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CHIRAL THEORY OF THE Kl_3 – DECAY FORM FACTORS

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CHIRAL THEORY OF THE K_{l3}-DECAY FORM FACTORS: Preprint ITEP 89-122/

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The form factors of K_{l3}-decay are evaluated in the framework of theory with effective chiral lagrangian incorporating $0^{\mp}$ and $1^{\mp}$ fields. In the model under consideration, a renormalization of mass of pseudoscalar mesons, arising in the standard approach due to mixing $\partial_\mu \pi$ with axial-vector field, is absent. We argue against the schemes with such a mixing. Our theory predicts for the parameter $\xi(0) = f_-(0)/f_+(0)$ the value $\xi(0) = -0.235$.

Fig. - , ref. - 16

In the standard theory of weak interaction, the matrix element of the K_{ℓ_3} decay has the form

$$\mathcal{M} = \frac{G}{\sqrt{2}} \sin\theta_c \; \bar{\ell}(P_\ell)\gamma_\mu(1+\gamma_5)\nu(P_\nu)\cdot$$

$$\cdot\left[f_+(q^2)(P_K+P_\pi)_\mu + f_-(q^2)(P_K-P_\pi)_\mu\right] \qquad (1)$$

where

$$q = P_e + P_\nu = P_K - P_\pi .$$

From the general properties of the theory, we have the next information on f+:

I. In the case of $K^0 \to \pi^- e^+ \nu$ decay, according to the Ademolo-Gatto theorem [1]

$$f_+(0) = 1 + o(\varepsilon^2) \qquad (2)$$

where ε the is parameter of SU(3) symmetry breaking. $|\varepsilon| \approx 0.1$.

2. The form factor $f_-(q^2)$ must be equal to zero in the limit of SU(3) symmetry. Consequently, it must be proportional to ε.

3. The algebra of currents and soft-pion technics give a relation between f+ and f- at $q^2 = m_K^2$ [2]:

$$f_+(m_K^2) + f_-(m_K^2) = F_K/F_\pi , \qquad (3)$$

where F_K and F_π are the constants of $K \to \mu\nu$ and $\pi \to \mu\nu$ decays respectively.

In the experimental works, the form factors of K_{ℓ_3} decays usually are approximated by a linear functions of q^2:

$$f_\pm(q^2) = f_\pm(0)\left[1 + \lambda_\pm q^2/m_\pi^2\right]. \qquad (4)$$

The other parameter usually measured is

$$\xi(0) = f_-(0)/f_+(0). \qquad (5)$$

In the last papers, the results are expressed often in the terms of the form factor

$$f(q^2) = f_+(0)\left(1 + \lambda_0 q^2/m_\pi^2\right) \qquad (6)$$

where

$$f(q^2) = f_+(q^2) + q^2(m_K^2 - m_\pi^2)^{-1} f_-(q^2) .$$

There are many works devoted to evaluation of $f_{\pm}$ in a framework of different models or additional assumptions on properties of theory (see, for example, the review [3]).

Of all the models, more interesting are the models incorporating the scalar mesons because such a mesons can give a contribution to $f_{-}(q^2)$. A preference of the models with $0^{\pm}$ mesons was demonstrated in the recent paper [4], where the origin of difference between the experimental magnitude of K_{e4} decay probability and the theoretical magnitude, calculated by soft-pion technics, was considered. It was shown that in the case of comparatively small mass of the isosinglet scalar meson, the amplitude increases considerably from the unphysical point m_{π}^2 (in the variable $(p_\pi + p'_\pi)^2$), corresponding to soft-pion limit, to the point $4m_{\pi}^2$, where physical region begins.

Formerly, it was shown that the model used in ref.[4] allows to calculate the masses of mesons of the scalar nonet using the known values of m_π, m_K and $R=F_K/F_\pi$.

In present work, we calculate the form factors of $K_{\ell 3}$ decay using above mentioned model with $0^{\pm}$ fields and extending it by incorporation of $1^{\pm}$ fields.

