}_{3\,Exp} = 4.415 \text{ GeV}$. For the first values $s_0^{(n)}$ we obtain

$$s_0^{(1)} = 5.738\ GeV^2, \quad s_0^{(2)} = -7.185\ GeV^2$$
$$s_0^{(3)} = 26.99\ GeV^2, \quad s_0^{(4)} = -168.99\ GeV^2 \qquad (44)$$

Thus, ψ_0 meson should be considered separately.

For the first four values of $s_1^{(n)}$ we obtain

$$s_1^{(1)} = 3.065\ GeV^2, \quad s_1^{(2)} = -1.06\ GeV^2$$
$$s_1^{(3)} = 1.174\ GeV^2, \quad s_1^{(4)} = -2.098\ GeV^2 \qquad (45)$$

Consequently, application og EPM starting from ψ_1 is valid. All the further calculations are made by the same formulae as in the Υ -meson family. Let us $m_c = 1.25\ GeV$ [3] The right- and left-handed parts of eq.(35) coincide for the first 60 moments up to 1%. The difference between the right- and left-handed of eq.(35) for n=65 is 10% and for n=70 is already 1263%.

Emphasize the difference between our approach and that of and that of ref. [2]. To obtain the moment from the experimental function $R_c(s)$ the authors of ref. [2] add the power corrections to the bare quark loop moment, i.e. the mean value from gluon condensate. In our approach, to obtain the moment from the experimental function $R_c(s)$ we add the edge terms arising from replacement of summation by EPM integration to the bare loop moment. These two approaches do not contradict each other if the edge terms for the forst 8 moments coincide with the power corrections.

Using eqs.(6),(39) we calculate at m_c= 1,25GeV the electron widths of ψ_0 , ψ_1 , ψ_3 , ψ_5 in the α_s zero in approximation. The α_s correction for the ψ -meson family is taken into account in the same way as for the

Υ -meson family: one should replace $R_b(\beta_K) \to R_c(\beta_K)$ and take for $\alpha_s = 0.2$. For the δ_K values we get:

$$\delta_0 = 0.39\,, \quad \delta_1 = 0.238\,, \quad \delta_2 = 0.198\,, \quad \delta_3 = 0.169$$

The values of the electron widths with the account of the α_s first correction are given in the Table. The calculated values of Γ^{ee}_{eth} and Γ^{ee}_{1th} agree with experiment. When comparing Γ^{ee}_{3T} and Γ^{ee}_{5T} with experiment it should be noted that we did *not* take into account a part of the ψ -meson in D-states. That is why when comparing with experiment one should replace

$$\Gamma^{ee}_{1Exp} \to \Gamma^{ee}_{1Exp} + \frac{3}{4}\Gamma^{ee}_{2Exp} = (2.3 \pm 0.2)\,KeV$$

$$\Gamma^{ee}_{3Exp} \to \Gamma^{ee}_{3Exp} + \frac{1}{4}\Gamma^{ee}_{2Exp} + \frac{2}{3}\Gamma_{4} = (1.33 \pm 0.2)\,keV$$

$$\Gamma^{ee}_{5Exp} \to \Gamma^{ee}_{5Exp} + \frac{1}{3}\Gamma^{ee}_{4Exp} = (0.75 \pm 0.15)\,keV$$

In addition, a part of the background should be added to Γ^{ee}_{5Exp}.

5. Light quarks

Consider the polarization operator $\Pi(Q^2)$ related to the light current quark $j^{I=1}_{\mu}(x)$

$$i\int d^4x\, e^{iqx} \langle 0|T\{j^{I=1}_{\mu}(x), j^{I=1}_{\nu}(0)\}|0\rangle = (q_\mu q_\nu - g_{\mu\nu} q^2)\,\Pi(Q^2) \tag{46}$$

$Q^2 = -q^2$ and

$$j^{I=1}_{\mu}(x) = \frac{1}{2}\left[\bar{u}(x)\gamma_\mu u(x) - \bar{d}(x)\gamma_\mu d(x)\right] \tag{47}$$

The dispersion relation for $\Pi(Q^2)$ is

$$\Pi(Q^2)=\frac{1}{12\pi^2}\int_{4m_\pi^2}^{\infty}\frac{R^{(I=1)}(s)\,ds}{s+Q^2} \tag{48}$$

In the model at hand we can use for $R^{(I=1)}(s)$ eqs. (3) and (6) with replasing $R_a(s)\to R^{(I=1)}(s)$ and $R_a^{(0)}\to\frac{3}{2}$. The moment approach used for heavy quarks is not suitable for light quarks therefore we will use operator expansion method.

