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RELATIVISTIC DYNAMICAL REDUCTION MODELS AND NONLOCALITY *

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ABSTRACT

We discuss some features of continuous dynamical models yielding state vector reduction and we briefly sketch some recent attempts to get a relativistic generalization of them. Within the relativistic context we analyze in detail the local and nonlocal features of the reduction mechanism and we investigate critically the possibility of attributing objective properties to individual systems in the micro and macroscopic cases. At the nonrelativistic level, two physically equivalent versions of continuous reduction mechanisms have been presented. However, only one of them can be taken as a starting point for the above considered relativistic generalization. By resorting to counterfactual arguments we show that the reason for this lies in the fact that the stochasticity involved in the two approaches has different conceptual implications.

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1. Introduction

Quantum mechanics is a theory which makes only conditional predictions about the results of prospective measurements of (in general) incompatible observables. Any attempt to apply quantum mechanics to the measurement process itself can only lead to further conditional predictions about what will be found if ... and so on. Here the problem of the quantum theory of measurement finds its origin.

There is now a widespread agreement about the fact that the standard solution to this problem, i.e. the adoption of the postulate of wave packet reduction (WPR), is inadequate. In particular WPR breaks the linear nature of the theory and it requires the acceptance of a splitting between system and apparatus, (or micro and macro, quantum and classical) which turns out to be basically shifty. Finally WPR gives rise to specific difficulties when one considers changes of reference frame.

An attempt to overcome the first two of the above mentioned difficulties has led to the consideration of Dynamical Reduction Models (DRM). The underlying philosophy is quite simple: one introduces a "small" stochastic modification of the standard dynamics that has a negligible impact for microsystems but which nevertheless leads to a dynamical suppression of macroscopic superpositions. That such a line can be consistently followed (in the nonrelativistic case) has been proved in a series of recent papers [1,2]. An attempt at a relativistic generalization of a particular DRM which is known[2] as CSL (Continuous Spontaneous Localizations) has also been presented[3]. Here we will be mainly concerned with a critical discussion of the implications of such relativistic models for nonlocality and with the possibility of a macro-objective interpretation of the formalism.

There are two physically equivalent versions of nonrelativistic CSL; only one of them, however, can be taken as the starting point for the relativistic generalization. This is due, as we will prove here by resorting to counterfactual arguments, to the fact that the two considered versions of CSL exhibit stochastic features which are conceptually different.

2. Nonrelativistic DRM: Some Remarks

The development of the DRM program necessarily requires a dynamical evolution which yields, under specific circumstances, a suppression of the off-diagonal elements of the statistical operator in an appropriate "preferred basis". It is important to stress, however, that this condition on the statistical operator is not sufficient to guarantee that any individual system must be associated to a state vector belonging to only one of the "preferred manifolds". Typically, there are stochastic dynamical evolution mechanisms which leave individual systems in superpositions of preferred states but which induce a randomization of the relative phases of such superpositions, thus leading to the above mentioned suppression. Following Stapp [4] we say that such mechanisms induce von Neumann reductions. We are not interested in such models since we want the reductions to occur at the individual level, or, in Stapp's language, to have Heisenberg reductions.

CSL models actually give such kinds of reductions. It is useful to give a sketchy description of the two distinct, physically equivalent, versions of CSL which have been mentioned above. The first is based on the consideration of the linear evolution equation with a skew hermitian stochastic term[2],

$$\frac{d\,|\psi_V, t\rangle}{dt} = \left[-iH - \lambda \int dx\, N^2(\mathbf{x}) + \int dx\, N(\mathbf{x})\, V(\mathbf{x},t)\right] |\psi_V, t\rangle \qquad (1)$$

where $N(\mathbf{x})$ is a number density operator averaged on an appropriate volume around $\mathbf{x}$, and $V(\mathbf{x},t)$ is a white noise in all variables with covariance

$$\langle\langle V(\mathbf{x}, t)\, V(\mathbf{x}', t')\rangle\rangle = \lambda\, \delta(\mathbf{x} - \mathbf{x}')\, \delta(t - t'). \qquad (2)$$

The second term in the equation is a counterterm guaranteeing the conservation of the stochastic average of the square norm. The physics is obtained by considering the normalized states $|\psi_v(t)\rangle / \| \,|\psi_v(t)\rangle \|$ and by assuming that the probability density of occurrence of a specific potential $V(\mathbf{x},t)$ is not the (Raw) one $P\,[V]$ associated with the white noise distribution, but the cooked one

$$P_C[V] = P\,[V]\, \|\, |\psi_V, t\rangle \|^2 . \qquad (3)$$

We will refer to this description as the *Raw + Cooking* scheme.

