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ON THE VALUE OF GLUONIC CONDENSATE IN QUANTUM CHROMODYNAMICS

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The value of gluonic condensate $\langle\frac{\alpha_s}{\pi}G^2\rangle$ is calculated using the QCD model with infinite number of vector mesons. For polarization operator connected with the vector current of c-quarks $\Pi^c(Q^2)$ within the interval $0<Q^2<\infty$ the Wilson operator expansion is used. In the operator expansion we remain only the unit operator and the operator connected with gluonic condensate. The requirement for $\Pi^c(Q^2)$ in the interval at hand to differ from the operator expansion vacuum expectation value not more than by 20% involves the restrictions to the value of gluonic condensate $0.04\ \text{GeV}^4 \leqslant \langle\frac{\alpha_s}{\pi}G^2\rangle \leqslant 0.12\ \text{GeV}^4$. Analogously, for the Υ-meson family it is supposed that the operator expansion is valid for the polarization operator Π^b in the interval $-4m_b^2 + 10\ \text{GeV}^4 < Q^2 < \infty$. In the operator expansion we remain only the unit operator and the operator connected with gluonic condensate. The restrictions to the value of gluonic condensate $0.05\ \text{GeV}^4 \leqslant \langle\frac{\alpha_s}{\pi}G^2\rangle \leqslant 0.1\ \text{GeV}^4$ follow from the requirement for $\Pi^b(Q^2)$ in this interval of Q^2 to differ from the operator expansion vacuum expectation value not more than by 20%. The electronic widths of the ground and excited states of ρ, ψ, Υ meson families are calculated.

Fig. -, ref. - 12

I. Introduction

The gluonic condensate $\langle \frac{\alpha_s}{\pi} G^2 \rangle$ is one of the basic feature of QCD vacuum. In their classical work Shifman, Vainshtein and Zakharov[1] have shown that the value of gluonic condensate is nonzero and have obtained

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle_{SVZ} \approx 0.012 \ \mathrm{GeV}^4 \tag{1}$$

When obtaining the value (1) the momentum method was used. Since then in a great number of papers[2-7] there were obtained various values of gluonic condensate significantly differing from the value (1) of ref.[1].

In ref.[8] the QCD model with infinite number of vector mesons was constructed. This model describes well experimental data on electronic widths of the known resonances of ρ, ψ, Υ-meson families. Consider, for example, the Υ -meson family. 6 vector resonances were observed experimentally, and then the curve R(s) becomes smooth. As was shown in ref.[8], if the vector meson widths obey the conditions

$$M_k^2 - M_{k-1}^2 \ll M_k \Gamma_k \ll M_k^2 \tag{2}$$

starting from k=5 (M_k - is the k-th resonance mass , Γ_k its total width), then after the 6-th resonance the function R(s) will be described by the smooth curve and all the formulae of the model at hand are applicable. The basis for this paper is the Wilson operator expansion. From requirement that in the validity region of the operator expansion the polarization operator $\Pi(Q^2)$

must be close to formulae obtained from the operator expansion we find the value of gluonic condensate. The region where it is required that $\Pi^c(Q^2)$ must be close to the operator expansion formulae for the ψ — meson family is the following: $0 < Q^2 < \infty$. This Q^2 region is $\sim 10\ \mathrm{GeV}^2$ distant from the cut $\Pi^c(Q^2)$. For the Υ-meson family we shall also require that Q^2 for which the operator expansion is applicable be more than 10 GeV^2 distant from the cut $\Pi^b(Q^2)$. This means that for the Υ-meson family we will require that the operator expansion in the region $-70\ \mathrm{GeV}^2 < Q^2 < \infty$, be close to formulae $\Pi^b(Q^2)$. For the value of gluonic condensate obtained from the family of ψ and Υ mesons we obtained similar values of gluonic condensate significantly larger than those of ref.[1]. Intial formula owing to which we have obtained all results of this paper is the formula which connects the electronic width of the resonance under consideration with its mass derivative $\frac{dM_K}{dK}$. For the ψ meson family

$$\Gamma_K^{ee} = \frac{2\alpha^2}{9\pi} R_c^{(p.T.)}(s_K) M_K^{(1)} \tag{3}$$

In eq.(3) and in the next formulae we use the notations

$$M_K^{(\ell)} \equiv \frac{d^\ell M_K}{dK^\ell}, \quad s_K = M_K^2, \quad s_K^{(\ell)} \equiv \frac{d^\ell s_K}{dK^\ell} \tag{4}$$

It can be shown[8] that function s_K considered as a function of variable K given at integer nonnegative K can be continued to analytical function $s(K)$ with the cut along negative χ taking the values s_k at the points k=0,1,... In this sense the derivatives (4) are defined. We have include into function $R_c^{(P.T.)}$ all the corrections to α_s in perturbation theory. In the zero order in α_s the function $R_c^{(0)}(s)$ is defined by the well known formula

$$R_c^{(0)}(s)=\frac{3}{2}Q_c^2\cdot v\cdot(3-v^2) \tag{5}$$

where $v=\sqrt{1-4m_c^2/s}$, m_c is the c-quark mass $Q_c=2/3$. Eq.(3) was derived in ref.[9] from requirement of asymptotic freedom only for large k . We shall apply it for all k. This is based on the fact that analogous formula in nonrelativistic quantum mechanics

$$|\psi_k(0)|^2=\frac{m^{3/2}}{\sqrt{2}\,\pi^2}\,E_k^{1/2}\,\frac{dE_k}{dk} \tag{6}$$

has for the considered in[9] potentials the accuracy of $\lesssim 2\%$ for the ground state and of a parts of per-cent for excited states. Eq.(3) can be considered as an ansatz which makes it possible to describe the polarization operator $\Pi^c(Q^2)$ via the masses of the known resonances so that in the region of the operator expansion applicability $\Pi^c(Q^2)$ will be slightly differ from the operator expansion. Besides, using eq.(3) one may calculate the electronic widths of the resonances under consideration which agree with experiment. The fact that it is possible to do allows one to state that eq.(3) is applicable for small k too.

2. Finding of Gluonic Condensate Value from the ψ-Meson Family.

Consider the polarization operator connected with the vector current of charmed quarks

$$i\langle 0|\int d^4x\, e^{iqx}\, T\{j_\mu^c(x) j_\nu^c(0)\}|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^c(q^2) \quad (7)$$

where

$$j_\mu^c(x) = \bar{c}\gamma_\mu c$$

The dispersion relation for $\Pi^c(Q^2)$, $Q^2 = -q^2$ is written as

$$\Pi^c(Q^2) = \frac{1}{12\pi^2 Q_c^2}\int_{M_0^2}^{\infty}\frac{R_c(s)\,ds}{s+Q^2} \tag{8}$$

The dispersion relation (8) is written without subtractions since, as will be shown below, the divergent part in $\Pi^c(Q^2)$ cancels.

