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**THE MULTIPHONON VERSION
OF THE QUASIPARTICLE-PHONON NUCLEAR MODEL:
APPLICATION TO THE PROBLEM
OF EXISTENCE
OF LOW-LYING TWO-PHONON STATES
IN DEFORMED NUCLEI**

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1. Introduction

The two-phonon components of low-lying states in even-even deformed nuclei have been discussed for a long time^{1-9/}. These components can essentially influence the properties of low-lying states such as the excitation energy, electromagnetic transitions, form-factors in transfer reactions, etc.. The question of two-phonon components became now very urgent due to a rich experimental information^{10-15/} on low-lying levels in ^{168}Er . This nucleus is considered now as a kind of a proving ground for theoretical models. Apparently, the experimental data in ^{168}Er can hardly be described without including two-phonon components into consideration.

One of the most obscure problems is the existence of low-lying states with dominating two-phonon components (we shall call them the two-phonon states) in deformed nuclei. Still there are no reliable experimental data undoubtedly testifying to the existence of these states. Theoretical models provide contradictory predictions. The QPNM asserts that low-lying two-phonon states should not exist in deformed nuclei since the violation of the Pauli principle in the wave function leads to the shift of the strength of these states towards higher excitation energies^{1-3/}. Other approaches^{4-9/}, in particular, the microscopic multiphonon model (MPM)^{8/} and the self-consistent collective coordinate method (SCCM)^{9/} admit or at least do not reject the existence of these states. It is to be emphasized that both schematic calculations in the MPM in which the multiphonon wave function is constructed of only two γ -vibrational phonons in the Tamm-Dancoff approximation (TDA) and more realistic calculations in the SCCM in which the wave function is constructed of many phonons of different multipolarity in the random phase approximation (RPA) provide similar results. The comparison of these models^{8/} shows that the discrepancy between the QPNM predictions and those of other models is mainly due to the fact that in contrast with the MPM and SCCM the QPNM disregards the interaction with multiphonon configurations which lowers the energies of low-lying two-phonon states.

In the present paper we show that the discrepancy between the models can be removed by taking into account in the QPNM the interaction with complex configurations. For this purpose a multiphonon version of the QPNM is constructed.

The analysis of the interaction of two-phonon states with other configurations faces troubles within the MPM and SCCM since the SCCM has a rather cumbersome structure and the MPM deals with the numerical diagonalisation of the Hamiltonian matrix without the secular equation. In this connection, the second aim of this paper is to derive such formulas, which are on the one hand as simple as possible and on the other hand clarify the coupling between different configurations and can be used for numerical estimates.

In sect. 2 the Hamiltonian and wave function are considered in a multiphonon version of the QPNM. In sect. 3 an analysis of the matrix elements between the components of the wave function with a different number of phonons is presented in detail. In sect. 4 the secular equation is derived, the influence of multiphonon configurations on the properties of low-lying two-phonon states is discussed and the comparison with other models is made. Summary and conclusions are expounded in sect. 5.

2. The Hamiltonian and wave function

The Hamiltonian has the form

$$H = H_{sp} + H_{pair} + H_{mm}, \quad (1)$$

where H_{sp} is the average field as the Saxon-Woods potential, H_{pair} is the monopole pairing interaction, H_{mm} are the isoscalar and isovector multipole-multipole forces. After calculating one-phonon excitations within the RPA, the normalised Hamiltonian (1) can be expressed through the phonon and quasiparticle operators

$$H = : H_Q + H_{QQ} + H_{\alpha Q} :, \quad (2)$$

where

$$H_Q = \sum_q \epsilon_q B(qq0) - 1/4 \sum_{\substack{\bar{g}=\lambda\bar{\mu}_1 \\ \bar{g}'=\lambda\bar{\mu}_1'}} L_{\bar{g}\bar{g}'} Q_{\bar{g}}^+ Q_{\bar{g}'}, \quad (3)$$

$$H_{00} = -1/8 \sum_{\substack{g=\lambda\mu, \\ g'=\lambda\mu'}} L_{gg'} (Q_{g'}^+ Q_g^+ + Q_{g'}^- Q_g^-), \quad (4)$$

$$H_{\alpha Q} = -1/4 \sum_{\substack{g=\lambda\mu \\ q_1, q_2 \in \tau}} \Gamma_{q_1, q_2}^{g\tau} ((Q_{q_1}^+ + Q_{q_2}^-) B(q_1, q_2, -\mu) + \text{h.c.}), \quad (5)$$

