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WESS-ZUMINO-WITTEN MODEL AS A THEORY OF FREE FIELDS



WESS-ZUMINO-WITTEN MODEL AS A THEORY OF PREE FIELDS. II. A PIECE OF GROUP THEORY: Preprint ITEP 89-70/ A.GERASIMOV, A.MARSHAKOV^{**}, A.MOROZOV, M.OLSHANETSKY, S.SHATASHVILI

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Some results from group theory are collected, which are important for free field representation of Kac-Moody algebra and WZW model on the lines of ref.1.

Fig. - 3, ref. - 5

3. A PIECE OF GROUP THEORY

Let us collect here some basic information from group theory [1], which seems relevant for future study of the free field realization of WZWM and coset models.

3.1. ROOT SYSTEMS

Let f_j be a vector space of finite dimension, $\dim f_j = r$. We shall give the definitions of root system and its Weyl group in f_j . Consider reflection Z_{α} with respect to the hyperplane, orthogonal to the vector $\alpha \in f_j$ and passing through the origin:

$$\mathcal{T}_{\alpha}(\lambda) = \lambda - (\lambda_{\alpha} \alpha^{\nu}) \alpha, \quad \alpha^{\nu} = \frac{2\alpha}{(\alpha, \alpha)}.$$
 (3.1.1)

We are interested in finite groups, generated by the reflections (3.1.1). Consider a finite set of non-vanishing vectors $\Delta = \{d\}$, satisfying the following conditions:

> 1. \triangle generates \int as a vector space; 2. $Z_{\mathbf{x}} \triangle = \triangle$ for every $\boldsymbol{x} \in \triangle$ (3.1.2)

3.
$$(\alpha',\beta) \in \mathbb{Z}$$
 for every $\alpha,\beta \in \Delta$.

The system of vectors Δ is reffered to as a root system, and the group W generated by all the reflections (3.1.1) for all $\alpha \in \Delta$ - as a <u>Weyl group</u> of Δ .

Let the space \mathcal{H} be a direct sum of k subspaces, $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \ldots \oplus \mathcal{H}_k$ and let \triangle_i be a root system in \mathcal{H}_i . Then the union $\Delta = \Delta_1 \cup \Delta_k$ will be a root system in \mathcal{H} . The Weyl group is identified with the product $W_1(\Delta_1) = W_k(\Delta_k)$. If no such expansion exists, the root system is called <u>irreducible</u>.

It may be shown, that if \mathcal{A} is a root, then only the root $-\mathcal{A}$ is collinear to it (this is obvious) and $2\mathcal{A}(-2\mathcal{A})$ may also be collinear. It turns out, that except for the latter case there is a one-to-one correspondense between irreducible root systems and complex simple Lie algebras.

Consider a hyperplane in the space \mathcal{H}_{j} , such that it does not contain any root. The roots, lying on one side from the hyperplane are said to be <u>positive</u> with respect to the ordering introduced. Let us denote these by Δ_{+} . Evidently, $\Delta = \Delta_{+} V (-\Delta_{+} = \Delta_{-})$. Among positive roots Δ_{+} one

can uniquely define a subset of roots \square with the property, that any $\alpha \in \Delta_+$ is a linear combination of roots $\beta \in \square$ with non-negative integer coefficients,

$$k = \sum_{\beta \in \Pi} h_{\alpha}^{\beta} \beta, \qquad (3.1.3)$$

The roots $\beta \in \Pi$ are reffered to as <u>simple</u> and Π is a system of simple roots. Evidently, simple roots form a basis of the space $\frac{1}{2}$. The number $ht_{\alpha} = \sum h_{\beta}^{\beta}$ is called the <u>height</u> of a root α .

Different ordering in the space defines another subsets Δ'_+ and Π' . Two subsets $\Delta_+(\Pi)$, $\Delta'_+(\Pi)$ are conjugate under the

action of Weyl group $W(\Delta)$. Moreover, any $d \in \Delta$ is conjugate to some simple root: $\mathbb{Z} d \in I7$, $\mathbb{Z} \in W$.

Root systems of rank r=2 are isomorphic to one of the following three types: A_2, C_2, G_2 (see Fig. 1).

In the set of positive roots one may find a <u>maximal</u> one, i.e. a root $\theta \in \Delta_+$, such that $\theta + \alpha \in \Delta$ for any $\alpha \in \Delta_+$. The number

$$h = ht_0 + 1 \tag{3.1.4}$$

is reffered to as <u>Coxeter number</u>. It is independent of the ordering in μ and thus of the choice of a system Δ_+ .

