An Empirical Scaling of the Thermal Diffusivity Applicable to Both L- and H-Mode Discharges

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AN EMPIRICAL SCALING OF THE THERMAL DIFFUSIVITY APPLICABLE TO BOTH L-AND H-MODE DISCHARGES

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Abstract. An empirical scaling of the thermal diffusivity has been constructed which fits well with most of the key observations of steady-state heat transport in tokamaks. This scaling also agrees with observed dynamical transport phenomena and suggests interesting future experiments.

Keywords : L-mode, H-mode, Energy Confinement

1. INTRODUCTION

Since experiments on TFTR demonstrated that the electron temperature profile was insensitive to the input power deposition profile [1], the concept of "profile consistency" has received much attention [2]. This concept suggests that tokamak energy confinement is determined globally by some mechanism rather than through the local thermal conductivity. Such a concept appears to be also supported by observation of H–mode discharge [3–6], in which improvement of the edge confinement triggers improved confinement in the core. Tearing modes have been suggested to be a candidate for affecting or restricting the electron profile [7,8]. It has also been argued that the observed profile corresponds to a minimum energy state with some restrictions, similarly to Taylor's argument on RFP [9–11].

In this report, we examine whether tokamak heat transport really requires a global constraint to be consistent with most of the key heat transport observations. We have found that a certain form of parameter dependence of the thermal conductivity agrees well with the steady-state transport observations. Furthermore, it is consistent with key observations of dynamical transport phenomena. This study indicates that tokamak transport probably is determined by local thermal conductivity and the proposed scaling may be close to a real transport scaling. This proposed scaling also suggests interesting future directions for experiments.

2. AN EMPIRICAL SCALING OF THE THERMAL CONDUCTIVITY

First we list the key observations of the steady-state heat transport.

For L–mode (limiter) discharges [12]:

1. The energy confinement deteriorates severely with adding beam power.
2. τ_{E} is insensitive to $\bar{n}$.
3. The shape of the observed electron temperature profile is insensitive to variation in the power deposition profile.
4. τ_{E} increases linearly with B_{T} for fixed q value at the limiter.
5. τ_{E} and hence W_{t} increases with I_{p} for fixed B_{T}. However, this is mainly due to broadening of the temperature profile. It is also observed that the majority of the heat content is stored within the q=2 surface.

Assuming that tokamak heat transport is described by thermal conductivity which depends only on local plasma parameters, we seek to find a certain functional form of the plasma parameters for the thermal conductivity which is consistent with the above observations. For simplicity we first assume that the thermal conductivity has the following functional form:

$$K_L = C[q(r), r](\nabla T)^{\alpha} T^{-\beta} (n)^{\gamma} B_{\mathrm{T}}^{\mu} \quad . \qquad (1)$$

We solve the steady-state heat balance equation,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r K_{\mathrm{L}}\,\frac{\partial T}{\partial r}\right)+P(r)=0 \quad . \tag{2}$$

Here we assume that the power deposition profile is given by:

$$P(r)=\frac{(\delta+2)}{2\pi}\,\frac{P_{\mathrm{total}}}{a^{\delta+2}}\,r^{\delta} \quad , \tag{3}$$

where δ is a parameter to specify the shape of the $P(r)$ profile. For simplicity, $C[q(r),r]$ is assumed to be $C_0 r^{\epsilon}$ within $r = a$ (q=2 surface) and to be very high outside q=2 surface, and hence, $T(q = 2)$ is set to be zero (Observation 5).

The following solution for the temperature profile is obtained:

$$\begin{aligned} T(r) = {} & \left(\frac{\alpha-\beta+1}{\alpha+\delta-\epsilon+2}\right)^{\frac{\alpha+1}{\alpha-\beta+1}}\left[\frac{P_{\mathrm{total}}}{2\pi\langle n\rangle^{\gamma}B_T^{\mu}C_0}\right]^{\frac{1}{\alpha-\beta+1}} \\ & \times\, a^{\frac{\alpha-\epsilon}{\alpha-\beta+1}}\left[1-\left(\frac{r}{a}\right)^{\frac{\delta+\alpha-\epsilon+2}{\alpha+1}}\right]^{\frac{\alpha+1}{\alpha-\beta+1}} \quad . \end{aligned} \tag{4}$$