In approximation of infinite number of narrow resonances with the masses M_K and electronic widths Γ_K the function $R_c(s)$ has the form

$$R_c(s) = \frac{9\pi}{\alpha^2}\sum_{K=0}^{\infty}\Gamma_K^{ee} M_K\,\delta(s - M_K^2) \tag{9}$$

$\alpha = 1/137$. With the help of eqs.(3),(9) rewrite eq.(8) in the form

$$\Pi^c(Q^2)=\frac{1}{12\pi^2 Q_c^2}\left\{\frac{R_c^{(P.T.)}(s_0)s_c^{(1)}}{s_c+Q^2}+\sum_{\kappa=1}^{\infty}\frac{R_c^{(P.T.)}(s_\kappa)s_\kappa^{(1)}}{s_\kappa+Q^2}\right\} \tag{10}$$

Transform the sum in eq.(10) into integral exploiting the Euler-McLoran formula. We get

$$\Pi^c(Q^2)=\Pi_I^c(Q^2)+\Delta\Pi^c(Q^2) \tag{11}$$

where

$$\Pi_I(Q^2)=\frac{1}{12\pi^2 Q_c^2}\int_{4m_c^2}^{\infty}\frac{R_c^{(P.T.)}(s)\,ds}{s+Q^2} \tag{12}$$

and

$$\Delta\Pi^c(Q^2)=\frac{1}{12\pi^2 Q_c^2}\left\{\frac{R_c^{(P.T.)}(s_0)s_0^{(1)}}{s_0+Q^2}-\right.$$
$$-\int_{4m_c^2}^{s_1}\frac{R_c^{(P.T.)}(s)\,ds}{s+Q^2}+\frac{1}{2}W_{P.T.}(Q^2)-\frac{1}{12}W_{P.T.}(Q^2)+ \tag{13}$$
$$\left.+\frac{1}{720}W_{P.T.}^{(3)}(Q^2)-\frac{1}{30240}W_{P.T.}^{(5)}(Q^2)+\dots\right\}$$

We introduce here the notations

$$W_{P.T.}(Q^2)=\frac{R_c^{(P.T.)}(s_1)s_1^{(1)}}{s_1+Q^2}$$

$$W_{P.T.}(Q^2)=\frac{d^{\ell}}{d\kappa^{\ell}}\left(\frac{R_c^{(P.T.)}(s_\kappa)s_\kappa^{(1)}}{s_\kappa+Q^2}\right)\Bigg|_{\kappa=1}$$

Replacing of summation by integration was made starting from $\kappa=1$ but not from k=0. This is due to that the $s_0^{(\ell)}$

derivatives fastly grow with the number ℓ and the Euler-McLoren formula appears to be ineffective. The point s_1 is farther from the cut and for this reason there is no such a grow with ℓ increase (see[8]).

The operator expansion for T-product of vector currents composed from c-quarks has the form[1]

$$i\int d^4x\, e^{iqx}\, T\{j_\mu^c(x) j_\nu^c(0)\} = (q_\mu q_\nu - q^2 g_{\mu\nu}) \cdot \left(C_I \cdot I + C_G\, O_G + \dots \right) \qquad (14)$$

where I is the unit operator

$$O_G = G_{\mu\nu}^a G_{\mu\nu}^a, \qquad C_I = \Pi_I^c(Q^2),$$
$$C_G = \frac{\alpha_s}{\pi}\rho(Q^2), \quad \rho(Q^2) = \frac{1}{48Q^4}\left\{ \frac{3(a+1)(a-1)^2}{a} \cdot \frac{1}{2\sqrt{a}} \ln\frac{\sqrt{a}+1}{\sqrt{a}-1} - 3 + 2/a - 3/a^2 \right\}, \qquad a = 4m_c^2/Q^2 + 1 \qquad (15)$$

Let us take the vacuum expectation value from the operator (15) and compare it with eq.(11). The term Π_I cancels and we get the basic formula of this paper:

$$\Delta\Pi^c(Q^2) = \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \cdot \rho(Q^2) \qquad (16)$$

In the r.h.side of eq.(16) the α_s corrections are disregarded, consequently, in the l.h.side too these corrections may be disregarded, i.e. one may replace $R_c^{(P.T.)}(s) \to R_c^{(0)}(s) = \frac{3}{2} Q_c^2 v(3-v^2)$. The value $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$ is a renormalization value. The l.h.side of eq.(16) is replaced as

$$\Delta \Pi^c(Q^2) = \frac{1}{8\pi^2}\left\{\frac{u_c}{s_c+Q^2} - 2\cdot\frac{4m_c^2}{Q^2}F + \frac{1}{2}W_1 - \frac{1}{12}W_1^{(1)} + \right.$$

$$\left. + \frac{1}{720}W_1^{(3)} - \frac{1}{30240}W_1^{(5)} + \cdots\right\} \qquad (17)$$

In eq.(17) we use the notations

$$u_o = v_o(3-v_c^2)s_c^{(1)}, \quad u_1 = v_1(3-v_1^2)s_1^{(1)}, \quad W_1 = \frac{u_1}{s_1+Q^2},$$

$$F = \frac{Q^2}{4m_c^2}\int_{4m_c^2}^{s_1}\frac{v(3-v^2)ds}{s+Q^2} = \frac{1}{a-1}\left[\ln\frac{1+v_1}{1-v_1} - \frac{(3-a)a}{2}\cdot\right. \qquad (18)$$

$$\left.\cdot\ln\frac{\sqrt{a}+v_1}{\sqrt{a}-v_1}\right] - v_1$$

The indices over W_1 is explained by the formula (13). At $Q^2\to\infty$ the r.h.s. of eq.(16) takes the form

$$\langle\frac{\alpha_s}{\pi}G^2\rangle P(Q^2) \to -\frac{1}{12}\langle\frac{\alpha_s}{\pi}G^2\rangle\frac{1}{Q^2} \qquad (19)$$

Consequently, the terms $1/Q^2$ in the l.h.side of eq.(16) must cancel and we obtain the equation

$$u_c - 2s_1v_1^3 + \frac{1}{2}u_1 - \frac{1}{12}u_1^{(1)} + \frac{1}{720}u_1^{(3)} - \frac{1}{30240}u_1^{(5)} + \cdots = 0 \qquad (20)$$