Substitute into eq. (48) $R^{(I=1)}(s)$ defined by eq. (3), make use of EFM and get

$$\Pi(Q^2)=\frac{1}{8\pi^2}\sum_{n=0}^{\infty}\frac{s_n^{(1)}}{s_n+Q^2}=\frac{1}{8\pi^2}\left\{\frac{s_0^{(1)}}{s_0+Q^2}+\right. \tag{49}$$
$$\left.+\int_0^\infty\frac{ds}{s+Q^2}-\int_0^{s_1}\frac{ds}{s+Q^2}+\frac{1}{2}\,\frac{s_1^{(1)}}{s_1+Q^2}-\frac{1}{12}\left[\frac{s_1^{(1)}}{s_1+Q^2}\right]^{(1)}+\ldots\right.$$

The dispersion relation (48), (49) is written without subtractions for simplicity since, as it will be seen below, divergent terms cancel.

The operator expansion in this case has the form[2]

$$\Pi(Q^2)=\Pi_q+\frac{C_2}{Q^2}+\frac{C_4}{Q^4}+\frac{C_6}{Q^6}+\ldots \tag{50}$$

where

$$\Pi_q=\frac{1}{8\pi^2}\int_0^\infty\frac{ds}{s+Q^2} \tag{51}$$

$$C_2=0 \tag{52}$$

$$C_4=\frac{\alpha_s}{24\pi}\langle 0|G_{\mu\nu}^{a}G_{\mu\nu}^{a}|0\rangle \tag{53}$$

$$C_6 = -\frac{16\cdot 7}{81}\pi\alpha_s\,|\langle 0|\bar q\, q|0\rangle|^2 \tag{54}$$

Adopt that [7]

$$\frac{\alpha_s}{\pi}\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle = (0.012\pm 0.005)\,GeV^4 \tag{55}$$

[7]

$$\alpha_s|\langle 0|\bar q\, q|0\rangle = \left(0.8\,{}^{+0.6}_{-0.4}\right)10^{-4}\ GeV^6 \tag{56}$$

At large Q^2 eq.(49) must coincide with the operator expansion. The divergent parts of both formulae cancel and thus we get

$$\frac{1}{8\pi^2}\left\{\frac{s_0^{(1)}}{s_0+Q^2} - \int_0^{s_1}\frac{ds}{s+Q^2} + \frac{1}{2}\left[\frac{s_1^{(1)}}{s_1+Q^2}\right] - \frac{1}{12}\left[\frac{s_1^{(1)}}{s_1+Q^2}\right]^{(1)} + \dots\right\} \tag{57}$$

$$= c_2/Q^2 + c_4/Q^4 + c_6/Q^6 + \dots$$

Expand the left-handed part of eq.(57) in $1/Q^2$ and assuming that derivatives $s_1^{(k)}$ do not grow too fastly with k increasing, remain the first four terms in the right-handed part of eq.(57). Equalling the terms of one order in in the right and left-handed parts of eq.(57), we obtain

$$s_0^{(1)} - s_1 + \frac{1}{2}s_1^{(1)} - \frac{1}{12}s_1^{(2)} = 0 \tag{58}$$

$$-s_0 s_0^{(1)} + \frac{1}{2}s_1^2 - \frac{1}{2}s_1 s_1^{(1)} + \frac{1}{12}\left(s_1 s_1^{(2)} + s_1^{(1)\,2}\right) = A_4 \tag{59}$$

$$s_0^2 s_0^{(1)} - \frac{1}{3}s_1^3 + \frac{1}{2}s_1^2 s_1^{(1)} - \frac{1}{12}\left(s_1^2 s_1^{(2)} + 2 s_1 s_1^{(1)\,2}\right) = A_6 \tag{60}$$

