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STUDY OF THE THREE–NUCLEON CONTINUUM WITH REALISTIC NN INTERACTIONS

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BADANIE KONTINUUM W UKŁADZIE TRZECH NUKLEONÓW PRZY UŻYCIU REALISTYCZNYCH ODDZIAŁYWAŃ NN

ИССЛЕДОВАНИЕ ТРИ-НУКЛОННОГО КОНТИНУУМА С ИСПОЛЬЗОВАНИЕМ РЕАЛИСТИЧЕСКИХ NN ВЗАИМОДЕИСТВИИ

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Nucleon-deuteron elastic scattering and nucleon induced deuteron breakup were studied in the energy range of the incoming nucleon E_{lab}=10÷70 MeV. Rigorous Faddeev three-nucleon continuum calculations were performed with realistic, meson-exchange based nucleon-nucleon interactions. Predictions gained with the Paris or Bonn potentials were compared with existing experimental data. For some elastic scattering observables very good quantitative description results. It was shown that carefull study of particular elastic scattering polarization observables will play a role to nail down unsettled nucleon-nucleon force properties, which remain open by present day 2N experimental data. To such properties belong f.e. the charge independence breaking of the NN interaction in ^3P waves or the proper strength of the $^3S_1-^3D_1$ tensor force. Kinematically complete experimental data for various breakup configurations have been analyzed. Significant discrepancies between theory and experiment found for some configurations can probably be attributed to the action of the 3-nucleon force.

Badano elastyczne rozpraszanie nukleon-deuteron oraz rozszczepienie deuteronu spowodowane oddziaływaniem z padajacym nukleonem dla energii nukleonu E_{lab}=10÷70 MeV. Wykonano scisłe obliczenia kontinuum 3-nukleonowego w ramach formalizmu Faddeeva używajac realistycznych oddziaływan nukleon-nukleon, opartych na modelu wymiany mezonowej. Wyniki teoretyczne uzyskane przy użyciu potencjałów Paryskiego lub Bonskiego porównano z dostepnymi danymi eksperymentalnymi. Otrzymano bardzo dobry ilosciowy opis niektorych obserwabli elastycznego rozpraszania. Pokazano, że dokładna analiza pewnych obserwabli polaryzacyjnych w elastycznym rozpraszaniu pozwoli na ustalenie tych własnosci oddziaływania nukleon-nukleon, ktore sa niedostatecznie wyznaczone poprzez obecne dane eksperymentalne w układzie 2N. Do takich własnosci naleza n.p. łamanie niezależnosci ładunkowej oddziaływania NN w falach czastkowych ^3P lub właściwa wielkość siły tensorowej $^3S_1-{}^3D_1$. Przeprowadzono analizę danych eksperymentalnych dla różnych, kinematycznie zupełnych konfiguracji rozszczepienia. Znalezione dla niektórych konfiguracji znaczące niezgodności pomiędzy teorią a eksperymentem wskazują prawdopodobnie na efekty spowodowane działaniem siły 3-nukleonowej.

Упругое рассеяние нуклона дейтроном и разрыв дейтрона падающим нуклоном были исследованы для энергии нуклона $E_{lab}=10\div70$ МэВ. Ригористические вычисления 3-нуклонного континуума проведено в рамках формализма Фаддеева при использовании реалистических, базирующих на модели обмена мезонов, нуклон-нуклонных взаимодействий. Теоретические результаты получены при использовании Парижского или Бонниского потенциялов сравнено с доступными экспериментальными данными. Получено очень хорошое количественное описание некоторых величин упругово рассеяния. Показано, что тщательное исследование некоторых поляризационных величин упругово рассеяния позволит получить информацию о этих свойствах нуклон-нуклонного взаимодействия, которые недостаточно определены существующими экспериментальными данными в системе 2N. К таким свойствам принадлежат например нарушение зарядовой независимости нуклон-нуклонного взаимодействия в парциальных состояниях ^3P или настоящая величина тензорной силы $^3S_1-{}^3D_1$. Проведено анализ экспериментальных данных для разных кинематически полных конфигураций процеса разрыва. Значительные несогласованности между теорией а экспериментом получены для некоторых конфигураций указывают вероятно на эффекты действия 3-нуклонной силы.

C O N T E N T S

I. Introduction

The three-nucleon (3N) system has aroused an interest over many years. In such a system the traditional approach to nuclear physics based on a nonrelativistic Hamiltonian in which the nucleons interact pairwise , can be tested in a nontrivial manner once the forces have been adjusted in the two-nucleon (2N) system. In order to generate internucleon interactions also mesons and isobars are allowed in this traditional approach in addition to the basic degrees of freedom - nucleons. Such meson-exchange based two-nucleon forces as given by the Paris[1] and the Bonn[2] potentials ,and purely phenomenological 2-nucleon interactions (as for instance the Reid soft-core potential [3]) found by applying nonrelativistic Schrödinger equation to nucleon-nucleon (NN) problem , describe the big amount of two-nucleon experimental data.

Assumption that only such two-nucleon forces are acting fixes the three-nucleon Hamiltonian. Few questions then naturally arise:

1. Does such a Hamiltonian explain the growing experimental realm of three nucleon observables ?
2. Does a study of three-nucleon systems offer a new information about NN interaction which couldn't be get by studying two-nucleon systems alone ?
3. Is a 3-nucleon interaction required in addition to the 2-nucleon one, in order to explain some 3-nucleon experimental data?

In recent years reliable solutions of the Faddeev equations[4] with realistic NN interactions have been achieved in momentum and configuration space, however only for the three-nucleon bound state[5-8].

All known realistic two-nucleon interactions do not describe the triton quantitatively (for a recent review see[9]). The triton is underbound by about ≃1 MeV (with the exception of the force model newly developed by the Bonn group[2]). Inclusion of the 2π-exchange three-nucleon force[10,11] provides additional binding energy[12-15]. However, ingredients of that force (for a recent review see[16]), such as the off-shell πN scattering amplitude for harder pions, where the soft pions techniques presently used are no longer valid, and the πNN-form factor, are not yet sufficiently well understood. As a result, very likely caused by the defects in the πN amplitude, the additional binding energy depends uncomfortable strongly on the cut-off parameter of the πNN-formfactor and definite conclusions about the contribution of the 2π-exchange three-nucleon force to the binding energy of the triton can not yet be drawn.

As triton calculations[17-19] have recently shown the force model of the Bonn group[2] can nearly account for the triton binding energy. The resulting difference between xperimental and theoretical binding amounts about 100 keV. Basically the reason for the almost proper binding energy of the triton in this force model is the weak $^3S_1-^3D_1$ tensor force[20,21]. The knowledge of this NN force component is however very bad as can be seen from values of ε_1 phase shift parameter which is essentially undetermined below about 150 MeV[22].

To resolve this apparent contradiction and to get additional information more three-nucleon observables are needed. Unfortunately, there are ones which scale with the triton binding energy like the radius of ^3H, the ratio of asymptotic D/S normalization, the doublet n-d scattering length (i.e., the Phillips line) and others[23]. Therefore, study of such observables will not yield additional information than that which is contained in the triton binding energy.

This, however, can be different in the three-nucleon system at positive energies where the emphasis on different force components can be varied from the ones which play the dominant role in the ^3H ground state. For instance the vector analysing power in elastic neutron-deuteron (nd) scattering depend sensitively on P-wave forces[49]. Especially the nucleon induced deuteron break-up seems to be a good source of important information concerning two- and three-nucleon forces. There very different geometrical configurations can be chosen for three outgoing nucleons. The study of kinematical configurations which are the most sensitive to 2-nucleon force properties could serve to distinguish between different 2N force models. For example, searching for configurations which are most sensitive to special 2N force components like the $^3S_1-^3D_1$ tensor force could shed some light on the problem which tensor force is the proper one. Contrary , finding the discrepancies between the theory based on 2N forces only and experiments in kinematical configurations which are most insensitive to the 2-nucleon force properties could be a signature of the action of three-nucleon forces. Study of such configurations could yield information about 3N-force effects. It should be also mentioned that study of the neutron induced deuteron break-up is up to now the only experimental source of information on neutron-neutron (nn) interaction , which is not accessible to study directly in nn scattering because of missing experimental facilities.

Thus information from three-nucleon scattering states should be used to test the dynamic of nucleon-nucleon interaction. However the theoretical results should not be obscured by uncontrolled and unphysical approximations. For instance the use of the separable form of NN interaction in Faddeev calculations is very tempting, reducing the 3-body problem to the solution of a set of coupled integral equations in one variable only[24,25]. The fact that 2N forces mediated by meson exchanges are basically local[64] provokes however to solve the 3-body Faddeev equations with local forces directly, without approximating them by unphysical separable form.

There is a rich history on the three nucleon continuum problem using 3-body equations of the Faddeev type. Finite rank two-body forces of various rank and sophistication , which simplify the solution of 3-body equations[24,25] , have been first applied. Qualitative studies with such simple forces have been carried out by many authors[28-37]. From comparison of theoretical results with experimental data for the elastic nucleon-deuteron (Nd) scattering and breakup processes some qualitative insight , like sensitivity of certain observables to the different force components (like $^1S_0$, tensor force etc.) has been gained. However no quantitative conclusion could be drawn because of the unrealistic nature of the NN interaction used in these calculations.

Some calculations in momentum space with local forces , which are considered to be more realistic , also emerged , first for pure S-wave forces[38,39] , then for the Reid potential[40-42] however treated pertubatively in higher partial waves. Also calculations in cofiguration space[43] for the de Tourreil-Rouben-Sprung potential[44] appeared. Again, no quantitative conclusion can be drawn because of the approximate nature of both, NN potential and numerical performance.

The inclusion of realistic , meson-exchange based NN interactions in such type of 3N-continuum calculations was also pursued on. Several years ago the Graz group initiated this program through designing separable representations of meson-theoretical force models[45,46]. The Graz-Osaka collaboration succeeded in deducing first results with such separable approximations for elastic Nd scattering[47,48].

However ,the rank of such sophisticated separable approximation required to reproduce the meson exchange-based NN interaction is rather high depending on the NN partial wave state. Even more important is the fact , that only the comparison with exact 3-body results, obtained without making any approximation about the form of the NN potential used, can decide if given separable approximation reproduces sufficiently accurately all the dynamical content of such NN interaction.

In the last years a considerable progress has been achieved in the calculation of the three-nucleon scattering observables[50-53].

In work of Takemiya[50] the Faddeev equations for elastic nucleon-deuteron scattering were solved with local NN potentials including higher partial waves nonpertubatively, however still using some " approximate averaging " in the integration over singularities.

Recently Witała- Glöckle- Cornelius collaboration succeded to solve 3N scattering equations of the Faddeev type for any sort of two-nucleon forces in a numerically precise sense[52,53]. Modified AGS-equations are solved in momentum space and in a partial wave basis without using any finite rank approximations of the potentials. This permits for the first time to perform three-nucleon continuum calculations using NN potentials based on realistic meson-exchange theory , e.g. Paris[1] and Bonn[2] potentials , without making any approximations. The Faddeev equations in continuum are in this way solved with the same degree of rigorousness as modern triton calculations so one can be sure that discrepancies with experimental data indicate a defect of the input nuclear interaction. This new situation is hard to overestimate. For the first time given nucleon-nucleon force model which has been adjusted in 2-nucleon system, can now be tested quantitatively on the 3-nucleon continuum level. It should be however mentioned , that these calculations are restricted to neutron-deuteron system , thus avoiding the still pending Coulomb problem in case of the pd system.

This new achievement is the subject of the present work. The theoretical formalism needed to perform 3N continuum calculations with realistic potentials will be presented together with specific numerical methods used. The possibility to perform such calculations opens quite new level of interpreting the experimental data in the three-nucleon continuum domain. As ilustration the first results of such rigorous 3N-continuum calculations with the Paris-[1] and Bonn-[2]potentials are discussed and compared with few existing 3N experimental data.

The theoretical formalism and some details of the numerical performance are described in chapter II.

Chapter III presents the meson-exchange based NN interactions used in the following. They form the dynamical input in such calculations.