Equalling the terms $1/Q^4$ at $Q^2\to\infty$ in both sides of eq.(16)

$$\langle\frac{\alpha_s}{\pi}G^2\rangle = \frac{3}{2\pi^2}\left\{u_c s_c - 2\cdot(4m_c^2)^2F_4 + \frac{1}{2}s_1u_1 - \right.$$

$$\left. -\frac{1}{12}(s_1u_1)^{(1)} + \frac{1}{720}(s_1u_1)^{(3)} - \frac{1}{30240}(s_1u_1)^{(5)} + \cdots\right\} \qquad (21)$$

where

$$F_4 = \frac{1}{2\cdot(4m_c^2)^2}\int_{4m_c^2}^{s_1} s\, v(3-v^2)\,ds = \frac{v_1(3-v_1^2)}{4(1-v_1^2)^2} - \frac{3}{8}\ln\frac{1+v_1}{1-v_1} \tag{22}$$

In all the above formulae the nonwritten terms are omitted because of smallness of coefficients at these terms. The parameters m_c, $s_1^{(1)}$, $s_1^{(2)}$, $s_1^{(3)}$, $s_1^{(4)}$, $s_1^{(5)}$, $s_1^{(6)}$ were varied in a way that the l.h.side and r.h.side of eq.(16) were maximally close to each other. The value $s_0^{(1)}$ is found from eq.(20).

It should be taken into account that the masses of neighbouring mesons $\mathcal{M}_0$ (3.096) and $\tilde{\mathcal{M}}_2$ (4.03) are expressed via varied parameters. We consider here only the mesons which are in s-state

$$\begin{aligned} s_0 &= s_1 - s_1^{(1)} + \frac{1}{2}s_1^{(2)} - \frac{1}{6}s_1^{(3)} + \frac{1}{24}s_1^{(4)} - \frac{1}{120}s_1^{(5)} + \frac{1}{720}s_1^{(6)} \\ \mathcal{M}_0 &= \sqrt{s_0} \\ \tilde{s}_2 &= s_1 + s_1^{(1)} + \frac{1}{2}s_1^{(2)} + \frac{1}{6}s_1^{(3)} + \frac{1}{24}s_1^{(4)} + \frac{1}{120}s_1^{(5)} + \frac{1}{720}s_1^{(6)} \\ \tilde{\mathcal{M}}_2 &= \sqrt{\tilde{s}_2} \end{aligned} \tag{23}$$

Varying the parameters we required that the values $\mathcal{M}_0$ and $\tilde{\mathcal{M}}_2$ slightly differ from experimental mass values. The error in the calculation of s_0 and $\tilde{s}_2$ is of order of the largest omitted term, i.e. $\frac{1}{720\cdot 7}s_1^{(7)}$.

For $s_1^{(7)}$ the estimate was $s_1^{(7)} \sim 10\ \mathrm{GeV}^2$.
The value $s_0^{(1)}$ may be calculated by two methods: using eq.(20) and using the formula

$$s_0^{(1)} = s_1^{(1)} - s_1^{(2)} + \frac{1}{2}s_1^{(3)} - \frac{1}{6}s_1^{(4)} + \frac{1}{24}s_1^{(5)} - \frac{1}{120}s_1^{(6)} \tag{24}$$

This term cancels with the term at the unit operator in the operator expansion. Repeating arguments of the preceding Section we obtain instead of eq.(16)

$$\Delta \Pi^{b}(Q^2) = \langle \frac{\alpha_s}{\pi} G^2 \rangle P(Q^2) \tag{30}$$

The l.h.side of eq.(30) at $Q^2 > 0$ may be written as

$$\Delta \Pi^{b}(Q^2) = \frac{1}{8\pi^2} \left\{ \frac{u_0}{s_0 + Q^2} + \frac{u_1}{s_1 + Q^2} - 2 \cdot \frac{4m_b^2}{Q^2} F + \right.$$

$$\left. + \frac{1}{2} W_2 - \frac{1}{12} W_2^{(1)} + \frac{1}{720} W_2^{(3)} - \frac{1}{30240} W_2^{(5)} \right\} \tag{31}$$

In eq.(31) we use the notations

$$u_k = v_k (3 - v_k^2) s_k^{(1)}, \quad k = 0, 1, 2$$

$$W_2 = u_2 / (s_2 + Q^2) \tag{32}$$

$$F = \frac{Q^2}{8m_b^2} \int_{4m_b^2}^{s_2} \frac{v(3 - v^2)\, ds}{s + Q^2} = \frac{1}{a-1} \left[\ln \frac{1 + v_2}{1 - v_2} - \frac{(3-a)\sqrt{a}}{2} \ln \frac{\sqrt{a} + v_2}{\sqrt{a} - v_2} - v_2 \right]$$

$$a = 1 + \frac{4m_b^2}{Q^2} \tag{33}$$

At $-4m_b^2 < Q^2 < 0$ for the function F there is the formula

$$F = \frac{1}{a-1} \left[\ln \frac{1 + v_2}{1 - v_2} - \frac{(3-a)\sqrt{a}\,\sqrt{4m_b^2 + Q^2}}{\sqrt{-Q^2}} \,\mathrm{arctg}\, \frac{v_2 \sqrt{-Q^2}}{\sqrt{4m_b^2 + Q^2}} \right] \tag{34}$$

Eq.(34) is obtained from eq.(33) by analytical continuation. Formula for $P(Q^2)$ at $Q^2 > 0$ is obtained from eq.(15) by replacing $m_c \to m_b$. At $-4m_b^2 < Q^2 < 0$ the function $P(Q^2)$

is obtained by analytical continuation and can be written as

$$P(Q^2)=\frac{1}{48Q^4}\left\{\frac{3(a+1)(a-1)^2}{a}R-3+\frac{2}{a}-\frac{3}{a^2}\right\}$$

$$R=-\sqrt{\frac{-Q^2}{4m_b^2+Q^2}}\ \mathrm{arctg}\sqrt{\frac{-Q^2}{4m_b^2+Q^2}} \qquad (35)$$

Eight parameters m_b, $s_1^{(1)}$, $s_2^{(1)}$, $s_2^{(2)}$, $s_2^{(3)}$, $s_2^{(4)}$, $s_2^{(5)}$, $s_2^{(6)}$ were varied so as to achieve the most closeness of the l.h. and r.h. sides of eq.(30) in the interval $-4m_b^2+10\ \mathrm{GeV}^2<Q^2<\infty$. This interval is chosen such that just as for the ψ-meson family it would be distant from the cut of the function $\Pi^b(Q^2)$ by $\geqslant 10\ \mathrm{GeV}^2$.