Observation 3 means that the temperature profile is insensitive to variation in the power deposition profile parameter (δ) (the range of δ in the experiment is $|\delta| < 1$) and thus $\alpha - \epsilon + 2$ must be much greater than 1. From this expression for $T(0)$, τ_{E} scaling can be estimated.

$$\tau_{\mathrm{E}} \propto \langle n\rangle^{\frac{\alpha-\beta+1-\gamma}{\alpha-\beta+1}} B_{\mathrm{T}}^{\frac{-\mu}{\alpha-\beta+1}} P^{\frac{-(\alpha-\beta)}{\alpha-\beta+1}} \left(\frac{I_{\mathrm{p}}}{B_T}\right)^{\frac{(\alpha-\epsilon)}{2(\alpha-\beta+1)}} \tag{5}$$

We use $a(q=2) \propto (I_\mathrm{p}/B_\mathrm{T})^{1/2}$.

The power scaling is $P^{-f/(f+1)}$ where $f = \alpha - \beta$. The value of f is 1 for Goldstone L–mode scaling [12] and is 2 for ISX–B scaling [13]. To be consistent with observations 2 and 4, $\gamma = -\mu = f + 1$.

Observation 5 means that $\alpha - \epsilon = 2f + 2$. From these arguments, a major feature of K_L thermal conductivity for L–mode is that α is fairly large, i.e., K_L increases rapidly with ∇T.

For H–mode discharges [3–6]:

6. τ_E in H–mode discharge is significantly higher than that of the limiter discharge at a fixed input power. A sudden and significant (more than an order of magnitude) improvement in the edge confinement is followed by a factor of approximately 2 improvement of the core confinement.

7. H–mode discharge obtained in the divertor discharge where the edge conditions (e.g., high magnetic shear near the separatrix, low recycling near the main plasma) are quite different from those of the limiter discharge.

Observation 6 is an important one in which an empirical scaling must be consistent with. We compare temperature profiles for L–mode (limiter) and H–mode discharges with the same conditions (except the input power and hence the temperature profile) as illustrated in Fig. 1. We do not compare the confinement at the edge between two types of discharges because particle recycling and existence of the separatrix itself could affect the transport (Observation 7). With increasing input power, the profile evolves from OH profile to a nearly saturated L–mode profile. We assume that the L–mode saturated profile is below the profile of the H–mode discharge just above the power threshold, i.e., at any radius, the temperature of the L–mode discharge can never exceed that of the H–mode discharge at power level just above the H–mode power threshold. This is not clearly presented in any report, even though data suggest this. The previously assumed functional form of K is not consistent with the existence of the H–regime, i.e., higher temperature gradient in the H–mode means higher K, contradictory to Observation 6. However, with an additional fitting parameter, a scaling, consistent with both modes can be found, since the temperature profile does not overlap between L– and H–profiles at any power level. This is illustrated in Fig. 2. L–mode regime in T and ∇T parameter space with fixed other parameters, including radius, is indicated by a shaded area which is separated from the H–regime. If two regimes overlap in the parameter space, then two significant different values of K exist with identical plasma parameters, a clear contradiction to our assumption that tokamak heat transport is described by thermal conductivity which depends only on local plasma parameters. To make the scaling consistent, we need to introduce an additional fitting parameter.

$$K = \frac{K_{\mathrm{L}}}{1 + F\left(\frac{T(r)}{T_{\mathrm{S}}}\right)} + K_{\mathrm{H}} \quad . \tag{6}$$

$T_S[B_{\mathrm{T}},\ n,\ q(r),\ r]$ is a saturated L–mode temperature profile which separates two regimes and $F(x)$ is a function which has a feature $[F(x) \gg 1$ for $x > 1$ and $F(x) \ll 1$ for $x < 1]$. The profile does not saturate with the power scaling factor in the range $f = 2 \sim 3$. But practically, the input power is limited and thus the profile is also limited. The main point here is that the improved confinement regime must exist in the high temperature regime to be consistent with the existence of the H–mode unless some kind of nonlocal effects are invoked. With the above type K scaling, K

is close to that of the L–mode given by Eq. (1) when $T(r) < T_S(r)$ and K is that of the H–mode, significantly lower than that of the L–mode and independent of T and ∇T when $T(r) > T_S(r)$. At the low temperature regime (ohmic), K_L becomes even smaller than K_H. If K_H is that of Alcator scaling (even though this is not essential for our argument), then the above scaling is applicable even to the ohmic regime. This is not consistent with τ_E scaling of the H–mode discharge ($\tau_E \propto I_p$). However, it is important to notice that global τ_E in the H–mode discharge is strongly influenced by degree of the temperature pedestal seen near the separatrix, occurrence of ELMs, and the high radiative loss. The JFT-2M experiment shows that τ_E in H–mode discharges is improved up to that of the Alcator scaling [14].