Theoretical cross sections and polarization observables for Nd elastic scattering and nucleon induced deuteron breakup were calculated in energy range of incoming nucleon $E_{lab} \cong 10 \div 70$ MeV. They are presented and compared with some experimental data for the elastic scattering and break-up processes in chapter IV and V , respectively.

The results are summarized and discussed in the last chapter VI.

II. Theoretical formalism

The correct treatment of the few-body aspects of the 3-body scattering was first given by Faddeev[4,54]. Based on its modified formulation, introduced by Alt, Grassberger and Sandhas[25], the three-body analogue of the two-body Lipmann-Schwinger equation for the transition operator was derived. These so called AGS-equations connect the transition operators $U_{\alpha\alpha}$ for elastic scattering and $U_{\gamma\alpha}$, $\gamma\neq\alpha$, for rearrangment processes ($\alpha,\gamma=1,2,3$). Here as in the following, the two-body fragmentation channel α contains freely moving particle α and the bound $\beta\gamma$ pair , with $\alpha\neq\beta\neq\gamma$ being a cyclical permutation of 1,2 and 3. Making restriction to the 3-nucleon problem one can treat neutrons and protons as identical particles by introducing the isospin quantum numbers. The AGS equations reduce in this case to the following equation for the nucleon-deuteron elastic scattering transition operator U [55]

$$U = PG_0^{-1} + PtG_0U. \tag{II.1}$$

Here t is the 2-body off-shell t-operator , G_0 the free 3-particles propagator and P the sum of two cyclical permutation operators. In case when t sums up two-body interactions in channel 1 , P is given by

$$P = P_{12}P_{23} + P_{13}P_{23} \tag{II.2}$$

with P_{ij} representing permutation of two nucleons.

Once U is known the transition operator U_0 for the break-up process can be calculated[55]

$$U_0 = (1+P)tG_0U. \tag{II.3}$$

The main problem in solving eq.(II.1) in momentum space is the presence of singularities in t and G_0. The action of P in (II.1) smears out the bound-state pole in t , which appears at the deuteron binding energy , into a logarithmic singularity. This latter difficulty can be avoided introducing new operator T[38,56,57)

$$T = tP + tG_0PT. \qquad (II.4)$$

Then the transition operators U for elastic Nd scattering and U_0 for the deuteron breakup are

$$U = PG_0^{-1} + PT$$
$$U_0 = (1+P)T. \qquad (II.5)$$

Starting from this standard formulation, introducing standard Jacobi momenta p,q [55) and partial-wave basis states (see fig.II.1)

$$| \, pq\alpha > \equiv | \, pq(ls)j(\lambda \, {}^1/_2)IJ(t \, {}^1/_2)T > \qquad (II.6)$$

infinite system of coupled , two-dimensional integral equations is obtained from eq.(II.4) for the $\langle pq\alpha|T|\Phi\rangle$ amplitudes

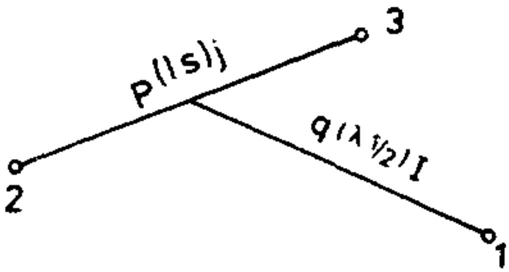
$$\langle pq\alpha|T(E)|\Phi\rangle = \langle pq\alpha|tP|\Phi\rangle$$

$$+ \sum_{\bar{T},\alpha',l_{\bar{\alpha}}} \int_0^\infty dq' q'^2 \int_{-1}^{1} dx \, \frac{\langle pl_\alpha|t^{(\alpha,\bar{\alpha})}(E-{}^3/_{4m}q^2)|\pi_1 l_{\bar{\alpha}}\rangle}{\pi_1^{\,l_{\bar{\alpha}}}} \qquad (II.7)$$

$$\times \frac{G_{\bar{\alpha},\alpha'}(q,q',x)}{\pi_2^{\,l_{\alpha'}}} \, \frac{\langle \pi_2 q'\alpha'|T(E)|\Phi\rangle}{E+i0-(q^2+q'^2+qq'x)/m} .$$

The ket $|\Phi\rangle=|q_0,m_n,\psi_{m_d}\rangle$ describes initial state with momentum q_0 of the neutron relative to the deuteron , and with spin

Fig.II.1 Choice of Jacobi momenta and definition of the angular-momentum coupling scheme.



projections m_n and m_d , respectively. The total center-of-mass energy E fixed by the binding energy of the deuteron E_d is

$$E = {}^3/_{4m}\, q_0^{\,2} + E_d \,. \qquad \text{(II.8)}$$

Two-nucleon interaction enters 3-body equations through the off-shell matrix elements $\langle p l_\alpha | t^{(\alpha,\bar\alpha)}(E-{}^3/_{4m}q^2) | \pi_1 l_{\bar\alpha}\rangle$ of the two-body t-operator. The geometrical coefficients $G_{\bar\alpha,\alpha'}(q,q',x)$ and the momenta $\pi_1=\sqrt{q'^2+0.25q^2+qq'x}$, $\pi_2=\sqrt{q^2+0.25q'^2+qq'x}$ arise from the matrix elements $\langle pq\bar\alpha|P|p'q'\alpha'\rangle$ of the permutation operator P[55]. The quantum numbers in the set $\bar\alpha$ differ from those in α only in the orbital angular momentum l of the pair and total isospin of the 3N system T (equal ${}^1/_2$ or ${}^3/_2$). The change in l occurs only when the tensor force is acting.

From the amplitudes $\langle pq\alpha|T|\Phi\rangle$ the transition amplitudes $\langle q_0', m_n', m_d'|U|\Phi\rangle$ for elastic and $\langle m_1 m_2 m_3 p_1 p_2 p_3|U_0|\Phi\rangle$ for the breakup processes with definite spin projections of the final particles are calculated by quadrature using eq.(II.5). Finally in a standard manner[58,59] the cross sections and polarization observables are obtained.

Special attention should be paid to the two-nucleon t-operator. It is well established[60] that neutron-proton (np) and proton-proton (pp) (or , assuming charge symmetry of the NN interaction, neutron-neutron (nn)) forces are different in the $^1S_0$ state. This is so called charge dependence of the 2-nucleon forces. Since e.g. the breakup cross sections in the so called final state interaction region of the 3-body phase-space are very sensitive to $^1S_0$ NN forces , this difference should be properly accounted for in the calculations. It was done in the following way.

For the case of nd system (two neutrons and one proton) the 2-nucleon t-operator has the general form

$$t = P_{nn} t_{nn} P_{nn} + P_{np} t_{np} P_{np} \tag{II.9}$$

where P_{nn} and P_{np} project on a nn and a np pair , respectively. Three isospin $^1/_2$ particles can form the 3-particle isospin states $|(t\,{}^1/_2)T\rangle$ with total isospin $T={}^1/_2$ or $^3/_2$ and subsystem isospins $t=0,1$. Neglecting the very weak transition between $t=0$ and $t=1$ states in the np system , isospin part of matrix element $\langle p l_\alpha | t^{(\alpha,\bar{\alpha})}(E-{}^3/_{4m}q^2) | \pi_1 l_{\bar{\alpha}}\rangle$ has the form (see also ref.[61] for details)

$$\langle (t_\alpha {}^1/_2) T_\alpha | t | (t_{\bar{\alpha}} {}^1/_2) T_{\bar{\alpha}} \rangle =$$

$$\delta_{t_\alpha t_{\bar{\alpha}}} \delta_{T_\alpha T_{\bar{\alpha}}} \delta_{T_\alpha {}^1/_2} \left[\delta_{t_\alpha 0}\, t_{np}^{t=0} + \delta_{t_\alpha 1} \left({}^2/_3\, t_{nn}^{t=1} + {}^1/_3\, t_{np}^{t=1} \right) \right]$$

$$+ \delta_{t_\alpha t_{\bar{\alpha}}} \bar{\delta}_{T_\alpha T_{\bar{\alpha}}} {}^{\sqrt{2}}/_3 \left(t_{nn}^{t=1} - t_{np}^{t=1} \right) \tag{II.10}$$

$$+ \delta_{t_\alpha t_{\bar{\alpha}}} \delta_{T_\alpha T_{\bar{\alpha}}} \delta_{T_\alpha {}^3/_2} \left({}^2/_3\, t_{np}^{t=1} + {}^1/_3\, t_{nn}^{t=1} \right)$$

where $\bar{\delta}_{ij} = 1 - \delta_{ij}$. Angular momentum partial wave t-matrix elements $t_{np(nn)}^{t=0(1)}$ are generated by Lipmann-Schwinger equation using appropriate NN interaction.

To solve eq.(II.7) the infinite partial wave basis α has to be truncated to a finite one. For a given energy E the short-range two-body interaction V can be considered to be negligible beyond a certain total angular momentum j_{max} in the two-body subsystem. Putting $t^{(\alpha)}=0$ for $j>j_{max}$ yields a finite number of channels for each total angular momentum J of the three-body system. Taking e.g. $j_{max}=2$ and distinguishing between np and nn forces in $^1S_0$ partial wave state only, one is left with 35 coupled 2-dimensional integral equations for $J\geq 5/2$.

As was mentioned above the main problem of treating eq.(II.7) is caused by the singularities of the two-body t-operator and the free propagator G_0. The 2-nucleon t-operator belonging to the $^3S_1-^3D_1$ interaction , which supports a bound state (the deuteron) , has a pole at the off-shell energy

$$E - {}^{3}/_{4m}\, q^2 = E_d \; . \qquad \text{(II.11)}$$

This pole occurs then at $q=q_0$ and in "deuteron"-like channels $\alpha_D=((l1)1(\lambda\ ^1/_2)IJ(0\ ^1/_2)^1/_2)$ with $l=0$ or 2. As is seen from eq.(II.7) that pole appears also in $\langle pq\alpha_D|T|\Phi\rangle$. This suggests the following definitions,

$$\langle pq\alpha|\hat{z}|\Phi\rangle \equiv \begin{cases} (E-{}^{3}/_{4m}\, q^2 - E_d)\ \langle pq\alpha|z|\Phi\rangle & \text{for } \alpha=\alpha_D \\ \langle pq\alpha|z|\Phi\rangle & \text{for } \alpha\neq\alpha_D \end{cases} \qquad \text{(II.12)}$$

where z = T, t or tP.
Putting

$$x_0 = \frac{mE - q^2 - q'^2}{qq'} \qquad \text{(II.13)}$$

eq.(II.7) can be rewrite as

$$\langle pq\alpha|\hat{T}(E)|\Phi\rangle = \langle pq\alpha|\hat{t}(E-{}^3/_{4m}\, q^2)P|\Phi\rangle$$

$$+\sum_{\bar{T},\alpha',l_{\bar{\alpha}}} \int_0^\infty dq' q'^2 \int_{-1}^{1} dx \frac{\langle p l_\alpha|\hat{t}^{(\alpha,\bar{\alpha})}(E-{}^3/_{4m}q^2)|\pi_1 l_{\bar{\alpha}}\rangle}{\pi_1^{l_{\bar{\alpha}}}}$$

$$\times \frac{G_{\bar{\alpha},\alpha'}(q,q',x)}{\pi_2^{l_{\alpha'}}} \frac{m}{qq'(x_0+i0-x)} \tag{II.14}$$

$$\times \left[\langle \pi_2 q'\alpha'\neq\alpha_D|\hat{T}(E)|\Phi\rangle + {}^{4m}/_3 \frac{\langle \pi_2 q'\alpha'=\alpha_D|\hat{T}(E)|\Phi\rangle}{q_0^2 - q'^2} \right]$$

This is the set of equations which was solved by generating its Neumann series and summing it up by the Pade' method using a specially written in this aim computer code.