The lagrangian of $0^{\pm}$ fields is of the form

$$L(0^{\pm}) = \frac{1}{2} Tr(\partial_\mu \hat{U} \partial_\mu \hat{U}^+) - c\, Tr\{\hat{U}\hat{U}^+ - A^2 t_0^2\}^2 - c\xi\left(Tr\{\hat{U}\hat{U}^+ - A^2 t_0^2\}\right)^2 + \frac{F_\pi}{2\sqrt{2}} Tr\{\hat{M}(\hat{U}+\hat{U}^+)\} + \Delta L_{PS}^{U(1)}, \qquad (7)$$

where $\hat{U} = (\sigma_B + i\pi_B) t_B$, σ_B and π_B are members of scalar and pseudoscalar nonets (B=0,1,2 ... 8) $M=\mathrm{diag}\{m_\pi^2; m_\pi^2; 2m_K^2 - m_\pi^2\}$, $F_\pi \cong 93$ MeV and

$$t_0 = \sqrt{1/3}\, I \quad , \quad t_{1,2,\dots 8} = \sqrt{1/2}\, \lambda_{1,2,\dots 8}.$$

The term $\Delta L_{PS}^{U(1)}$ solves the U(1) problem in the sector of 0^- mesons.

The parameter A introduces the spontaneous breaking the chiral symmetry by the quark condensate. At M=0 the lagrangian (7) is invariant under global chiral flavor transformations

$$\hat{V} \to \exp(i a_B t_B)\, \hat{V}\, \exp(i b_c t_c). \qquad (8)$$

Because of $V_B \sim \bar{q}_R t_B q_L$ in our model (that is, the scalar and pseudoscalar mesons are treated as the bound states of quark-antiquark pairs), such an invariance is equivalent to the invariance of the fundamental QCD lagrangian in respect to independent global transformation under the left and right components of the quark fields

$$q_L \to q_L \exp(i b_c t_c) \quad , \quad q_R \to q_R \exp(-i a_B t_B). \qquad (9)$$

It follows from the lagrangian (7) that all effects of symmetry breaking in such a model take place due to non-zero v.e.v.of the σ_0 and σ_8 fields, which can be expressed [4-6] through F_π and F_K :

$$\langle \sigma_0 \rangle = \frac{F_\pi}{\sqrt{6}}(2R+1) \quad , \quad \langle \sigma_8 \rangle = -\frac{2F_\pi}{\sqrt{3}}(R-1) \qquad (10)$$

where

$$R = F_K / F_\pi \quad .$$

In the model (7)

$$m^2_{\sigma_\pi} = m^2_\pi + (m^2_K - m^2_\pi)(R-1)^{-1}(2R-1)^{-1} \qquad (11)$$

$$m^2_{\sigma_K} = m^2_K + (m^2_K - m^2_\pi)(R-1)^{-1} \qquad (12)$$

where σ_π and σ_K are the scalar partners of the π and K mesons.

The relation (12) coincides with the result of evaluation of m_{σ_K} by Matsuda and Oneda [7] outside the chiral effective lagrangian theory. They used unsubstracted dispersion relations for K_{e3} form factors and the hypotheses of PCAC and PCVC.

An identification of σ_π with the known resonance $a_0(980)$ gives

$$R = 1.176 . \qquad (13)$$

Such a magnitude of R lays in the middle of the experimental values changing from R - 1.160 $\pm$ 0.017

[8] to R = 1.220±0.015 [9].

The details of determination of R may be found in ref.[6].