Alternatively one can consider the nonlinear, stochastic, norm conserving, evolution equation[2]

$$\frac{d\,|\psi_V, t\rangle}{dt} = \Big\{-iH - \lambda \int dx\, \left[N(\mathbf{x}) - \langle N(\mathbf{x})\rangle\right]^2 + \qquad (4)$$

$$+\lambda \int dx\, \left[\langle N^2(\mathbf{x})\rangle - \langle N(\mathbf{x})\rangle^2\right] + \int dx\, \left[N(\mathbf{x}) - \langle N(\mathbf{x})\rangle\right] V(\mathbf{x},t) \Big\}\, |\psi_V, t\rangle$$

and assume that V(**x**,t) is distributed according to the raw distribution P [V] . We will refer to this description as the *Nonlinear-Equation* scheme. For a detailed mathematical study of the relations between the two schemes see ref.(5).

The two schemes are equivalent in the following precise sense: for any given initial state $|\psi,0\rangle$ and, e.g., for a given $V^{\sim}(\mathbf{x},t)$ to be used in the *Raw + Cooking* scheme, there exists a V(**x**,t) to be used in the *Nonlinear- Equation* scheme such that the evolved states at any time t coincide. Moreover $V^{\sim}(\mathbf{x},t)$ and V(**x**,t) occur with the same probability.

3. Relativistic DRM

Within a stochastic framework one has to formulate in an appropriate way the invariance requirement. Actually what is needed is only stochastic invariance, i.e., while individual processes can look different to different observers, the ensemble of possible individual processes must turn out to be the same. We remark that CSL, as presented in Section 2, exhibits Galileian invariance.

Trying to get a relativistic theory yielding Heisenberg reductions we consider the Tomonaga-Schwinger (T-S) picture in a quantum field theory framework. We assume that the fields evolve according to Heisenberg equations deriving from a hermitian lagrangian density L_0 while the state vector obeys the T-S equation

$$\frac{\delta\,|\,\psi_V(\sigma)\rangle}{\delta\,\sigma\,(x)} = \left[L_I(x)\,V\,(x) - \lambda L_I^2\,(x)\right]|\,\psi_V(\sigma)\rangle \tag{5}$$

where $L_I(x)$ is a hermitian function only of the fields and V(x) is a c-number white noise with covariance given by eq.(2).

We remark that we are working in the *Raw+Cooking* scheme, and that the equation corresponding to eq.(3) is

$$P_C[V] = P\,[V]\,\|\,|\psi_V(\sigma)\rangle\|^2 \tag{6}$$

The lagrangian density L_0 can contain interaction terms. We point out that hermitian terms in L_0 which do not depend on the field derivatives can be shifted from the Heisenberg to the T-S equation, while the reducing dynamics can only be described in terms of the T-S equation, due to the skew hermitian coupling of the noise to $L_I(x)$.

One would like the dynamics to induce localization of massive fermions . This cannot be obtained directly by specifying the above formalism only to fermion fields[3]. One can then try to get localization for fermions by coupling their field to a meson field and then introducing a reducing dynamics for this auxiliary field. In refs.(3) the following choice has been proposed

$$L_0 = \text{Free fermions} + \text{Free mesons} + \eta\,\overline{\Psi}(x)\,\Psi(x)\,\Phi(x)$$

$$L_I\,(x) = \Phi\,(x). \tag{7}$$

As a quantum field theory the model presents additional divergences with respect to the standard ones, arising from the white noise character of the stochastic potential

V(x). However, one can study the nonrelativistic limit for fermions in a sector with a fixed number of fermions and one gets, with some approximations, a theory which is very similar to CSL. We refer the reader to refs.(3) for a detailed discussion of this model.

4. Objectivism and Nonlocality

Without going into specific details about the proposed model we want to investigate here the consequences of the adoption of the previously outlined relativistic framework for describing dynamical reductions. We will be particularly concerned to discuss the nonlocal features of the scheme and to investigate whether it allows a macro-objective description of natural phenomena. In this discussion we shall make use of concepts like *elements of physical reality, chance* and *determinism* which we examine now. It has to be stressed that this investigation requires, to be meaningful, the consideration of the individual level of description of physical processes.

In standard quantum mechanics, when consideration is given to an observable A, one states that a physical system posesses* an element of physical reality a associated to the observable A iff the system is in an eigenstate of A corresponding to the eigenvalue a. Calling P_a the projection operator on the eigenmanifold associated to a one has, in such a case,

$$<\psi,t \mid P_a \mid \psi,t> =1 \ . \tag{8}$$

We remark that the condition that $<\psi,t \mid P_a \mid\psi,t>$ be extremely close to one has to be considered sufficient for the attribution of the element of physical reality. In fact since the recording of the result of any measurement involves the reading of the position of some pointer, and since the wave functions associated to different pointer positions unavoidably overlap, the probability that a pointer be in a certain interval is never exactly equal to 1.