In fig.II.2 the shaded area comprises the q, q'-values for which $|x_0|\leq 1$. Thereby the maximal q (or q') value leading to a singularity is $q_{max}=\sqrt{{}^4/_3 mE}$. That singularity is treated by subtracting the integrand at $x=x_0$. Doing that throughout $0<q,q'\leq q_{max}$ leads to numeriqal inaccuracies since for small q-values x_0 can be very large and the Legendre polynomials $P_k(x_0)$ entering $G_{\bar{\alpha},\alpha'}$ blow up with increasing total angular momentum J-values. Therefore the subtraction was performed only in the region $|x_0|\leq 1.2$ which is inside the solid line in fig.II.2. For $q\leq q_{max}$ the integral over q' in eq.(II.14) is broken up into 6 parts (see fig.II.2)

$$\int_0^\infty dq' = \int_0^{q_1} dq' + \int_{q_1}^{q_2} dq' + \int_{q_2}^{q_3} dq' + \int_{q_3}^{q_4} dq' + \int_{q_4}^{q_{max}} dq' + \int_{q_{max}}^{\infty} dq'. \tag{II.15}$$

The regularized integrand with respect to x can be integrated using 10 Gauß-Legendre quadrature points. The x-Integral for

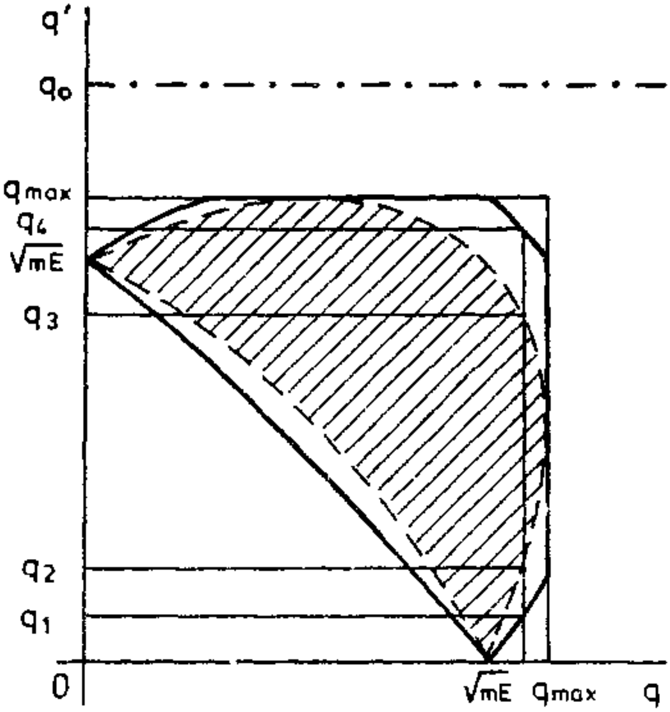


Fig. II.2 The singularities in the q-q' plane together with the choice of integration intervals. Logarithmic singularities lie on the dashed lines. The pole at $q'=q_0$ arising from the deuteron bound state is shown by the dashed-dotted line. The regularizing in the x-integration was made in the region limited by the thick solid line.

the subtracted term can be performed analytically and leads to logarithmic singularities along the dashed line ($|x_0|=1$) in fig.II.2. That singularity was also treated by the subtraction method. The q'-integral for $0 \le q' \le q_{max}$ was performed using the proper number of quadrature points. Since the lengths of the subintervals and the behaviour of the integrands in these intervals are different for different q-values , the distribution of quadrature points has to be chosen appropriately.

In interval $q_{max} < q' < \infty$ a high enough cut-off at

$q'_{max}=25$ fm^{-1} was introduced with sufficient number of quadrature points within $q_{max}\leq q'\leq q'_{max}$. The deuteron pole lying in this interval at $q'=q_0$ and occuring for channels $\alpha'=\alpha_D$ was also treated by the subtraction technique.

Iterating the set (II.14) those integrals lead to terms in the Neumann series evaluated at certain p- and q-values. Due to very many breakup configurations the matrix elements finally resulting have to be interpolated. For that purpose it is important to choose a good set of grid points, which includes certain special points as q=0 and $q=q_{max}$. The second case corresponds exactly to a configuration of a final state interaction where two nucleons have identical momenta. Besides these special points a sufficient number of q points was choosen in the interval $(0,q_{max})$ with some of them lying in the vicinity of q_{max}. This is necessary to map out the very sharp slope in the amplitudes $\langle pq\alpha|\hat{T}|\Phi\rangle$ and $\langle pl_\alpha|t^{(\alpha,\bar{\alpha})}|\pi_1 l_{\bar{\alpha}}\rangle$ for $q\to q_{max}$ and channels α including the 2-nucleon partial-wave state $^1S_0$. That slope is caused by the virtual-state pole of the $^1S_0$ 2-nucleon t-matrix. Based on such set of grid points interpolation of the amplitude $\langle\pi_2 q'\alpha|\hat{T}|\Phi\rangle$ needed for the next iteration is made at the required q'-quadrature points using spline technique [95].

The discretization in the subsystem momentum p was done with sufficient number of points distributed over the interval (0,25) fm^{-1}. Beyond p=25 fm^{-1} the 2-nucleon t-matrix is negligibly small. The required interpolation of the amplitudes at the values π_1 and π_2 was performed also by splines.

For further details of performance see also ref.[53,82].

This unique numerical algorithm enabling to perform 3N continuum calculations with realistic NN interactions has been confronted to a completely different one [96] which uses complex contour deformation and Lovelace-type equations for finite rank potentials. Using these finite rank potentials very good agreement results between the two codes[97]. The deviations for various nd elastic scattering observables were a few percent in the worst cases.

Three nucleon dynamics is a consequence of assumed NN interaction. Some details concerning the two-nucleon potentials used in this paper are presented in the next chapter.

III. Two-nucleon force input

The nucleon-nucleon interaction V forms a physical input for model of 3-nucleon dynamics presented in chapter II. This interaction is summed up to infinite order into the 2-nucleon transition operator t by Lipmann-Schwinger equation

$$t = V + VG_0 t. \tag{III.1}$$

The meson theoretical methods, based on the original Yukawa's idea[63] that the exchange of mesons is responsible for the internucleonic forces, were pursued to derive NN interaction. These methods provide clear insight into the interaction mechanism itself. They can be divided into the techniques using dispersion relations and into field-theoretic methods essentially based on covariant perturbation theory. These two approaches results in the Paris- and Bonn- potential models, respectively, both being adjusted in the two-nucleon system. These NN potentials describe the large amount of experimental 2-nucleon data fairly well.

In the following the momentum space representations of the Paris and Bonn potentials will be used. For the Paris potential the parametrization of ref.[1] will be taken and for the Bonn potential parametrizations in simple one-boson-exchange terms (OBEPQ) of refs.[2,64] will be used.

Such parametrization of Bonn model, as given in ref.[2], called OBEPQ(A) in the following has a low deuteron d-state probability P_d=4.38% and a substantially smaller ϵ_1 phaseshift parameter for E_{lab}≤150 MeV than the Paris potential with P_d=5.77% . Another parametrization, called in the following OBEPQ(B)[64], with P_d=5% has ϵ_1 values very close to the Paris ones[64]. Also the ^3P phases of OBEPQ(B) are closer to the Paris ones than for OBEPQ(A)[62,64].

Allowing for the charge independence breaking in t=1 two-body states increases drastically the number of coupled integral equations (II.14) due to nonvanishing $T=3/2$ amplitudes $\langle pq\alpha|\hat{T}|\Phi\rangle$. Since the permutation operator P cannot change the total isospin T which takes on the value $T=1/2$ for the initial state $|\Phi\rangle$, and since the coupling in $t=1$ two-body waves between $T={}^1/_2$ and $T={}^3/_2$ states is caused by nonvanishing small term propotional to $t_{nn}^{t=1}-t_{np}^{t=1}$, (see eq.(II.10)), so it is very probable that reliable predictions for some observables could be gained neglecting total isospin $T={}^3/_2$ states altogether. In such a case a very simple recipe of accounting for the charge independence breaking of the NN interaction in $t=1$ two-body states results. As an effective two-body t-operator one should use

$$t_{eff}^{t=1} = {}^2/_3\, t_{nn}^{t=1} + {}^1/_3\, t_{np}^{t=1}. \qquad \text{(III.2)}$$

This special linear combination for $t=1$ is refered to as the ${}^2/_3-{}^1/_3$ rule for the t-operator[81]. The importance of using such simple prescription follows from the fact that the number of coupled integral equations is then significantly reduced.

The OBEPQ potentials are adjusted to the np system and have consequently a stronger $^1S_0$ force than the Paris potential, which is adjusted to the pp system. Consequently , in the charge dependent calculations performed in the following according to 2/3-1/3 rule described above, the effective t-matrix for the $^1S_0$ state was obtained with t_{nn} (or t_{pp}) and t_{np} generated by the Paris- and OBEPQ-potentials, respectively.

In the fol.owing two chapters 3N scattering observables obtained with such realistic two-nucleon dynamics as presented above will be compared to some scattering and breakup experimental data. Such a comparison could yield a new information about the NN interaction itself or/and could indicate the presence of 3-nucleon force effects.

IV. Elastic nucleon-deuteron scattering

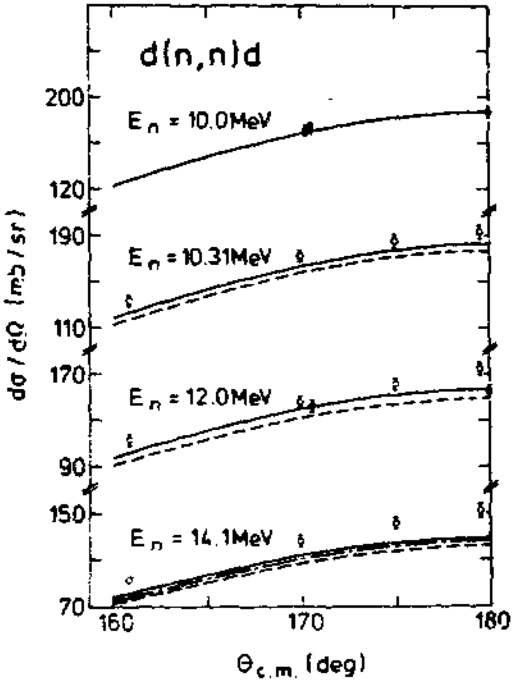
Elastic nucleon-deuteron scattering observables have been studied experimentally and theoretically with the hope to learn more about NN forces than what can be obtained from studying the two-nucleon systems only. However the theoretical studies up to now have been based on separable 2-nucleon forces of different ranks and different types of form-factors. The results obtained are obscured by the unclear physical relevance of the separable NN interactions used.

With the possibility to include exactly the dynamic of the two-nucleon subsystem while calculating three-nucleon observables, the way is now open to test quantitatively different physical dynamical pictures on the three-body level. So the natural question arise: how well the realistic NN interactions do describe the various elastic nucleon-deuteron scattering observables.

In fig.IV.1 the differential nd cross section at extreme backward angles at a few low neutron energies is shown. Measurements specifically performed for these angles region[65)] and data from[66)] are shown together with the theoretical predictions obtained with all $j \leq 2$ two-nucleon partial wave states. The agreement between the experimental data of ref.[65)] and the Paris potential prediction is perfect whereas the OBEPQ(A) prediction lies something below the data. There are systematic discrepancies when a comparison with the data from ref.[66)] is made. However, at higher energies these data have been corrected by a second measurement[67)] and agree then with experimental results of ref.[65)].