The following notation has been used in (3)-(5) :

$$Q_{\bar{q}}^{\pm} = 1/2 \sum_{q_1, q_2} (\psi_{q_1, q_2}^{\bar{q}} A^+(q_1, q_2, \bar{\mu}) - \phi_{q_1, q_2}^{\bar{q}} A(q_1, q_2, -\bar{\mu})), \quad (6)$$

where $A^+(q_1, q_2, \bar{\mu})$ and $B(q_1, q_2, \bar{\mu})$ are operators of the type $\alpha_{q_1}^+ \alpha_{q_2}^+$ and $\alpha_{q_1}^+ \alpha_{q_2}^-$ with $\bar{k}_1 + \bar{k}_2 = \bar{\mu}$; α_q^+ is the creation operator of the one-quasiparticle state with quantum numbers q and energy ϵ_q ; $\bar{q} = \sigma q$, $\bar{k} = \sigma K$, $\bar{\mu} = \sigma \mu$, $K \geq 0$, $\mu \geq 0$; K is the angular momentum projection onto the nuclear symmetry axis; $\sigma = \pm 1$; $g = \lambda \mu$ are the multipolarity and number of the RPA phonon; $\sum_{q_1, q_2 \in \tau}$ is the summation only over neutron ($\tau = N$)

or only over proton ($\tau = Z$) one-quasiparticle states. The expressions for the functions $L_{gg'}$ and $\Gamma_{q_1, q_2}^{g\tau}$ can be determined from ref. 2/. These functions are the larger the higher is the collectivity of phonons. If the phonons are close to two-quasiparticle states these functions approximately equal zero.

The term H_Q generates quasiparticle and phonon excitations. The quasiparticle-phonon interaction $H_{\alpha Q}$ couples the wave function components differing by an odd number of phonons. The term H_{0Q} couples the components differing by an even number of phonons. This term has earlier been neglected in the QPNM. By analogy with the principle of cancellation of dangerous diagrams^{16/} one can show that in the one-phonon approximation H_{0Q} will be compensated by the first term of H_Q . Further, we shall see that with the inclusion of the multiphonon wave function and Pauli principle the interaction H_{0Q} has not to be neglected.

The multiphonon wave function is taken in the form

$$\psi_{\rho}(\bar{K}_0^n) = (R^{(0)}) + \sum_{g_1} R^{(1)} \delta_{\bar{\mu}_1, \bar{K}_0 g_1} Q_{g_1}^+ + \sum_{g_1 g_2} p_{\mu_1 \mu_2}^{K_0} R^{(2)} \delta_{\bar{\mu}_1 + \bar{\mu}_2, \bar{K}_0} Q_{g_1}^+ Q_{g_2}^+ + \dots + \sum_{g_1 \dots g_n} p_{\mu_1 \dots \mu_n}^{K_0} R^{(n)} \delta_{\bar{\mu}_1 + \dots + \bar{\mu}_n, \bar{K}_0} Q_{g_1}^+ \dots Q_{g_n}^+ | \rangle, \quad (7)$$

where $R_{g_1 \dots g_n}^{(n)}$ is the n-phonon amplitude; $| \rangle$ is the RPA vacuum, i.e. $Q_{\bar{g}} | \rangle = 0$; ρ is the number of the state with given \bar{K}_0^n . The Kronecker symbols connect the projections of phonon moments onto \bar{K}_0 . The coefficients $p_{\mu_1 \dots \mu_n}^{K_0}$ are chosen so that the normalisation condition of the wave function is

$$\begin{aligned} (\psi_{\rho}^*(\bar{K}_0^n) \psi_{\rho}(\bar{K}_0^n)) &= (R^{(0)})^2 + \sum_{g_1} (R_{g_1}^{(1)})^2 N_{g_1}^{K_0} + \sum_{g_1 \geq g_2} (R_{g_1 g_2}^{(2)})^2 N_{g_1 g_2}^{K_0} \\ &+ \dots + \sum_{g_1 \geq g_2 \geq \dots \geq g_n} (R_{g_1 \dots g_n}^{(n)})^2 N_{g_1 \dots g_n}^{K_0} = 1, \end{aligned} \quad (8)$$

where

$$N_{g_1 \dots g_n}^{K_0} = 1 + \mathcal{X}^{K_0}(g_n \dots g_1 | g_1 \dots g_n). \quad (9)$$

The function $\mathcal{X}^{K_0}(g_n \dots g_1 | g_1 \dots g_n)$ appears only if the Pauli principle is violated in components with $n \geq 2$. One can easily be convinced that

$$p_{\mu_1 \dots \mu_n}^{K_0} = \bar{p}_{\mu_1 \dots \mu_n}^{K_0} n^{-1/2}, \quad (10)$$

where the coefficient $\bar{p}_{\mu_1 \dots \mu_n}^{K_0}$ takes into account the cases when nonzero moment projections of some phonons are coupled to the total projection equal to zero (for instance, for the states with $K^n = 4^+$ composed of four γ -vibrational phonons we have $\bar{p}_{gggg}^{K_0} = 1/\sqrt{2}$). For simplicity, we have omitted indices K_0 and ρ in the amplitudes $R_{g_1 \dots g_n}^{(n)}$ and the Kronecker symbols in (9). Further, the index K_0 will be omitted in the functions in (9)-(10).