Again within the standard quantum formalism let us consider a system which is in a linear superposition of two eigenvectors of the observable A. In such a case no element of physical reality corresponding to A can be attributed to the system. Suppose that a measurement of A is performed, and that we use the WPR postulate. One can then say that the definite response of the macroscopic measuring apparatus *is due to chance*. On the contrary, if the system is in an eigenstate of A, one can say that the definite response of the macroscopic measuring apparatus *is deterministically implied* by the pre-existing element of physical reality of the system.

4.1 WPR, Nonlocality and Changes of Reference Frame.

Consider an EPR-Bohm like set up for the singlet state involving a measurement of a spin component of particle 1 by a measuring apparatus M_1 (see Fig.1). In the figure 1 and 2 represent the "world lines" of the two particles. C is the space-time point (region) in which the measurement occurs. Note that if one assumes that WPR takes places instantaneously in the reference frame in which M_1 is at rest, according to the previous definition, the event in C implies the instantaneous emergence of an element of physical reality associated to the same spin component for particle 2 and, in particular, that this

*For better appreciating what follows it might be useful to recall the obvious fact that in the standard quantum framework the entanglement of wave functions for composite systems forbids the attribution of elements of physical reality to the constituents and that the persistence of linear superpositions of far away states forbids the attribution of objective local properties even to a single particle.

element of physical reality can be attributed to the particle at the space-time point B which is space-like separated from C.

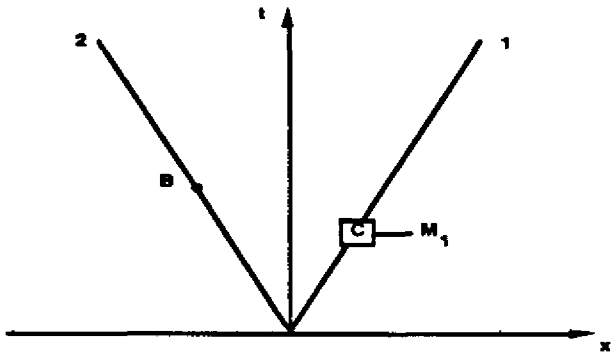

FIG. 1

We must consider the possibility of changes of reference frame. Since the separation $(B\text{-}C)^2$ can be arbitrarly large and negative, *even changes to an extremely slowly moving frame* can reverse the time order of B and C. Let us call O' such a reference frame. If one assumes that WPR takes place instantaneously also for O', the fact that for him B occurs earlier than C implies that (for him) the mean value of the relevant projection operator for the spin component of the particle at B is 1/2. This ambiguity about the mean values of the considered projection operator reflects an ambiguity in the possibility of attributing to particle 2 the corresponding element of physical reality at the given (objective) space-time point B. Note that if consideration is given to a space-time point in the future of C, such an ambiguity does not occur*.

Let us consider the analogous situation in the case in which, beside the apparatus at C, there is an apparatus M_2 at B devised to measure the same spin component. The ambiguity in the attribution of the element of physical reality to the particle at B, in accordance with the considerations of the previous subsection, has as a consequence that while according to O the result of the measurement at B is deterministic, according to O' it is due to chance.

Non relativistic DRM models exhibit analogous ambiguites. It has however to be remarked that the fact that the statement "*the outcome at B is due to chance*" is not covariant does not constitute a dead end for the theory and, in particular, it does not forbid its relativistic generalization. In fact the ambiguity does not involve the response of the macroscopic apparatus but only the nontestable (for space-like separation B-C) modalities yielding this unambiguous response.

4.2 Macro-Objectivism and Nonlocality in Relativistic Reduction Models

We will perform now a critical investigation of relativistic CSL, which parallels the one of the previous Subsection. Such a theory has precise implications, differing from those of standard quantum mechanics, about the behaviour of macroobjects. It is then important to analyze whether these implications are compatible with the adoption of a macroobjective position.

*A detailed discussion of the difficulties met by WPR in connection with changes of reference frame has been presented in refs.(6). B. d'Espagnat[7] has stressed that in trying to generalize DRM one should face analogous difficulties.

To this purpose we start by considering local observables

$$A_I(\sigma) = \int dx\, f_\alpha(x)\, F[\Phi_I(x), \partial_\mu \Phi_I(x)] \tag{9}$$

where $f_\alpha(x)$ is a function having as its support α a compact subset of the space-like surface σ and is of class C^∞ on σ. In eq.(9) the index I is added to recall that we are working in the (T-S) Interaction Picture .