The roots $a \in \Delta$ generate a root lattice $Q = \mathbb{Z}\Delta$ in $\frac{b}{2}$. The dual lattice Q^{\vee} is generated by <u>weights</u> λ , related to roots by the bracket product,

$$\langle \lambda, d \rangle = \frac{2(\lambda, d)}{(d, d)} = (\lambda, d^{V}).$$
 (3.1.5)

The basis in Q^V is formed by the set of <u>fundamental</u> weights, satisfying

$$\langle \lambda_j, \kappa_k \rangle = \delta_{jk} , \quad \forall \kappa \in \Pi.$$
 (3.1.6)

Let ρ be a half-sum of all positive roots,

$$p = \frac{1}{2} \sum_{d \in \Delta_+} \alpha . \qquad (3.1.7)$$

Then the reflection (3.1.1) with $a_j \in \Pi$ acts on ρ as

$$\mathcal{Z}_{d_i} \rho = -\frac{1}{2} d_i + \frac{1}{2} \sum_{d \in \Delta_+} d_i = \rho - d_i \qquad (3.1.8)$$

The first equality follows from the fact, that $\mathcal{Z}_{\mathcal{A}_i}$ permutes positive roots $\mathcal{A}_i(\mathcal{A} \neq \mathcal{A}_i)$ Comparison of (3.1.8) and (3.1.1)

leads to the following relations:

$$(p, \alpha_i^{V}) = 1$$
, $p = \Sigma \lambda_j$ (3.1.9)
 $(p, \alpha) = \Sigma \left(\frac{\alpha_{i,j} \alpha_i}{2} n_{\alpha}^{i}\right)$ (3.1.10)

3.2. GAUSS DECOMPOSITION

Let O_j be a simple Lie algebra. This means, that $\dim O_j > 1$ and the adjoint representation of O_j is irreducible. Consider an element $X \in O_j$ with the minimal possible dimension of the zero-mode eigenspace of the corresponding adjoint operator, $\operatorname{cld}_X Y = [X, Y]$. A maximal commutative subalgebra $\mathfrak{h} \subset O_j$ which contains X is called <u>Cartan subal-</u> <u>gebra</u>. Because of commutativity it is possible to consider common eigenspaces $O_{j,k}$ for all $h \in \mathfrak{h}$. The corresponding eigenvalues are linear functionals on \mathfrak{h} ,

$$[h, g]_{\lambda} = \chi(h) c g_{\lambda}, \chi(h) = (\chi, h).$$
 (3.2.1)

The eigenspaces $\mathcal{O}_{\mathcal{A}}$ are called <u>root subspaces</u> of $\mathcal{O}_{\mathcal{J}}$. In accordance with (3.2.1) there is a decomposition

$$q = \frac{h}{2} + \sum_{n=1}^{\infty} q_{\lambda}. \qquad (3.2.2)$$

Dimension r of f_j is called the <u>rank</u> of f_j . (Complex) dimensions of all root subspaces c_{f_d} are equal to unity. ($c_{f_d} = a e_d$, a is a complex number, e_d is a step generator.)

Let G be a simple Lie group with the Lie algebra G . G acts by adjoint representation on G:

$$Ad_{g} : y = y = y = y = y$$
 (3.2.3)

By means of (3.2.3) one can "diagonalize" ${\cal G}$:

$$o_{j} = \mathcal{A}d_{g} f_{j} \qquad (3.2.4)$$

Thus f_j parametrizes the set of conjugancy classes of \mathcal{G} . Consider the subgroup M generated by transformations, preserving Cartan subalgebra f_j .

$$M = \{ g \in G \mid ghg' = h_1, h, h_1 \in H \}.$$
 (3.2.5)

In other words, M is normalizator of \mathcal{H} . It is worthwhile to note, that M is a subgroup of maximal compact subgroup Uof G. Let M' be a subgroup generated by transformations, commuting with \mathcal{H} (M' is centralizator of \mathcal{H}),

$$M' = \{g \in M \mid ghg' = h, h \in j\}$$
. (3.2.6)

The factor group M/M' is finite and acts on the set of root subspaces $\mathcal{O}_{\mathcal{A}}$ by permutations.