Perfect agreement for the Paris potential could not be however taken as the argument against OBEPQ(A) dynamics. Namely, in the scattering of neutrons on the deuteron one encounters simultaneously both nn and np forces in the isospin $t=1$ states. As mentioned in chapter II, np force in $^1S_0$ state is stronger than the pp (or assuming charge

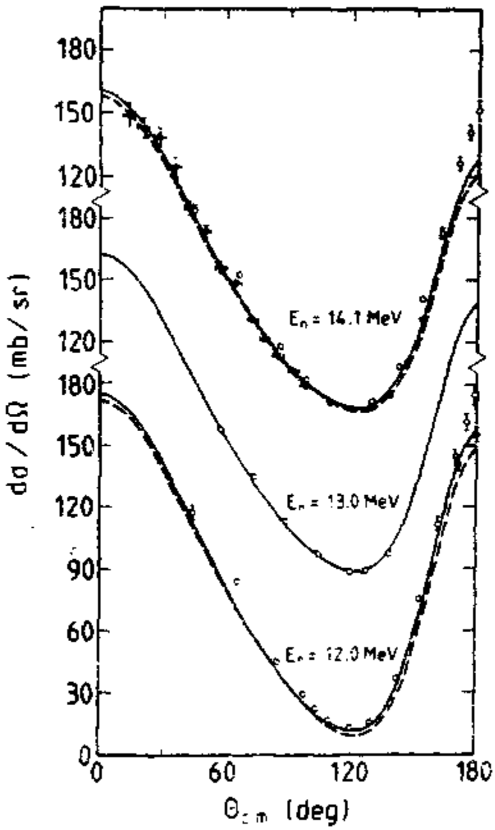
Fig. IV.1 Backward-angle differential cross sections for elastic nd scattering. Full points are data from ref.[65], open circles data from ref.[66]. The Paris and the OBEPQ(A) predictions are given by the solid and dashed lines, respectively. The dashed-dotted line at E_n=14.1 MeV is the result of the charge-dependent calculation for the state $^1S_0$ where the Paris potential is used for all other force componets.



symmetry, nn) force , as is evident from the values of the scattering lengths[64]. In order to study the influence of this effect on nd elastic scattering a calculation was performed at E=14.1 MeV using the t-matrix in the form given by 2/3-1/3 rule for $^1S_0$ state and the Paris potential t-matrix for all other states. For t_{nn} in $^1S_0$ state Paris potential (fitted to pp-data) was used whereas for t_{np} the version OBEPQ(B) (fitted to np data) was taken. The resulting curve lies between the Paris and OBEPQ(A) potentials predictions, closer to the Paris one. So taking properly into account the charge independence breaking of the NN interaction in $^1S_0$ state the differential cross section at extreme backward angles is appropriately reproduced by the Paris and Bonn dynamics.

In fig.IV.2 full angular distributions of the nd elastic scattering for three low energies are shown. The experimental data at 13 MeV netron energy[68] are normalized

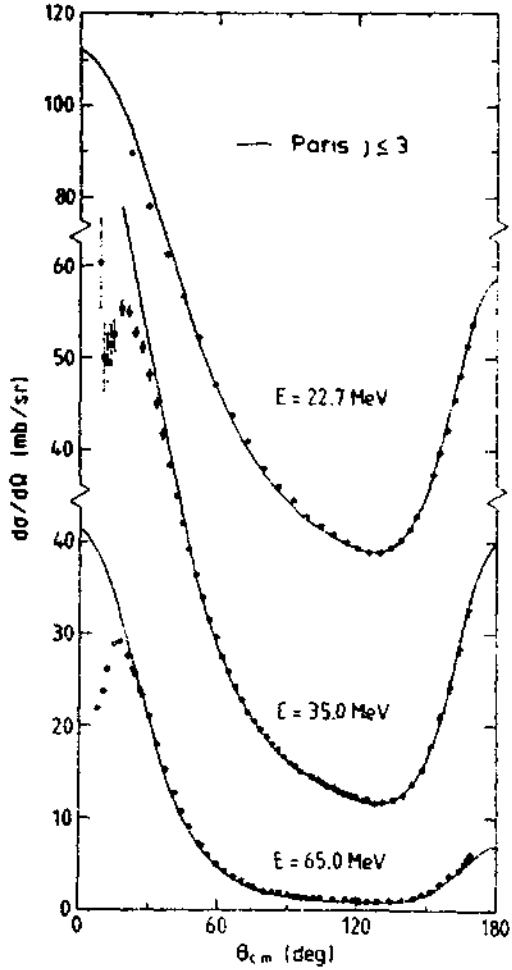
Fig. IV.2 Differential cross section for the elastic nd scattering. Open circles are data from: ref.[66] for E_n=12 and 14.1 MeV and from ref.[68] for E_n=13 MeV. Full points represent data from: ref.[65] for E_n=12 MeV and ref.[69] for E_n=14.1 MeV. For description of different curves giving theoretical predictions see fig.IV.1.



to theoretical predictions with the Paris potential by minimizing χ^2 deviations and agree then very well in shape with the theory. At E=14.1 MeV there is a nice agreement between theoretical predictions and experimental data taken at 14.3 ∓ 0.2 MeV[69]. The agreement with data from ref.[66] is not quite satisfactory but the same comment as made for the extreme backward-angle region applies here.

To see if such a nice agreement for the differential cross section is obtained also at higher nucleon energies a calculations with the Paris potential were made at E=22.7, 35, and 65 MeV. All j≤3 partial wave states were taken into account. In fig.IV.3 resulting angular distributions are shown together with experimental pd data. The agreement between theory and experiment is excellent for angles

Fig. IV.3 Differential cross section for the elastic pd scattering. The points are experimental data taken from : ref.[70] for E=22.7 MeV, ref.[71] for E=35 MeV, and ref.[72] for E=65 MeV. The solid lines are predictions of the Paris potential including all j≤3 force components.



$\theta_{cm} \geq 45^{\circ}$. Only at extreme forwards angles distinct discrepancies appear caused by Coulomb forces which are totally neglected in our theoretical calculations.

To resume, the experimental angular distribution of Nd elastic scattering differential cross section is appropriately reproduced by the Paris and Bonn dynamics when charge independence breaking of the NN interaction in $^1S_0$ state is properly taken into account.

It is known from the nucleon-nucleon scattering that the spin observables are much more sensitive to the details of the NN interaction than the cross section. In fig.IV.4 the

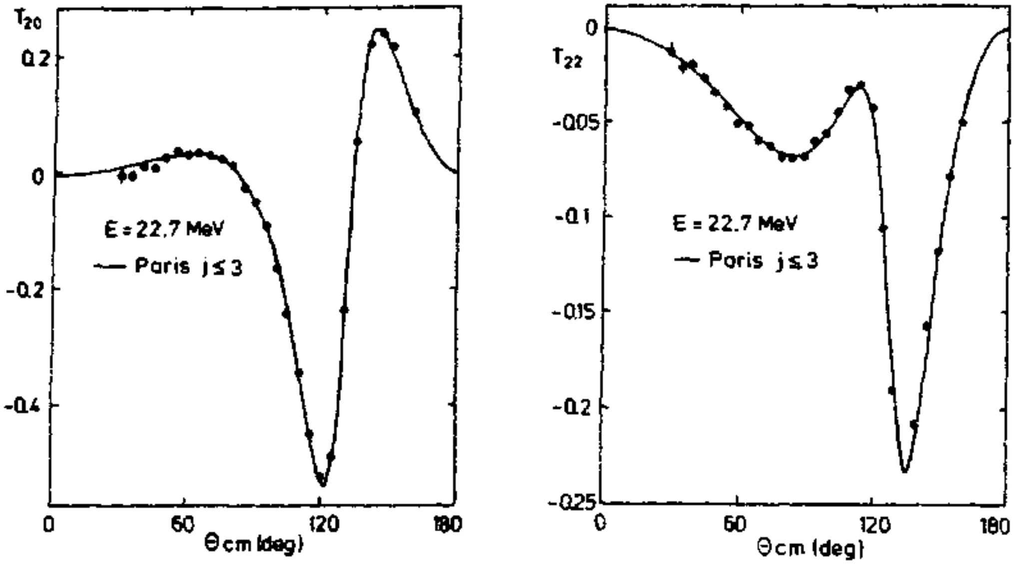


Fig. IV.4 The tensor analysing powers T_{20} and T_{22} for pd elastic scattering. Open circles are data from ref.[70]. The solid line is the Paris potential prediction obtained with all j≤3 partial wave states.

tensor analysing powers T_{20} and T_{22} for pd elastic scattering at E_p=22.7 MeV together with the Paris potential predictions are shown. The curves obtained with OBEPQ(A) and OBEPQ(B) potentials are hardly distinguishable. Thus all three potential predictions agree spectacularly well with the experimental data. The agreement of different potential predictions indicates that the tensor analysing powers at these energy are not sensitive observables to determine the details of NN interactions.

A striking discrepancy, however, shows up at these and lower energies in the nucleon and deuteron vector analysing powers A_y and iT_{11}. In figs.IV.5,6 theoretical predictions and experimental data for A_y at E=10 and 12 MeV[49] are shown. The theory deviates from experimental data especially in and around the maximum of A_y , near θ_{cm}≅120°.

Fig. IV.5 Analysing power A_y for nd elastic scattering at E_n=10 MeV. Open circles are nd experimental data taken from ref.[49]. The Paris potential predictions with j≤3, j≤2 and $^1S_0+^3S_1-^3D_1$ force components are given by solid, dashed and dashed-dotted lines, repectively. The dashed-double dotted curve is the Paris potential prediction (j≤2) with $^3P_0$-wave strength reduced by a factor λ=0.95.

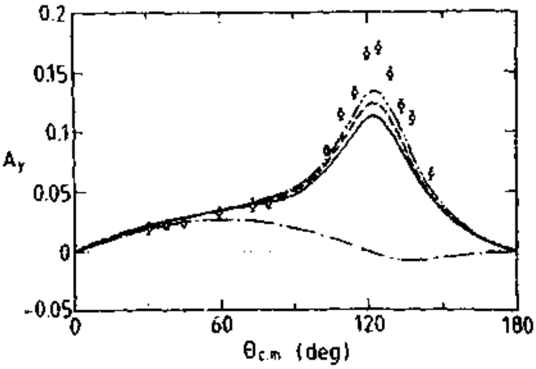
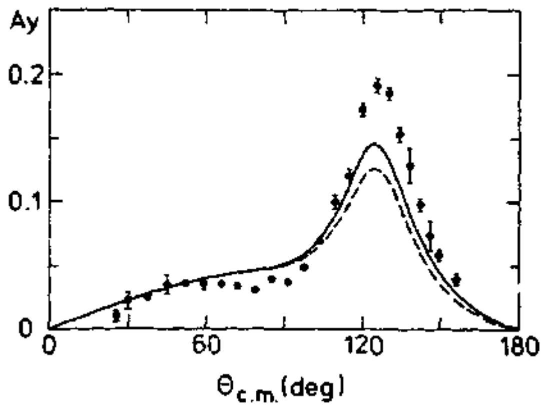


Fig. IV.6 Analysing power A_y for nd elastic scattering at E_n=12 MeV. Full points are nd experimental data taken from ref.[49]. The solid and dashed lines are predictions for the Paris and OBEPQ(A) potentials, respectively, obtained with j≤2 partial wave states.



Theoretical predictions shown in fig.IV.5 by dashed and continuous lines were obtained with the Paris potential taking partial wave states j≤2 and j≤3, respectively, into account. As can be seen the inclusion of j=3 states does not increase the maximum of A_y but to the contrary, it increases slightly the discrepancy with the experimental data.

It was shown already in [49] that maximum in A_y at low energy results mainly from an interplay between ^3P waves. Really, neglecting all ^3P states gives no maximum at all (dashed-dotted curve in fig.IV.5). The theoretical maximum of A_y increases with decreasing strength of the $^3P_0$ force[49].

This explains the stronger discrepancy in maximum of A_y for the OBEPQ(A) (see fig.IV.6), which is very likely caused by its $^3P_0$ phase shifts which are larger than those obtained with the Paris potential [62].

In order to explain the discrepancy in maximum of A_y one could think about decreasing the strength of $^3P_0$ force so much as allowed by the experimental two-nucleon np data. Multipliyng the potential matrix element for the $^3P_0$ wave by a factor $\lambda<1$ one can reduce the λ in case of the Paris potential to the value of 95%[62]. Further reduction of λ when keeping the remaining potential components constant is forbidden by 2-nucleon np data. Such reduction increases, as expected, the values of A_y in the maximum but by far not sufficiently (see dashed-double dotted curve in fig.IV.5). One can further lift the maximum of A_y by increasing in magnitude the $^3P_1$ phase shifts , what, however, would also lead to increase of polarisation for the free np scattering and this is definitely ruled out by the experimental 2-nucleon data. The similar effect exists also for the $^3P_2$ partial wave. One cannot expect that some specific interplay of $^3P_0$ and $^3P_1$ (or/and $^3P_2$) (a weaker decrease of $^3P_0$ strength and some increase of $^3P_1$ (or/and $^3P_2$) strength), as allowed by 2-nucleon data, would remove the discrepancy between experimental A_y data and theoretical prediction.