Let us consider now two arbitrary space-like surfaces σ_1 and σ_2 containing α (see Fig.2). In standard quantum field theory the mean value of A_I does not depend on

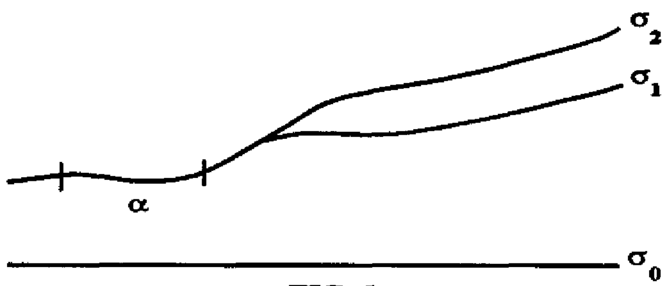

FIG. 2

which one of the two states $|\psi(\sigma_i)\rangle$, i = 1,2, associated to the considered surfaces, is chosen for its evaluation. In relativistic CSL this is no more true. In fact, in such a case, the state vector describing an individual physical process for a specific realization of the stochastic potential V(x) is $|\psi_V(\sigma)\rangle / \| |\psi_V(\sigma)\rangle \|$. If we denote by $S_V(\sigma_2, \sigma_1)$ the evolution operator from σ_1 to σ_2, we have

$$\frac{\langle \psi(\sigma_1) | S^+_V(\sigma_2, \sigma_1)\, A_I\, S_V(\sigma_2, \sigma_1) | \psi(\sigma_1)\rangle}{\| S_V(\sigma_2, \sigma_1) | \psi(\sigma_1)\rangle \|^2} \neq \frac{\langle \psi(\sigma_1) | A_I | \psi(\sigma_1)\rangle}{\| |\psi(\sigma_1)\rangle \|^2} \tag{10}$$

even though

$$[A_I, S_V(\sigma_2, \sigma_1)] = 0 \tag{11}$$

since $S^+_V(\sigma_2, \sigma_1)\, S_V(\sigma_2, \sigma_1) \neq 1$. It follows from this that if one relates, as before, the attribution of objective local properties to individuals to the mean value of a projection operator referring to a local observable, one meets difficulties.

We stress, however[3], that no such surface dependence exists for the ensemble average of the mean value of local observables. This lack of ambiguity at the ensemble level is necessary for the consistency of the theoretical scheme.

In analyzing the dependence, at the individual level, of the mean value of a local observable A_I upon the space-like surface over which it is evaluated (among those coinciding on its support), it is useful to discuss separately the ambiguities for microscopic and macroscopic systems.

We start by considering a microscopic system S and two local observables A_1 and A_2 of S having space-like separated supports α_1 and α_2. Suppose that there is a macroscopic measuring apparatus M_1 devised to measure A_1. The T-S equation describing the unfolding of the physical process is then

$$\frac{\delta |\psi_V(\sigma)>}{\delta \sigma(x)} = \left[i L_{1\text{-}S}(x) + L_1(x) V(x) - \lambda L_I^2(x)\right] |\psi_V(\sigma)> \qquad (12)$$

where $L_{1\text{-}S}$ is the local hermitian system-apparatus interaction which describes the triggering of the apparatus by the microsystem and L_I is the skew hermitian term inducing reductions. Both $L_{1\text{-}S}$ and L_I are assumed to be different from zero only in the region C indicated in Fig.3.

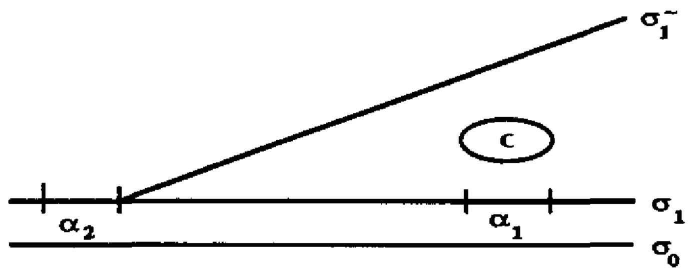

FIG.3

The initial state $|\psi(\sigma_0)>$ is supposed to be

$$|\psi(\sigma_0)> = \frac{1}{\sqrt{2}} [|\psi_1> + |\psi_2>] |\chi_1> \qquad (13)$$

with

$$A_i |\psi_j> = \delta_{ij} |\psi_j>; \qquad i, j = 1, 2 \qquad (14)$$

and $|\chi_1>$ describing the untriggered apparatus state. Suppose that for the individual case under consideration the specific realization of V(x) is one of those "yielding the result 1" for the measurement of A_1. Let us consider the two space-like surfaces σ_1 and $\sigma^{\sim}_1$ which contain the support α_2 of A_2 but such that, in going from σ_1 to $\sigma^{\sim}_1$, one crosses the space-time region C (see Fig. 3). According to eqs.(12) and (13) one has

$$<\psi_V(\sigma_1)| A_2 |\psi_V(\sigma_1)> = 1/2 \qquad (15.a)$$

while

$$<\psi_V(\sigma^{\sim}_1)| A_2 |\psi_V(\sigma^{\sim}_1)> \cong 0. \qquad (15.b)$$

