KFKI-1990-25/A

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WHITHER HAS THE NONTHERMAL TAIL GONE IN NONRELATIVISTIC HEAVY ION COLLISION?

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CENTRAL
RESEARCH
INSTITUTE FOR
PHYSICS

BUDAPEST

KFKI-1990-25/A
PREPRINT

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HU ISSN 0368 5330

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ABSTRACT

In a simplified model we follow the evolution of the originally nonthermal momentum distribution. Our result is, that, although the final entropy deficiency may be quite substantial at 800 Mev/nucleon beam energy for smaller nuclei, the detected process does not reflect the extent of deviation from equilibrium (quite substantial). The reason is the addition of flow velocities, different in different volume elements.

Б. Лукач, А. Рац: Объяснение отсутствия нетеплового хвоста в нерелятивистских столкновениях тяжелых ионов. KFKI-1990-25/A

АННОТАЦИЯ

С помощью упрощенной модели прослеживается эволюция распределения моментов с нетепловым начальным распределением. Выясняется, что, хотя недостаток энтропии в конечном состоянии в случае меньших ядер при энергиях столкновения 800 МэВ/нуклов может быть значительным, экспериментальные данные не свидетельствуют о таком отклонении от равновесного распределения. Это можно объяснить, приняв во внимание скорости течения, которые в различных элементах объема являются разными.

Lukács B., Rácz A.: Hová tűnt a nem-termikus farok a nemrelativisztikus nehézion ütközésekben? KFKI-1990-25/A

KIVONAT

Egy egyszerűsített modellben végigkövetjük egy kezdetben nem-termikus eloszlásfüggvény evolúcióját. Eredményül kapjuk, hogy bár a végső entrópiahiány 800 Mev/nukleon ütközőenergián, kis magok esetében még elég jelentős marad, ez az eltérés a detektálási folyamat során nem jelentkezik. Mindez megmagyarázható, ha figyelembe vesszük a folyási sebességet is, amely különböző a különböző térfogatelemekben.

1 INTRODUCTION

Central heavy ion collisions in the 0.4-0.8 GeV/nucleon beam energy regime have been exhaustively studied in experiments, and they seem to have been clarified quite well from theoretical viewpoint. According to our present knowledge no exotic state of matter is created in such collisions, and the remaining problems are rather technical. Economical macroscopic formalisms such as hydrodynamics and thermodynamics are not a priori applicable here, where the systems are small and short-lived; as estimations for the applicability, the result is that the situation is out of the borders of the above disciplines.

For example, it is very doubtful if *thermal* equilibrium can be built up in the system. Let us see some (rather crude) estimations for an 800 MeV Ar+Ar collision. One can get a characteristic time between subsequent collisions as

$$\tau \simeq \frac{1}{n\sigma v_{rel}} \tag{1.1}$$

At the hot dense stage of the collision $n \simeq 2n_o$, $T \simeq 120$ MeV, therefore, with the usual 60 mb total σ one gets

$$\tau \simeq 0.7\ fm/c.$$

However, the total cross section includes very forward collisions as well, which are negligible for equilibration. For this purpose it is better to use a "transport" relaxation time [1] which can be obtained by averaging $\sigma(1-cos\theta)$ instead of σ. Even for isotropic cross section this results in a factor 2, therefore

$$\tau_{tr} \simeq 1.4\ fm/c.$$

At the end of the collision however $n \simeq 0.4\ n_o$ and the temperature is lower as well, so then the characteristic collision time is longer by cca. a factor 7.

Now, the length of the expansion stage can be measured between the maximal density ("total overlap"), and the end of the continuum regime ("breakup") [2]. For this time t_{br} different calculations give different results [3], [4], [5], because of different possible definitions of the breakup moment. Anyway,

$$t_{br} \simeq 4 \ldots 8\ fm/c.$$

Nevertheless, during the expansion the temperature of the system varies widely, and the collisions have to equilibrate the momenta to a changing pattern. Therefore the time to be compared to the collision time is rather the one characterizing the cooling

$$\tau_{cool} \equiv -T/\dot{T} \simeq 2.7\ fm/c.$$

By comparing the second and fourth time scales, we indeed are not in the regime of asymptotic complete thermalisation.

The situation is even worse, as will be seen later. However, it is pointless to argue further here, since no one seriously has argued *for* complete equilibrium. This is just the reason for having elaborated methods solving the whole Boltzmann equation such as BUU and VUU.

Still, in the above mentioned energy range the *detected* particle spectrum does not suggest serious deviations from thermal equilibrium. This fact is rather unexpected and needs some discussion in itself. But let us mention in advance that this controversy will be the object of the present paper; namely: why are the expected nonthermal deviations unseen in the detected spectrum?

As for the nature of the detected spectrum: of course, it is never an ideal thermal distribution (say Boltzmann). But this is not expected either, because of the relative motions of different volume elements. Performing the proper Lorentz transforms just after breakup and summing up for volume elements, a transformed *thermal* spectrum is already in agreement with the detected one in 800 MeV Ar+Ar collision within the errors of measurement [5]. Even at higher energy and smaller nucleus (1.8 GeV Ne+Ne) the produced K^+'s have an essentially thermal distribution [6] (see also Ref. 7 for kaons). We close this list with Ref. 8, which explicitly compared spectra from thermal comoving distributions with those obtained from an exact solution of the Boltzmann equation [9] for finite time. That solution approaches the asymptotic Boltzmann shape as $e^{-t/6\tau_0}$. The factor 1/6 means that the equilibration is even slower by a factor 3 which was the more pessimistic estimation, therefore serious deviations should be seen. Still, *complete* equilibrium solution is superior to partial ones when comparing to the detected spectra.

In addition we note that experiments show approximate isotropy at the end of central collisions [6-7], [10-12].

In the present paper we try to examine what *is* indicated by the detected spectrum and what is *not*. Therefor we assume a deviation of *prescribed shape*, and study its evolution up to detection. (Notice that to answer our present question we do not have to know the detailed *shape* of the deviation, its *extent* is enough.) It turns out that in spite of the serious entropy deficiency even at breakup, from Ar upwards the nonthermalized distributions lead to practically the same *detected* spectra as the ideal thermalized ones.

Sect. 2 recapitulates the model in which such a calculation can be economically performed. If the shape of deviation is prescribed, then the momentum distribution, and with it the behaviour of nuclear matter is completely determined by the extensive densities, and as many higher momenta as the number of free parameters in the shape. These latter quantities, called pseudoextensives, are analogous to the lower momenta (density and energy density), but vanish in equilibrium. Having included them, the *structure* of dynamical equations remains the same as in thermal equilibrium, which is very convenient for hydrodynamical calculations.

Sect. 3 compares the evolution of some global quantities (density, temperature, etc.) to that obtained in a VUU calculation, in order to check the approximation.

The calculated momentum distributions are discussed in Sect. 4, with the result that the *detected* spectra are less sensitive on deviations from thermal equilibrium than e.g. the entropy.

The reason is given in Sect. 5, with the conclusion that up to 1 GeV/nucleon the detected spectrum is not the proper information about the exact degree of thermalization.

The Appendix gives some detailed dynamical equations of the model.

2. THE MODEL

For studying the process of thermalization of the momentum distribution during the collision a BUU or VUU type calculation would be the proper way, since such calculations solve the Boltzmann equation directly. However, to perform such detailed calculations for a sequence of energies and atomic numbers, substantial computer time would be needed. Here we are not interested in the detailed *shape* of the momentum distribution but only in the *extent* of its deviation from a thermal distribution. Therefore it would be enough to prescribe an Ansatz for the distribution, with a free parameter for the deviation, and to follow the evolution of this parameter. However, in general, this shape would not be kept by the Boltzmann equation.