The wave functions (7) are not orthogonal because of nonorthogonality of n-phonon components $Q_{\bar{g}_1}^+ \dots Q_{\bar{g}_n}^+ | \rangle$. Indeed, at $\bar{g}_1 \dots \bar{g}_n \neq \bar{g}'_1 \dots \bar{g}'_n$ we have

$$\langle | Q_{\bar{g}_n} \dots Q_{\bar{g}_1} Q_{\bar{g}'_1}^+ \dots Q_{\bar{g}'_n}^+ | \rangle \sim \mathfrak{K}(g_n \dots g_1 | g'_1 \dots g'_n) \neq 0. \quad (11)$$

The above nonorthogonality will not effect the results given below since in what follows we shall use approximations at which nondiagonal quantities of the type (11) are neglected.

Note that the wave function (7) and Hamiltonian (1) are in fact the same as in the MPM. Therefore, the basic results given below are valid also for the MPM. There are several essential differences between our approach and the MPM: i) instead of RPA the MPM uses the TDA, ii) the Hamiltonian in the MPM in contrast with (2)-(5) is not expressed through phonon operators, iii) in the MPM a direct diagonalisation of the Hamiltonian matrix is performed (the secular equation is not used).

3. Matrix elements

Now we derive expressions for the matrix elements

$$M_{\bar{g}'_1 \dots \bar{g}'_n; \bar{g}_1 \dots \bar{g}_n} = p_{\mu_1} \dots p_{\mu_n} p_{\mu'_1} \dots p_{\mu'_n} \sum_{\substack{\sigma_1 \dots \sigma_n \\ \sigma'_1 \dots \sigma'_n}} \delta_{\bar{\mu}_1 + \dots + \bar{\mu}_n, R_0} \delta_{\bar{\mu}'_1 + \dots + \bar{\mu}'_n, R_0} \times \langle | Q_{\bar{g}_n} \dots Q_{\bar{g}_1} H Q_{\bar{g}'_1}^+ \dots Q_{\bar{g}'_n}^+ | \rangle, \quad (12)$$

$$N_{\bar{g}'_1 \dots \bar{g}'_n; \bar{g}_1 \dots \bar{g}_n} = 1 + \mathfrak{K}(g_n \dots g_1 | g'_1 \dots g'_n) = p_{\mu_1} \dots p_{\mu_n} p_{\mu'_1} \dots p_{\mu'_n} \sum_{\substack{\sigma_1 \dots \sigma_n \\ \sigma'_1 \dots \sigma'_n}} \delta_{\bar{\mu}_1 + \dots + \bar{\mu}_n, R_0} \delta_{\bar{\mu}'_1 + \dots + \bar{\mu}'_n, R_0} \times \langle | Q_{\bar{g}_n} \dots Q_{\bar{g}_1} Q_{\bar{g}'_1}^+ \dots Q_{\bar{g}'_n}^+ | \rangle,$$

taking account of the Pauli principle. For this purpose we use exact commutation relations (taking account of the quasiparticle structure of phonons) between the operators in (3)-(5) and (7). As

an example, we give the commutation relations for the phonons with $g=20i$ (for simplicity, we use index i instead of g):

$$[Q_{i_1}, [Q_{i_2}, Q_{i_3}^+]] = \sum_{i_1'} \{ \mathfrak{X}(i_1, i_2 | i_3, i_1') Q_{i_1'} + \bar{\mathfrak{X}}(i_1, i_2 | i_3, i_1') Q_{i_1'}^+ \}, \quad (14)$$

$$[B(q_1, q_2), Q_{i_1}^+] = \sum_{i_1'} \{ b_{q_2, q_1}^{i_1 i_1'} Q_{i_1'}^+ + c_{q_2, q_1}^{i_1 i_1'} Q_{i_1'} \}, \quad (15)$$

where

$$\mathfrak{X}(i_1, i_2 | i_3, i_4) = -1/2 \sum_{q_1, q_2} a_{q_1, q_2}^{i_2 i_3} b_{q_2, q_1}^{i_1 i_4}, \quad (16.1)$$

$$\bar{\mathfrak{X}}(i_1, i_2 | i_3, i_4) = -1/2 \sum_{q_1, q_2} a_{q_1, q_2}^{i_2 i_3} d_{q_2, q_1}^{i_1 i_4}, \quad (16.2)$$

$$z_{q_1, q_2}^{i_1 i_1'} = \sum_{q_3} (\psi_{q_3, q_1}^i \psi_{q_3, q_2}^{i'} - \phi_{q_3, q_2}^i \phi_{q_3, q_1}^{i'}), \quad (17.1)$$

$$b_{q_1, q_2}^{i_1 i_1'} = \sum_{q_3} (\psi_{q_3, q_1}^i \psi_{q_3, q_2}^{i'} + \phi_{q_3, q_2}^i \phi_{q_3, q_1}^{i'}), \quad (17.2)$$

$$c_{q_1, q_2}^{i_1 i_1'} = \sum_{q_3} (\psi_{q_3, q_1}^i \phi_{q_3, q_2}^{i'} + \phi_{q_3, q_2}^i \psi_{q_3, q_1}^{i'}), \quad (17.3)$$

$$d_{q_1, q_2}^{i_1 i_1'} = \sum_{q_3} (\psi_{q_3, q_1}^i \phi_{q_3, q_2}^{i'} - \phi_{q_3, q_2}^i \psi_{q_3, q_1}^{i'}), \quad (17.4)$$

It is seen from (16.1)-(17.4) that the functions $\mathfrak{X}(i_1, i_2, i_3, i_4)$ and $\bar{\mathfrak{X}}(i_1, i_2, i_3, i_4)$ are of an order of ψ^4 and $\psi^2\phi$, respectively.