In this interval there is analytical freedom and operator expansion. When varying it was required that the following 7 equalities would be fulfilled. The value

$M_0=\sqrt{s_0}$ where

$$s_0=s_2-2s_2^{(1)}+2s_2^{(2)}-\frac{4}{3}s_2^{(3)}+\frac{2}{3}s_2^{(4)}-\frac{4}{15}s_2^{(5)}+\frac{4}{45}s_2^{(6)} \qquad (36)$$

would be close to $M_{0\,exp}$ = 9.46 GeV;

The value

$M_1=\sqrt{s_1}$, where

$$s_1=s_2^{(1)}-s_2^{(1)}+\frac{1}{2}s_2^{(2)}-\frac{1}{6}s_2^{(3)}+\frac{1}{24}s_2^{(4)}-\frac{1}{120}s_2^{(5)}+\frac{1}{720}s_2^{(6)} \qquad (37)$$

would be close to $M_{1\,exp}$ = 10.0234 GeV;

The value $M_3=\sqrt{s_3}$, where

$$s_3 = s_2 + s_2^{(1)} + \frac{1}{2} s_2^{(2)} + \frac{1}{6} s_2^{(3)} + \frac{1}{24} s_2^{(4)} + \frac{1}{120} s_2^{(5)} + \frac{1}{720} s_2^{(6)} \quad (38)$$

would be close to M_{3exp} = (10.577±0.004) GeV;
The value $M_4 = \sqrt{s_4}$, where

$$s_4 = s_2 + 2 s_2^{(1)} + 2 s_2^{(2)} + \frac{4}{3} s_2^{(3)} + \frac{2}{3} s_2^{(4)} + \frac{4}{15} s_2^{(5)} + \frac{4}{45} s_2^{(6)} \quad (39)$$

would be close to M_{4exp} = 10.856 GeV;
and the value $M_5 = \sqrt{s_5}$ where

$$s_5 = s_2 + 3 s_2^{(1)} + \frac{9}{2} s_2^{(2)} + \frac{9}{2} s_2^{(3)} + \frac{27}{8} s_2^{(4)} + \frac{81}{40} s_2^{(5)} + \frac{81}{80} s_2^{(6)} \quad (40)$$

would be close to M_{5exp} = 11.019 GeV.

The value $s_0^{(1)}$ may be obtained using the equation

$$u_0 + u_1 - 2 s_2 u_2 + \frac{1}{2} u_2 - \frac{1}{12} u_2^{(1)} + \frac{1}{720} u_2^{(3)} - \frac{1}{30240} u_2^{(5)} = 0 \quad (41)$$

which follows from the requirement of absence of the terms $1/Q^2$ at $Q^2 \to \infty$ in $\Delta \Pi^{\delta}(Q^2)$. The same value can be obtained from the formula

$$s_0^{(1)} = s_2^{(1)} - 2 s_2^{(2)} + 2 s_2^{(3)} - \frac{4}{3} s_2^{(4)} + \frac{2}{3} s_2^{(5)} - \frac{4}{15} s_2^{(6)} \quad (42)$$

When varying it was required that for $s_0^{(1)}$ obtained from eqs. (41),(42) close values would be obtained.

The accuracy of all these formulae is defined by the neglected term with $s_1^{(7)} \sim 10$ GeV^2. Analogously, it was required that the varied parameter $s_1^{(1)}$ would be close to the value

$$s_1^{(1)} = s_2^{(1)} - s_2^{(2)} + \frac{1}{2} s_2^{(3)} - \frac{1}{6} s_2^{(4)} + \frac{1}{24} s_2^{(5)} - \frac{1}{120} s_2^{(6)} \tag{43}$$

From requirement that the l.h. and r.h.sides of eq.(30) would differ not more than by 20% in the interval $-4\, m_b^2 + 10$ GeV$^2 <$ $< Q^2 < \infty$ there follow the restrictions on the value of gluonic condensate

$$0.05\ GeV^4 \leq \langle \frac{\alpha_s}{\pi} G^2 \rangle \leq 0.1\ GeV^4 \tag{44}$$

The value of gluonic condensate was determined by the formula

$$\langle \frac{\alpha_s}{\pi} G^2 \rangle = \frac{3}{2\pi^2} \Big\{ u_0 s_0 + u_1 s_1 - 2\cdot(4m_b^2)^2 F_4(\tfrac{v}{2}) + \frac{1}{2} s_2 u_2 - \frac{1}{12}(s_2 u_2)^{(1)} + \frac{1}{720}(s_2 u_2)^{(3)} - \frac{1}{30240}(s_2 u_2)^{(5)} \Big\} \tag{45}$$

At the magnitudes of varied parameters

$m_b = 4.544$ GeV, $s_1^{(1)} = 9.032$ GeV^2, $s_2^{(1)} = 5.053$ GeV^2

$s_2^{(2)} = -2.14$ GeV^2, $s_2^{(3)} = 4.152$ GeV^2, $s_2^{(4)} = -3.598$ GeV^2 (46)

$s_2^{(5)} = -5.318$ GeV^2, $s_2^{(6)} = 10$ GeV^2

the value of gluonic condensate is $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.056$ GeV^4 and the difference between l.h. and r.h.sides of eq.(3) in the interval -72.6 GeV$^2 < Q^2 < \infty$ does not exceed 15%. The ratio of the r.h.side of eq.(30) to the l.h.side with Q^2 growing tends fastly to unity (see graph (2)).

Making use of formulae (36)-(46) we obtain the mass values

$$\mathcal{M}_0 = 9.4542 \text{ GeV}, \quad \mathcal{M}_1 = 10.0232 \text{ GeV}, \quad \mathcal{M}_3 = 10.5767 \text{ GeV},$$
$$\mathcal{M}_4 = 10.855 \text{ GeV}, \quad \mathcal{M}_5 = 11.384 \text{ GeV}$$

and of the values

$$(s_0^{(1)})_{41} = 11.477 \text{ GeV}^2, \qquad (s_0^{(1)})_{42} = 11.905 \text{ GeV}^2$$

The value $(s_1^{(1)})_{43} = 9.024 \text{ GeV}^2$ is close to the value of the parameter $s_1^{(1)} = 9.032 \text{ GeV}^2$.