In [73] the influence of the mixing parameter ε_1 on the analysing power A_y was pointed out. "Experimental" ε_1 values are still plagued by very large error bars[22]. However freedom given by the large error bars in the mixing parameter ε_1 also cannot cure discrepancy in maximum of A_y. A very drastic change of ε_1 ($\varepsilon_1 \cong 4$ for E<60 MeV) increases the maximum of A_y by about 7% only[73].

Other possibility to account for the discrepancy in maximum of A_y offers the charge dependence of the NN interaction in the $^1S_0$ wave. Calculation made at E=10 MeV using simple 2/3-1/3 rule has not a noticeable effect (see also ref.[74]). However ,due to the strong dependence of the A_y on ^3P force components one can argue that such a simple treating of the charge dependence in $^1S_0$ state is insufficient. The possibility exists that a proper charge dependent calculation which include the 3-body channels with isospin $T=3/2$ for $^1S_0$ 2-body subsystem would result in quite different A_y values. However , such calculation made at E=10 MeV gives A_y values which are not distinguishable from the Paris potential prediction on scale of fig.IV.5. Thus charge dependence in $^1S_0$ force component could not be responsible for the discrepancy in maximum of A_y.

Before seeking other possibilities to explain low energy A_y behaviour let us see how the problem looks like at higher energies. In fig.IV.7 the theoretical predictions for A_y at E=22.7, 35, 50 and 65 MeV are shown and compared with experimental pd data[70-72,75]. With increasing energy the disagreement between theory and experiment gets smaller. For energies E≥50 MeV very good description of experimental data results. It is also interesting to note that with increasing energy the sensitivity of A_y to the ^3P force components gets smaller. Switching-off these force components while calculating A_y does not change for energies E≥35 MeV the values of this observable so drastically as at lower energies. This indicates that the solution of A_y puzzle maybe should be sought mainly in behaviour of ^3P force components.

Neutron analysing power A_y is not the only spin observable in nucleon-deuteron scattering which exhibits sensitivity to ^3P waves at low nucleon energies. A study of nearly complete set of pd polarization data at E_p=10 MeV[76] revealed other observables showing definite discrepancies between theory and experiment and also sensitive to ^3P interactions[74]. Especially interesting are, besides A_y , the deuteron analysing power iT_{11} and the proton to deuteron polarization transfer coefficients $K_y^{x'z'}$ and $K_z^{y'z'}$.

It is interesting to note that changes of $K_y^{x'z'}$,$K_z^{y'z'}$ in comparison to A_y and iT_{11} when turning-off the three ^3P state contributions are different. For A_y and iT_{11} all ^3P components are important to create the maximum at $\theta_{cm}\cong 120^\circ$.

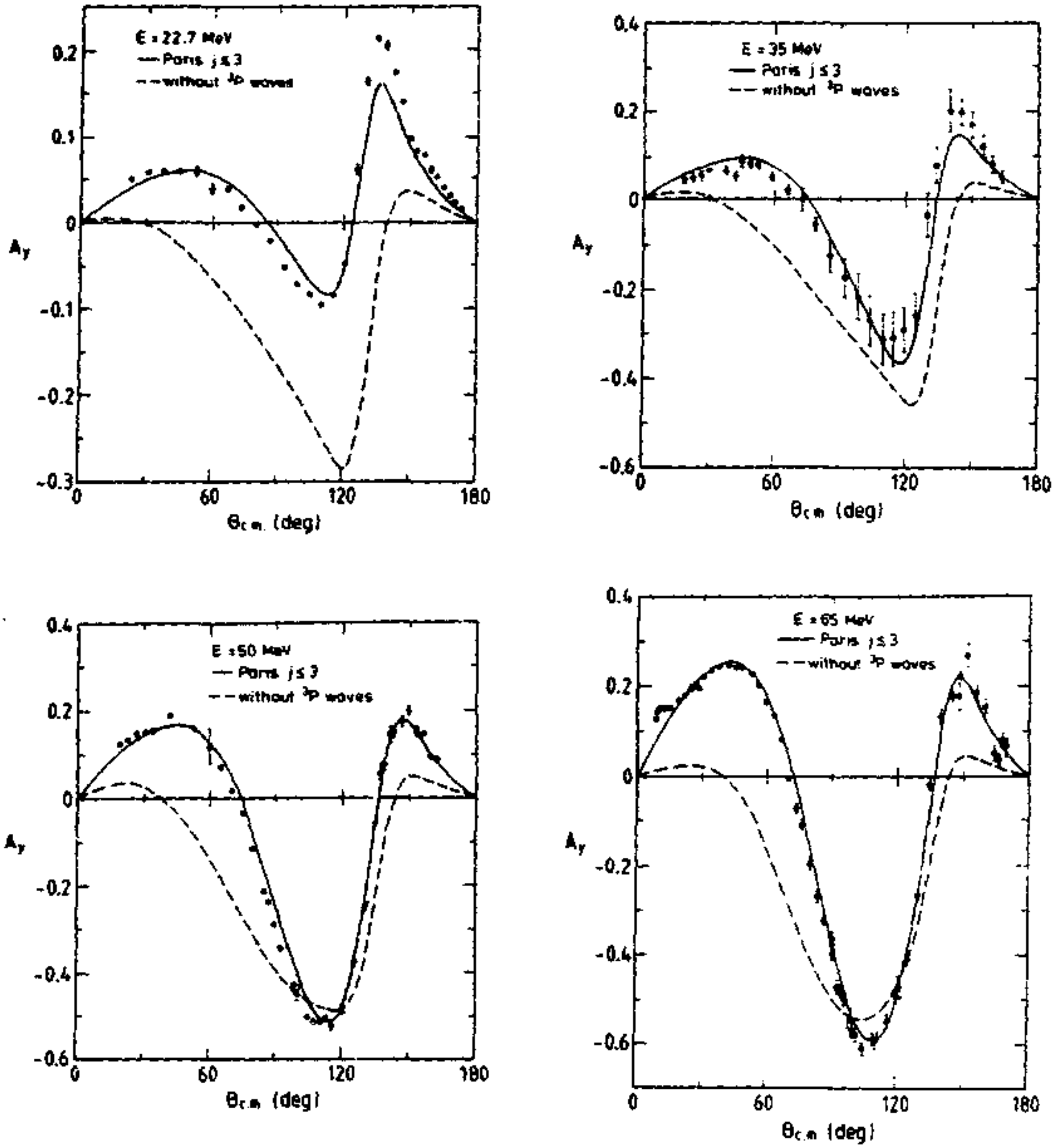


Fig. IV.7 Analysing power A_y for pd elastic scattering. The points are experimental data taken from : ref.[70] for E=22.7 MeV, ref.[71] for E=35 MeV, ref.[75] for E=50 MeV, and ref.[72] for E=65 MeV. The solid lines are predictions of the Paris potential including all $j \leq 3$ force components. The dashed curves are obtained neglecting $^3P_0$, $^3P_1$ and $^3P_2$-$^3F_2$ partial wave contributions while calculating A_y.

Turning off the $^3P_0$ wave increases the maximum, while opposite effects come from $^3P_{1,2}$ components. A similar dependence is found for $K_z^{y'z'}$ in the maximum region around 120°. However the $K_z^{y'z'}$ values at backward angles ($\theta_{cm} \geq 150°$) are mainly influenced by $^3P_1$ and $^3P_2$ partial waves, increasing when turning off those componets. In the case of $K_y^{x'z'}$ its minimum at about 120° is affected predominantly by $^3P_0$ and $^3P_1$ waves and turning them off deepens this minimum. For angles $\theta_{cm} \geq 150°$ $K_y^{x'z'}$ is practically sensitive only to the $^3P_0$ wave components. In this region turning off the $^3P_0$ component increases the $K_y^{x'z'}$ values. This quite different behaviour is observed also at E=22.7 MeV.

The strong sensitivity to ^3P wave components seen in these observables together with the discrepancies between the experimental data and the theory force one to study the possible effects of charge independence violation (np nuclear interaction not equal to nn and/or pp interactions) in the ^3P wave force components. The difference between the predictions of the Paris (pp) and the OBEPQ(A) (np) potential , caused by slightly different ^3P wave force components, did not explain the existing discrepancies[62,74) (see also fig.IV.6). Both lie well below the experimental values while a charge dependent calculation for ^3P wave force components based on these two potentials would lie between these predictions. However, there are indications coming from very precise polarization measurements in low energy pp scattering and from energy independent phase shift analysis of these data[77,78) that low energy ^3P-phase shifts should have other values than those given in the energy dependent phase shift analysis of[22) or predicted by the Paris or OBEPQ(B) potentials (see table 1). Also recent energy dependent phase-shift analysis of np and pp scattering data below E_{lab}=30 MeV[79) revealed definite breaking of charge independence in the πNN coupling constants, resulting in np-^3P phase shifts quite different from earlier analyses.

As a first step in investigation of the possibility, that the proper description of the low energy ^3P pp phase

Table 1 Phase shifts in ^3P waves.

E_{lab} (MeV)	ref.[77]	ref.[78]	ref.[22] np	ref.[22] pp	partial wave
6.14		1.65±0.16 1.64±0.28			$^3P_0$
10.0	2.62±0.40		3.65±0.04	3.38±0.04	$^3P_0$
6.14		-1.14±0.04 -1.19±0.07			$^3P_1$
10.0	-1.94±0.10		-2.18±0.02	-2.04±0.02	$^3P_1$
6.14		0.31±0.03 0.27±0.05			$^3P_2$
10.0	0.64±0.09		0.70±0.01	0.62±0.01	$^3P_2$

E_{lab} (MeV)	potential					partial wave
	Paris	OBEPQ(B)	OBEPQ(B)			
			λ_{pp}	λ_{np}	λ_{nn}	
6.14	2.47	2.44	1.49	2.05	1.36	$^3P_0$
10.0	4.25	4.24	2.57	3.54	2.35	$^3P_0$
6.14	-1.37	-1.36	-1.20	-1.28	-1.48	$^3P_1$
10.0	-2.31	-2.31	-2.05	-2.19	-2.52	$^3P_1$
6.14	0.36	0.34	0.30	0.34	0.30	$^3P_2$
10.0	0.74	0.71	0.62	0.70	0.62	$^3P_2$

Table 2 The strength factor λ used to construct different interactions in ^3P waves starting from OBEPQ(B) potential.

partial wave state	λ		
	pp interaction	np interaction	nn interaction
$^3P_0$	0.65	0.88	0.60
$^3P_1$	0.87	0.94	1.10
$^3P_2$-$^3F_2$	0.90	0.99	0.90

shifts might remove or at least decrease the existing descrepancies in elastic 10 MeV pd scattering, lets start from a potential, which acts only in $j \leq 2$ states and is equal to OBEPQ(B) in the ^3P states and to the Paris potential in the remaining states. The solid curves in figs.IV.8-11 are the predictions of that potential. Then let us change the strength in the ^3P wave force components with a factor λ in order to obtain the pp low energy phase shifts from[77,78]. The resulting potential with the factor λ_{pp} from table 2 is used in the following for the pp system. In a similar manner the potential for the np system is constructed by the use of the factors λ_{np} of table 2, which are chosen so that the low energy np ^3P phase shifts[22] are reproduced. With these two potentials charge dependent calculations with respect to the ^3P forces were performed (according to the 2/3-1/3 rule). The dashed curves in figs.IV.8-11 show the result of this calculations. A remarkably good description of the pd data results. In case of $K_y^{x'z'}$ the discrepancy left at backward angles (fig.IV.11) could be removed by adding the $j=3$ force components[74].