Now, this problem does not arise if, instead of the Boltzmann equation, we consider some momenta of the distribution function, and evolution equations for these momenta, compatible with the Boltzmann equation. In Ref. 13 such a formalism was given, and now we apply it to the present problem. Some formulae will be recapitulated first, partly for convenience, partly because of mistypes of some numerical constants there.

Our model system is an ideal (Boltzmann) gas; the deviation from equilibrium is described by the simplest possible polynomial, and the distribution is assumed to be isotropic, to permit hydrodynamics. The calculations are done upto quadratic terms in the deviation. We accept a relaxation time approximation.

We accept the three-step approach of Refs. 3, 4, which fairly reproduces the observables up to Berkeley energies. There the collision is treated in three disjoint steps:

i) ignition, in which the two clusters of nuclei interpenetrate with the original velocity;

ii) expansion, with spherical hydrodynamics;

iii) breakup.

For simplicity, we do not explicitly calculate the ignition stage (which would be a straightforward but laborous task) but estimate its final point by some initial conditions of the expansion stage, detailed later.

, For hydrodynamics, first note that Ref. 2 presented analytic solutions for an ideal Boltzmann gas with special initial shape and spherical expansion, (which will remain valid for our case). The details will not be needed here; however the main point is that the radius of the growing sphere changes as

$$R(t) = R_o \frac{\sqrt{t^2 + t_o^2}}{t_o} \tag{2.1}$$

where R_o is the initial radius (cca. at maximal compression) and t_o is a time scale constant. The flow velocity v is proportional to the radial distance, thus

$$v = r\frac{\dot{R}}{R} \tag{2.2}$$

Now, previous calculations have demonstrated that satisfactory final results can be obtained even by homogeneous local thermodynamic parameters (as density, temperature, etc.) and with linear velocity field, and by using only the *global* balance laws. This will be done below.

For thermodynamic description we follow the formalism of Ref. 13, and specially the simplest model system elaborated there in Sect. 5. For details cf. Ref. 13; here we collect only the final equations. Assume that for the distribution function $f(\mathbf{p})$

$$f(\mathbf{p}) = f_o(k^2)G(k^2)$$
$$\mathbf{k} = \mathbf{p} - \tilde{\mathbf{p}} \tag{2.3}$$

(that is, isotropic in comoving coordinates), where f_o is the ideal Boltzmann distribution, $\tilde{\mathbf{p}} = m\mathbf{v}$ is the flow momentum and $G\{k^2\}$ is a polynomial of minimal order, preserving the energy density and particle density of f_o (Eckart matching condition [15]). Then $G\{k^2\}$ is quadratic, containing only one multiplicative free parameter as [13]

$$G(k^2) = 1 + \delta\Big[k^4 - \frac{20m\epsilon}{3n}k^2 + \frac{20}{3}\big(\frac{m\epsilon}{n}\big)^2\Big] \tag{2.4}$$

Therefore the deviation from equilibrium is characterised by the single unequilibrium parameter δ; consequently momenta above the second one will be determined by the zeroth momentum n, the second, $2m\epsilon$, and δ. By other words, δ is known if in addition to n and ϵ, the fourth momentum is known as well. From the fourth momentum one can form a *pseudoextensive* z as

$$z = \frac{40}{3}m^2\epsilon^2 n^{-7/3}\big(\frac{4m\epsilon}{3n}\big)^2\delta \tag{2.5}$$

with balance law similar to that of n and ϵ but possessing an equilibration source too [13]. Since the function H is a functional of f, the entropy density s depends on n, ϵ and z, and evaluating it up to terms z^2 we get [13]

$$s = n\Big(\frac{3}{2} - \ln(n) - \frac{3}{2}\ln\big(\frac{3n\kappa}{4\pi m\epsilon}\big)\Big) - \frac{1}{2}\frac{K}{m^4}n^{17/3}\epsilon^{-4}z^2 \tag{2.6}$$

$$\kappa = (2\pi\hbar)^2 \cdot \exp\Big\{-\frac{5 + ln(16)}{3}\Big\}$$

$$K = \frac{27}{640}$$

where the value of κ, connected to the chemical constant (cf. Ref. 14), comes from quantum statistics. (Higher terms in z are negligible not too far from equilibrium; here we shall not discuss the approximation.)

Still there is the problem of the evolution equation of z. The relaxation time approximation suggests [13]

$$\dot{z} = (\mathbf{v}\nabla)z = -z\nabla\mathbf{v} - \frac{1}{\tau}z \tag{2.7}$$

and with such a balance law the hydrodynamic pressure P can be made equal to the thermodynamic pressure p obtained from s [13]. This approximation is insufficient for very fast expansion [16], when the deviation from rigid motion causes dethermalisation [17]. In these stages a hydrodynamical calculation would result in anomalies. First, at low densities the mean free path is longer than the linear size of the sphere, so the particles evaporate. Second, remaining too far in the continuum description the density may decrease so much that the system enters the unstable region below cca. 0.4 normal nuclear density [18]. Third, the notion of a common temperature parametrizing the distribution is physically meaningless in states when the time between subsequent particle collisions is longer than the time scale of the expansion. However, here this stage is treated by a *breakup process* as e.g. in Ref. 2; at the moment when the equalibration processes are already negligible, we switch over to Liouville equation and perform a Lorentz transformation into CM system [19] (i.e. sum up the flow and thermal momenta and then integrate over the whole volume). This is to be done anyway since there is no more equilibration if the time scale of cooling is already shorter than the collision time, because then there is no more physical possibility to keep the step with the changing f_o. Note that the change of the *shape* of a completely thermal distribution happens on the time scale $(\dot{T}/T)^{-1}$, and the collisions must perform a double role: to thermalize and to readjust the temperature. This suggest

$$\frac{1}{\tau} \Longrightarrow \frac{1}{\tau} + \frac{\dot{T}}{T} \simeq \frac{1}{\tau} - 2\frac{\dot{R}}{R_o} \tag{2.8}$$

with τ roughly the *transport* time scale:

$$\tau \simeq \left(n\sigma_{tr}v_{th}\right)^{-1} \tag{2.9}$$

In the spirit of our model, eq. (2.8) yields the breakup time, namely the one when τ becomes infinite. (If one extended the calculation beyond that time, then the relaxation coefficient would become negative, indicating departure from thermal shape, this is just the dethermalisation seen in Ref. 17. Nevertheless it is better not to use relaxation time approximation there.)

Then the model is defined. The equation of state is contained in the entropy function $s = s(n, \epsilon, z)$ given by eq. (2.6). In order to follow the evolution one has to solve the balance and dynamical equations, collected in the Appendix. To this end one needs initial conditions and the integration is to be continued until breakup defined above. In the initial conditions we follow Refs. 3, 4 and start from a supposed total overlap stage but for simplicity, with double nuclear density. (For the possible error see again Refs. 3, 4.) There the temperature can be calculated from the beam energy; for the extra density z the proper initial value should be taken from Monte Carlo simulations for the development of the Maxwell tail. However, one may use a simpler estimation. If the thermalisation is not complete (and therefore the tail is not quite Maxwellian), then the entropy is lower, and the relative difference is roughly a function of the number of collisions. As an order of magnitude estimation, in what follows, we use 30% initial entropy shortage, roughly conform to one collision per particle until overlap. At the end of the expansion stage we apply the breakup prescription [2].

3. COMPARISON TO VUU CALCULATIONS

Having integrated the evolution equations one can get the quantities characterizing the degree of thermalisation of the system. The simplest such quantity is the entropy loss. One expects a more complete thermalization for larger nuclei, because of more collisions during the expansion. This is seen on Fig. 1, where the relative final entropy loss (with the assumed initial 30%) is displayed as a function of the atomic mass at 800 MeV/nucleon beam energy.