The lagrangian (7) gives the next expression for the current generating the $K^0 \to \pi^- e^+ \nu$ decay:

$$V_\mu^{4-i5} = -i\left(\partial_\mu K^0 \pi^+ - K^0 \partial_\mu \pi^+\right) - i\sqrt{3/2}\,\langle\sigma_8\rangle\,\partial_\mu \sigma_{K^+} \,. \tag{14}$$

Using it for calculation of the amplitude described by the diagrams of Fig.1, we could obtain

$$f_+(q^2) = 1 \text{ and } f_-(q^2) = (m_K^2 - m_\pi^2)/(m_{\sigma_K}^2 - q^2). \tag{15}$$

In calculation, the relation [4,6] was used

$$g_{\sigma_K - K^0 \pi^+} = -(m_K^2 - m_\pi^2)/(\sqrt{2}\,F_\pi (R-1)) \tag{16}$$

following from the lagrangian (7).

Note that form factors (15) satisfy to the relation (3), but the sign of $\xi(0)$ is positive and $f_+(q^2)$=const in contradiction with the observations.

The situation will change if we consider $L(0^\pm)$ as not only effective lagrangian, but as a lagrangian of quantum theory and take into account all possible loop corrections of type shown in fig.2. Such loops will give correction of order of q^2 to f_+ and correction of order of $(p_K^2 - p_\pi^2)$ to f_-, which has the value of order of $m_K^2 - m_\pi^2$ itself. Then, the value of $f_-(q^2)$ can change considerably. It is not a simple task to carry out a program of evaluating many-loop corrections and we resort to assumption that the main contribution from loop corrections arises when intermediate states form strongly-bounded vector-like states. Such a supposition is in a sence of the vector dominance hypotheses and allows to calculate $f_\pm(q^2)$ if the model (7) will be extended by adding a part $L(1^\pm)$ incorporating the vector and axial-vector fields.

A procedure of this kind was realized in many papers (see, for example, the references [10,11]) where the $L(1^\pm)$ was taken in the form of the chiral-invariant Yang-Mills lagrangian written in terms of

$$A_\mu^L = V_\mu + A_\mu \quad , \quad A_\mu^R = V_\mu - A_\mu \tag{17}$$

$$L(1^{\pm}) = -\frac{1}{8} T_2 \{(\hat{F}^{R}_{\mu\nu})^2 + (\hat{F}^{L}_{\mu\nu})^2\} =$$

$$= -\frac{1}{4} T_2 \{\partial_\mu \hat{V}_\nu - \partial_\nu \hat{V}_\mu + \sqrt{2} g f^{abc} t^c (V^a_\mu V^b_\nu + A^a_\mu A^b_\nu)\}^2 - \quad (18)$$

$$-\frac{1}{4} T_2 \{\partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + \sqrt{2} g f^{abc} t^c (V^a_\mu A^b_\nu - V^b_\nu A^a_\mu)\}^2.$$

In papers [10-12], an interaction between $1^{\pm}$ and $0^{\pm}$ fields arised as result of replacement of the derivative $\partial_\mu U$ by the covariant derivative $D_\mu U$:

$$\partial_\mu \hat{U} \Rightarrow D_\mu \hat{U} = \partial_\mu \hat{U} - ig \hat{A}^R_\mu \hat{U} + ig \hat{U} \hat{A}^L_\mu . \quad (19)$$

Though such a substitution seems to be ordinary and going without saying, we have serious objections against using it in the theory based on the effective chiral lagrangian.

First of all, the substitution (19) leads to the scheme of theory possesing (at M=0) more deep <u>local flavour chiral symmetry</u> in comparison with the basic QCD lagrangian possesing only the <u>global flavor symmetry</u> because of existence of the non-invariant kinetic term $\bar{q} i \gamma_\mu \partial_\mu q$. For this reason, the substitution (19) would lead to scheme of theory with properties different from ones of QCD and consequently it seems to be unreasonable.

Besides, the substitution (19) causing the transitions $\partial_\mu \pi \to$ $\to A_\mu$ [12] would break the relations (11) and (12) which are in accordance with the experimental data [5].

It should be reminded that these relations are not ones following from (7) only [7]. It is necessary to mention also that the vertex function $\pi\pi V$ must turn into zero at q^2_V =0 in accordance with our discussion on nature of the vector field in the theory based on the lagrangian (7). Such property would be impossible in the case of the substitution (19).