Eqs.(15.a) and (15.b) show that there is an ambiguity in the mean value of the local observable A_2 associated to the microsystem S. This is not surprising, it constitutes simply the analogy within relativistic DRM of the situation discussed in Subsection 4.1 for standard quantum mechanics with the WPR postulate. In fact the surface $\sigma^{\sim}_1$ can be approximately identified with a t' = const. hyperplane for a boosted observer O' which moves with a very slow velocity with respect to O but for whom, however, the interaction (and the consequent reduction) at C has taken place at a time earlier than t'.

We pass now to discuss the analogous problem, i.e. the possible occurrence of ambiguities in the mean values of local observables, in the case of a macroobject. To this purpose, we consider a situation strictly analogous to the previous one, in which however a further macroscopic measuring apparatus M_2 devised to measure A_2 is supposed to be present. With obvious meaning of the symbols we have as the analogy of eq.(12) the following T-S evolution equation

$$\frac{\delta |\psi_V(\sigma)>}{\delta\sigma(x)} = \Big[i\,L_{1\text{-}S}(x) + i\,L_{2\text{-}S}(x) + L_{I1}(x)\,V(x) + L_{I2}(x)\,V(x) -$$

$$- \lambda L_{I1}^2(x) - \lambda L_{I2}^2(x)\Big] |\psi_V(\sigma)> . \qquad (16)$$

With reference to Fig.4 we specify that the system-apparatus interactions $L_{1\text{-}S}$ and $L_{2\text{-}S}$ are supposed to be different from 0 in C_1 and B_1, respectively, and the reducing terms L_{I1} and L_{I2} in C_2 and B_2, respectively. The initial state is now

$$|\psi(\sigma_0)> = \frac{1}{\sqrt{2}}\,\big[|\psi_1> + |\psi_2>\big]\,|\chi_1>|\chi_2> \qquad (17)$$

where $|\chi_2>$ is the untriggered state of the apparatus M_2. As in the previous case we assume that V(x) is such as to lead to the result A_1=1.

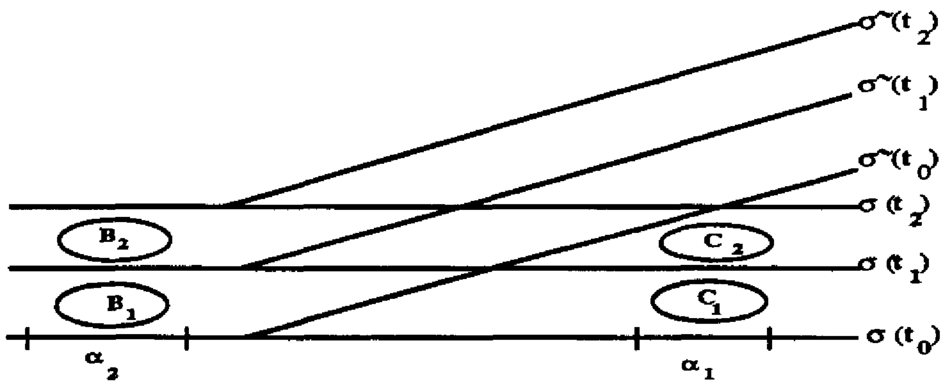

FIG.4

We remark that according to eq.(16), for the considered initial state, we have the following situation.

i) For $t<t_0$, the states associated to $\sigma(t)$ and $\sigma^{\sim}(t)$ are factorized, one of the factors being $|\chi_2>$.

ii) For $t=t_1$

$$|\psi_V(\sigma(t_1))> = \frac{1}{\sqrt{2}}[|\psi_1>|\chi_1^1>|\chi_2^0>+|\psi_2>|\chi_1^0>|\chi_2^1>] \tag{18.a}$$

while

$$|\psi_V(\sigma^{\sim}(t_1))> \equiv |\psi_1>|\chi_1^1>|\chi_2^0> \ . \tag{18.b}$$

At the r.h.s. of eqs.(18a-b) the superscripts in the apparatuses states refer to the eigenvalues of the observables A_1 and A_2.

iii) For $t > t_2$ the states associated to $\sigma(t)$ and $\sigma^{\sim}(t)$ are factorized, one of the factors referring to the apparatus M_2 and corresponding to having registered the definite outcome 0 in the measurement of A_2.