For the values $s_3^{(1)}$ and $s_4^{(1)}$ there are the formulae

$$s_3^{(1)} = s_2^{(1)} + s_2^{(2)} + \frac{1}{2} s_2^{(3)} + \frac{1}{6} s_2^{(4)} + \frac{1}{24} s_2^{(5)} + \frac{1}{120} s_2^{(6)}$$
$$s_4^{(1)} = s_2^{(1)} + 2 s_2^{(2)} + 2 s_2^{(3)} + \frac{4}{3} s_2^{(4)} + \frac{2}{3} s_2^{(5)} + \frac{4}{15} s_2^{(6)} \qquad (48)$$

At the magnitudes of the parameters (34) we obtain

$$s_3^{(1)} = 4.789 \text{ GeV}^2, \quad M_3^{(1)} = \frac{s_3^{(1)}}{2\mathcal{M}_3} = 0.227 \text{ GeV}$$
$$s_4^{(1)} = 7.718 \text{ GeV}^2, \quad M_4^{(1)} = \frac{s_4^{(1)}}{2\mathcal{M}_3} = 0.355 \text{ GeV} \qquad (49)$$

4. Light Quarks

Consider the polarization operator $\Pi(Q^2)$ connected with the vector current of light quarks $j_\mu^{I=1}(x)$

$$i \int d^4x \, e^{iqx} \langle 0 | T \{ j_\mu^{I=1}(x), j_\nu^{I=1}(0) \} | 0 \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(Q^2) \qquad (50)$$

where $Q^2 = -q^2$ and

$$j_\mu^{I=1}(x) = \frac{1}{2}\left[\bar{u}(x)\gamma_\mu u(x) - \bar{d}(x)\gamma_\mu d(x)\right]$$

The dispersion relation for $\Pi(Q^2)$

$$\Pi(Q^2) = \frac{1}{12\pi^2}\int_{4m_\pi^2}^{\infty}\frac{R^{(I=1)}(s)\,ds}{s+Q^2} \qquad (51)$$

In the model at hand one may use for $R^{(I=1)}(s)$ eqs.(8) and (3) replacing $R_c(s) \to R^{(I=1)}$ and $R_c^{(c)} \to \frac{3}{2}$.

Substitute into eq.(51) the function $R^{(I=1)}$ defined by eq.(8) and exploit formula (3) and the Euler-McLoran formula. We obtain

$$\Pi(Q^2) = \frac{1}{8\pi^2}\sum_{k=0}^{\infty}\frac{s_k^{(1)}}{s_k+Q^2} = \frac{1}{8\pi^2}\left\{\frac{s_0^{(1)}}{s_0+Q^2} + \int_{(m_u+m_d)^2}^{\infty}\frac{ds}{s+Q^2} - \int_{(m_u+m_d)^2}^{s_1}\frac{ds}{s+Q^2} + \frac{1}{2}\frac{s_1^{(1)}}{s_1+Q^2} - \frac{1}{12}\left[\frac{s_1}{s_1+Q^2}\right]^{(1)}\right\} \qquad (52)$$

In eq.(52) summation is replaced by integration starting from k=1 and all nonwritten terms are neglected. This can be done since $s_1 = 2.89\ \mathrm{GeV}^2$, i.e. it is significantly smaller than the parameter s_1 for the family ψ and Υ.

The operator expansion in this case allows one to write the formula

$$\Pi(Q^2) = \Pi_q + \frac{C_2}{Q^2} + \frac{C_4}{Q^4} + \frac{C_6}{Q^6} + \frac{C_8}{Q^8} \qquad (53)$$

where

$$\Pi_q = \frac{1}{8\pi^2}\int_{(m_u+m_q)^2}^{\infty}\frac{ds}{s+Q^2} \tag{54}$$

$$c_2 = 0 \tag{55}$$

$$C_4 = \frac{1}{24}\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle + \frac{1}{2}\left(m_u\langle 0|\bar u u|0\rangle + m_d\langle 0|\bar d d|0\rangle\right) \tag{56}$$

$$\begin{aligned} C_6 = &-\frac{1}{2}\pi\alpha_s\langle 0|(\bar u\gamma_\mu\gamma_5 t^a u - \bar d\gamma_\mu\gamma_5 t^a d)^2|0\rangle - \\ &-\frac{1}{9}\pi\alpha_s\langle 0|(\bar u\gamma_\mu t^a u + \bar d\gamma_\mu t^a d)\sum_{q=u,d,s}\bar q\gamma_\mu q|0\rangle \end{aligned} \tag{57}$$

At large Q^2 eqs.(53) and (52) must coincide and therefore we expand eq.(52) in $1/Q^2$ and equal the terms of one and the same order in $1/Q^2$ in eqs.(53) and (52):

$$\begin{aligned} &s_0^{(1)} s_1 + 1/2\, s_1^{(1)}\; 1/12\, s_1^{(2)} = 0 \\ &-s_0 s_0^{(1)} + 1/2\, s_1^2 - 1/2\, s_1 s_1^{(1)} + 1/12\left(s_1 s_1^{(2)} + s_1^{(1)^2}\right) = A_4 \\ &s_0^2 s_0^{(1)} - 1/3\, s_1^3 + 1/2\, s_1^2 s_1^{(1)} - 1/12\left(s_1^2 s_1^{(2)} + 2 s_1 s_1^{(1)^2}\right) = A_6 \\ &-s_0^3 s_0^{(1)} + 1/4\, s_1^4 - 1/2\, s_1^3 s_1^{(1)} + 1/12\left(s_1^3 s_1^{(2)} + 3 s_1^{(2)} + 3 s_1^2 s_1^{(1)^2}\right) = A_8 \end{aligned} \tag{58}$$

where $A_i = 8\pi^2 C_i$.

Let us use the first of eqs.(58) to find $s_1^{(2)}$. After simple manipulations we get

$$\begin{aligned} &(s_1 - s_0)s_0^{(1)} + 1/12\, s_1^{(1)^2} = 1/2\, s_1^2 + A_4 \\ &(s_1^2 - s_0^2)s_0^{(1)} + 1/6\, s_1 s_1^{(1)^2} = 2/3\, s_1^3 - A_6 \\ &(s_1^3 - s_0^3)\, s_0^{(1)} + 1/4\, s_1^2 s_1^{(1)^2} = 2/3\, s_1^3 - A_6 \end{aligned} \tag{59}$$

Excluding $s_0^{(1)}$ and $s_1^{(1)}$ from eqs.(59), let us express A_6 through A_4 and A_8

$$A_6=\frac{s_1^3(s_1-4s_0)}{12(2s_1+s_0)}-\frac{s_1(s_1+2s_0)}{2s_1+s_0}A_4-\frac{1}{2s_1+s_0}A_8 \tag{60}$$

Neglecting the last term in (60), substituting $s_0=m_\varsigma^2$, m_ς = 0.77 GeV, $s_1=m_{\varsigma'}^2$, $m_{\varsigma'}$ = 1.7 GeV we get

1) At $\langle\frac{\alpha_s}{\pi}G^2\rangle$ = 0.04 GeV^4 C_6 = -0.0010 GeV^6

2) At $\langle\frac{\alpha_s}{\pi}G^2\rangle$ = 0.056 GeV^4 C_6 = -0.0022 GeV^6 (61)

3) At $\langle\frac{\alpha_s}{\pi}G^2\rangle$ = 0.12 GeV^4 C_6 = -0.0072 GeV^6

The value C_6 obtained with the help of factorization hypothesis and vacuum insert [1] is equal to

$$(C_6)_{svz}=-\frac{16\cdot 7}{81}\pi\alpha_s|\langle 0|\bar{q}q|0\rangle|^2 = -7.9\ 10^{-4}\,\mathrm{GeV}^6 \tag{62}$$

Comparison of the results (61) and (62) shows that the hypothesis of factorization and vacuum insert gives an incorrect result. The accuracy of this hypothesis was analysed in ref.[3]. Let us find from eq.(59) the parameters $s_0^{(1)}$ and $s_1^{(1)}$

$$s_0^{(1)}=\left(s_1^3/3+2s_1A_4+A_6\right)/(s_1-s_0)^2 \tag{63}$$

$$s_1^{(1)}=\sqrt{6\left[s_1^2+2A_4-2(s_1-s_0)s_0^{(1)}\right]} \tag{64}$$

The values of $M_k^{(1)}=s_k^{(1)}/2M_k$ are given in the Table.