For all these reasons, following the authors of the refs.[13-15] (proceeding, however, from somewhat different considerations), we introduce the interaction of $1^{\pm}$ and $0^{\pm}$ fields with the aid of the effective lagrangian

$$L^{int}(1;0) = \frac{1}{2}\Big(\frac{ig}{\sqrt{2}M_\rho^2}\,Tr\{\hat{F}^R_{\mu\nu}D_\mu\hat{U}D_\nu\hat{U}^+ + \hat{F}^L_{\mu\nu}D_\mu\hat{U}^+D_\nu\hat{U}\} + H.c.\Big) \tag{20}$$

the part of which describing $\rho\pi^+\pi^-$ and $K^{*-}K^0\pi^+$ vertexes has the form

$$-\frac{ig}{M_\rho^2}\Big[\rho^{(0)}_{\mu\nu}(\partial_\mu\pi^+\partial_\nu\pi^- - \partial_\mu\pi^-\partial_\nu\pi^+) + \frac{1}{\sqrt{2}}K^{*-}_{\mu\nu}(\partial_\mu K^0\partial_\nu\pi^+ - \partial_\mu\pi^+\partial_\nu K^0)\Big].$$

In the momentum space we obtain:

$$A\big(\rho^{(0)}(q)\to\pi^+(p_1)+\pi^-(p_2)\big) = -g\,\frac{q^2}{M_\rho^2}(p_1-p_2)_\mu\,\epsilon^\lambda_\mu(q)$$

$$A\big(K^{*-}(q)\to K^0(p_K)+\pi^-(p_\pi)\big) = \frac{g}{\sqrt{2}}\Big[-\frac{q^2}{M_\rho^2}(p_K-p_\pi)_\mu + (p_K^2-p_\pi^2)M_\rho^{-2}(p_K+p_\pi)_\mu\Big]\epsilon^\lambda_\mu(q). \tag{2I}$$

It follows from (2I), that our definition of the constant g coincides with the ordinary definition of the $\rho\pi\pi$ constant because of our expression for the vertex $\rho\pi\pi$ turns into usual one at $q^2 = M_\rho^2$. In connection with our rejection from the substitution (I9), a question may arise, how a splitting between the masses of 1^+ and 1^- mesons appears in our scheme. In the schemes exploiting the substitution (I9) such a splitting arised from

$$\frac{1}{2}Tr\{D_\mu\hat{U}D_\mu\hat{U}^+\} \Rightarrow g^2\langle\sigma\rangle^2A_\mu^2 + 0\cdot V_\mu^2 + \ldots$$

But this (rejected by us) mechanism really is not the only mechanism of splitting between the masses of 1^+ and 1^- mesons, because of besides term having the form

$$\frac{h_1}{4}Tr\{\hat{A}^L_\mu\hat{A}^L_\mu\hat{U}^+\hat{U} + \hat{A}^R_\mu\hat{A}^R_\mu\hat{U}\hat{U}^+\} \Rightarrow \frac{h_1}{2}(V_\mu^2 + A_\mu^2)\langle UU^+\rangle + \ldots \tag{22}$$

one can add another global-invariant but gauge non-invariant term

$$-\frac{h_2}{2}T_2\{\hat{A}^L_\mu\hat{U}^+\hat{A}^R_\mu\hat{U}\}\Rightarrow$$
$$\Rightarrow\frac{h_2}{2}(-V_\mu^2+A_\mu^2)\langle UU^+\rangle+\ldots \qquad (23)$$

Really, the transformation properties of the fields A^L_μ and A^R_μ coincide with the ones for the quark structures $\bar{q}_L\gamma_\mu q_L$ and $\bar{q}_R\gamma_\mu q_R$ under the global transformations (9). Namely, the fields A^L_μ and A^R_μ transform as

$$\hat{A}^L_\mu\Rightarrow exp(-ib_B t_B)\hat{A}^L_\mu exp(ib_{B'}t_{B'})$$
$$\hat{A}^R_\mu\Rightarrow exp(ia_c t_c)\hat{A}^R_\mu exp(-ia_{c'}t_{c'}).$$

As a result, in combination with the transformation (8), the construction (23) turns out to be invariant under the global chiral transformations.