In general, expressions for the matrix elements (12)-(13) are rather cumbersome. It is more convenient to consider these expressions for the wave function (7) formed by phonons of only one type with $g=20i$. Then, the state (7) has quantum numbers $K_0^n=0^+$, and the matrix elements (12)-(13) conserving all the basic properties of the general case acquire a simple and clear form convenient for analysis and numerical estimates:

$$M_n^n = N_n (n \omega + \Delta_n) + \delta(LX)_{n \geq 1}, \quad (18)$$

$$M_n^{n+1} = N_{n+1} n \sqrt{(n+1)/2} U + \delta(UX)_{n \geq 1} + \delta(\Gamma c), \quad (19)$$

$$M_n^{n+2} = N_{n+2} \frac{\Delta_n^{n+2}}{n+2} + \delta(\omega X, LX)_{n \geq 1}, \quad (20)$$

$$M_n^{n+3} = \delta(\Gamma c), \quad (21)$$

$$M_n^{n+4} = \delta(LX) \quad (22)$$

where $\Delta_n = -1/8 LX(n-1)n$, (23)

$$U = 1/N_2 < |Q_g H Q_g^+| >. \quad (24)$$

Expressions (18)-(22), except for the terms with index $n \geq 1$, hold at $n \geq 0$. The forms of N_n and Δ_n^{n+2} for some values of n , we shall need in what follows, are given in table 1. In (18)-(24) and table 1

we use the notation: $M_m^n \equiv \underbrace{M_{\underbrace{g \dots g}_m}^{\underbrace{g \dots g}_n}}$, $N_n \equiv N_{\underbrace{g \dots g}_n}$, $\omega = \omega_g$, $L = L_{gg}$.

Table 1. Expressions for N_n and Δ_n^{n+2} for some values of n in the approximation $g_1 = \dots = g_n = 201$.

n	N_n	Δ_n^{n+2}
0	1	$-1/(8\sqrt{2!}) LX$
1	1	$-1/(8\sqrt{3!}) LX \ 3(1+X)$
2	$1/2! (2!+X)$	$-1/(8\sqrt{2!4!}) LX \ 6(2+5X+3X^2)$
3	$1/3! (3!+9X+3X^2)$	---
4	$1/4! (4!+72X+66X^2+18X^3)$	---

$\chi = \chi(gg|gg)$, $\bar{\chi} = \bar{\chi}(gg|gg)$, $\Gamma = \Gamma_{q_1 q_2}^{g g}$, $c = c_{q_1 q_2}^{g g}$, $\delta(F)$ is a value of an order of F . As is seen from (18)-(24) and table 1, in the approximation $g_1 = \dots = g_n = 201$ all the functions arising due to the inclusion of the Pauli principle into consideration can be expressed through χ .

In (18)-(24) we have written down only the terms that are dominating in both the RPA and TDA. For the other terms which contain inverse phonon amplitudes ϕ and are absent in the TDA we give only the order of their magnitude. It is seen that in the TDA the Hamiltonian (2) couples the wave function components differing from each other not more than by two phonons. For collective one-phonon states, when the inverse amplitudes ϕ can be compared in magnitude with the direct ones ψ , the RPA should be used instead of the TDA. In this case, there arises coupling between components differing more than by two phonons. Also, additional terms including the functions $\bar{\chi}$ and σ appear in the matrix elements (18)-(20). For instance, in the RPA

$$M_n^e = 1/2! (2 + \chi)(2\omega - 1/4 L\chi - 1/2 L\bar{\chi}), \quad (25)$$

where the addition $1/2 L\bar{\chi} \sim 1/2 L\psi^3\phi$ is comparable in value with the leading term $1/4 L\chi \sim 1/4 L\psi^4$.

The functions Δ_n and Δ_n^{n+2} appear only if the Pauli principle is taken into account ($\chi \neq 0$). Since $-2 \leq \chi \leq 0$, for low-lying collective phonons (for which always $L > 0$) the functions Δ_n and Δ_n^{n+2} may take only positive values. The function Δ_n increases with n , $|\chi|$ and L . It implies the shift of the strength of the n -phonon configuration towards higher excitation energy. Just the positive shift Δ_n under the violation of the Pauli principle made it possible to conclude within the QPNM¹⁻³ that low-lying two-phonon states should not exist in deformed nuclei.

The underlined terms in (18), (20) and (22) are generated by the interaction H_{00} . This interaction, earlier disregarded in the QPNM, couples the wave function components differing by an even number of phonons and results in the considerable additions in the diagonal matrix elements M_n^0 .