However, the figure shows that for small nuclei (until cca. Ar) the final entropy loss is still quite substantial (above 10%). Then one might expect serious deviations from thermal spectra at breakup. Still, this is just the deviation *not* seen in the detected spectrum. We should rather understand this contradiction.

Our present model is rather hand-made and approximative. Therefore one should first check it via some comparison to more detailed and better founded calculations. The most obvious candidate for an etalon is a VUU or BUU calculation. However, admitting that only such a calculations would give the complete energy spectrum, here we are not really interested in the details of the spectrum, only in the extent of deviation from equilibrium at breakup. For this purpose it seems natural to compare global data of the fireball (such as density, temperature and entropy), from VUU and from our simplified model calculation.

For this comparison we select Ref. 20 (cf. also Refs. 21 and 22). It gives the results of a VUU calculation with Skyrme equation of state, mainly for Nb+Nb collision at 1.05 GeV/nucleon beam energy. That calculation starts at first touching and extends for cca. 40 fm/c. Consider first the evolution of the average density in a 2 fm central sphere. It reaches its maximum $2.7n_0$ cca. at $t - t_{touch} = 6\ fm/c$. This must be the moment of the total overlap and henceforth we measure the time from here. Then the density starts to decrease and passes the normal nuclear density at cca. 9 fm/c. Now, our model calculation gives the curve on Fig. 2. (For the dynamical equations cf. the Appendix.) In what follows, we display always two curves, a solid one for thermal equilibrium and a dashed one for 30% initial entropy shortage. However on Fig. 2 the two curves coincide since up to z^2 terms the deviation does not influence the dynamics in the present model [13].

The curve shows that the breakup happens at cca. 4 fm/c after total overlap. Our density starts from $2n_0$ as an initial condition of the model and drops by cca. $1n_0$ during that time. Since the same decrease in the VUU calculation took cca. 6 fm/c, one can conclude that the dynamical time scales of the models differ. However, the difference does not seem vital, and notice that our n is rather an average density of the whole sphere.

The next quantity to be compared is the temperature. In Ref. 20 at total overlap it is cca. 80 MeV, still very slowly increasing. After cca. 1 fm/c it reaches the maximum at 80 MeV, and at 4 fm/c (which is our breakup) it is cca. 75 MeV. Henceforth it seems to follow a power law in the density. Now, our results are displayed on Fig. 3. As seen, at total overlap $T \sim 30\ MeV$, nevertheless its *maximal* value is 80 MeV at cca. 2 fm/c, and it decreases to cca. 70 MeV at breakup. The agreement with the VUU results is not bad. On the other hand, the calculation with complete thermal equilibrium gives quite different temperatures, some 50% too high at breakup. This difference should be obvious when comparing equilibrium calculations to experimental data, and it is not, as told in the Introduction. (We shall see the reason in the next Chapter.) One could also question the serious increase of temperature in the first 2 fm/c of the *expansion*, because this might have been a signal for negative specific heat. Nevertheless the second derivative $S_{,EE}$ does have the negative sign required for stability. Therefore one can conclude that the source of the temperature increase is the thermalisation still going; anyways, Ref. 20 found a (much smaller) temperature increase as well. The difference indicates that our approximate off-equilibrium equation of state may be incorrect so far from equilibrium.

The next two curves in Ref. 20 gave the pion yield and flow angle; such quantities are not included in our model. The fifth calculated quantity there was the average transverse momentum. Its final value is 120 MeV/c, and at 4 fm/c (which is our breakup time) it is already greater than 2/3 of this final value. In contrast, our model yields a value independent of time

$$< p_{\perp}^2 >^{1/2} = \sqrt{\frac{1}{3} m_n \frac{E_{Lab}}{A}}.$$

(It is pointless to compare the numerical values, because our calculation is meant for a central collision, while in this point Ref. 20 took an impact parameter 3 fm, comparable to the size of the whole Nb nucleus. Then the majority of the nucleons are spectators there with negligible $p_{\perp}$, so decreasing the average value.) That is, *for this quantity* our model is quite inadequate, although not very bad for the detected value.

The reason for this difference is transparent enough. The transverse momentum measures not the thermal shape but the isotropy of the momentum distribution. Now, in our simplified model the deviation from equilibrium *is* isotropic. Then the comparison shows that at this point the approximation is an oversimplification. However, as we have seen, the isotropisation is already quite developed at breakup, the detected spectra are isotropic [10] - [12], and we are interested only in the *extent* of the deviations, not in their detailed forms. If necessary, the present model can be improved by including anisotropies in the ways of Refs. 23 and 24.

The next quantity is the specific entropy. In Ref. 20 its value at total overlap is cca. 3, while the asymptotic value is cca. 6. Our results are displayed on Fig. 4. The starting value is 4 (because of our initial condition of 30% entropy shortage); the final value at breakup is 5.6, some 5% less than in thermal equilibrium. One can conclude that the final entropy is well reproduced in our model.

The model calculation also yields the evolution of our deviation parameter z; this is given in Fig. 5. One can see that between total overlap and breakup it decreases by a factor of 2.5, according to the decrease by a factor of 6 in the *entropy* deviation.

Ref. 20 gives one more curve for global data which can be calculated in our model. It is the final specific entropy vs. bombarding energy for an Au+Au collision. There S/N is 3 at 100 MeV/nucleon, and cca. 5.5 at 1 GeV. Our result is Fig. 6. The values are practically the same as well as the shape of the curve; in addition one can see that thermalisation is almost complete for Ar at breakup even for the highest calculated energy.

Since at breakup our model reproduces more or less all the global quantities which can be calculated within its framework, our conclusion is that with sufficient caution it can be used to estimate the *extent* of the detectable deviations from a thermal spectrum. This will be done in the next Chapter.

4. MOMENTUM SPECTRA

Now we are in the position to calculate momentum spectra in our model for collisions at 400 and 800 MeV/nucleon beam energies, for three different nuclei, Ne, Ar and U, respectively, with or without 30% entropy deficiency at total overlap. The detected spectrum is obtained by adding up thermal and flow momenta just after breakup in the same way as in Ref. 2; the distribution function is determined by the 3 independent densities n, ϵ and z. But first, before the spectra, we display the final entropy vs. beam energy for Ne+Ne and Ar+Ar. (Figs. 7 and 8; for U+U the curve would be very similar to Fig. 6.) One can see a deficiency of cca. 15% at 1 GeV for Ne and 10% for Ar. Being ΔS quadratic in z, the deviation in the momentum distribution might be quite serious, but in the same time thermal equilibrium models give roughly good final results. This is just the problem which triggered the present search.

Now, Fig. 9 gives various momentum spectra for an Ne+Ne collision at 800 MeV/nucleon beam energy: label "no" stands for total overlap, "nb" for just before breakup *in the comoving system*, while "fb" labels the momentum distribution just after breakup, which will be detected, including the contribution of the flow. On the horizontal axis the variable is the dimenionless momentum $\tilde{k}$, where

$$\tilde{k} = \frac{3}{2}\frac{n}{\epsilon}\frac{t_b}{t_o^2 + t_b^2} r|\mathbf{k}|$$

Again, solid lines mean distributions in thermal equilibrium while the initial entropy deficiency is 30% for dashed ones.

Consider first the solid lines. The rightmost one is, of course, a thermal distribution with the highest temperature, as expected at maximal density. During the expansion T decreases and reaches its minimum at breakup; this is the leftmost curve. However, the addition of the flow velocities shifts the curve back in the direction of the first one. This final third curve is not a thermal distribution, but, as seen, mimics it quite well, with an effective temperature somewhere between the maximal and minimal ones. This is well known since Ref. 2; the reason is that during expansion some part of the initial thermal energy has been converted into kinetic one, but after breakup they are added up again.