We have constructed a functional form of the thermal conductivity which agrees with the key steady-state transport observations listed above. If the heat transport is determined through local thermal conductivity, a real scaling is expected to be close to the one described above, except for detailed functional form. We have neglected the effects of local density and density gradient in our argument. For L–mode discharges, there is a general trend that the energy confinement decreases with flatter density profile. Since the density profile in the H–mode discharge is very flat, flatter than that of the L–mode discharge, the effects of local density gradient do not change our main conclusion.

3. DISCUSSION

The next logical step is to find a plasma turbulence mechanism which causes this type of scaling. However, this is a difficult task and beyond the scope of this report. Because of the difficulty in identifying the responsible turbulence, one may argue that the constructed scaling is just a restatement of the experimental observations in terms of K. Nonetheless, this type of scaling predicts dynamic heat transport behaviors which have been observed experimentally.

For example, a local temperature perturbation should propagate more rapidly than expected from the steady-state value of K. A sudden local temperature perturbation, e.g., caused by sawtooth drop, has a factor of 2 to 3 higher local temperature gradient than the equilibrium temperature gradient and thus the thermal conductivity at the perturbed location can be an order of the magnitude higher, as long as the scaling parameter (α) is greater than 3. Therefore, the temperature perturbation is expected to propagate very rapidly toward the edge. Indeed, the very rapid heat pulse propagation has been observed after the sawtooth drop [15], consistent with the prediction from the scaling.

The H–mode regime is observed to start from the very edge and propagate inward. Our model explains as follows: if the temperature exceeds the saturated L–mode temperature (T_S) in the central region, as illustrated in Fig. 3, there must exist an L–mode region with high temperature gradient. But in this region, the transport is very high and thus the high temperature region (H–regime) is expected to shrink. On the other hand, if the temperature exceeds the saturated L–mode temperature at the edge, the sign of the temperature gradient is positive in the transition region and thus the temperature increases regardless of the value of K, leading to inward expansion of the high temperature region (H–mode region). In experiments, this effect is clearly seen when an H–mode is triggered by a sawtooth heat pulse [16].

Our empirical scaling also provides useful information on how to improve energy confinement. In our scaling, H–mode regime can be achieved by raising the edge temperature. Divertor configuration appears to provide a favorable condition for improvement of the edge confinement and hence higher edge temperature. Another approach is strong plasma heating with high beam power to exceed the saturated L–mode temperature (T_S). T_S is most sensitive to q value and it decreases rapidly with increasing q value. This approach may become practical at high q discharge, assuming that K has an upper bound, e.g., Bohm diffusion. This may correspond to "super shot" in TFTR [17], which is obtained only in the high q discharge. However, our model also predicts that supershot can be obtainable even in lower q discharge if significantly higher power is injected.

One of the important areas in the present tokamak research is to optimize H–mode discharge. The energy confinement in H–mode discharge is limited either by ELMs, which destroy the energy confinement in the outer region intermittently (effects of the ELMs extend deeply into the core region for DIII–D discharges), or by impurity radiation. ELMs are believed to be some kind of MHD instability caused by an excessive pressure gradient or current density gradient due to extremely good edge confinement. If profile consistency is valid within H–mode discharges, achieving higher core temperature requires higher edge temperature and hence a higher edge pressure gradient. If ELMs limit the edge pressure gradient, they may also limit the core temperature. According to our scaling based on local transport, the good confinement regime exists as long as the temperature exceeds the saturated L–mode temperature. Thus, excessively good edge confinement, characterized by the H–mode discharge is not necessary. It may be advantageous to maintain the edge temperature just above the threshold by external means to avoid ELMs. This may also help to solve or reduce other serious problems of the H–mode discharge which excessive good edge confinement appears to cause:

1. H–mode plasma tends to disrupt around $q = 3$, resulting in lower beta limit. This is believed to be due to steep current density gradient at the edge.