Relation between the previously considered theory of root systems and the theory of simple Lie algebras is based on the observation, that the set of linear functionals $\alpha(h) \in \mathcal{G}^{\vee}$ is a root system in \mathcal{H}^{\vee} with respect to the Weyl group, which is isomorphic to the factor-group M/M'.

Choosing an ordering in $\frac{1}{2}$ one can rewrite (3.2.2) as

$$o_{j} = h + \sum_{\alpha \in \Delta} o_{j\alpha} = h + o_{j}^{\dagger} + o_{j}^{\dagger},$$

$$o_{j}^{\pm} = \sum_{\alpha \in \Delta^{+}} o_{j\pm \alpha} .$$
(3.2.7)

This decomposition is called Gauss decomposition of the algebra, and it may be integrated for almost all elements g of the group G:

$$\begin{aligned} y &= g_{L} g_{D} g_{U} &= g_{L} e^{2xp} g^{T} = G_{L} g_{D} e^{exp} g^{T} &= G_{U} \\ g_{U} e^{exp} e^{T} = G_{U} . \end{aligned}$$

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Let $\mathcal{C}_{\mathcal{A}}$ be generators, corresponding to $\mathcal{G}_{\mathcal{A}}$. In terms of decomposition (3.2.2) the commutation relations take the following form:

$$[h, e_{d}] = \alpha(h) e_{d}$$

$$[e_{d}, e_{-d}] = h_{d} \qquad \alpha(h_{d}) = 2 \qquad (3.2.9)$$

$$[e_{d}, e_{\beta}] = \begin{cases} 0 & \alpha + \beta \in \Delta \\ e_{\alpha + \beta} & \alpha + \beta \in \Delta \end{cases}$$

If G = sl(n,C), then D is the subgroup of diagonal matrices, $G_L(G_u)$ is the subgroup of lower (upper) triangular matrices; M'=D, M/M' is generated by permutation matrices of the form $diag(1, 1, 01...101 \ 1) + (E_{ij} - E_{ji})$

(E_{ij} is a matrix with zero entries, except for the element (i,j), which is equal to 1.) If is subgroup of traceless diagonal matrices, $f = \{ diag(h_1, \dots, h_h), \Sigma h_j = 0 \}$ the root generators are $e_{d} = E_{ij}(i < j), e_{-d} = E_{ij}(i > j)$ the roots $h_d = diag(0, 10, 0 - 1, 0, 0)$ For other examples see next ss.3.3.

Note now, that by means of decomposition (3.2.7) one can construct the maximal compact subalgebra $\mathcal{U} \subset \mathcal{O}_{\mathcal{J}}$. Let us remind, that $\mathcal{J}_{\mathcal{J}}$ is an r-dimensional complex space $\mathcal{J} = \mathcal{J}_{\mathcal{I}}^{\mathcal{R}} \mathcal{L}_{\mathcal{J}}^{\mathcal{R}}$ and $\{\ell_{\mathcal{A}}\}$ are generators of $\mathcal{O}_{\mathcal{J}}$ ($\mathcal{O}_{\mathcal{A}} = \mathcal{O}_{\mathcal{C}}$). Then $\mathcal{L}_{\mathcal{J}}^{\mathcal{R}}$ is the Cartan subalgebra of $\mathcal{U}_{\mathcal{J}}$, and

$$i(e_{d}+e_{-d}), (e_{d}-e_{-d})$$

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are generators of \mathcal{W} . The particular case is compact subalgebra su(2) of $\mathcal{O}_{j} = sl(2,C)$, with $\int_{-\infty}^{\infty} c_{s_{d}} c_{s_{d}} c_{s_{d}} c_{s_{d}}^{-\infty}$. Let us introduce now an invariant non-degenerate scalar product on G . Invariance means, that

$$([z,x],y)+(x,[z,y])=0$$
 for any $z \in \mathcal{G}$.

Such product is known under the name of Cartan-Killing form. Its existence is equivalent to semisimplicity of the algebra. Explicit expression for this invariant form is

$$(x, y) = tr (ad_x \cdot ad_y)^{1}_{V} ad_x = [x, J].$$
 (3.2.10)

In terms of structure constants

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$$g_{ab} = C_{ac}^{d} C_{bd}^{c} \frac{1}{C_{V}}$$
(3.2.11)

which is obviously invariant and non-degenerate for any simple Lie algebra.