The comparison of the nd and pd A_y data in fig.IV.8 shows some difference especially in the region of the maximum. One reason for this difference could be the Coulomb force effects. However since no rigorous method to include the Coulomb force into the 3-body continuum calculations has been worked out and applied up to now, the theoretical estimation of the magnitude of these effects is not known. If we assume that these effects are small one is tempted to explain this difference by a charge symmetry breaking (nn interaction not equal to pp one) in the ^3P wave force components.

In [80] a motivation was given to differentiate between nn and pp force. It was claimed that the up and down quark mass differences lead to especially large effects in the $^3P_0$ and $^3P_1$ states.

Guided by the known dependence of A_y on the strength factor λ of the $^3P_0$ and $^3P_1$ force components, we chose the

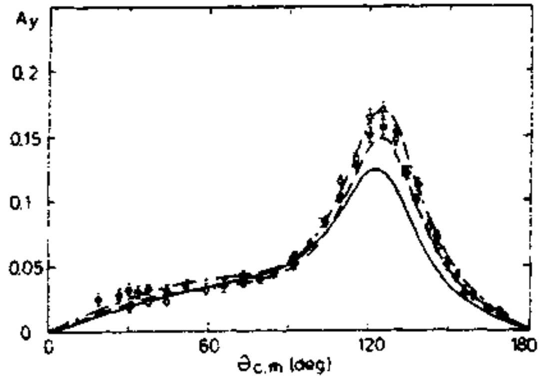


Fig.IV.8 Nucleon analysing power A_y in Nd elastic scattering at E=10 MeV. Open circles are nd data from ref.[49], full points - pd data from ref.[76]. Theoretical predictions obtained with different interactions in ^3P waves are given by curves. The solid line results with a "mixed" potential equal to the OBEPQ(B) potential in ^3P force components and to the Paris potential in the rest. Predictions of the charge dependent (in ^3P waves) calculations for the pd and nd systems are given by dashed and dashed-dotted curves, respectively. In all calculations the forces were restricted to act only in j≤2 partial wave states.

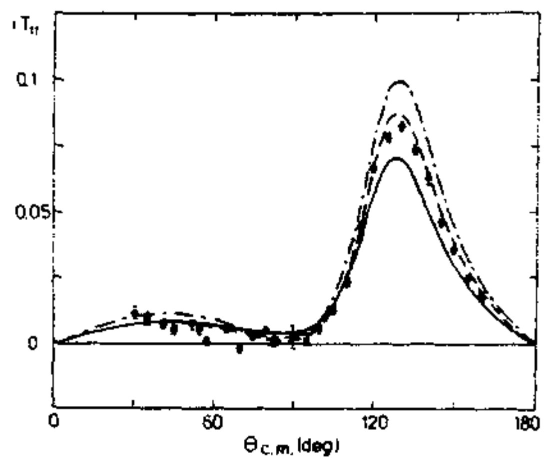


Fig.IV.9 The same as in fig.IV.8 but for iT_{11}.

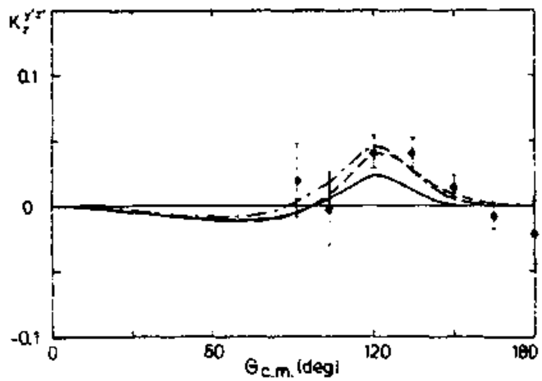


Fig. IV.10 The same as in fig. IV.8 but for $K_z^{y'z'}$.

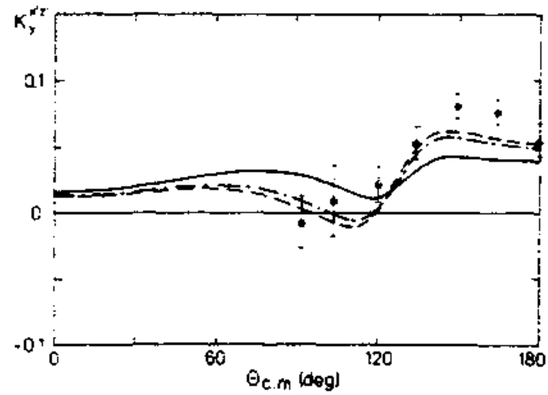


Fig. IV.11 The same as in fig. IV.8 but for $K_y^{x'z'}$.

λ_{nn} values of table 2. Taking such a potential as a nn interaction in a charge dependent calculation with respect to the ^3P wave forces, the dashed-dotted curves of figs. IV.8-11 result. A remarkably good description of the nd A_y data is thereby achieved in the region of maximum. Furthermore interesting enough a large difference between the iT_{11} for nd and pd scattering is predicted within this approach.

It should be noted that the required changes in the $^3P_0$ and $^3P_1$ nn phase shifts are relatively large (see table 1). For $^3P_0$ they go in the direction claimed in ref.[80]. For $^3P_1$, however, the change needed to get a good description of nd A_y data is opposite to that in[80].

The results presented above indicate that probably in order to explain existing discrepancies in polarization observables of low energy elastic Nd scattering the Paris- and Bonn-^3P force components have to be readjusted. The natural question arises, whether the meson exchange theory of nuclear forces can cope with such a modification of specific force components. This interesting question as well as influence of the total isospin T=3/2 ^3P states require futher studies. In any case it seems to be quite plausible that study of elastic Nd scattering will be helpful in establishing the isospin symmetry breaking effects in NN interaction, thus contributing significantly to fix the details of 2-nucleon dynamics.

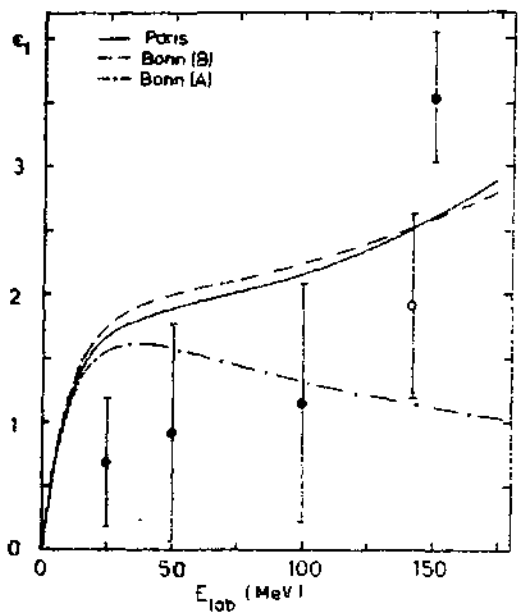


Fig.IV.12 The $^3S_1-^3D_1$ phase shift parameter ϵ_1 as given by [22] (full points) and [81] (open circle) and predicted by the Paris, OBEPQ(A) and OPEPQ(B) potentials.

Another example of the 2N force property which is rather poorly known is the strength of the tensor force causing the coupling of the $^3S_1$ and $^3D_1$ 2N states. Phase shift parameter ϵ_1 is a measure for the strength of that coupling. The "experimental ϵ_1 values"[22,81] for $E_{lab}\leq150$ MeV are shown in fig.IV.12 together with the predictions given by different potential models. The OBEPQ(A) potential has a significantly weaker tensor force than the Paris and OBEPQ(B) ones. Neutron-proton scattering data measured up to now are not sensitive enough to determine ϵ_1 sufficiently accurately in order to distinguish between different tensor forces.

The weaker tensor force goes with a larger central force which provides the larger triton binding energy[20,21] in case of OBEPQ(A) potential. This reduces the discrepancy between the theoretical and experimental triton binding energy and leaves only a few hundred keV discrepancy to be filled by the contribution of 3-nucleon force effects. The important question arises which potential gives the proper tensor force.

Analysis of the nearly complete set of polarization pd data at E=10 MeV[74] revealed a few spin observables which show some sensitivity to the tensor force. The most promising ones were the nucleon to nucleon polarisation transfer coefficients $K_y^{y'}$ and $K_z^{x'}$. However the comparison of Paris and OBEPQ(A) potentials predictions and the $K_z^{x'}$ data was not conclusive, since the inclusion of j=3 force components puts the predictions from Paris and OBEPQ(A) potentials on the lower and upper side of the experimental error bars, respectively. In case of $K_y^{y'}$ the situation is also unclear since data for angles $\theta_{cm}\leq90°$ favoured OBEPQ(A) potential and two points at larger angles with larger error bars favoured Paris potential. There is also some scatter in the experimental data of $K_y^{y'}$ which weakens any conclusions.

Since the sensitivity of $K_y^{y'}$ to $^3S_1-^3D_1$ tensor force appears also at higher energies, new measurement of this observable was performed at an incident proton energy of 22.7 MeV[82]. The measured and calculated $K_y^{y'}$ values are shown in fig.IV.13. The difference between the Paris and OBEPQ(A) predictions can be shown to be just a consequence of the

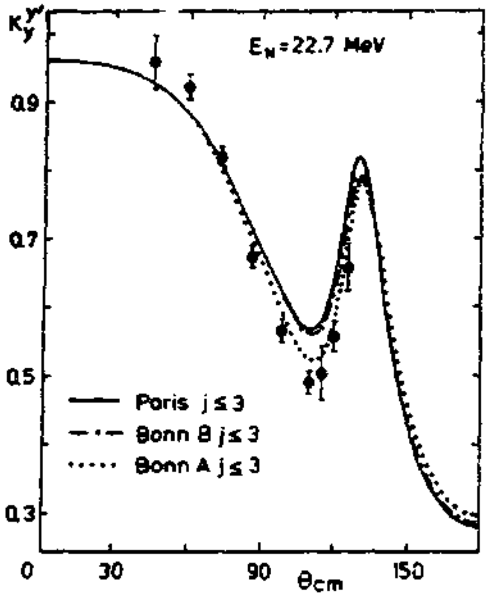


Fig.IV.13 The experimental nucleon spin transfer coefficient $K_y^{y'}$ [82] in pd scattering in comparison to different theoretical predictions. The Paris-, OBEPQ(A)- and OBEPQ(B)-potential predictions (for j≤3 partial wave states) are given by the solid, dotted and dashed-dotted lines, respectively.

different ${}^3S_1-{}^3D_1$ 2N forces. Eventually rather strong modifications of ${}^3P$ phase shifts required to remove the discrepancy between theory and experiment for vector analysing powers and $K_z^{y'z'}$ and $K_y^{x'z'}$ coefficients at E=10 MeV do not lead to a noticeable shift of the theoretical curve of $K_y^{y'}$.

Very good agreement between nd theory and pd data for differential cross section (fig.IV.3) and tensor analysing powers T_{20} and T_{22} (fig.IV.4) suggests that at this energy Coulomb force effects are not very important. These new data clearly favour the OBEPQ(A) potential with the weaker tensor force. Precise measurements at other energies and especially for nd system, would be very useful to confirm that result.

V. Nucleon induced deuteron breakup

Breakup of the deuteron by incoming nucleon seems to be a fruitfull source of information on two- and/or three-nucleon forces. The most important feature in this respect is the possibility to chose the specific geometrical configurations for three outgoing nucleons. It allows to study in detail possible momentum dependent effects of nuclear dynamics, which due to unavoidable momentum averaging procedure are more difficult to see in elastic Nd scattering.

Both two- and three-nucleon aspects of underlying dynamics can be studied in such a process. The study of kinematical configurations which are most sensitive to two-nucleon force properties could serve to test different 2N force models. On the other side , finding the discrepancies between the theory based on 2N forces only and experimental data in kinematical configurations, which are insensitive to special 2-nucleon force properties, could be a signature of the action of 3-nucleon force. Thus detailed study of these configurations could yield information about 3N force effects.