Substituting $\alpha_s(m_{\varsigma}^2) = 0.39$ and $\alpha_s(m_{\varsigma'}^2) = 0.27$ we obtain Γ_{κ}^{ee} . The comparison of the calculated electronic widths with the observed ones is given in the Table.

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Table

Comparison of calculated values of electronic widths $\Gamma^{ee}_{\kappa\, Theor}$ with experimental values $\Gamma^{ee}_{\kappa\, exp}$ for ψ, Υ, ρ -meson families

Mesons	mass MeV	Γ_{tot} MeV	$M_\kappa^{(1)}/GeV$			$\Gamma^{ee}_{\kappa\, Theor}/keV$			$(\Gamma^{ee}_\kappa)_{exp}/keV$
			I	II	III	I	II	III	
ψ_0	3096.9±0.1	0,068±0.010	0,58	0,58	0,66	4.13	4,10	4,57	4.72 ± 0.35
ψ_1	3686.0±0.1	0.243±0,043	0,44	0,46	0,42	2,81	2,91	2,63	2,15±0,21
ψ_2	3769.9±2.5	25±3							0,26±0,05
ψ_3	4040±10	52±10	0,31	0,26	0.32	1.94	1,61	1,98	0,75±0,15
ψ_4	4159±20	78±20							0,77±0,23
ψ_5	4415±6	43±20							0,47±0,10
Υ_0	9460.3±0.2	0,052±0.003	0.57	0.57	0.60	0.99	0.99	1.02	1.34±0,05
Υ_1	10023.3±0.3	0,044±0.009	0.45	0.45	0.44	0.63	0.63	0.62	0.60±0,04
Υ_2	10355.3±0.5	0,026±0,006	0.25	0.24	0.24	0.35	0.35	0.35	0,44±0,03
Υ_3	10580.0±3.5	24±2	0.23	0.23	0.23	0.33	0.32	0.33	0,24±0,05
Υ_4	10865±8	110±13	0.34	0.36	0.34	0.50	0.51	0.49	0,31±0.07
Υ_5	11019±8	79±16							0,13±0,03
	770±3	153±2	1.07	1,10	1,20	6,82	6,99	7,63	6,9±0.3
ρ'	1770±20	235±50	0,73	0.70	0,58	4,46	4,29	3.56	

I. - The set of parameters in GeV

a) for the ψ -meson family

$m_c = 1.323$, $s_1^{(1)} = 3.266$, $s_1^{(2)} = -1.665$, $s_1^{(3)} = 0.622$

$s_1^{(4)} = 3.753$, $s_1^{(5)} = 16.56$, $s_1^{(6)} = -15$ at this set of the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.03882$;

b) for the Υ -meson family:

$m_b = 4.544$, $s_1^{(1)} = 8.961$, $s_2^{(1)} = 5.157$, $s_2^{(2)} = -2.036$

$s_2^{(3)} = 3.789$, $s_2^{(4)} = -0.727$, $s_2^{(5)} = -4.269$, $s_2^{(6)} = 9.747$

at this set of the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.053$;

c) for the ρ meson family $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.04$.

II. - the set of parameters in GeV

a) for the ψ -meson family:

$m_c = 1.322$, $s_1^{(1)} = 3.383$, $s_1^{(2)} = -1.411$, $s_1^{(3)} = -0.655$

$s_1^{(4)} = 1.361$, $s_1^{(5)} = 7.072$, $s_1^{(6)} = -11.71$, at this set of the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.0562$

b) for the Υ -meson family:

$m_b = 4.544$, $s_1^{(1)} = 9.032$, $s_2^{(1)} = 5.053$, $s_2^{(2)} = -2.14$

$s_2^{(3)} = 4.152$, $s_2^{(4)} = -3.598$, $s_2^{(5)} = -5.318$, $s_2^{(6)} = 10$,

at this set of the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.0567$.

c) for the ρ -meson family $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.056$.

III. - the set of the parameters in GeV

a) for ψ -meson family:

$m_c = 1.277$, $s_1^{(1)} = 3.07$, $s_1^{(2)} = -1.057$, $s_1^{(3)} = 0.737$,

$s_1^{(4)} = -3.204$, $s_1^{(5)} = 16.82$, $s_1^{(6)} = -0.052$, at this set of the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.12$.

b) for the Υ -meson family: $m_b = 4.53$, $s_1^{(1)} = 8.917$, $s_2^{(1)} = 5.028$, $s_2^{(2)} = -2.295$, $s_2^{(3)} = 4.047$, $s_2^{(4)} = 0.936$, $s_2^{(5)} = -3.962$, $s_2^{(6)} = 8.45$, at this set of the parameters $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.1$.

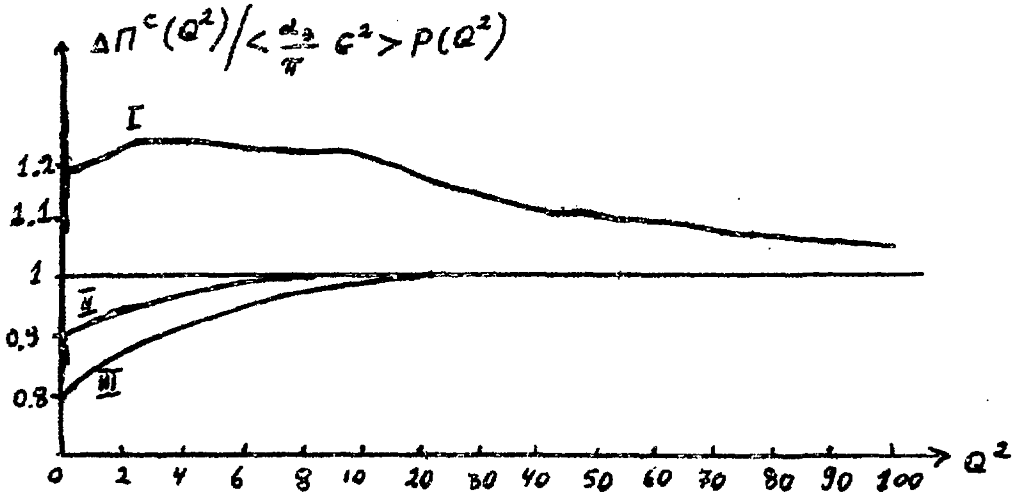
c) for the ρ -meson family $\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.12$.