However, the situation is more complicated when one turns to the off-equilibrium dashed lines. At the beginning the distribution is far from thermal, with a very expressed shoulder at $\tilde{k} \simeq 1$. During the expansion the equilibration works, but still there is a serious deviation from equilibrium just before breakup in the $\tilde{k} \simeq 1$ region. However, this deviation is further washed out by adding the flow velocities, and the detected spectrum, while not quite of thermal shape, is not too far from that even for the smallest nucleus investigated. ("Effective" thermalisation.)

Note that the entropy is determined by the distribution function in the local comoving frame, i.e. by the curve "nb", while the detected one is "fb". Therefore there is a possibility to have a fairly thermal detected spectrum together with a still substantial entropy shortage. It seems that *this* happens in the experiments.

While on Fig. 9 the tail is quite thermal, still the global curve could be distinguished from such a one. The difference is much smaller for Ar+Ar (Fig. 10), while for U+U (Fig. 11) the after-breakup off-equilibrium curve is halfway between two equilibrium curves, mimicking both except for temperature. The differences are even less for 400 MeV/nucleon beam energy (not displayed), as can be expected.

5. CONCLUSIONS

In a simplified model we have investigated the thermalisation during expansion in various nucleus-nucleus collisions at the upper end of the nonrelativistic regime. Our results can be summarized as follows.

For smaller nuclei, as Ne and Ar, the thermalisation is far from being complete in entropy; therefore one expects a rather nonthermal momentum distribution at breakup. This is shown indeed by the model calculations. However, the flow velocities contribute to the detected spectrum, and their addition leads to a further "effective" thermalisation in the same way as to the increase of the effective temperature [2]. The model calculations predict that the *detected* spectra will be fairly thermal from Ar upwards, in agreement with the lack of clearly nonthermal characteristics when trying to reproduce the experimental spectra by calculations.

This effective thermalisation via breakup might suggest a substantial change of the distribution function in a regime almost without further collision (the late stages of VUU calculation). Still, the entropy does not increase anymore, which facts are seemingly in contradiction. However, we are speaking of two *different* distribution functions. Namely, the distribution function, from which the entropy density is calculated, is defined at fixed location and time, and later the entropy density is integrated over volume. In contrast, the detected distribution is meant at a fixed place and integrated over time or over the oncoming matter elements. Generally the two distribution functions do not coincide even in the Liouville regime. The problem deserves further investigation.

Therefore it seems that the detected spectra do not display too much *directly* of the expected nonthermal nature of the momentum distributions. Therefore, the fair agreement between thermal equilibrium calculations and experimenal spectra up to 1 GeV/nucleon beam energy does not indicate even approximative equilibrium during expansion. An *indirect* signature for or against equilibrium would be the final temperature. However, in several cases the breakup moment is determined by fitting the detected temperature to the slope of energy spectrum, and then even this signature is lost. The lack of equilibration would be revealed if the entropy could be measured independently, e.g. from produced particle ratios [25], [26]. Nevertheless, at 800 MeV/nucleon beam energy even the pion production is moderate and heavier mesons are ruled out. Similarly, hadrochemical processes are sensitive to the temperature during the whole collision, but at such energies hyperon and resonance production is almost nil.

One can conclude that below cca. 1 GeV/nucleon beam energy the final observables of a heavy ion collision give rather poor and indirect insight into the nonthermal behaviour of the system, which is probably substantial enough.

ACKNOWLEDGEMENTS

The authors acknowledge that the topics had previously been mentioned them by Dr. K. Martinás. In the same time they would like to thank Dr. J. Zimányi for illuminating discussions. Partly supported by the Department of Applied Reactor Physics of CRIP.

APPENDIX

Our starting balance equations, from Ref. 13, are as follows:

$$(\partial_t + \mathbf{v}\nabla)n + n\nabla\mathbf{v} = 0 \tag{A.1}$$

$$(\partial_t + \mathbf{v}\nabla)\epsilon + \epsilon\nabla\mathbf{v} = -p\nabla\mathbf{v} \tag{A.2}$$

$$(\partial_t + \mathbf{v}\nabla)z + z\nabla\mathbf{v} = -\frac{1}{\tau}z \tag{A.3}$$

$$(\partial_t + \mathbf{v}\nabla)s + s\nabla\mathbf{v} = \frac{1}{\tau}\alpha z^2 \tag{A.4}$$

$$\alpha = K m^{-4} n^{14/3} \epsilon^{-4} \tag{A.5}$$

In addition, of course, the total energy of the sphere is constant, i.e.

$$E \equiv \int (\epsilon + \frac{1}{2}mnv^2)dV = \text{const.} \tag{A.6}$$

Now, notice that (in the present z^2 approximation) the z equation is decoupled, i.e. the dynamics is unaffected by z. Then we can turn to the analytic solution of Ref. 2. It has a limit of *homogeneous* density and energy distributions, which we will use. Using eq. (2.2) $\nabla\mathbf{v}$ can be expressed via R and $\dot{R}$

$$\dot{n} + 3\frac{\dot{R}}{R}n = 0 \tag{A.1'}$$

$$\dot{\epsilon} + 3(\epsilon + p)\frac{\dot{R}}{R} = 0 \tag{A.2'}$$

$$\dot{z} + 3\frac{\dot{R}}{R}z = -\frac{1}{\tau}z \tag{A.3'}$$

$$\dot{s} + 3\frac{\dot{R}}{R}s = \frac{1}{\tau}\alpha z^2 \qquad (A.4')$$

Then, by using the homogeneity assumption, the conservation equation for the total energy can be written into the form

$$E = \frac{4\pi}{3}R^3\epsilon + \frac{3}{10}Nm\dot{R}^2 = const. \qquad (A.6')$$

where N is the total particle number:

$$N = \frac{4\pi}{3}nR^3 \qquad (A.7)$$

By comparing eqs. (A.6') and (A.2') one obtains

$$\ddot{R} = \frac{5p}{Rnm}$$

Now, up to z^2 [13]

$$p = \frac{2}{3}\epsilon$$

Hence eq. (2.1) is obtained according to Ref. 2 with

$$t_o = \sqrt{\frac{3}{10}m\frac{n(0)}{\epsilon(0)}} \qquad (A.8)$$

where we have used the initial conditions

$$\begin{aligned} R(0) &= R_o \\ \dot{R}(0) &= 0 \end{aligned} \qquad (A.9)$$

By assuming no compression, from particle and energy conservation between beam pulse and total overlap one gets

$$\begin{aligned} n(0) &= 2n_o \\ \left(\frac{\epsilon}{n}\right)_{t=0} &= \frac{1}{4}\frac{E_{lab}}{A} \end{aligned} \qquad (A.10)$$

Now we turn to the decoupled z equation. Write

$$z(t) = \zeta(t)R^{-3} \qquad (A.11)$$

Then the solution can be written as

$$\zeta(t) = \zeta_o e^{-\int_o^t dt'\,\tau^{-1}(t')} \qquad (A.12)$$

With the form (2.8-9) for τ and with the above n and $\sigma \sim v^{-1}$ the integral gets an analytic form

$$z = z_o f^{-1}(x) \cdot \exp\left\{ -\gamma \left[\frac{x}{1+x^2} + \arctan(x) \right] \right\} \qquad (A.13)$$

with

$$x \equiv t/t_o$$
$$\gamma \equiv \frac{1}{2} n_o v_o t_o \sigma_o . \qquad (A.14)$$

Here v_o is to be taken from the temperature at total overlap, which, in turn, comes from the initial conditions for n, ϵ (discussed above) and for z (see the end of Sect. 2).