Such a study requires rich and precise data base, obtained in kinematically complete measurements. Study of Nd elastic scattering indicates that especially interesting would be polarization data. However the experimental data for nucleon induced deuteron breakup are much more poor than in case of Nd elastic scattering. Especially neutron induced deuteron breakup data are rarity up to now. Only recently few kinematically complete nd breakup measurements (also with polarized neutrons) were performed or are planned[83-86].

In searching for three-nucleon force (3NF) effects in break-up configurations one can follow the suggestions found in the first model calculation[87] including a 3NF. In this calculation simple two-nucleon force model (the S-wave Malfliet-Tjon[88] potentials MT I,III in a unitary pole approximation) together with 2π-exchange 3NF, averaged over spin and isospin, was used. There the space star, the

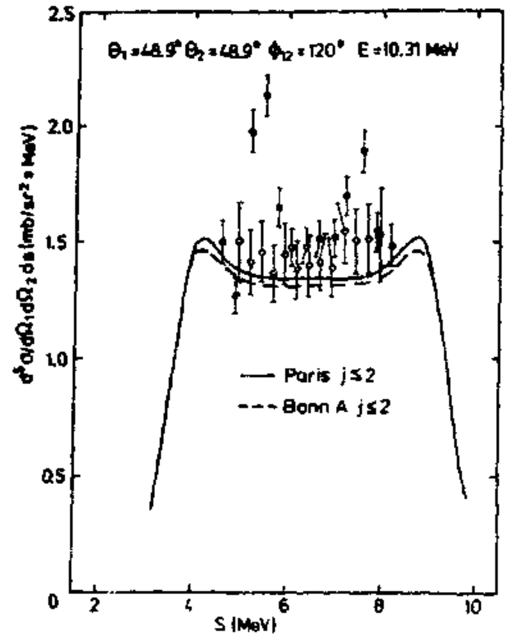


Fig. V.1 The cross section $d^5\sigma/d\Omega_1 d\Omega_2 dS$ as a function of arc length S along 3-body kinematics for the space star configuration at E=10.3 MeV, $\Theta_1=\Theta_2=48.9°$ and $\phi_{12}=120°$. The nd data of refs.[83)] and [84)] are given by open circles and full points, respectively. The solid and dashed lines present the Paris- and OBEPQ(A)-potential predictions, respectively, with 2-nucleon forces restricted to act in the $j\leq 2$ partial waves.

collinear, quasi free scattering (QFS) and final state interaction (FSI) configurations turned out to be the most promising.

In the space star configuration the momenta of the three nucleons form an equilateral triangle which is perpendicular to the beam in the c.m. system. For this configuration in model calculation[87)] an enhancement of the cross section due to 3NF was found. In fig. V.1 the theoretical break-up cross sections for the Paris and OBEPQ(A) potentials are compared to two recent nd measurements at 10.3 MeV[83,84)]. As expected from model studies with a set of pure S-wave forces[38,39)] the

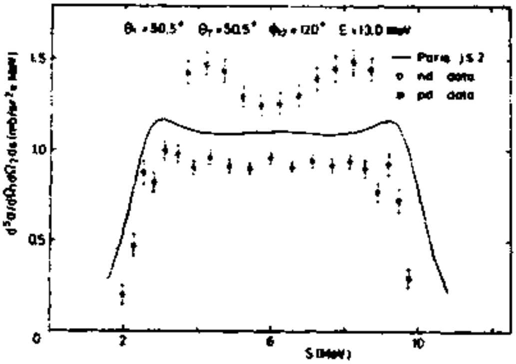


Fig. V.2 The same as in fig. V.1 but for the space star at E=13 MeV. Open circles are nd experimental data from ref.[85]; full points - pd data from ref.[89] The line is the prediction of the Paris potential obtained with all $j \le 2$ partial wave states.

space star configuration turns out to be insensitive to the choice of 2N forces (a necessary requirement to see 3NF effects) and the two theoretical curves are close together. Though the data have a tendency to lie above the theory, they do not agree sufficiently well between themselves to draw a conclusion. In fig. V.2 another example of space star configuration at 13 MeV is shown. Here the nd data[85] lie far above the theory while the pd data[89] lie below. It is very unlikely that Coulomb forces cause such a big difference. It is seen from figs. V.1,2 that the energy dependence of the theoretical star cross sections is strong, whereas nd data of[84,85] are nearly energy independent. That controversial situation prohibits a conclusion about a significant discrepancy between experiment and theory which could be a signature of the action of a 3NF. To clarify the situation independent measurements would be very desirable.

The **collinear configuration** for which one nucleon is at rest in the c.m. system, has been under investigation for quite some time[91]. In model study[87] an enhancement of the cross section caused by 3NF was found in the collinear

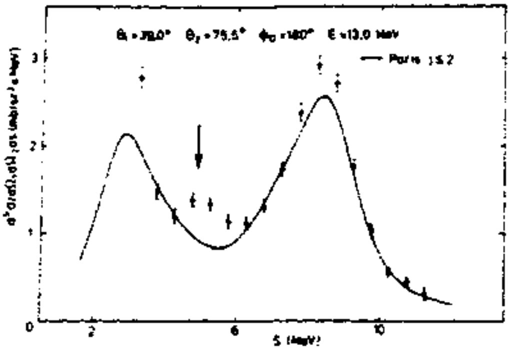


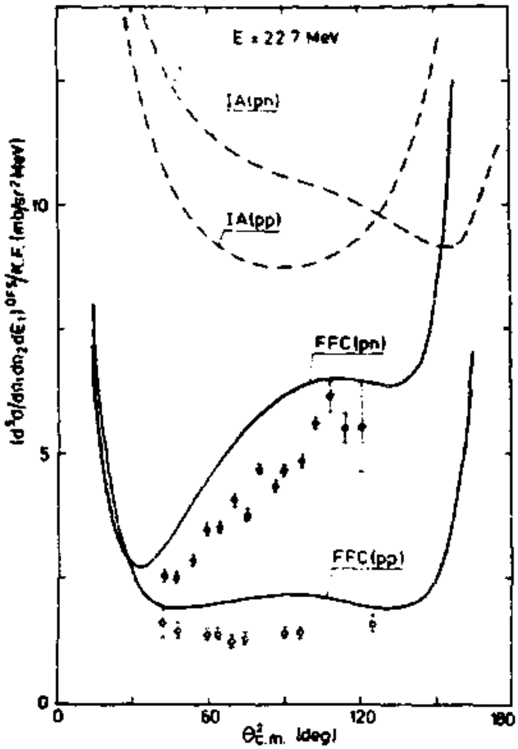
Fig. V.3 The same as in fig. V.1 but for the collinear configuration at $\Theta_1=39^\circ$, $\Theta_2=75.5^\circ$ and $\phi_{12}=180^\circ$. The collinearity point is indicated by the arrow.

region. In fig. V.3 experimental data from nd measurement at 13 MeV[85] are compared to theory. The point of collinearity is indicated by an arrow. The bump present in the experimental data is absent in the theoretical curves. It is interesting to note that cross section in collinear region is quite insensitive to variations in NN interactions[61]. Similar discrepancy between theory and experiment in the collinear region is found at 14.1 MeV[61]. In that case the breakup process of the deuteron was induced by protons.

It is tempting to associate these discrepancies with the effect of a 3NF so additional measurements concentrating on the collinearity condition would be very welcome.

The special geometrical configuration with one of three outgoing nucleons at rest in laboratory system (quasi free scattering condition) aroused since a long time an interest as a tool to study free NN interaction (see [92] and references therein). Such a possibility results when underlying mechanism is predominantly given by the interaction of incoming nucleon with only one of the substituent particles in the deuteron what is more probable with increasing energy due to smaller wavelength of incoming nucleon. For sufficiently high energy the polarization

Fig. V.4 Comparison of impulse approximation (IA) and full Faddeev calculation (FFC) predictions with QFS pn (full points) and pp (open circles) experimental angular distributions [93] at E=22.7 MeV. Theoretical results are obtained with the Paris potential restricted to act in $j \leq 2$ partial wave states.



observables in such configuration are then equal to free scattering ones[92].

If this simple mechanism is correct then the solution of a three nucleon scattering problem is reduced to calculation of leading term $\langle pq\alpha | tP | \Phi \rangle$ in integral equation (II.14) (so called impulse approximation (IA)). All numerical difficulties associated with generation of the Neumann series (multiple scattering series (MSS)) for eq.(II.14) disappear in this case.

The rescattering terms in the MSS are very important for QFS configuration for nucleon laboratory energies E≤100 MeV. Even at 140 MeV the correction to the IA under QFS condition is nonnegligible. In this case, however, rescattering of second order in the 2-body t-matrix is essentially sufficient[92].

The rescattering for identical nucleon pairs under QFS condition is strickingly different from the case of nonidentical nucleon pairs. In fig. V.4 this is shown in case of the angular distribution of QFS scattering at E=22.7 MeV[93]. At this energy there is the strong , even qualitative disagreement of the IA with the data, whereas the full Faddeev calculation (FFC) follows nicely the shape of the data. In fig. V.5 another example, analysing power under QFS condition for pp and np pairs at 65 MeV[94], is shown. While the full 3-body process leads at least to the same sign of A_y as for free NN scattering in the case of np, the sign and magnitude is different in the case of pp. Clearly the full Faddeev calculation describes the experimental data rather well.

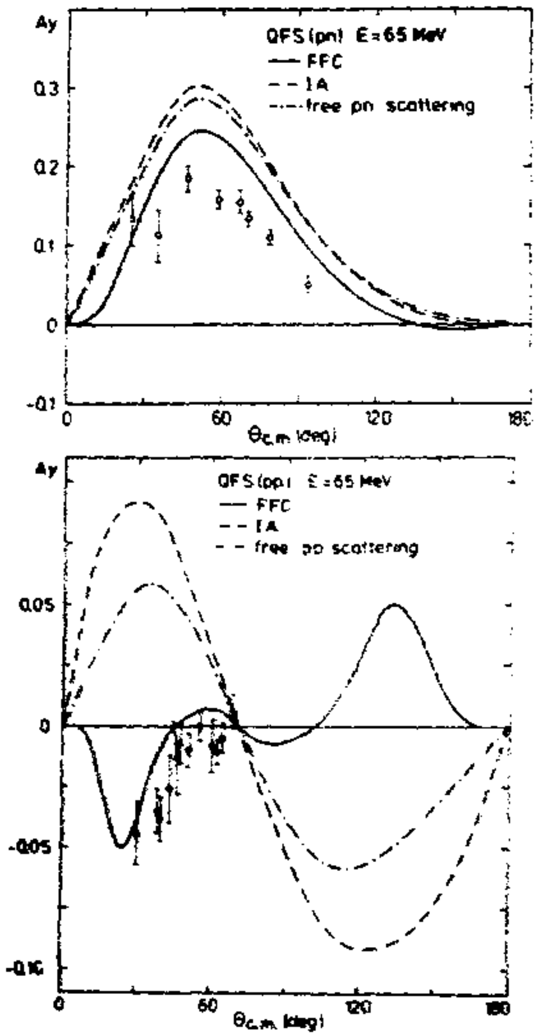
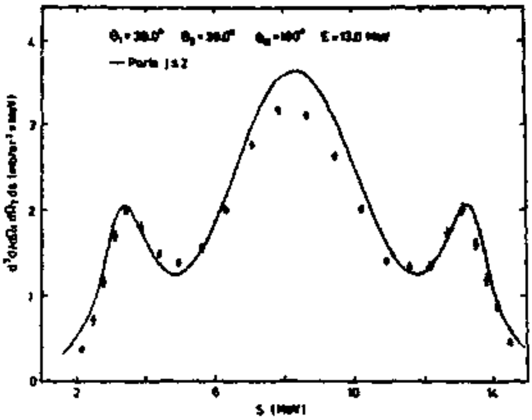


Fig. V.5 Impulse approximation and full Faddeev calculation predictions compared with QFS pn and pp experimental data for nucleon analysing power at E=65 MeV[94]. For description of continuous and dashed curves see fig. V.4. The dashed - dotted line represents the free pn (pp) scattering.

Fig.V.6 The same as in fig.V.1 but for QFS configuration at $\Theta_1=\Theta_2=39^\circ$ and $\phi_{12}=180^\circ$. Closed circles are preliminary pd data of ref.[89]. The solid line is the prediction of the Paris potential obtained with all j≤2 force components.