$$-s_0^3 s_0^{(1)}+\frac{1}{4}s_1^4-\frac{1}{2}s_1^3 s_1^{(1)}+\frac{1}{12}\left(s_1^3 s_1^{(2)}+3 s_1^2 s_1^{(1)2}\right)=A_8 \tag{61}$$

where

$$A_i=8\pi C_i\,,\quad A_4=(0.04\pm 0.02)\,GeV^4,\quad A_6=(0.03^{+0.02}_{-0.01})\,GeV^6$$

Let us employ eq.(58) to find $s_1^{(2)}$. After simple manipulations we obtain

$$(s_1-s_0)\,s_0^{(1)}+\frac{1}{12}s_1^{(1)2}=\frac{1}{2}s_1^2+A_4 \tag{62}$$

$$(s_1^2-s_0^2)\,s_0^{(1)}+\frac{1}{6}s_1 s_1^{(1)2}=\frac{2}{3}s_1^3+A_6 \tag{63}$$

$$(s_1^3-s_0^3)\,s_0^{(1)}+\frac{1}{4}s_1^2 s_1^{(1)2}=\frac{3}{4}s_1^4+A_8 \tag{64}$$

Take that $M_1=m_{g'}(1600)=(1.59\pm 0.02)\,GeV$, then $s_1=2.53\,GeV^2$ and all A_k can be neglected compared with the corresponding degrees of s in eqs.(62)-(64). A_8 is unknown but we suppose that $A_8 \ll s_1^4$. We will use three equations (62)-(64) to find three values

$$s_0=\frac{1}{4}s_1 \tag{65}$$

$$s_0^{(1)}=\frac{1}{3}\,\frac{s_1^3}{(s_1-s_0)^2} \tag{66}$$

$$s_1^{(1)}=\sqrt{6s_1^2-12(s_1-s_0)s_0^{(1)}} \tag{67}$$

After simple transformations we get the final answer

$$m_s = \frac{1}{2} m_{s'} = 0.795 \text{ GeV} \tag{68}$$

$$M_0^{(1)} = \frac{16}{27} m_{s'} = 0.94 \text{ GeV} \tag{69}$$

$$M_1^{(1)} = m_{s'}/\sqrt{6} = 0.65 \text{ GeV} \tag{70}$$

Let us check up selconsistency of the method. We obtain from eq. (58)

$$s_1^{(2)} = 12\left(s_0^{(1)} - s_1 + \frac{1}{2} s_1^{(1)}\right) = 12\left(\frac{16}{27} - 1 + \frac{1}{\sqrt{6}}\right) m_{s'}^2 = 0.026 \text{ GeV} \tag{71}$$

i.e. $s_1^{(2)} \ll s_1^{(1)}$.

Value $s_1^{(3)}$ can be found using the formula

$$s_0 = s_1 - s_1^{(1)} + \frac{1}{2} s_1^{(2)} - \frac{1}{6} s_1^{(3)} \tag{72}$$

We omit in eq. (72) $\frac{1}{24} s_1^{(4)}$, $-\frac{1}{120} s_1^{(5)}$ and next terms. It follows from eq. (72) that

$$s_1^{(3)} = 6\left(s_1 - s_0 - s_1^{(1)} + \frac{1}{2} s_1^{(2)}\right) = -0.93 \text{ GeV}^2 \tag{73}$$

The smallness of $s_1^{(2)}$ and $s_1^{(3)}$ indicates the self-consistency of the method.

The electronic width with the account of the first correction is determined by

$$\Gamma_K^{ee} = \frac{\alpha^2}{3\pi}\left(1 + \frac{\alpha_s(M_K^2)}{\pi}\right) M_K^{(1)} \tag{74}$$

Substituting $\alpha_s(m_\varsigma^2)=0.34$ and $\alpha_s(m_{\varsigma'}^2) = 0.25$ we obtain $\Gamma_0^{ee}=5.3\ keV$ and $\Gamma_1^{ee}=3.37\ keV$.