REFERENCES

[1] Sir R. Peierls,
Lecture Notes in Physics, "Transport Phenomena" , 31 1974)

[2] J. Bondorf, S. A. I. Garpman and J. Zimányi,
Nucl. Phys. **A296**, 320 (1978)

[3] I. Montvay, J. Zimányi,
Nucl. Phys. **A346**, 490 (1979)

[4] N. L. Balázs, B. Lukács and J. Zimányi,
Nucl. Phys. **A415**, 530 (1984)

[5] T. S. Bíró, H. W. Barz, B. Lukács and J. Zimányi,
Phys. Rev. **C27**, 2695 (1983)

[6] T. S. Biró, B. Lukács, J. Zimányi and H-W. Barz,
Nucl. Phys. **A386**, 617 (1982)

[7] F. Asai, H. Sato and M. Sano,
Phys. Lett. **98B**, 19 (1981)

[8] J. Zimányi,
Act. Phys. Hung. **51**, 139 (1981)

[9] M. Krook and T. T. Wu,
Phys. Rev. Lett. **36**, 1107 (1976)

[10] H. A. Gustafsson et al.,
Phys. Lett. **142B**, 141 (1984)

[11] R. E. Renford et al.,
Phys. Rev. Lett. **53**, 763 (1984)

[12] R. Stock,
Nucl. Phys. **A434**, 537c (1985)

[13] B. Lukács, K. Martinás,
Annln. Phys. **45**, 102 (1988)

[14] Ya. B. Zeldovich and I. D. Novikov,
Stars and Relativity , The University of Chicago Press (1971)

[15] C. Eckart,
Phys. Rev. **58**, 919 (1940)

[16] J. Ehlers,
Relativity, Astrophysics and Cosmology , ed. W. Israel, D. Reidel, Dordrecht, 1973, p. 1

[17] B. Lukács, K. Martinás and T. Pacher,
Astron. Nachr. **307**, 171 (1986)

[18] P. Danielewicz,
Nucl. Phys. **A314**, 465 (1979)

[19] A. S. Eddington,
Mathematical Theory of Relativity , Cambridge University Press, Cambridge 1924

[20] J. J. Molitoris, H. Stöcker and B. L. Winer,
Phys. Rev. **C36**, 220 (1987)

[21] H. Kruse, B. Jacak and H. Stöcker,
Phys. Rev. Lett. **54**, 289 (1985)

[22] J. J. Molitoris et al.,
Phys. Rev. **C33**, 867 (1986)

[23] I. Lovas, Gy. Wolf and N. L. Balazs,
Phys.Rev. **C35**, 141 (1978)

[24] B. Kämpfer et al.,
Yad. Fiz. **51**, 366 (1990)

[25] P. Lévai, B. Lukács and J. Zimányi,
J. Phys. **G16**, 1 (1990)

[26] P. J. Siemens and J. I. Kapusta,
Phys. Rev. Lett. **43**, 1386 (1979)

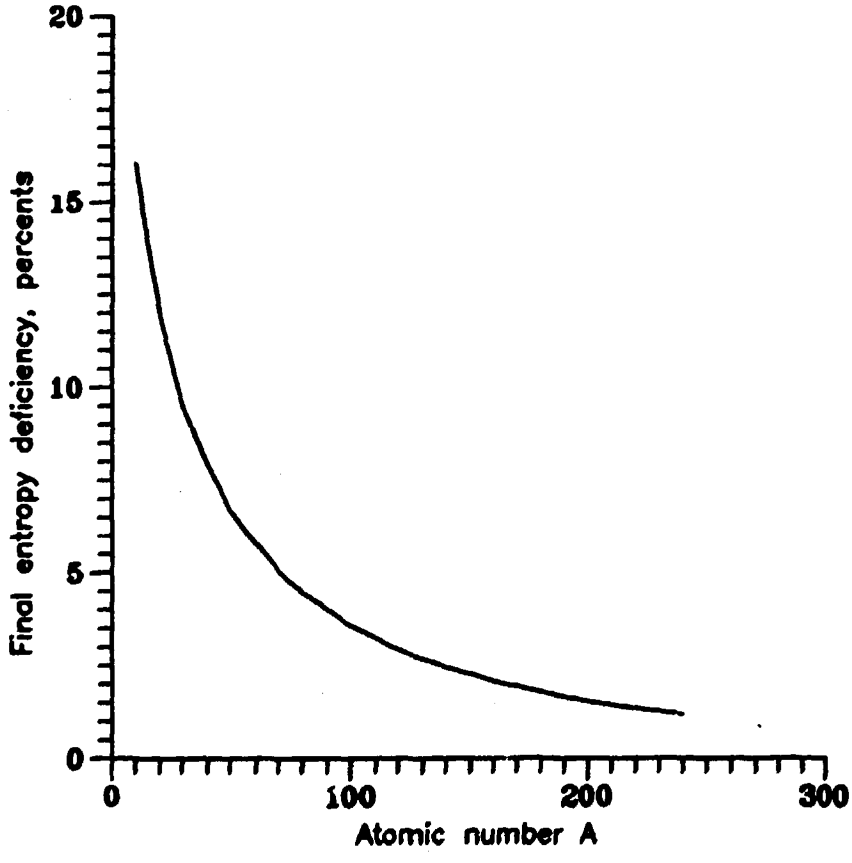

Fig. 1

Relative final entropy deficiency vs. atomic number with an assumed 30% initial one at 800 MeV/nucleon beam energy

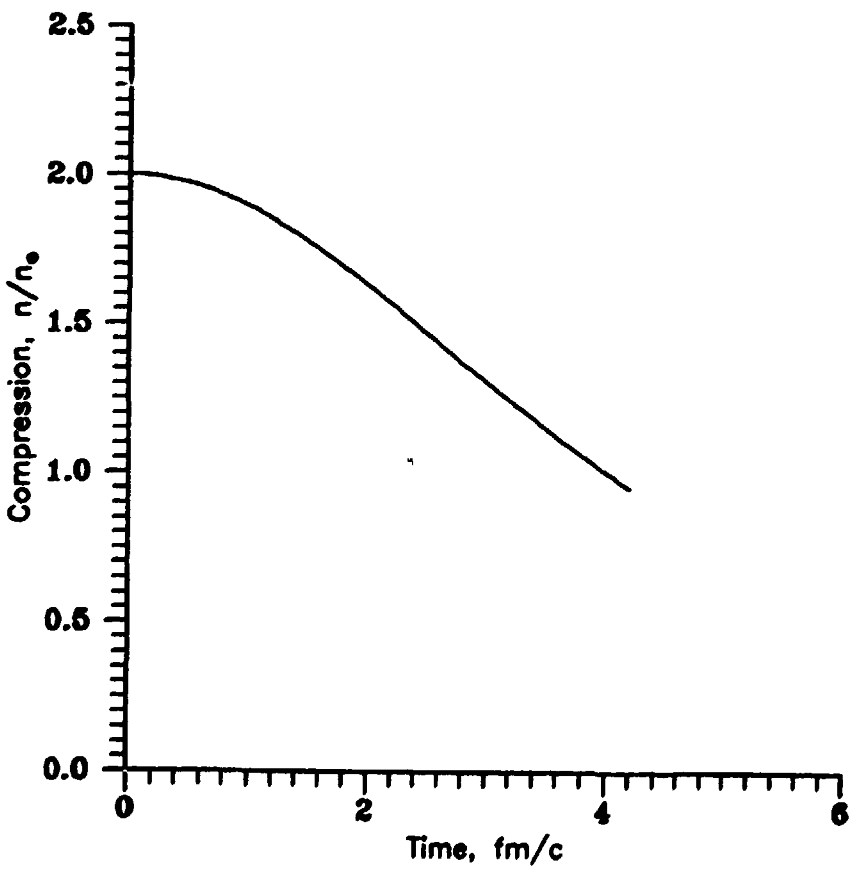

Fig. 2

Density n (in units of normal nuclear density) vs. time in our model calculation for Nb+Nb at 1050 MeV/nucleon beam energy