In fig.V.6 a comparison of the Paris potential predictions with preliminary pd data at E=13 MeV[89] arround QFS condition is shown. It is interesting to note that the theory is a bit higher in the QFS peak than experiment. It would be interesting to compare theory also with QFS peaks from nd experiments which would remove the uncertainties caused by possible Coulomb-effects. If that discrepancy would be confirmed also at higher energies, this could indicate an effect of 3NF like it has been found in the model calculation of ref.[87].

In case when two of three outgoing nucleons have equal momenta (final state interaction condition), the process is dominated by $^1S_0$ interaction. It has been shown by Kloet and Tjon[39] that the heights of FSI peaks depend very sensitively on the $^1S_0$ scattering lengths and on other 2N force components and their properties. This is confirmed by rigorous calculation at E=14.1 MeV[61] using the Paris potential (fitted to the pp system) and OBEPQ (fitted to the np system). This calculation showed that the heights of the FSI peaks were different according to the different scattering lengths of the two potentials. It follows that charge-dependent $^1S_0$ forces should be used to analyze the FSI peaks.

Charge dependent calculation according to 2/3-1/3 rule

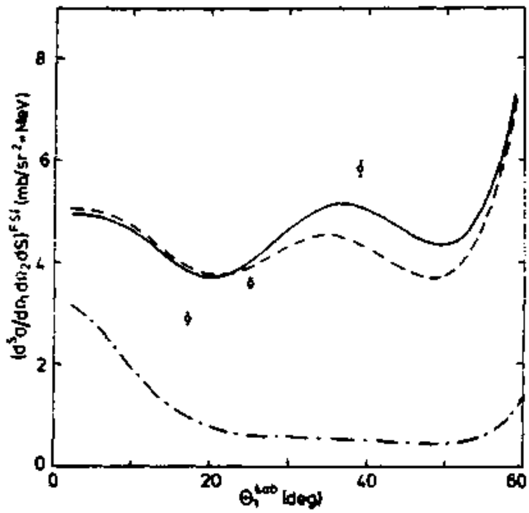


Fig. V.7 Dependence of the cross section at the FSI condition for particles 1-3 ($p_1^{lab}=p_3^{lab}$) on the production angle Θ_1^{lab}. The solid and dashed curves are the predictions of the charge dependent calculations including the 2N forces in all $j\leq 2$ partial wave states and in ${}^1S_0$ + ${}^3S_1-{}^3D_1$, respectively. The dashed-dotted curve derives from solid one, when all kinematical factors are taken out. Open circles are E=13 MeV nd experimental data of ref.[85].

with the Paris potential as ${}^1S_0$ nn interaction and OBEPQ(B) potential as ${}^1S_0$ np interaction has been performed and compared with experimental nd FSI data at E=13 MeV[90]. Discrepancies between theory and data, not only in the peak heights but also in the shapes, has been found. The experimental and theoretical FSI peak heights are presented in fig. V.7 as a function of the laboratory production angle of the interacting n-p pair. The differences between theory and experiment are reminiscent of a trend found in a model study[87], where it was found that the contribution of the 3NF at E=14.4 MeV increased (decreased) the FSI peak for production angles above (below) $\theta_{lab}\cong 20^\circ$. It is therefore tempting to associate these discrepancies with the absence of 3NF in presented calculations. Before making such a conclusion, however, a fully charge-dependent study of FSI case including the total isospin $T=3/2$ ${}^1S_0$ states is unavoidable.

In fig. V.7 also the influence of higher partial waves on height of FSI peak is shown. The dashed curve result when only $^1S_0$ and $^3S_1-^3D_1$ force components are taken into account. Inclusion of P- and D-wave forces decreases a bit the peak height at the small production angles, but increases the peak height substantially at the larger ones. This indicates that neglecting the P- and D-wave force components in analyzing FSI peaks can quantitatively be quite misleading at certain angles.

One point concerning the comparison between theory and experimental data in case of breakup process should be emphasized. The theoretical predictions belong to point geometry defined by a reaction point in the center of the target and two points in the center of the two detectors. The experimental points for kinematically complete measurements result from projecting the raw experimental events, located in a band arround kinematically allowed curve in the 2-dimensional energy plane of the two detected nucleons. Such a band arises because of the finite angular acceptance of the detectors, the finite size of the target, energy uncertainties of the beam particles and uncertainties in the energies of the detected particles. These raw events are collected, according to some projection method, on the ideal curve for point geometry.

Ideally, the theoretical reaction rates should also be calculated in the same kinematical band as covered by the experimental data taking into account finite experimental resolutions and then projected in the same manner as the experimental data onto the ideal curve for point geometry. Such procedure requires however a large amount of computer time. But even without performing such complex calculations some estimation of the effects induced by projection prodecure on theoretical points can be done performing calculations with independent variations of both detectors angles and beam particles energy as allowed by experimental resolutions.

As was shown in ref.[61,90] changes of the theoretical cross sections obtained in this way are not large enough that a projection of theoretical cross sections could remove significant discrepancies between theory and experiment found for FSI peak heights and collinear configurations. Also the space-star cross section is very stable to such variations[90].

VI. Conclusions

In this paper a study of three-nucleon continuum, based on nonrelativistic model with three nucleons interacting pairwise, was made. As underlying nuclear dynamics, realistic NN interactions based on meson-exchange model were used. With such local potentials modified AGS equations were solved for the first time exactly without making any approximation to underlying two-nucleon dynamics. In this way quite a new domain is opened, in which dynamical models of nucleon-nucleon interactions can be for the first time investigated quantitatively on the 3-nucleon continuum level. Comparison of such theoretical predictions with 3-nucleon experimental data, both for elastic Nd scattering as well as for the nucleon induced deuteron breakup, can thus yield a new information about NN interaction or/and 3N-force effects.

Some examples of such a comparison for elastic Nd scattering and breakup processes were presented.

Very good agreement between experiment and theory is obtained for the angular distribution of the Nd elastic scattering differential cross section. Especially satisfactory is the agreement with precise nd measurements in backward angle region. Both Paris and Bonn nucleon-nucleon potentials give equally good description of existing experimental data, when charge dependence breaking of the NN interaction in $^1S_0$ state is taken into account. In case of pd data for angles $\theta_{cm} \leq 45^\circ$ distinct discrepancy between theory and data ,caused by Coulomb forces,appears.

Spectacularly good agreement between both force models predictions and pd experimental values was found for the tensor analyzing powers T_{20} and T_{22} at E=22.7 MeV. This suggests that at this and higher energies the Coulomb force effects are probably not too important.

Study of the nucleon analysing power A_y in elastic Nd scattering revealed a big discrepancy between theory and experimental data in the region of A_y maximum at angles $\theta_{cm} \cong 120^\circ$ and for energies $E \leq 28$ MeV. For energy $E \geq 50$ MeV this discrepancy disappears and nice agreement with A_y experimental data results.

It was found that the maximum of A_y for E≤25 MeV is practically a result of complicated interplay between ^3P waves contributions. This dominance of ^3P states decreases with increasing energy.

Study of pd polarization data at E=10 MeV revealed that definite discrepancies between experimental data and theoretical predictions exist also for the deuteron vector analysing power iT_{11} and proton to deuteron polarization transfer coefficients $K_y^{x'z'}$ and $K_z^{y'z'}$. These observables also show strong dependence on ^3P-wave forces. Interesting enough the changes in $K_z^{y'z'}$, $K_y^{x'z'}$ and vector analysing powers induced by putting individually the three ^3P state contributions equal to zero are different. Therefore these 3-nucleon spin observables can play a significant role to pin down the ^3P force properties.

Modifying the strength of ^3P NN forces such that the low energy pp phase shifts from ref.[77,78] are obtained, one can explain discrepancies found in E=10 MeV pd data. Thereby charge independence in ^3P waves has to be broken. If one in addition assumes that Coulomb forces are not responsible for the d fference in the experimental A_y values for elastic nd and pd scattering, charge symmetry breaking in ^3P wave force components is a possible explanation. The calculations performed revealed that this explanation leads at the same time to a large difference between iT_{11} values for the nd and pd systems at E=10 MeV. It would therefore be very interesting to measure analyzing powers with polarized deuterons in nd scattering.

Further study requires precise experimental data base for polarization observables in Nd elastic scattering. They should be measured at different energies for nd as well as pd systems. From the theoretical side exact solutions of 3-body equations incorporating Coulomb interaction are required in order to fix the magnitude of such force effects in case of pd system. Also estimation of 3N force effects in Nd elastic scattering by performing calculations with some model 3NF should be made. Comparing theoretical predictions with precise Nd polarization data will allow to establish the details of ^3P force components in np, nn and pp systems. Changes in both Paris- and Bonn- force models probably will be required in order to reproduce the behaviour of ^3P components.

It should be pointed out that the Nd elastic scattering is especially preferred to study charge independence breaking in the ^3P forces as compared to nucleon-nucleon scattering. Below 30 MeV the ^3P phase shifts are small (a few degrees) and consequently the analyzing powers in NN elastic scattering are quite small, requiring extremely high precision measurements in order to observe charge independence breaking. The vector analysing powers in the Nd elastic scattering are much larger and depend sensitively on ^3P force components.

Another example, which show the importance of 3N scattering in fixing 2N force properties is given by study of nucleon spin transfer coefficient $K_y^{y'}$. This observable is most promising to distinguish between weak and strong $^3S_1-^3D_1$ tensor forces. The difference between the Paris and OBEPQ(A) predictions can be shown to be just a consequence of the different $^3S_1-^3D_1$ 2N tensor forces. The data at E=22.7 MeV clearly favour the weaker tensor force. As a consequence of the weaker tensor force the theoretical triton binding energy would increase significantly[21] and the discrepancy to the experimental value would get rather small. The 3N force effects needed then would be only of about 100 keV in comparison to 1 MeV required in case of stronger tensor force as given by the Paris potential. However, precise measurements of $K_y^{y'}$ at other energies, in pd as well as nd systems, are required in order to confirm that result.

Comparison of rigorous 3N-continuum calculations with experimental breakup cross section data revealed definite discrepancies for some kinematical configurations. The most significant one appears in case of nd- as well as pd-breakup processes for collinear configuration. The collinear region turns out to be insensitive to the 2N potentials used. The observed discrepancy could be thus a signature of the action of a three-nucleon force.

This configuration together with the QFS, space star and FSI was proposed in model study of [87] as most promising ones to investigate 3NF effects. Comparison of 13 MeV pd breakup data with theory reveals indeed a discrepancy in QFS peak. Also pd QFS analysing power data at 65 MeV could suggests some discrepancy with theory. Before however these discrepancies could be established as possible 3NF effects further cases of such disagreements should be investigated, especially in QFS peaks from nd experiments.

The controversial experimental situation prohibites a conclusion about a significant discrepancy with theory in case of space-star configuration. To clarify the situation independent measurements, also at higher energies, should be performed.

It is interesting to note that in case of FSI configuration the inclusion of 3NF will, according to model study of [87], change the break-up cross section in a direction which would decrease the discrepancies between experimental data and the theoretical predictions. However before making any quantitative conclusion an analysis of the FSI peak regions by fully charge-dependent calculation, including total isospin T=3/2 $^1S_0$ states is unavoidable.

From the present state of nucleon-deuteron continuum study one can conjecture that the dynamics of free 2N forces, improved in its ^3P components to account for possible charge independence breaking, is essentially sufficient for a quantitative description of the elastic scattering process of three interacting nucleons. 3N observables will play a role to nail down the unsettled up to now 2N force properties, such as proper strength of $^3S_1-^3D_1$ tensor force, which remain open by present day 2N observables.

Some special breakup configurations may show effects of the action of 3N-forces. Such cases should be identified by careful studies in which rigorous continuum calculations as presented in this paper ,however with inclusion of model 3NF, such as 2π-exchange 3NF, will play an important role.

In order these studies would be succesfull very close collaboration between experimentalists and theoreticians will be required.

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