6. Conclusion

The model of infinite number of narrow resonances at first sight contradicts experiment. Consider, for example, the Υ - meson family. Experimentally, there are 6 resonances and then is a smooth background where there is no resonance structure. Consider eq.(36) at large k. At k $\gg$ 1 it follows from eq.(36) that

$$s_{k+1} - s_k \approx \frac{a}{2\sqrt{k}} \tag{75}$$

i.e. the distances between resonances decreases with k increasing. Taking into account the resonance widths one should replace

$$\delta(s-s_k) \to \frac{1}{\pi}\frac{M_k\Gamma_k}{(s-s_k)^2+M_k^2\Gamma_k^2} \tag{76}$$

and tha formula for R_θ takes the form

$$R_6^{(1)} = \frac{1}{\pi}\sum_{k=0}^{\infty} R_6^{(0)}(s_k)\frac{M_k\Gamma_k s_k^{(1)}}{(s-s_k)^2+M_k^2\Gamma_k^2} \tag{77}$$

If at k > 5 the widths obey inequalities

$$s_k - s_{k-1} \ll M_k\Gamma_k \ll s_k \tag{78}$$

then $R_6(s)$ at $s>s_0$ will be asmooth function and all formulae of our paper will not change.

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Table. Comparison of computed values of electronic widths $(\Gamma_K^{ee})_{th}$ with the experimental values $(\Gamma_K^{ee})_{exp}$ for the families of Υ-, ψ-, ρ-mesons.

Mesons	Mass, MeV	Γ_{tot}, MeV	$M_k^{(1)}$, GeV	$(\Gamma_K^{ee})_{th}$, keV	$(\Gamma_K^{ee})_{Exp}$, keV
Υ_0	9460.0±0.2	0.043±0.03	0.94	1.14	1.22±0.05
Υ_1	10023.4±0.3	0.30±0.007	0.40	0.50	0.54±0.03
Υ_2	10355.5±0.5	$0.012^{+0.010}_{-0.004}$	0.29	0.37	0.40±0.03
Υ_3	10577 ± 4	24± 2	0.23	0.30	0.24±0.05
Υ_4	10865 ± 8	110± 13	0.20	0.26	0.31±0.07
Υ_5	11019 ± 9	79± 16	0.18	0.23	0.13±0.03
ψ_0	3096.9±0.1	0.063±0.009	0.93	5.07	4.7 ± 0.3
ψ_1	3686.0±0.1	0.215±0.040	0.42	2.34	2.1 ± 0.2
ψ_2	3769.9±2.4	25± 3			0.26±0.15
ψ_3	4030 ± 5	52±10	0.29	1.64	0.75±0.15
ψ_4	4159 ± 20	78±20			0.77±0.23
ψ_5	4415 ± 6	43±20	0.22	1.25	0.47±0.10
ρ	770 ± 3	153± 2	0.94	5.3	6.9 ± 0.3
ρ'	1590 ± 20	260±100	0.65	3.97	7.5 ± 1.5

When comparing $(\Gamma_1^{ee})_{Exp}$, $(\Gamma_3^{ee})_{Exp}$, $(\Gamma_5^{ee})_{Exp}$ with $(\Gamma_1^{ee})_{th}$, $(\Gamma_3^{ee})_{th}$, $(\Gamma_5^{ee})_{th}$ for the ψ meson family one should replace

$$\Gamma^{ee}_{1Exp} \to \Gamma^{ee}_{1Exp} + 3/4\,\Gamma^{ee}_{2Exp} = (2.3 \pm 0.2)\ \text{keV},$$

$$\Gamma^{ee}_{3Exp} \to \Gamma^{ee}_{3Exp} + 1/4\,\Gamma^{ee}_{2Exp} + 2/3\,\Gamma^{ee}_{4Exp} = (1.33 \pm 0.2)\ \text{KeV}$$

$$\Gamma^{ee}_{5Exp} \to \Gamma^{ee}_{5Exp} + 1/3\,\Gamma^{ee}_{4Exp} = (0.73 \pm 0.15)\ \text{KeV}.$$

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