So that

$$M_V^2=(h_1-h_2)\langle UU^+\rangle$$
$$M_A^2=(h_1+h_2)\langle UU^+\rangle. \qquad (24)$$

The described mechanism of mass splitting is in accordance with the general idea that the only source of arising all masses is a break (spontaneous or explicit) of the chiral symmetry due to *non-zero vacuum expectation values of* σ_0 *and* σ_8 *fields.*

Taking into account the mass terms (22) and (23), we get the new expression for strangeness-changing vector current

$$V_\mu^{4-i5}=-i(\partial_\mu K^0\pi^+-K^0\partial_\mu\pi^+)-i\sqrt{3/2}\langle\sigma_8\rangle\partial_\mu\sigma_{K^+}+M^2_{K^*}g^{-1}\sqrt{2}K^{*+}_\mu. \qquad (25)$$

As a result, the new diagram drown in fig. 3 is added to the diagrams of fig. I and the form factors become of the forms

$$f_+(q^2)=1+q^2/[M_\rho^2(1-q^2/M^2_{K^*})]$$
$$f_-(q^2)=(m_K^2-m_\pi^2)/(m^2_{\sigma_K}-q^2)-(P_K^2-P_\pi^2)/[M_\rho^2(1-q^2/M^2_{K^*})] \qquad (26)$$
$$f(q^2)=1+q^2/(m^2_{\sigma_K}-q^2).$$

From the formulae (26), one can see that f_+ and f_- obey the relation (3), because of

$$f_+(m_K^2) + f_-(m_K^2) = R + m_\pi^2 / [M_\rho^2 (1 - m_K^2/M_{K^*}^2)] \qquad (27)$$

and in the limit $m_\pi^2 = 0$ the formula (27) coincides with the relation (3), also obtained in the limit $m_\pi^2 = 0$.

Expanding our expressions in powers of q^2 and neglecting the terms of order of q^4 and higher, we obtain for the parameters λ_+ and λ_0:

$$\lambda_+ \cong m_\pi^2 / M_\rho^2 = 0.0328 \qquad (28)$$

$$\lambda_0 \cong m_\pi^2 / m_{\sigma_K}^2 = 0.0128$$

The parameter $\xi(0)$ has the value

$\xi(0) = -(m_K^2 - m_\pi^2)(M_\rho^{-2} - m_{\sigma_K}^{-2}) \cong -0.235$

The experimental data [16] for these parameters are:

$$\lambda_+^{exp} = \begin{array}{ll} 0.028 \pm 0.004 & \text{from } K^+_{e3} \text{ decays} \\ 0.030 \pm 0.0016 & \text{from } K^0_{e3} \text{ decays} \\ 0.033 \pm 0.008 & \text{from } K^+_{\mu 3} \text{ decays} \\ 0.034 \pm 0.005 & \text{from } K^0_{\mu 3} \text{ decays} \end{array}$$

$$\lambda_0^{exp} = \begin{array}{ll} 0.004 \pm 0.007 & \text{from } K^+_{\mu 3} \text{ decays} \\ 0.025 \pm 0.006 & \text{from } K^0_{\mu 3} \text{ decays} \end{array}$$

$$\xi(0)^{exp} = -0.35 \pm 0.15 .$$

From comparison of these data with our predictions (28) and (29) we conclude that our chiral model is available one for description of $K_{\ell 3}$ decay.