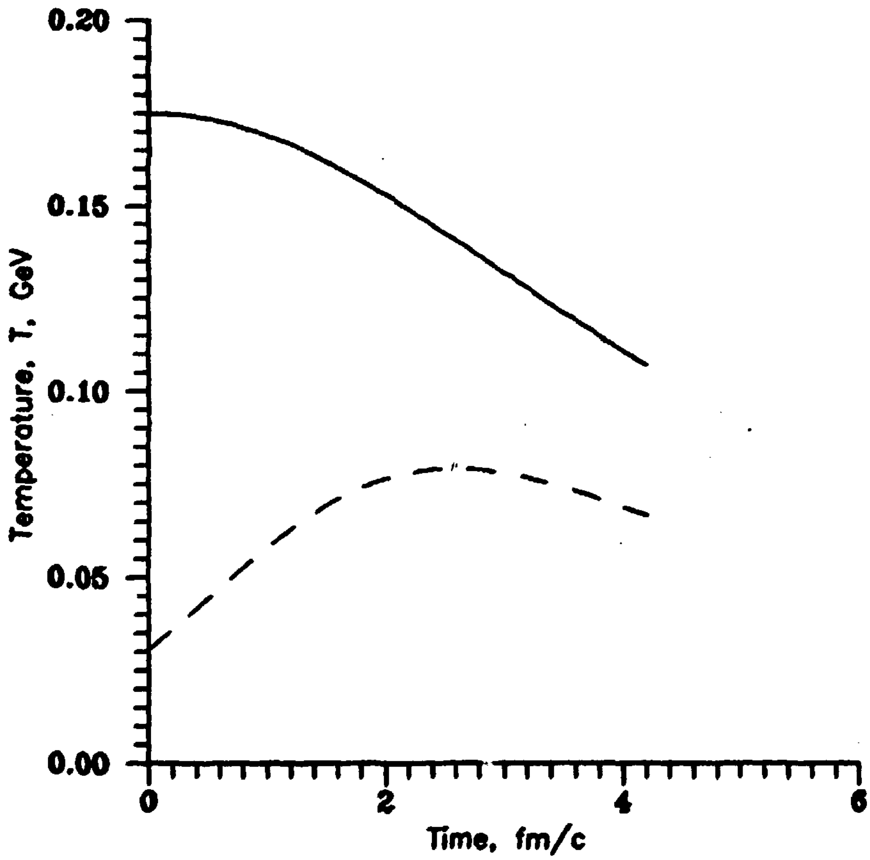

Fig. 3

As above, but *for temperature* T

Solid lines are valid for total thermal equilibrium

Dashed ones for our unequilibrium model with 30% initial entropy loss

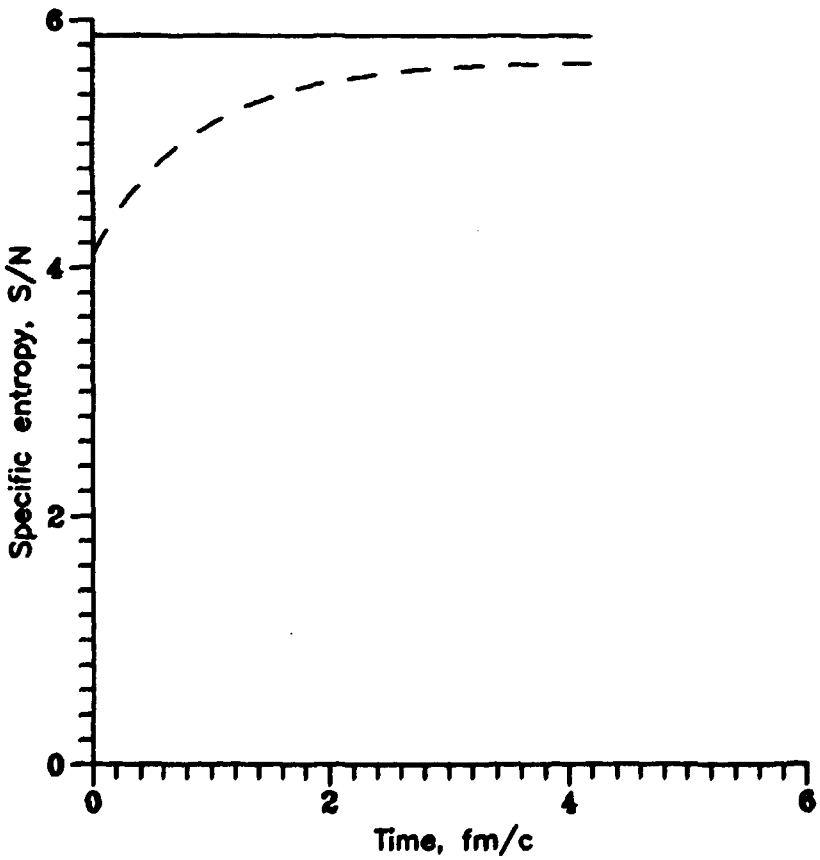

Fig. 4
Specific entropy S/N versus time

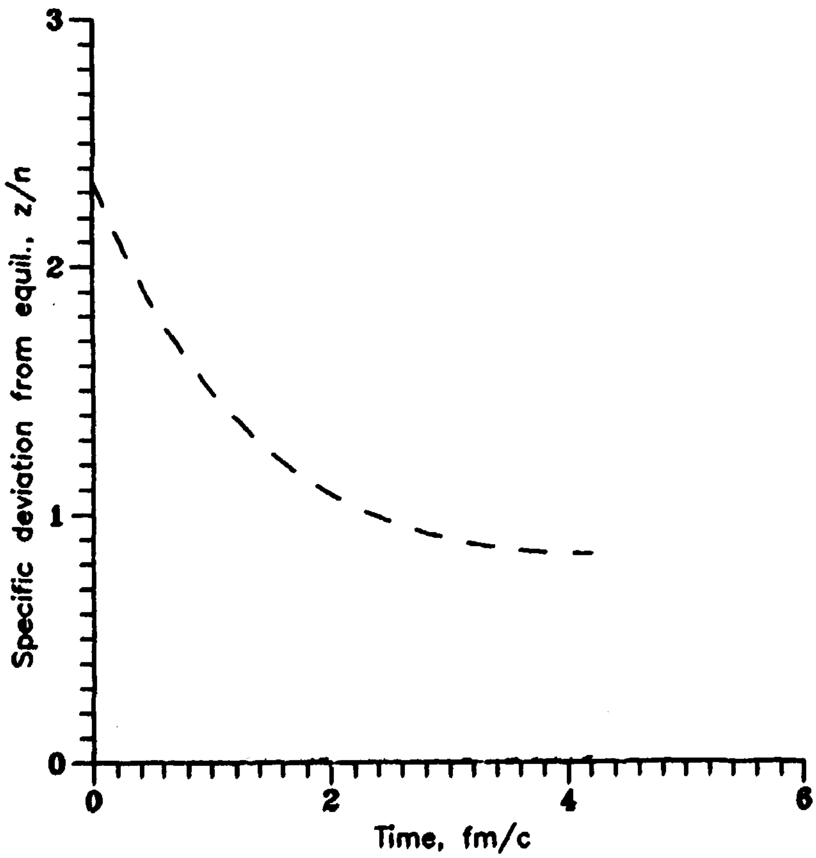

Fig. 5

Specific deviation parameter z/n vs. time

The initial value belongs to the assumed 30% entropy loss

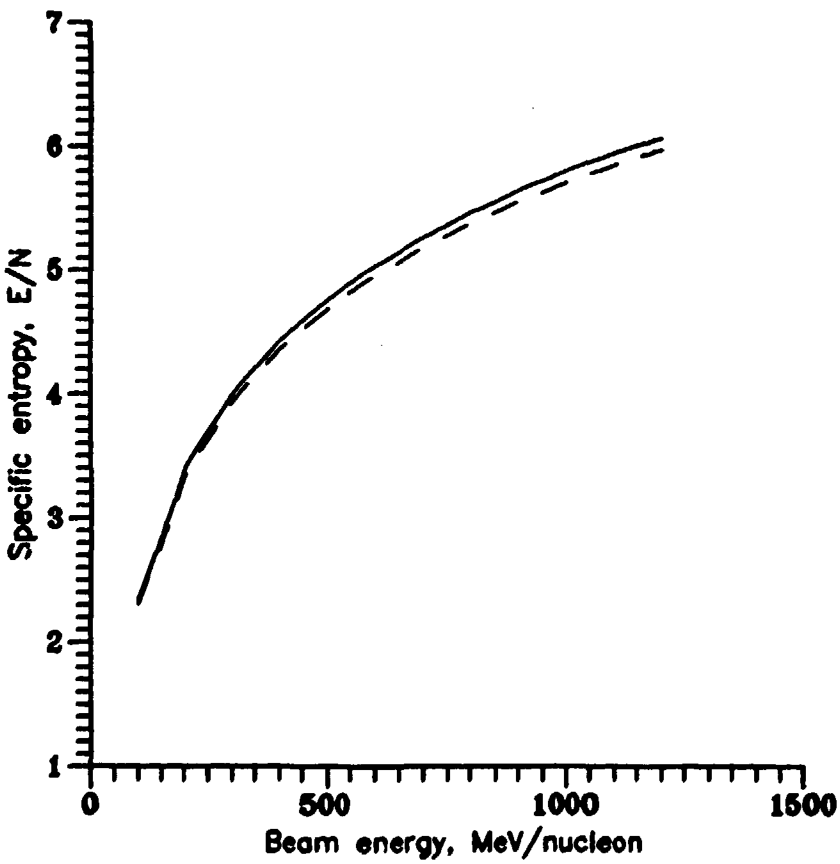

Fig. 6

Final specific entropy vs. beam energy for an Au+Au collision (henceforth, if not indicated otherwise, the beam energy is 800 MeV/nucleon)