From (3.2.10) and (3.2.1) we deduce, that for any

$$(h_1, h_2) = \frac{1}{c_V} \sum_{\alpha \in \Delta_+} (\alpha, h_1) (\alpha, h_2).$$
 (3.2.12)

The universal coefficient C_V is in fact quadratic Casimir eigenvalue in the adjoint representation (it will be discussed later, in ss. 3.5).

Root subspaces $\mathscr{G}_{\mathcal{L}}$ are mutually orthogonal, except for

In fact eq.(3.2.12) describes the form (3.2.10) on \mathcal{G} in coordinates (3.2.7).

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3.3. SOME EXAMPLES OF GAUSS DECOMPOSITION

In the previous ss.3.2 we have already discussed the form of Gauss decomposition for the group sl(n). Here we generalize this construction to other classical groups. The key fact, allowing consideration of arbitrary simple groups is that there is a basis in adjoint representation of the Lie algebra \mathcal{O}_{i} in which the Cartan subalgebra has a diagonal form, and the subalgebras \mathcal{O}_{i}^{\pm}

may be realized in terms of the corresponding triangular matrices. Let us proceed to concrete examples.

3.3.1. $O_{f} = so(2n+1)$ (B_n):

Group elements satisfy the condition $g \int g^T = J$ where J is non-degenerate symmetric matrix. For an element of algebra we have: x J + J x = 0. In the above mentioned basis matrix J has the form of h

$$J = \begin{pmatrix} 0 & j \\ i \\ j & 0 \end{pmatrix} \qquad \text{where} \quad j = \begin{pmatrix} 0 & i \\ j & 0 \end{pmatrix}. \qquad (3.3.1)$$

Then XE SO(2h+1)11

$$\begin{aligned} x &= \begin{pmatrix} A & \widetilde{\xi} & B \\ 2 & 0 & -\widetilde{\xi} \\ C & -\widetilde{\ell} & -\widetilde{A} \end{pmatrix} & \begin{array}{c} \widehat{\mathcal{A}} &= j & \mathcal{A}^{\mathsf{T}} j \\ \widetilde{\mathcal{A}} &= j & \mathcal{A}^{\mathsf{T}} j \\ \widetilde{\mathcal{A}} &= j & \mathcal{A}^{\mathsf{T}} j \\ \widetilde{\mathcal{A}} &= j & \mathcal{A}^{\mathsf{T}} \end{pmatrix}, \begin{array}{c} \mathcal{B} &= -\widetilde{B} \\ \mathcal{B} &= -\widetilde{C} \\ \widetilde{\mathcal{B}} &=$$

Cartan element is

$$h = diag(h_1, \dots, h_n, 0, -h_n, \dots -h_1).$$
 (3.3.3)

(3.3.4)

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The root subspaces, corresponding to the roots $e_i - e_j$ $(4 \le i, j \le h, i \ne j)$ belong to the matrix \mathcal{A} (and $\widetilde{\mathcal{A}}$), those corresponding to $e_i + e_j$ - to \mathcal{B} , $C_i, e_j \sim \frac{1}{2}, \frac{1}{2}$ Generic element of subgroup G_L is $(\mathcal{A} = 0) = 0$ ($(\mathcal{C}A^{-1}) + \mathcal{A} = \frac{1}{2}\mathcal{D}\mathcal{D}\mathcal{A} + \mathcal{C}\mathcal{A} = 0$

$$\begin{pmatrix} \widetilde{Z} & i & 0 \\ C & -\widetilde{A} & \widetilde{Z} & \widetilde{A}^{-1} \end{pmatrix} \quad \widetilde{Z} = \widetilde{Z}^T J \quad \mathcal{A}_{ii} = 1$$

and $G_u = G_L^T$.

3.3.2. $O_{f} = sp(n)$ (C_{n}):

Defining relations are now

$$xJ + Jx = 0 \qquad J = \begin{pmatrix} 0 & J \\ -j & 0 \end{pmatrix}. \tag{3.3.5}$$

Thus

$$x = \begin{pmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & -\widetilde{\mathcal{A}} \end{pmatrix} \qquad \begin{array}{c} \mathcal{B} = \widetilde{\mathcal{B}} \\ \mathcal{C} = \widehat{\mathcal{C}} \\ h = diag (h_{1}, \dots h_{h_{1}}, -h_{h_{1}}, \dots -h_{1}) \end{array}$$
(3.3.6)