The results (18.a) and (18.b) show that, at t_1, an ambiguity is present even in the mean value of the local observable corresponding to a definite macroscopic outcome of M_2. It has to be stressed, however, that according to iii), this ambiguity lasts only for the typical reduction time of the macroscopic measuring device M_2.

Let us summarize the situation. The consideration of the individual level of description of physical processes within standard quantum mechanics with the WPR postulate gives rise to difficulties when one takes into account changes of reference frame, in particular for what concerns the possibility of attributing to physical systems elements of physical reality corresponding to local observables. Such difficulties find their formal counterpart within relativistic DRM in the fact that, for a given realization of the stochastic potential, the mean values of local observables are ambiguous in the above specified sense. As we have shown, there is however a fundamental difference in the microscopic and the macroscopic cases. For microsystems such ambiguities may last, under appropriate circumstances, for arbitrarly large time intervals, while in the macroscopic case they last only for extremely short intervals. We can then take advantage of this nice feature to overcome the puzzling situation we have discussed in this section.

The analysis of the formal aspects of relativistic CSL leads naturally to the introduction of a more appropriate criterion for the attribution to physical systems of elements of physical reality corresponding to local observables: we say that an individual system has an element of physical reality associated to the local observable A_I with support α iff the mean value of the projection operator on one of the eigenmanifolds of A_I is extremely close to one for all space-like surfaces containing α.

With reference to the previous discussion we stress that, according to this criterion:

i) Macroscopic objects have " almost always" definite local macroproperties. This means that relativistic CSL is compatible with macroobjectivism.

ii) For what concerns micro-objects , contrary to what happens when the standard attitude is taken, no objective local property can emerge as a consequence of a "measurement process" occurring in a space-like separated region. Such property emerges only in the future light cone of the region in which the measurement is performed. This has to be so since the observer performing the measurement could send a message by a light signal , containing the information about the definite outcome of the appropriate prospective measurement .

From the above discussion it also turns out that in a situation like the one sketched in Fig.4, it is impossible to establish objectively whether the outcome of the measurement of A_2 is due to chance or is deterministic.

5. Counterfactual Arguments and the Stochastic Features of the Two Versions of Nonrelativistic CSL.

As discussed in Section 2 there are two distinct, physically equivalent versions of nonrelativistic CSL which are based on the use of the linear equation (1) plus the cooking prescription (3) for the distribution of the stochastic potential and on the use of the nonlinear eq.(4), respectively. The two schemes are physically equivalent in the precise sense specified in the last sentence of Section 2. However, in the remainder of this paper we will show that, by resorting to counterfactual arguments, one can prove that the two schemes exhibit conceptually different stochastic natures, so that the first scheme is suitable for relativistic generalization, while the second scheme is not.

The use of counterfactual arguments requires the consideration of possible worlds. One then makes this notion more precise by imposing appropriate restrictions on these worlds. We do not aim for great generality and thus we feel free to make the following precise assumptions:

a) The "laws of nature" embodied in the specific CSL scheme under consideration (Raw + Cooking or Nonlinear Equation) are assumed to hold for all possible worlds.

b) With reference to the actual and the alternative worlds (among the possible ones) we will consider, we remark that we will always deal with situations involving physical processes in which there are microsystems interacting with macroscopic measuring devices. We assume that the switching on and off of the interactions between the microsystems and the macro apparatuses are free variables whose different choices characterize the actual and the alternative worlds.

5.1 Nonrelativistic Context

Within such a context let us consider the following process taking place within the time interval (t_i,t_f) : we have a system S consisting of one free spin 1/2 particle which at time t_i is in the eigenstate of σ_x belonging to the eigenvalue +1, and two macroscopic measuring apparatuses M_1 and M_2 devised to measure the observable σ_z. As usual, in a certain time interval around t_1 ($t_i < t_1 < t_f$), an appropriate interaction between S and M_1 governed by a coupling constant g_1 yields the triggering of M_1 , and, subsequently, the CSL dynamics leads very quickly to a definite outcome, e.g. $\sigma_z = +1$. Analogously around t_2 ($t_1 < t_2 < t_f$), due to an S- M_2 coupling governed by g_2 the measurement of σ_z is repeated . The specific individual process we have described constitutes the actual world.