Let us get numerical estimates for N_n and M_n^n . For simplicity, we assume that $\phi_{q_1, q_2}^{\sigma} \ll \psi_{q_1, q_2}^{\sigma}$, that means in fact the TDA. As a result, we may neglect terms of the type $\delta(F)$ and coupling between the components differing by more than two phonons. Since the aim of the present paper is the study of two-phonon states, it is sufficient to conserve only the components with $n \leq 4$ in the wave function (7). For the quantities L , U and X we use typical values obtained in the microscopic calculations within the QPNM for ^{168}Er and given in table 2 (in these calculations the wave function contained only one- and two- phonon components and a large phonon basis was taken into account).

Table 2. The values of L , U and X for some two-phonon states

State	L , MeV	U , MeV	X
$0_{\beta\beta}^+$	4	0,2	-0,5
$0_{\gamma\gamma}^+$	20	0,5	-0,3
$4_{\gamma\gamma}^+$	20	0,5	-0,7

The quantities N_n , M_n^{n+1} and M_n^{n+2} as a function of X are given in figs. 1a-1d. It is seen from fig. 1a that for a certain value of X the quantity N_n vanishes, i.e. the corresponding n -phonon component disappears. The larger n , the smaller the violation of the Pauli principle is needed for total disappearing of the component. The four-phonon component disappears approximately at the value of X obtained in the realistic calculations (see table 2). This result confirms that in real nuclei the states composed of $n \geq 4$ phonons may be forbidden due to the violation of the Pauli principle.

Figure 1b represents the shifts Δ_n which take the large values. Figures 1c and 1d present the matrix elements M_n^{n+1} and M_n^{n+2} . It is shown that if the Pauli principle is slightly violated, the interaction between complex components is stronger than between the simple ones. In particular, the two-phonon component interacts more strongly with the three-phonon component than with

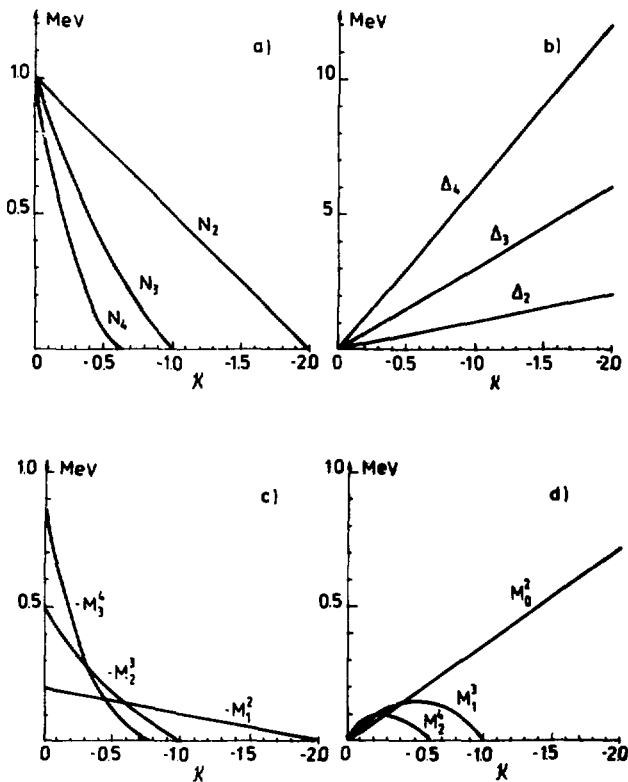


Fig. 1. The quantities N_n , Δ_n , M_n^{n+1} and M_n^{n+2} (figs. a), b), c) and d), respectively) as a functions of χ . The calculations have been performed with formulas presented in table 1 and with (19), (20) and (23).

the one-phonon one. This indicates that in studying two-phonon states one cannot use only the components with $n \leq 2$ in the wave function (7). With increasing $|\mathcal{X}|$ complex components are suppressed more rapidly than the simple ones. As a result, the situation is reverse: the coupling between simple components becomes stronger than between complex ones.

It is seen from figs. 1c and 1d that the matrix elements M_n^{n+2} , existing only if the Pauli principle is violated, are comparable in magnitude with M_n^{n+1} . This indicates once more the necessity of taking account of the interaction H_{QQ} .

Figures 1a-1d, obtained for the case when the configurations are composed of the same phonons, provide a general and somewhat crude picture. However, this picture should be similar also in the case when the configurations are composed of different phonons if they are low-lying ones. It is known that the principal two-quasiparticle components of low-lying phonons are formed by a small number of quasiparticles close to the Fermi level. This fact leads to approximately the same violation of the Pauli principle as in the case considered above. Note that in figs. 1a-1d the dependence of L on \mathcal{X} is neglected, which is incorrect in general. For instance, for two-quasiparticle states (when $\mathcal{X} \rightarrow 2$) we have $L \rightarrow 0$. Thus, in the general case at $\mathcal{X} \rightarrow 2$ there should be a downward bend of the straight lines describing Δ_n and M_0^0 and some change of the curves for other matrix elements.

4. Basic equations of the multiphonon version of the QPNM

Let us consider the multiphonon version of the QPNM with the Hamiltonian (2) and wave function (7) including the components with $0 \leq n \leq 4$ phonons of a different type. Then, in contrast with the accepted QPNM¹⁻³, the wave function (7) contains the components with $n=3$ and 4 and phononless ($n=0$) component. The latter will be shown to influence the properties of low-lying states essentially. Let us show that the inclusion of configurations with $n=3$ and 4 leads to the appearance of terms in the secular equation which can be treated as a shift of the strength of low-lying two-phonon states towards lower excitation energies.