When comparing $\Gamma^{ee}_{1\,exp}$ and $\Gamma^{ee}_{3\,exp}$ with $\Gamma^{ee}_{1\,theor}$ and $\Gamma^{ee}_{3\,theor}$ for the ψ -meson family one should substitute

$$\Gamma^{ee}_{1exp} \to \Gamma^{ee}_{1exp} + \frac{3}{4}\Gamma^{ee}_{2exp} = (2.35 \pm 0.31)\ \text{keV}$$

$$\Gamma^{ee}_{3exp} \to \Gamma^{ee}_{3exp} + \frac{2}{3}\Gamma^{ee}_{4exp} = (1.33 \pm 0.31)\ \text{keV}$$

Graph I.



$$\Delta\Pi^c(Q^2)=\Pi^c(Q^2)-\Pi_0^c(Q^2)$$

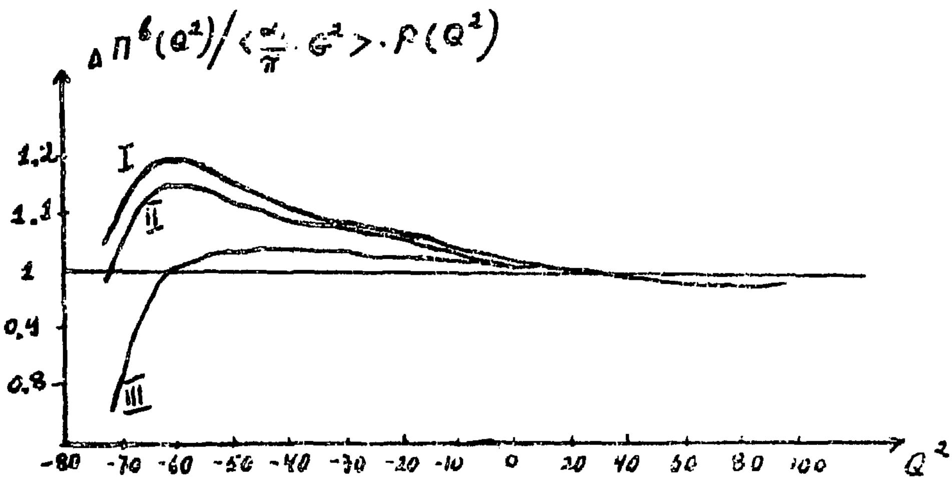
$\Pi^c(Q^2)$ is the polarization operator connected with the vector current of charmed quarks. $\Pi_0^c(Q^2)$ is the free polarization operator. $\langle\frac{\alpha_s}{\pi}G^2\rangle P(Q^2)$ is the gluonic condensate contribution into the mean vacuum value from the operator expansion (14).

I. - the set of parameters Ia (see the Table) $\langle\frac{\alpha_s}{\pi}G^2\rangle =$ $= 0.0382\ GeV^4$

II- - the set of the parameters IIa(see the Table) $\langle\frac{\alpha_s}{\pi}G^2\rangle =$ $= 0.0562\ GeV^4$

III- the set of the parameters IIIa (see the Table) $\langle\frac{\alpha_s}{\pi}G^2\rangle =$ $= 0.12\ GeV^4$

Graph 2.



$$\Delta\Pi^{b}(Q^2)=\Pi^{b}(Q^2)-\Pi_0^{b}(Q^2)$$

$\Pi^{b}(Q^2)$ is the polarization operator connected with the vector current of b-quarks. $\Pi_0^{b}(Q^2)$ is the free polarization operator. $\langle\frac{\alpha_s}{\pi}G^2\rangle P(Q^2)$ is the gluonic condensate contribution into the vacuum mean value from the operator expansion (14) ($c\to b$)

I - the set of the parameters Ib (see the Table) $\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.053\ GeV^4$

II - the set of the parameters IIb (see the Table) $\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.0567$

III- the set of the parameters IIIb (see the Table) $\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.1\ GeV^4$

R e f e r e n c e s

1. Shifman M.A., Vainshtein A.I., Zakharov V.I.// Nucl.Phys. 1979.V.B147.P.385.
2. Eidelman S.I., Kyrdadze L.M., Vainshtein A.I.// Phys.Lett. 1979.V.82B.P.279.
3. Lanner G., Narison S., Tarrach R.// Z.Phys.C.- Particles and Fields.1984.V.26.P.433.
4. Marrow J., Chaw G.// Z.Phys.C.- Particles and Fields. 1986. V.33.P.237.
5. Gimenez V., Penarrocha J.A., Bordes J.// Phys.Lett.1988.V.214B. P.247.
6. Bertlmann R.A. et al.// Z.Phys.C.- Particles and Fields.1988. V.39.P.231.
7. Mennessier G. Preprint PM/89-03 France.
8. Geshkenbein B.V.// Yad.Fiz.1989.V.49.P.1138.
9. Geshkenbein B.V.// Yad.Fiz.1985.V.42.P.991.
10. Abramovitz M., Stigan I.- Manual on Special Functions. M.: Nauka,1979.
11. Rev.of Part.Prop.// Phys.Lett.1988.V.204B.P.1.
12. Novikov V., Okun L., Shifman M., Vainshtein A., Voloshin M., Zakharov V.// Phys.Rep.1978.V.41.P.1.