The root subspaces, corresponding to the roots $\mathcal{C}_i \pm \mathcal{C}_j$ $(i \neq j)$ are the same that those in the case of B_n . For the roots $2\mathcal{C}_j$ $(j=1,\ldots,n)$ the root subspaces are matrix elements (j,n-j) of matrices β and C. Subgroup \mathcal{G}_{ζ} consists of the matrices

$$\begin{pmatrix} A & O \\ C & \widehat{A}^{-1} \end{pmatrix} \qquad \begin{array}{c} \mathcal{A}_{ii} = 1 \\ \widetilde{\mathcal{A}} C = \widetilde{C} \mathcal{A} \end{array} \qquad (3.3.7)$$

and $G_{ij} = G_{L}^{T}$.

3.3.3 $\mathcal{T} = so(2n)$ (D_n):

The form of matrices is just the same as in the case of so(2n+1). It is necessary only to omit the central row and column.

3.4. CHARACTERS OF LIE ALGEBRAS

Irreducible finite-dimensional representations of a Lie algebra $\mathcal{O}_{\mathcal{J}}$ may be described in terms of the weight lattice Q^{\vee} . There is a correspondence between a vector $\lambda \in Q^{\vee}$ and an irreducible representation $\overline{\mathcal{I}}_{\lambda}$ in a space \mathcal{R}_{λ} . Representations $\overline{\mathcal{I}}_{\lambda}$ and $\overline{\mathcal{I}}_{\mathcal{U},\lambda}$ are isomorphic for any element u' of Weyl group W'. In other words λ , which is called the <u>highest weight</u> of representation $\overline{\mathcal{I}}_{\lambda}$ is defined by λ up to conjugations from the Weyl group. Representations $\overline{\mathcal{I}}_{\lambda_{j}}$. with λ_{j} given by eq.(3.1.6) are called fundamental.

Representation $\overline{\mathscr{A}_{\lambda}}$ is completely characterized by the following formulae:

$$\overline{J}_{\lambda}(h) = \lambda(h) = \overline{\xi} \in R_{\lambda} \quad n \in \underline{J}$$

$$\overline{J}_{\lambda}(e_{\lambda}) = 0 \quad \alpha \in \Delta_{+} \quad (3.4.1)$$

The finite-dimensional space \mathcal{R}_{λ} is spanned by the vectors:

$$\pi_{\lambda}(h) \eta = \mu(h) \eta$$
 (3.4.3)

where eigenvalues $\mu(h)$ are given by the following expression:

$$\mu(h) = \lambda(h) - \sum_{i=1}^{\infty} \mu_i \chi_i(h) \qquad (3.4.4)$$

which is in accordance with (3.4.1).

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Evidently, $\mathcal{M} \in \mathcal{Q}^{V}$ and the set of weights $\mathcal{P}_{\lambda} = \{\mathcal{M}\}$ (the weight diagramm) is invariant under the action of Weyl group.

Let us define now the characters of representations of \mathcal{G} . The character of representation \mathcal{F}_{λ} is a complex function, defined by

$$ch_{\lambda}(x) = t z \bar{\pi}_{\lambda}(x), \ x \in \mathcal{G}.$$
 (3.4.5)

In view of (3.2.4) we can restrict X under the trace sign to elements of Cartan subalgebra, $h = g \times g^{-1}$. Then due to (3.4.4) we can rewrite (3.4.5) as follows:

$$ch_{\lambda}(h) = \sum_{\mu \in P_{\lambda}} \dim V_{\mu} e^{\mu(h)}$$
(3.4.6)

where clim μ stands for multiplicity of μ .

It follows from (3.4.2), that $\mathcal{O}_{\lambda}(h)$ should naively look like

$$e^{\lambda} \prod (1 + e^{-d} + e^{-2d}) = \frac{e^{-2\pi}}{\prod (e^{-d/2} - e^{-d/2})} (3.4.7)$$

This formula is not correct, since it contains the infinite seria of descendants. In fact $\mathcal{A}_{1,}(h)$ is invariant under the action of Weyl group. This property is a consequence of its invariance with respect to the action of $\mathcal{A}_{1,2}$. Since the denominator in (3.4.7) is Weyl-antiinvariant,

$$\prod_{\substack{d \in \Delta^{t} \\ \text{is necessary to take antiinvariant combination in the nume-}} \sum_{s \in \Delta} dets \overset{sp}{e}$$

rator of (3.4.7) as well. After this correction we immediately obtain a finite combination of exponentials $e^{\mathcal{M}(h)}$. This is the celebrated Weyl formula for characters.