We raise now the following counterfactual question: *In the considered individual case, if we would have not performed the measurement at t_1, would the result of the measurement at t_2 have still been +1?* The question requires the consideration, besides the actual world, of an alternative world characterized by the choice $g_1 = 0$.

5.1.1 Nonlinear Equation Scheme

Let us take the position that the "laws of nature" are given by the Nonlinear Equation version of CSL. The considered individual physical process which has taken place in the actual world is governed by the evolution equation

$$\frac{d\,|\psi_V, t>}{dt} = \Big\{ - i\, g_1 H_{S\text{-}M_1} - i g_2 H_{S\text{-}M_2} - \lambda \int dx\, [N(x) - <N(x)>]^2 \qquad (19)$$

$$+ \lambda \int dx\, [<N^2(x)> - <N(x)>^2] + \int dx\, [N(x) - <N(x)>]\, V(x,t) \Big\} \,|\psi_V, t>$$

in which a specific V(x,t) occurs among those yielding the result +1 for both measurements of σ_z . Obviously we do not know which V(x,t) has actually occured, but this does not forbid the analysis which follows. Since, in this scheme, the distribution of V(x,t) is independent of the physical situation under consideration and, in particular, is independent of whether g_1 is zero or not , in the alternative world the same realization of the potential occurs. This being the situation one can think of solving eq.(19) explicitly with the given V(x,t) and with $g_1 = 0$ (alternative world) and consequently, from the conceptual point of view, one can give a precise answer to the counterfactual question. Let us examine what the answer can be. One can easily convince oneself (see Appendix A) that there are specific realizations of the stochastic potential which, when substituted in eq.(19) with g_1 and g_2 different from zero, lead to the results +1 in both measurements, while, when substituted in the same equation with $g_1 = 0$, $g_2 \neq 0$, lead to the result -1 for the measurement by M_2. Moreover such potentials have an appreciable probability of occurrence. If in the considered individual case a potential of this type has occurred the answer to the counterfactual question raised above is : *no* !

Let us confine our attention to the set of all those individual cases, among all the possible ones, for which the above situation occurs. Obviously the identification of such a set is only conceptually and not practically feasible. For all these individual cases one would be allowed to state that there is a cause-effect relation between the switching on of the apparatus M_1 and the result of the apparatus M_2. In fact, if we denote by *a* and *b* the two following events

a = switching on of g_1
b = registering the result +1 by M_2

we have, within the above set

i) a can be made to happen at will

ii) If a is made to happen then b happens

iii) If a is not made to happen b does not happen either.

The above analysis can be repeated with reference to an EPR-Bohm like set -up, leading to the conclusion that, for an appropriate subset of all possible realizations of the stochastic potentials, one is allowed to assert that a measurement at a space time point "causes" the particular result obtained in another measurement taking place in a space-like separated region.

5.1.2 Raw + Cooking Scheme

We take now the position that the "laws of nature" are embodied in the Raw+Cooking version of CSL. It is important to remark that, since in this scheme the probability of occurrence of the stochastic potential depends on the specific physical situation under consideration and in particular, for the considered case, on whether g_1 is zero or not, there is no way of relating the realization of the potential in the alternative world to the one occuring in the actual world. As a consequence there is no possibility of identifying, even conceptually, occurrences for which the considered counterfactual question admits an answer, and specifically a negative answer.

5.2 Relativistic Context

Let us consider an EPR-Bohm like set-up involving, with reference to Fig.1, the measurement by two macro-apparatuses M_1 and M_2 of the same spin component at the space time points C and B, respectively. We consider now the problem in the context of the Nonlinear Equation scheme. Suppose that the actual world is characterized by a realization of the stochastic potential which would lead to the result +1 for the measurement at C and which would also lead to the result +1 for the measurement at B when (alternative worlds) only the measurement at C or only the one at B is performed, respectively. As in the case discussed in Subsection 5.1.1 such potentials have a finite probability (1/4) of occurring. In the case in which both measurements are performed (actual world) in the reference frame O of Fig. 1, the final result would be +1 at C and -1 at B. Let us analyze now the situation from a very slowly moving reference frame O' for which, however, B occurs earlier than C. We can then argue as follows

i) Consistency requirements about the definite outcomes of the macroscopic measuring apparatuses for the considered individual case imply that, when only one measurement (at C or at B respectively) is performed, the result +1 for the measurement at C and for the one at B should occur for O', just as it occurs for O. Analogously one must require that when both measurements are performed the results +1 at C and -1 at B, respectively, are found by O'. Then, since also for O' it is the result at B which changes when one chooses to perform both measurements, for him an event occurring later can be considered "to cause" an earlier event.

ii) If the theory is stochastically Lorentz invariant the recognition of the above fact implies that also for O such a situation should occur in some specific cases . However, according to the Nonlinear Equation, in no situation can O assert that an event occurring later could be considered "cause" of an event occurring earlier.