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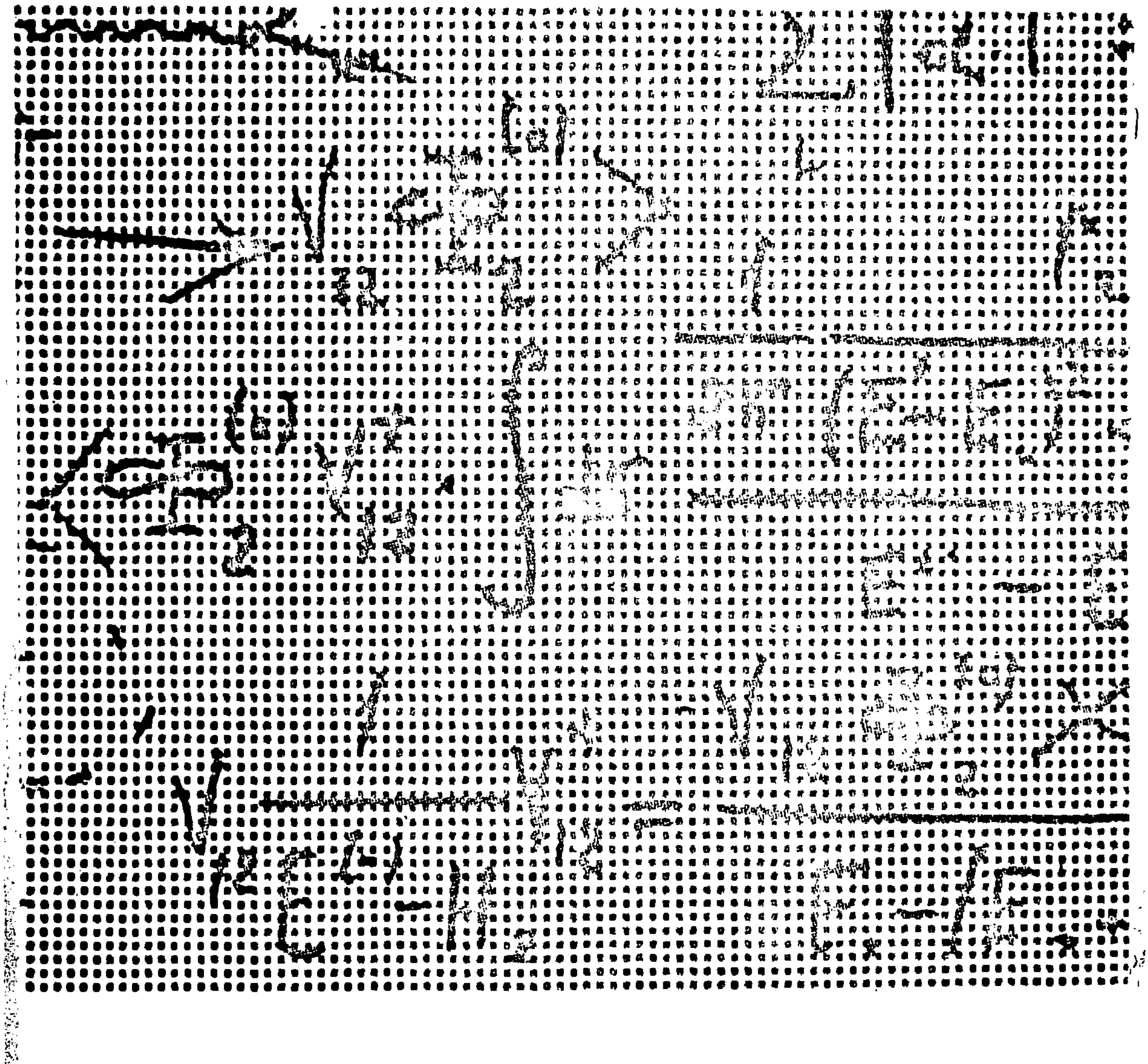

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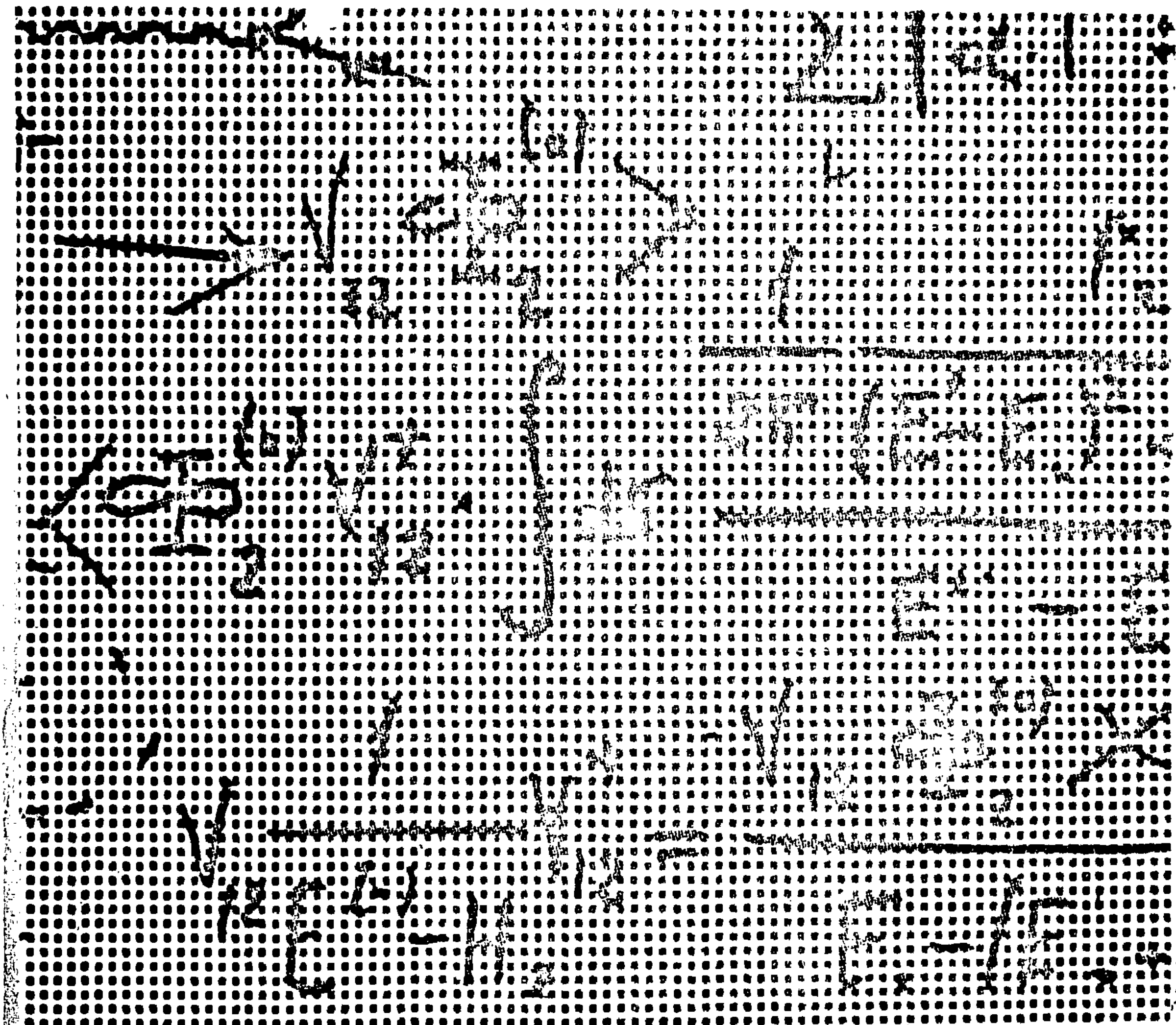

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