Using the variational procedure

$$\delta((\Psi^*(\bar{K}_0^n) H \Psi(\bar{K}_0^n)) - \eta((\Psi^*(\bar{K}_0^n) \Psi(\bar{K}_0^n)) - 1)) = 0, \quad (26)$$

where η is the energy of the state with K_0^n , we get the system of equations for the amplitudes $R_{g_1 \dots g_n}^{(n)}$:

$$\eta R_{g_1 g_2}^{(0)} + \sum_{g_1' g_2'} R_{g_1' g_2'}^{(2)} M_{g_1 g_2}^0 = 0, \quad (27.1)$$

$$R_{g_1}^{(1)} P_{g_1}(\eta) + \sum_{g_1' g_2'} R_{g_1' g_2'}^{(2)} M_{g_1}^{g_1' g_2'} + \sum_{g_1' g_2' g_3'} R_{g_1' g_2' g_3'}^{(3)} M_{g_1}^{g_1' g_2' g_3'} = 0, \quad (27.2)$$

$$R_{g_1 g_2}^{(2)} P_{g_1 g_2}(\eta) + R^{(0)} M_{g_1 g_2}^0 + \sum_{g_1'} R_{g_1'}^{(1)} M_{g_1}^{g_1' g_2} + \sum_{g_1' g_2' g_3'} R_{g_1' g_2' g_3'}^{(3)} M_{g_1 g_2}^{g_1' g_2' g_3'} + \sum_{g_1' g_2' g_3' g_4'} R_{g_1' g_2' g_3' g_4'}^{(4)} M_{g_1 g_2}^{g_1' g_2' g_3' g_4'} = 0, \quad (27.3)$$

$$R_{g_1 g_2 g_3}^{(3)} P_{g_1 g_2 g_3}(\eta) + \sum_{g_1'} R_{g_1'}^{(1)} M_{g_1}^{g_1' g_2 g_3} + \sum_{g_1' g_2'} R_{g_1' g_2'}^{(2)} M_{g_1 g_2}^{g_1' g_2' g_3} + \sum_{g_1' g_2' g_3' g_4'} R_{g_1' g_2' g_3' g_4'}^{(4)} M_{g_1 g_2 g_3}^{g_1' g_2' g_3' g_4'} = 0, \quad (27.4)$$

$$R_{g_1 g_2 g_3 g_4}^{(4)} P_{g_1 g_2 g_3 g_4}(\eta) + \sum_{g_1' g_2'} R_{g_1' g_2'}^{(2)} M_{g_1 g_2}^{g_1' g_2' g_3 g_4} + \sum_{g_1' g_2' g_3'} R_{g_1' g_2' g_3'}^{(3)} M_{g_1 g_2 g_3}^{g_1' g_2' g_3' g_4} = 0, \quad (27.5)$$

where

$$P_{g_1 \dots g_n}(\eta) = N_{g_1 \dots g_n}(\omega_{g_1} + \dots + \omega_{g_n} + \Delta_{g_1 \dots g_n} - \eta). \quad (28)$$

From (27.1)-(27.5) one can easily derive the system of equations for $R_{g_1 g_2}^{(2)}$. For this purpose, the sums of products of matrix elements of the same type are considered in the coherent approximation that, for instance, for the amplitudes $R_{g_1 g_2 g_3 g_4}^{(4)}$ has the form

$$\sum_{g_1''g_2''g_3''g_4''} R^{(4)}_{g_1''g_2''g_3''g_4''} M_{g_1''g_2''g_3''g_4''}^{g_1g_2g_3g_4} M_{g_1''g_2''g_3''g_4''}^{g_1''g_2''g_3''g_4''} \approx R^{(4)}_{g_1g_2g_3g_4} (M_{g_1''g_2''g_3''g_4''}^{g_1g_2g_3g_4})^2. \quad (29)$$

As a result we have

$$\sum_{g_1'g_2'} R^{(2)}_{g_1'g_2'} F_{g_1'g_2'}^{g_1g_2}(\eta) = 0, \quad (30)$$

where

$$F_{g_1'g_2'}^{g_1g_2}(\eta) = P_{g_1'g_2'}(\eta) (\delta_{g_1, g_1'} \delta_{g_2, g_2'} + \delta_{g_1, g_2'} \delta_{g_2, g_1'}) + M_0^{g_1g_2} M_0^{g_1'g_2'} \eta^{-1} - \sum_g M_g^{g_1g_2} M_g^{g_1'g_2'} P_g^{-1}(\eta) - \sum_{j_1j_2j_3} \bar{M}_{g_1g_2}^{j_1j_2j_3} \bar{M}_{g_1'g_2'}^{j_1j_2j_3} \bar{P}_{j_1j_2j_3}^{-1}(\eta) \quad (31)$$