2. Impurities accumulate in the core plasma. Good energy confinement generally accompanies a good impurity confinement.

We have proposed to create a fine ergodic structure in the edge region to eliminate the excessively good edge confinement [18]. The thermal conductivity is controlled by the field strength of the resonance helical field in order to avoid excessive temperature gradient and hence ELM.

In summary, an empirical scaling of the thermal conductivity has been constructed, which fits well with most of the key observations of both steady-state and heat transports in tokamak discharges. The scaling also suggests a new scheme to optimize H–mode discharge.

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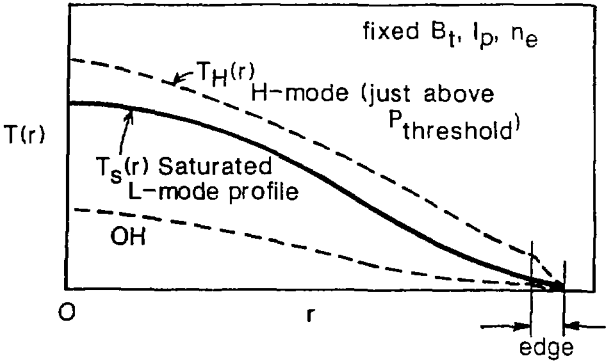

FIG. 1. Schematic comparison of temperature profile between limiter discharge [saturated L-mode profile $T_S(r)$] and H-mode discharge at power level just above the H-mode threshold power. Parameters $(B_T, I_P, \bar{n}_e)$ are fixed in this comparison.

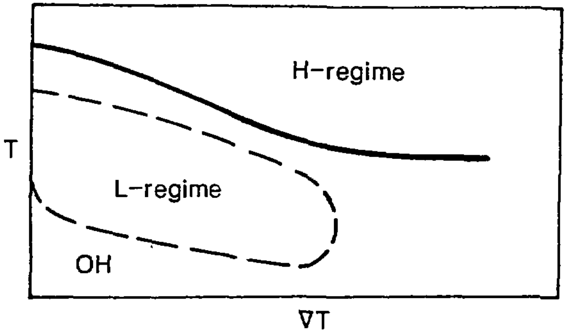

FIG. 2. Parameter space $(T, \nabla T)$ for L- and H-regimes with fixed parameters including radial location.

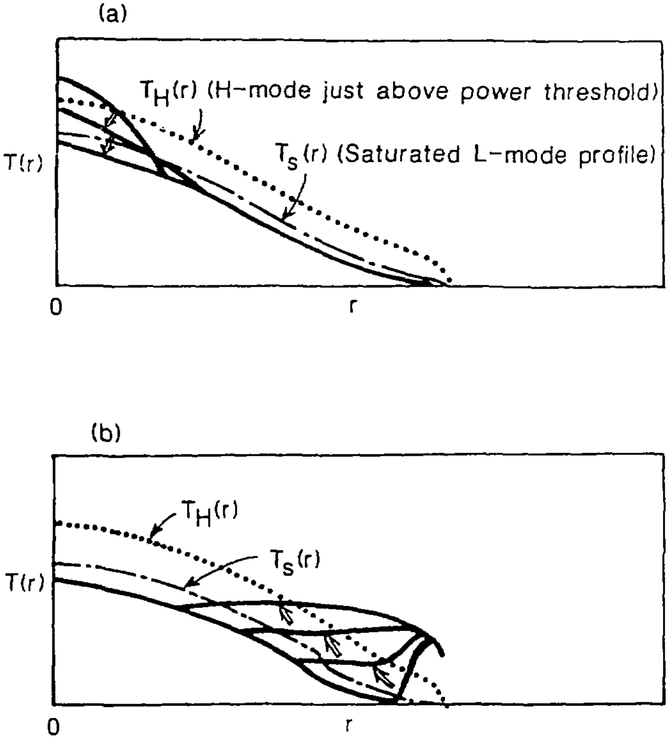

FIG. 3. Schematic temperature profile evolution during L–mode to H–mode phase transition: (a) shrinkage of the H–regime if the H–regime starts from the center, and (b) expansion of the H–regime if the H–regime starts from the very edge.