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$$ch_{s}(h) = \frac{\sum_{s \in W} dets \ e^{S_{s}(h) - S_{s}(h)}}{\sum_{s \in W} dets \ e^{S_{s}(h)}}.$$
 (3.4.8)

As follows from (3.4.5), in the limit $\not \to O$ (3.4.8) turns into a formula for dimension of representation

$$\dim \overline{\mathcal{I}}_{\lambda} = \frac{17}{\alpha \in \Delta +} \frac{(\lambda + p, \lambda)}{(p, \alpha)}.$$
(3.4.9)

Consider representation $\overline{\mathcal{J}}_{\lambda}$ of \mathcal{O}_{j} . There are some polinomial combinations of generators of \mathcal{O}_{j} which commute with all the generators in representation space. They are reffered to as <u>Casimir operators</u>. In accordance with Schur lemma they become c-numbers within any irreducible representation. The algebra of Casimir operators is finite generated. The orders of Casimir operators in original generators of \mathcal{O}_{j} are called <u>invarients</u> of the Lie algebra \mathcal{O}_{j} . The lowest order is equal to two, while the highest one - to the Coxeter number h. Let $\mathcal{C}_{\lambda}^{\mathcal{A}}$ be generators of the algebra \mathcal{O}_{j} in $\overline{\mathcal{A}}_{\lambda}$. Then the second-order Casimir operator has the form of

$$c_2^{\ \lambda} = g_{ab} e_{\ \lambda}^{a} e_{\ \lambda}^{b} \qquad (3.5.1)$$

where g_{ab} is the Cartan-Killing form (3.2.10). It is easy to check, that $[C_{\lambda}^{\lambda}, e_{\lambda}^{q}] : O$. Omitting the sign of representation, we may rewrite (3.5.1) in terms of orthonormal basis (h_{1}, \dots, h_{q}) in h_{j} and step generators e_{λ} (see (3.2.9)): $C_{\lambda}^{\lambda} = \sum_{j=1}^{2} h_{j}^{2} + \sum_{\lambda \in \Delta^{+}} e_{\lambda} e_{-\lambda} + e_{-\lambda} e_{\lambda}$. (3.5.2)

Using relations (3.2.9), one gets the expression:

$$c_{2}^{\lambda} = \sum h_{j}^{2} + \sum (e_{-\lambda} e_{\lambda} + h_{\lambda}).$$
 (3.5.3)

Thus, according to (3.4.4) the eigenvalue of C_2^{λ} is equal to

$$c_{\lambda}^{\lambda} \rightarrow (\lambda, \lambda + 2\rho)$$
 (3.5.4)

Note, that the highest weight of the adjoint representation coincides with the maximal root θ . Therefore for this representation

$$c_v = (\theta, \theta + 2\rho) \frac{1}{2}$$
(3.5.5)

Let g be a dual Coxeter number,

$$g = 1 + \sum_{j=1}^{2} M_{j}$$
 (3.5.6)

where integers m_i are defined from decomposition

$$\frac{\theta}{(\theta,\theta)} = \sum_{j=1}^{2} m_j \frac{d_j}{(d_j,d_j)} \qquad : \qquad (3.5.7)$$

$$m_j = \frac{2(\theta, \lambda_j)}{(\theta, \theta)}$$
(3.5.8)

Now from (3.5.5) we obtain the important identity:

$$C_{v} = (\underline{\theta}, \underline{\theta}) (1 + \frac{2(\theta, p)}{(\theta, \theta)}) = 1 + \sum_{j=1}^{2} M_{j} = g. \qquad (3.5.9)$$

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3.6. KIRILLOV-KOSTANT CONSTRUCTION [3]

3.6.1. The structure of Lie algebra of gives rise to symplectic structures on some representations R of of. In order to have Poisson brackets on functions on R, which satisfy Jacobi identities, one needs at least an invariant entisymmetric tensor of the third rank. Such tensor obviously exists in the case of adjoint or coadjoint representation and consists of structure constants o_{κ}^{ij} of the algebra \mathcal{J} . Whenever Cartan-Killing form on \mathcal{J} is non-degenerate (i.e.

of is semisimple) adjoint and coadjoint representations are equivalent. It is no longer the case for non-semisimple Lie algebras.For example, when there U(1)-factors, the action of group in coadjoint representation is preferable, since U(1)-generators do not act in adjoint representation at all. In what follows we shall take \mathcal{R} to be coadjoint representation and consider semisimple Lie algebras \mathcal{O}_{J} .