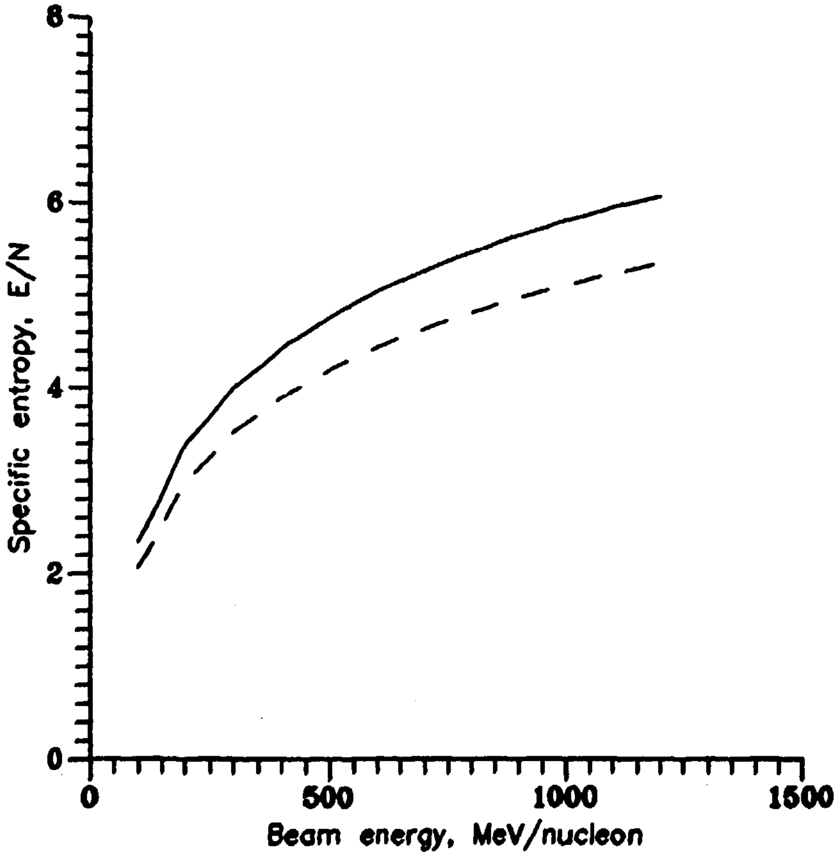

Fig. 7
As Fig. 6, but for Ne+Ne

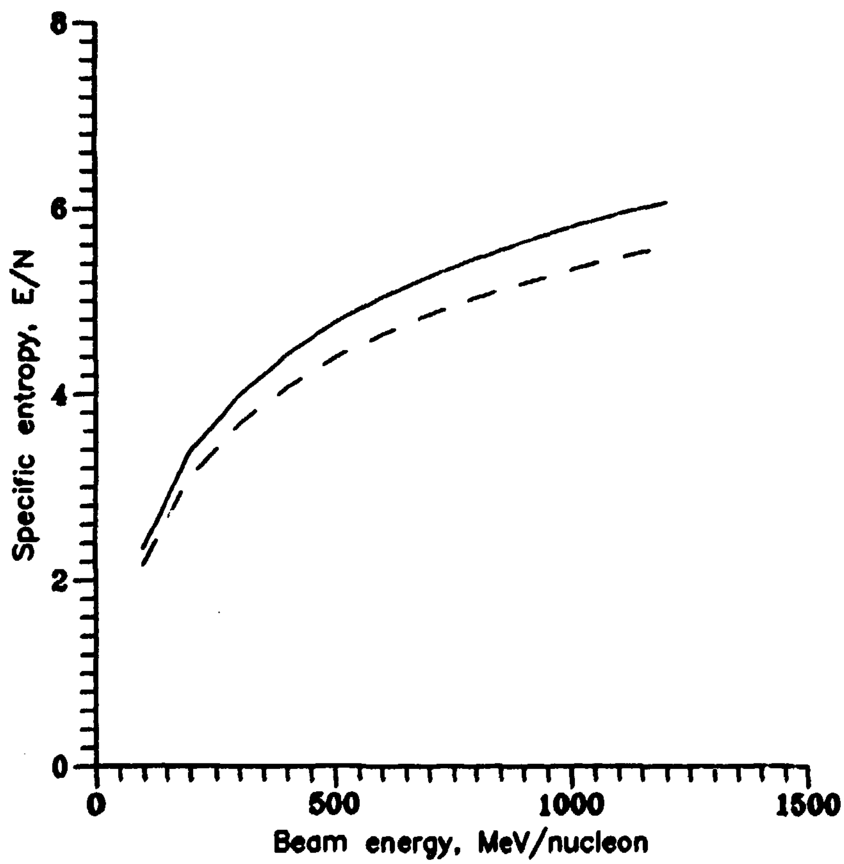

Fig. 8

As Figs. 6 and 7, but for Ar+Ar

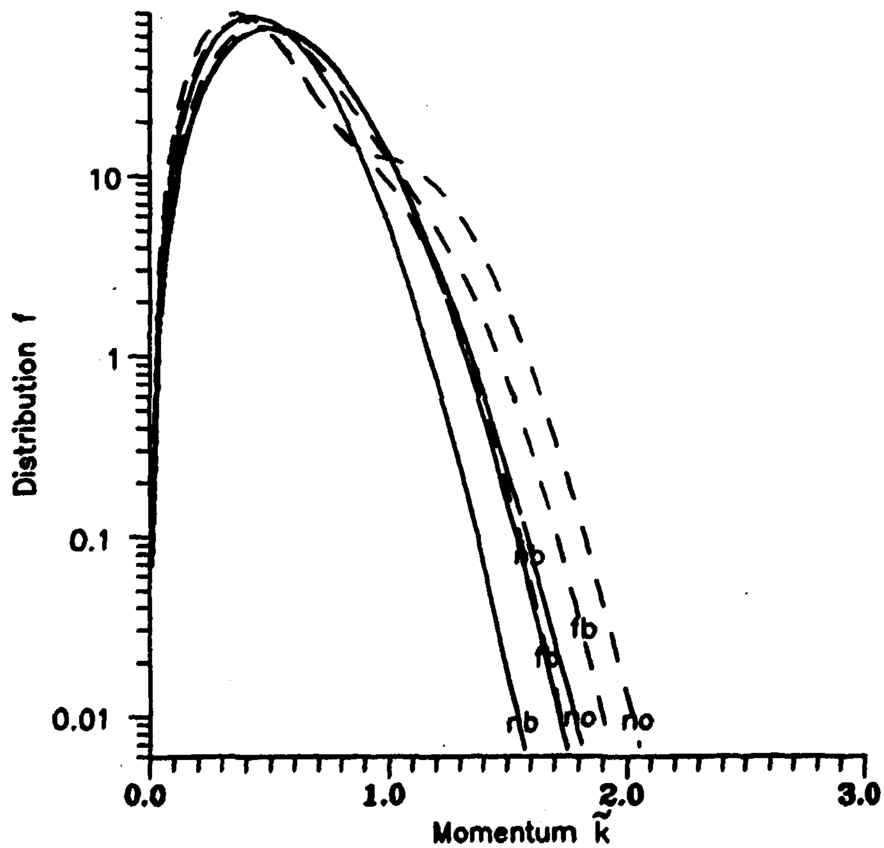

Fig. 9

Momentum spectra during an Ne+Ne collision at 800 MeV
The dimensionless momentum $\tilde{k}$ is defined in eq.(4.1)
Labels indicate different distributions as follows