A peculiarity of our model consists in that it gives a picture of exact vector dominance only in the case of ρ meson. For K^* the vertex $K^* K \pi$ is proportional to $g_{K^*}^2 / M_\rho^2$ instead of $g_{K^*}^2 / M_{K^*}^2$ required by the model of vector dominance (VDM). It is just a reason that our result (28) is in more consent with the data (30) then the result of VDM predicting $\lambda_+ = m_\pi^2 / M_{K^*}^2 = 0.0245$.

A complete check-up of our predictions needs more accurate data, in particular, for λ_o.

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Figures

Fig.I

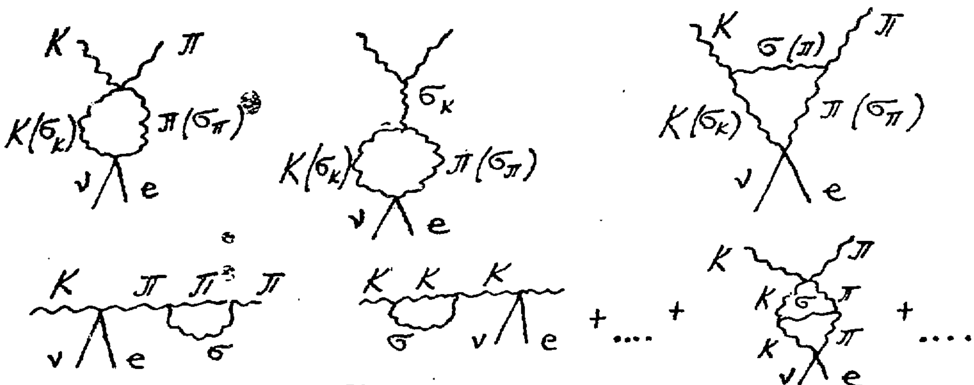

Fig.2

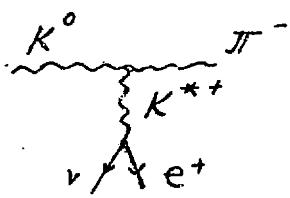

Fig.3

References

I. Ademolo M., Gatto R. // Phys.Rev.Lett., I965, vol.I3, p.264. Bouchiat C., Meyer Ph. // Nuovo Cim. I964, vol. 34, p.II22.

2. Callan C.G., Treiman S.B. // Phys.Rev.Lett., I966, vol.I6, p.I53.
Suzuki M. // Phys.Rev.Lett., I966, vol.I6, p.3I2.
Mathur V.S., Okubo S., Pandit L.// Phys.Rev.Lett., vol.I6, p. 37I.

3. Gaillard M.K., Chounet L.M.// "K form factors" CERN 70-I4, May I970.

4. Shabalin E.P. // Yadernaya Fiz. , I989, vol. 49, p.588.

5. Shabalin E.P. // Yadernaya Fiz. , I985, vol. 42, p.260.

6. Shabalin E.P. // Yadernaya Fiz. , I988, vol. 48, p.272.

7. Matsuda S., Oneda S.// Phys.Rev., I969, vol.I85, p.I887.

8. Bourquin M.et al. // Z.Phys., I982, vol.I2, Ser.C, p.307.

9. Paschous E.A., Turke V. // Phys.Lett., I982, vol.II6.

IO. Bhargava A.K., Dass T. // Phys.Rev., I970, vol. I, Ser.D, p.I49.

II. Majumdar D.P. // Phys.Rev., I970, vol.I, Ser.D, p.684.

I2. Gasiorowicz S., Geffen D.A.// Rev.Mod.Phys., I969, vol.4I, p.53I.

I3. Ivanov A.N. // Yadernaya Fiz. I98I, vol. 33, p. I679.

I4. Gasser J., Leutwiller H. // Ann. of Phys., I984, vol.I58, p.I42.

I5. Ecker G. et al.Preprint CERN-TH.5I85/88.

I6. Particle Data Group // Phys.Lett., I988, vol.204, ser.B, April I988.