$$- \sum_{k_1k_2k_3k_4} M_{g_1g_2}^{k_1k_2k_3k_4} M_{g_1'g_2'}^{k_1k_2k_3k_4} P_{k_1k_2k_3k_4}^{-1}(\eta),$$

$$\bar{M}_{g_1g_2}^{j_1j_2j_3} = M_{g_1g_2}^{j_1j_2j_3} - \sum_g M_g^{j_1j_2j_3} M_g^{g_1g_2} P_g^{-1}(\eta) \quad (32)$$

$$- \sum_{k_1k_2k_3k_4} M_{j_1j_2j_3}^{k_1k_2k_3k_4} M_{g_1g_2}^{k_1k_2k_3k_4} P_{k_1k_2k_3k_4}^{-1}(\eta),$$

$$\bar{P}_{j_1j_2j_3}(\eta) = P_{j_1j_2j_3}(\eta) - \sum_g (M_g^{j_1j_2j_3})^2 P_g^{-1}(\eta) \quad (33)$$

$$- \sum_{k_1k_2k_3k_4} (M_{j_1j_2j_3}^{k_1k_2k_3k_4})^2 P_{k_1k_2k_3k_4}^{-1}(\eta).$$

In (31)-(33) indices j and k run the same values as index g . If the Pauli principle is neglected, the coupling of the two-phonon component with the $n=0$ and 4 components disappear whereas with the $n=1$ and 3 ones is conserved:

$$F_{g_1'g_2'}^{g_1g_2}(\eta) = P_{g_1'g_2'}(\eta) (\delta_{g_1, g_1'} \delta_{g_2, g_2'} + \delta_{g_1, g_2'} \delta_{g_2, g_1'}) - \sum_g M_g^{g_1g_2} M_g^{g_1'g_2'} P_g^{-1}(\eta) - \sum_{j_1j_2j_3} \bar{M}_{g_1g_2}^{j_1j_2j_3} \bar{M}_{g_1'g_2'}^{j_1j_2j_3} \bar{P}_{j_1j_2j_3}^{-1}(\eta). \quad (34)$$

The secular equation has the form

$$\det ||F_{g_1 g_2}^{g_1 g_2}(\eta)|| = 0. \quad (35)$$

In the QPNM^{1-3/}, instead of (30) the system of equations for the amplitudes $R_g^{(1)}$ is considered and the quantity $P_{g_1 g_2}(\eta)$ is shown to be a pole of the secular equation. In the present paper, since we investigate two-phonon states, the final system of equations is written for the amplitudes $R_{g_1 g_2}^{(2)}$. Note that due to approximation of the type (29), some details of the secular equation will depend on the type of the amplitudes for which the final equations are written. However, the basic properties of the secular equation to be considered below will not change.

The secular equation (35) obtained in the approximation (29) may have extraneous roots. So, this equation is inapplicable for realistic calculations but can be used for analysis and numerical estimates.

The diagonal part of the secular equation at $N_{g_1 g_2} > 0$ can be written in the form

$$\begin{aligned} F_{g_1 g_2}^{g_1 g_2}(\eta) = & N_{g_1 g_2} (\omega_{g_1} + \omega_{g_2} + \Delta_{g_1 g_2} - \eta) \quad (36) \\ & + N_{g_1 g_2}^{-1} \left((M_0^{g_1 g_2})^2 \eta^{-1} - \sum_g (M_g^{g_1 g_2})^2 P_g^{-1}(\eta) - \sum_{J_1 J_2 J_3} (\bar{M}_{g_1 g_2}^{J_1 J_2 J_3})^2 \bar{P}_{J_1 J_2 J_3}^{-1}(\eta) \right. \\ & \left. - \sum_{k_1 k_2 k_3 k_4} (M_{g_1 g_2}^{k_1 k_2 k_3 k_4})^2 P_{k_1 k_2 k_3 k_4}^{-1}(\eta) \right). \end{aligned}$$

Earlier, in the refs.^{1-3/} only the terms

$$N_{g_1 g_2} (\omega_{g_1} + \omega_{g_2} + \Delta_{g_1 g_2} - \eta - N_{g_1 g_2}^{-1} \sum_g (M_g^{g_1 g_2}) P_g^{-1}(\eta)) \quad (37)$$

have been taken into account in the diagonal part.

Let us consider the two-phonon component $g_1 g_2$ with the strength mainly concentrated in the state with energy η . It is seen from (36) that the interaction of the component $g_1 g_2$ with the other components leads to the additional terms which can be treated as shifts with respect to the energy $\omega_{g_1} + \omega_{g_2} + \Delta_{g_1 g_2}$. The

signs of the shifts are determined by the energy positions of these components with respect to η . All the n-phonon states lying above (below) the state $g_1 g_2$ in the excitation energy will push out the strength of this state towards lower (higher) energies. Here, one can easily see an analogy with the well-known quantum mechanical example of mutual pushing apart of two interacting levels.