"no" stands for total overlap (therefore no flow)
"nb" is measured just before breakup in the comoving system (so again without flow effects)
"fb" is valid between breakup and the detector, including flow

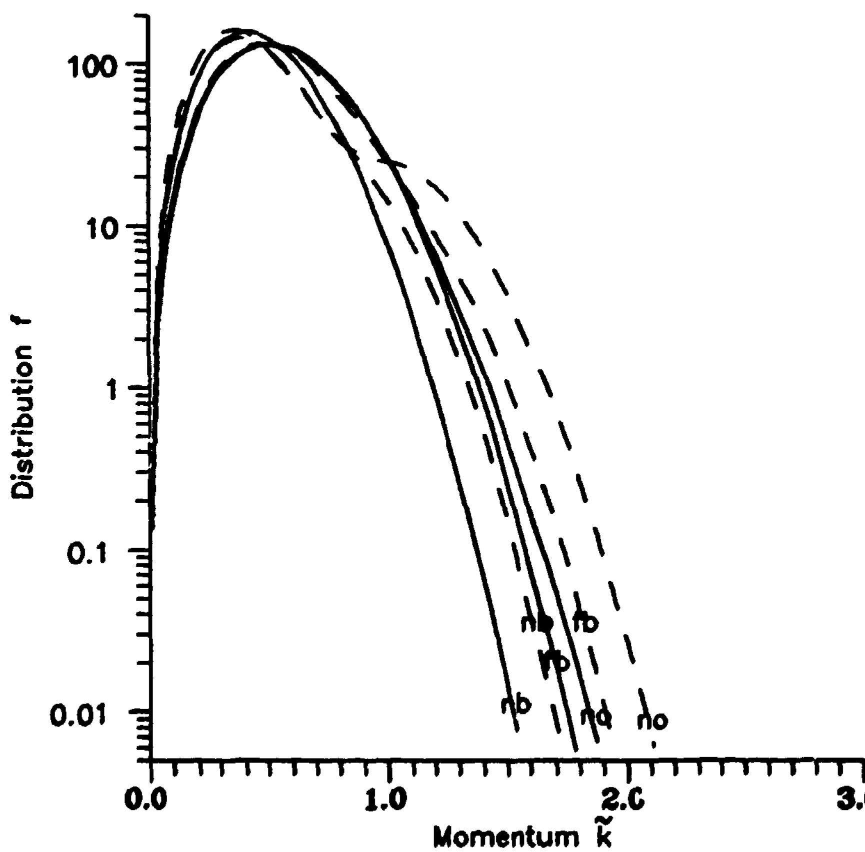

Fig. 10
As Fig. 9, but for Ar+Ar

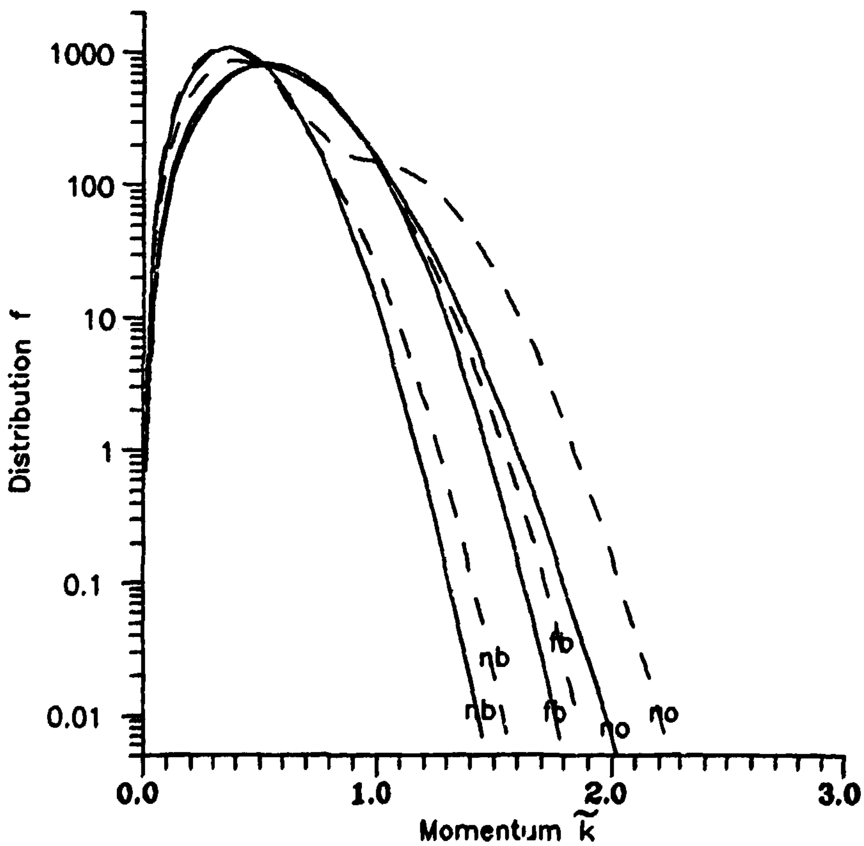

Fig. 11

As Figs. 9 and 10, but for U+U

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