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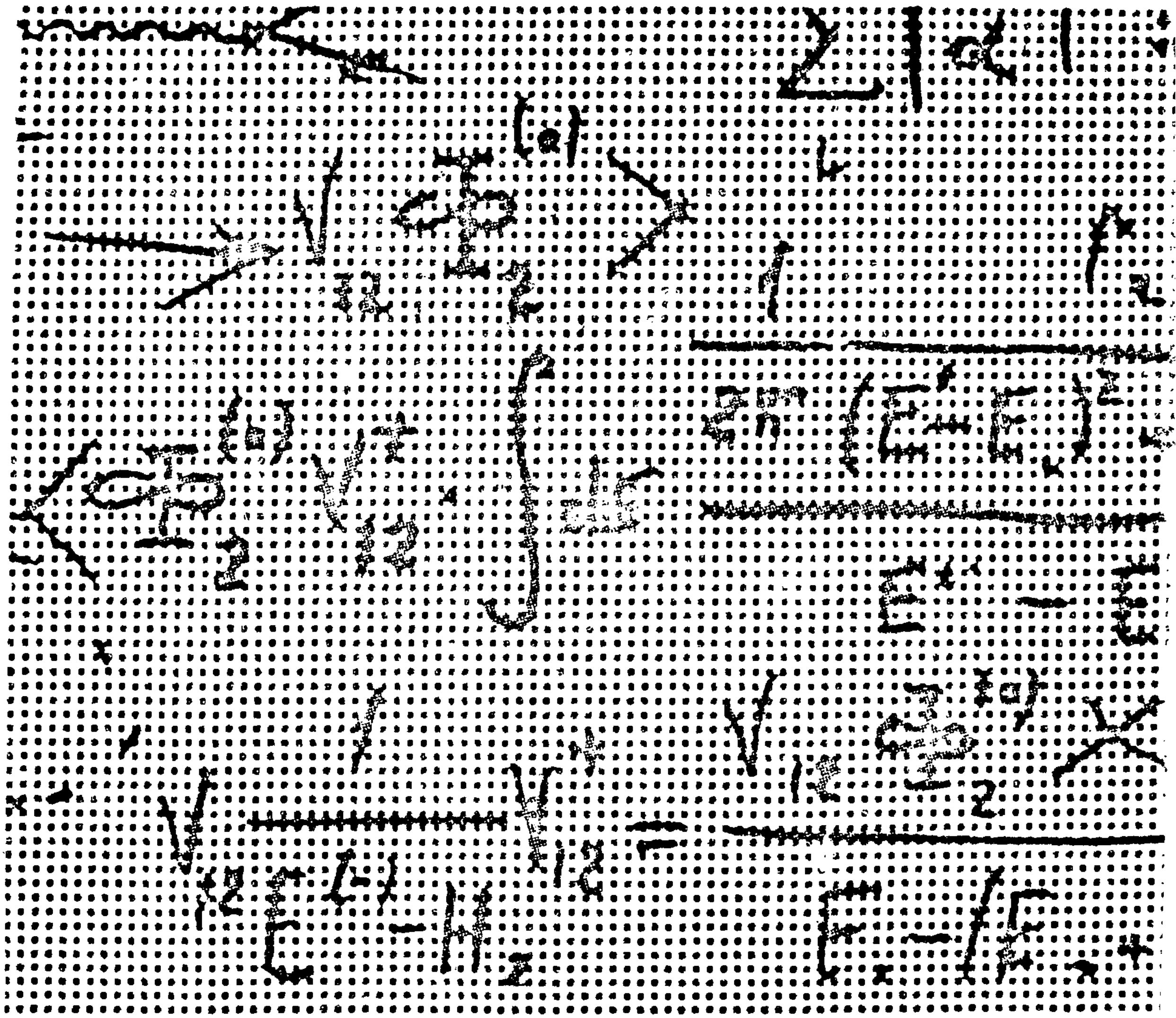

M., Preprint ITEP, 1989, N 122, p.1-12