If the two-phonon component considered is a low-lying one, the majority of components interacting with it lie higher in energy. These components will lead to a general coherent shift of the strength of $g_1 g_2$ towards lower energies and this shift will cancel to a certain extent the shift $\Delta_{g_1 g_2}$ arising due to the violation of the Pauli principle. Thus, the statement¹⁻³ that deformed nuclei should not contain low-lying two-phonon states which is based on the effect produced by the violation of the Pauli principle without taking into account the coupling with complex configurations needs revision.

It is to be mentioned that equation (36) allows one to analyse some results obtained in the MPM and SCCM. For instance, the calculations within these models systematically provide that the $0_{\gamma\gamma}^+$ state is higher than the $4_{\gamma\gamma}^+$ state. This result is somewhat difficult to explain within the MPM and SCCM but it can easily be interpreted with the use of eq. (36). Indeed, if the wave function (7) is composed only of γ -vibrational phonons, then the 0^+ state will contain components with $n=0, 2$ and 4 whereas the 4^+ state --components with $n=2$ and 4 . Equation (36) will be

$$F_{g_1 g_2}^{g_1 g_2}(\eta) = N_{g_1 g_2} \{ \omega_{g_1} + \omega_{g_2} + \Delta_{g_1 g_2} - \eta \} \quad (38)$$

$$+ N_{g_1 g_2}^{-1} \{ (M_0^{g_1 g_2})^2 \eta^{-1} - \sum_{k_1 k_2 k_3 k_4} (M_{g_1 g_2}^{k_1 k_2 k_3 k_4})^2 P_{k_1 k_2 k_3 k_4}^{-1}(\eta) \}.$$

It is seen from (38) that the interaction of the component $g_1 g_2$ with the phononless one that occurs only in the 0^+ -states, shifts the strength of the $0_{\gamma\gamma}^+$ state towards higher excitation energy, which explains the above result. An analogous situation is expected for two-phonon 0^+ and 2^+ levels constructed of octupole phonons

with $\lambda\mu=311$ and for two-phonon 0^+ and 4^+ levels constructed of phonons with $\lambda\mu=321$.

5. Summary and conclusions

The multiphonon version of the QPNM^{1-3/} takes into account not only the Pauli principle but also other effects important for the properties of low-lying two-phonon states (coupling with multiphonon configurations and phonon correlations in the ground state). Simple equations characterising the coupling between different components of the wave function and secular equation are derived. In virtue of the approximations made in this paper these equations cannot be used for realistic calculations but they are useful for clarifying the considered effects and for numerical estimations. They can serve as a starting point in considering a complicated nature of the interaction of configurations with a different number of phonons.

Numerical estimates are obtained for the matrix elements coupling different configurations. The Hamiltonian term Q^+Q^+ , which has earlier been disregarded in the QPNM while considering two-phonon excitations, was shown to be important. This interaction coupling configuration differing by two phonons arises if the Pauli principle is violated.

It is shown that if the violation of the Pauli principle shifts the strength of low-lying two-phonon states toward higher excitation energies, the coupling with multiphonon configurations can give an opposite effect of the same order. As a result, the multiphonon version of the QPNM admits, in principle, the existence of low-lying two-phonon states in deformed nuclei, which is in agreement with the results of other microscopic models^{8,9/}. However, these states have not yet been observed experimentally. Apparently, in real nuclei the majority of low-lying two-phonon states, owing to the interaction with other configurations, are distributed over many levels. Then, the low-lying states will have, as a rule, small two-phonon components. The absence of low-lying two-phonon states may hold for many deformed nuclei but it will be caused more by the fragmentation of two-phonon states than by their pushing due to the violation of the Pauli principle.

This situation would not contradict the conclusions of the available models and the experimental data on the transfer reactions¹³⁻¹⁵.

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Многофононный вариант КФМЯ: приложение к проблеме существования низколежащих двухфононных состояний в деформированных ядрах

Предложен многофононный вариант квазичастично-фононной модели ядра /КФМЯ/, удобный для качественного анализа взаимодействия конфигураций с разным числом фононов. Подробно обсуждаются свойства матричных элементов, связывающих такие конфигурации. Показано, что если нарушение принципа Паули приводит к сдвигу силы низколежащих двухфононных состояний вверх по энергии возбуждения, то связь с более сложными конфигурациями дает обратный эффект такого же порядка. В результате, многофононный вариант КФМЯ допускает существование низколежащих двухфононных состояний в деформированных ядрах, снимая тем самым противоречие, имевшее место по этому вопросу между КФМЯ и другими микроскопическими моделями.

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The Multiphonon Version of the Quasiparticle-Phonon Nuclear Model: Application to the Problem of Existence of Low-Lying Two-Phonon States in Deformed Nuclei

The multiphonon version of the quasiparticle-phonon nuclear model (QPNM) convenient for an analysis of an interaction between configurations with a different number of phonons is proposed. The properties of the matrix elements coupling these configurations are discussed in detail. It is shown that if violation of the Pauli principle shifts the strengths of low-lying two-phonon states towards higher excitation energies, the interaction with complex configurations (three-phonon, etc.) provides the opposite effect of the same order. As a result, the multiphonon version of the QPNM admits the existence of low-lying two-phonon states in deformed nuclei thus removing the discrepancy between the QPNM and other microscopic models.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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