Let us denote generators of the algebra O_j by e^j $j=1,\ldots,D=dimG$, and coordinates in coadjoint representation by X^{K} . Let us consider the set of functions on the dual space O_j^{K} . Then there is a Lie-Berezin bracket [4].

$$\{f(x), \varphi(x)\} = C_{\mu}^{ij} x^{\mu} \partial_{i} f \partial_{j} \varphi \qquad (3.6.1)$$

which satisfies Jacobi identity, since it is satisfied by the tensor of structure constants C_{K}^{ij} . (Note, that eq. (3.6.1) may be rewritten in terms of vector fields $V_{f}^{j} = C_{K}^{ji} \chi^{K} \partial_{ji} f$: $[V_{f}, V_{ij}] = V_{ff, ij} f$ + possible cocycles (3.6.2)

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where $[V_f, V_{\mathcal{G}}]$ stands for ordinary commutator of differential operators.) However, the matrix $\omega^{J}(x) = C_{\kappa}^{J} x^{\kappa}$ in (3.6.1) is degenerate at any X, since there are invariant functions $\int_{\Omega} (X)$ such that

$$\{x^{k}, f_{a}(x)\} = 0.$$
 (3.6.3)

Independent functions of this kind are labeled = rank Oj by the subscript a , running through r integer values, which lie between 2 and Coxeter number h, and are related to Casimir operators. If a Casimir operator looks like

$$C_a = g_{i_1 \dots i_a} e^{i_1 \dots e^{i_a}} \quad \text{then corresponding}$$

$$f_a(X) = g_{i_1} \quad i_a X^{i_1} \dots X^{i_a}. \quad (3.6.4)$$

Other invariant functions, which satisfy (3.6.3) are arbitrary functions of these independent f_{e} .

If the values of all independent functions are fixed,

$$f_a(\mathbf{X}) = \mathcal{J}_a \tag{3.6.5}$$

the whole space of functions becomes restricted, and in this restricted space the Lie-Berezin form $\omega^{1^{\kappa}}$ from (3.6.1) is non-degenerate. Since functions $f_{\alpha}(x)$ are invariant, conditions (3.6.5) define orbits of coadjoint representation of G. The inverse of non-degenerate restriction of Lie-Berezin form on a coadjoint orbit, $\omega_{1^{\kappa}\kappa}$, is known as Kirillov-Kostant form.

Let us consider two infinitesimal variations of point X within the same orbit, $\delta_1 X^{e}$, $\delta_2 X^{b}$. Since G acts transitively on the orbit, these may be represented as

$$\delta X = ad_Y^* \cdot X \,. \tag{3.6.6}$$

Kirillov-Kostent form allows one to construct two invariant functions on the orbit:

$$\hat{\Omega} = \omega_{j\kappa}(x) \, \delta_1 \, \chi^J \, \delta_2 \, \chi^{\kappa} \qquad (3.6.7)$$

and

$$\Omega = \omega_{jk}(X) Y^{j} Y^{k} = C_{ijk} X^{i} Y^{j} Y^{k} \qquad (3.6.8)$$

We shall reffer to the second one, Λ , as Kirillov-Kostant form, since in ss.4.6 we shall demonstrate, that Λ (but not $\hat{\Lambda}$) appears related to the WZW action.

3.6.2. We shall often consider generators of algebra \mathcal{O}_j and elements of the dual space \mathcal{O}_j^* as elements of matrices (see the end of ss.3.2). For example, in the case of $\mathcal{O}_j = sl(n)$ these are nxn traceless matrices. Relation between \mathcal{O}_j and \mathcal{O}_j^* is dictated by the pairing

$$\langle x, y \rangle = t_2(xy).$$
 (3.6.9)

Adjoint action of the group G an Of is defined by

$$Ad_y y = g y g^{-1}$$
 (3.6.10)

and it is convenient to define coadjoint action by

$$Adg^* x = g^{-1} x g.$$
 (3.6.11)

Invariant Casimir operators look like

$$T_{z} x^{k}$$
. (3.6.12)

and the second second

5-1

The orbit of coadjoint representation, $Tr(X)^k = \mathcal{M}_k$, may be alternatively defined by pointing out one of its points, X_0 . Then any other point on the orbit is

$$X = g^{-1} X_{c} g$$
 (3.6.13)

for some
$$g \in G$$
. The 1-form Y in (3.6.6) is given by
 $\delta X = \delta g^{-1} X_0 g + g^{-1} X_0 \delta g = \alpha c l_g^{-1} j_g X$, i.e.
 $Y = g^{-1} \delta g$
(3.6.14)

Kirillov-Kostant 2-form 12 in (3.6.8) is equal to

$$\Omega = \langle x, [Y, Y] \rangle = \langle x, [g' \delta g g' \delta g] \rangle = (3.6.15) = \langle x_0 [\delta g g' , \delta g g' \rangle$$

Generic orbit of coadjoint representation may be naturally parametrized with the help of Gauss decomposition

$$g = g_L g_D g_U = g_L g'_U g_D$$
 $g_L = g_L(Y), g_L = g_L$

$$\Omega = T_{2}(x_{c} g_{L}^{-1} \delta g_{L}(p) \delta g_{\mu}^{-1}(\tilde{\psi}) g_{\mu}^{-1}(\tilde{\psi}) J^{-1}). \qquad (3.6.17)$$

(For particular choices of X_0 this form is still degenerate, and the orbit has lower dimension and is parametrized by some subset of \mathcal{J} 's and $\widetilde{\mathcal{V}}$'s.)

D-r). Kirillov-Kostant form in this parametrization is

Since two-form Ω is closed, it is possible to define a 1-form \mathcal{A} , $\mathcal{A} = \mathcal{A}^{-1}\Omega$, and \mathcal{A} gives rise to the action

$$\mathcal{A} = \int d \qquad (3.6.18)$$

which is generalization of the short action

arising in the case of Heisenberg group. The form of Kirillov--Kostant form, arising when Gauss decomposition is used, in fact appears very close to representation (3.6.19) after appropriate choice of variables. Before we proceed to detailed discussion of this point in the next section, let us note, that Gauss decomposition is not valid at some manifolds of non-vanishing co-dimension on the orbit. Therefore appropriate boundary conditions should be specified at these points. See the first paper of ref.3 for detailed discussion of bounda--dimensional ry conditions in the case of finite/groups G. In the case of infinite chiral algebras the problem of these boundary conditions is closely related to accurate construction of Felder's projection operators [5].

In 88.4.6 we shall briefly discuss generalization of Kirillov-Kostant construction to the case of KM algebra. For this purpose group elements g should be considered as functions of z, and all formulae should be accurately central extended.

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Fig.3

a) The roots of algebra
$$sl(3) \cong A_2$$

 $d_1 = e_1 - e_2$; $d_2 = e_1 - e_3$; $d_3 = e_2 - e_3$; $d_1, d_3 \in \Pi$; $g = d_2 = e_1 - e_3$;
 $\lambda_1 = e_1$, $\lambda_2 = -e_3$; $h_{1,2} = h_{1,2} = h_{1,2} = 1$, $h_{1,3} = 2$

Correspondense between the fields \widetilde{W}_{j} and positive root subspaces: $\widetilde{W}_{j} \rightarrow \Im_{d,j}$



b) The roots of algebra $sp(2) \cong C_2$ $d_1 = e_1 - e_2$; $d_2 = 2e_1$, $d_3 = e_1 + e_2$, $d_4 = 2e_2$; d_1 , $d_4 \in \Pi$; $P = 2e_1 + e_2$; $\lambda_1 = e_1$; $\lambda_2 = e_1 + e_2$; $h_{tal_1} = h_{tal_2} = 1$, $h_{tal_3} = 2$, $h_{tal_2} = 3$ $h_{tal_3} = 9d_3$; 

c) The roots of algebra G₂.

 $d_{1} = -2e_{3} + e_{1} + e_{2}d_{2} = e_{1} - e_{3}d_{3} = 2e_{1} - e_{2}d_{4} - e_{1} - e_{2}$ $d_{5} = -2e_{2} + e_{1} + e_{3}d_{6} = e_{3} - e_{2}d_{1}d_{6} \in \Pi_{2}g = Sd_{6} + Sd_{1}$ $\lambda_{1} = d_{3}, \quad \lambda_{2} = d_{4}$

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М., Препринт ИГЭФ, 1989, № 70, с.1-20