Concluding, requirements i) and ii) are incompatible. This shows that one could not take as a starting point for a relativistic generalization the non relastivistic CSL in the Nonlinear Equation version.

No similar argument can be developed, as discussed in Subsection 5.1.2, for the Raw + Cooking scheme. Actually the relativistic DRM presented in Section 3 has been based on such a scheme. In the next Subsection we discuss the specific formal difficulties that one meets in trying to get a relativistic generalization of the Nonlinear Equation CSL scheme. They represent the mathematical counterpart of the conceptual difficulties we have just discussed.

5.3 Difficulties of a Nonlinear Equation Scheme

We remark that there is a standard way, at the non relativistic level, to go from the Raw+Cooking scheme to the Nonlinear Equation scheme. If one follows this line, starting from the model presented in Section 3, within the relativistic T-S context, one is led to consider the following nonlinear equation

$$\frac{\delta \mid \psi_V(\sigma) >}{\delta \sigma(x)} = \Big\{ [\Phi(x) - <\Phi(x)>] \, V(x) - \lambda \, [\Phi(x) - <\Phi(x)>]^2 +$$

$$+ \lambda \, [<\Phi^2(x)> - <\Phi(x)>^2] \Big\} \mid \psi_V(\sigma) > \qquad (20)$$

where

$$< F(x) > = < \psi_V(\sigma) \mid F(x) \mid \psi_V(\sigma) > \, . \qquad (21)$$

It turns out that this equation is non integrable. This corresponds to the fact that, since the theory is built in such a way as to be stochastically relativistically invariant , it must necessarily violate requirement i) of Subsection 5.2.

Let us sketch here the proof that eq.(20) is non integrable. Let us consider a one parameter family of space-like surfaces $\sigma(\eta)$, labelled by a real continuous parameter η, $0 \leq \eta \leq 1$ such that $\sigma(0) = \sigma_0$ and $\sigma(1) = \sigma$. We choose an initial state $|\psi(\sigma_0)>$ and we associate through eq.(20) with a given realization V(x) of the stochastic potential a state vector $|\psi_V(\eta)>$ to any surface of the family. Following the lines of ref.(2) it is easy to prove that, for this V(x), there exists a corresponding $V^{\sim}(x)$ to be put in the raw equation, given by

$$V^{\sim}(x) = V(x) + 2\lambda < \psi_V(\eta) | \phi(x) | \psi_V(\eta) > \qquad (22)$$

(η being the value of the parameter characterizing the surface passing through x) , such that the states associated to the surface $\sigma(\eta)$, for any η, in the two schemes coincide. If we now consider another one parameter family of space-like surfaces $\sigma'(\tau)$ we can play the same trick. However, since eq.(22) shows that the $V^{\sim}(x)$ to be put in the raw equation depends, besides x, on the particular family of surfaces which is considered, the potential $V^{\sim}(x)$ associated to the same space time point when the family $\sigma'(\tau)$ is considered, turns out to be different from the one associated to the family $\sigma(\eta)$. Since, however, raw equations with different $V^{\sim}(x)$ lead to different final states from the same initial state, this shows that the non linear equation is not integrable or equivalently that the final state depends on the specific way which one follows to go from σ_0 to σ.

6. Conclusions

We have presented a relativistic CSL formalism which describes a stochastically Lorentz invariant reducing dynamics. We have discussed specific problems like nonlocality and macro-objectivism within such a relativistic scheme and we have pointed out the necessity of introducing an appropriate criterion for the attribution of elements of physical reality to physical systems. The considered theoretical scheme makes use of a T-S linear equation with a skew-hermitian coupling to a stochastic potential and of an appropriate rule to evaluate the probability of occurrence of the potential. By making use of counterfactual arguments we have shown that, contrary to the nonrelativistic case in which one has two equivalent versions of CSL, the relativistic generalization must be based on one of them. If one tries to get it from the alternative scheme one meets insurmountable difficulties.

Appendix A

With reference to the situation considered in Subsection 5.1.1 we remark that the values of the potential which are relevant for the reductions are those in very narrow neighbourhoods of t_1 and t_2. Let us consider the probability of occurence of a potential $V_1(\mathbf{x},t)$ for $t \cong t_1$ and of a potential $V_2(\mathbf{x},t)$ for $t \cong t_2$. Due to the white noise character of the distribution of V, the joint probability of occurrence of the two considered potentials is the product of their probabilities

$$P\,[V_1\&V_2] = P\,[V_1]\,P\,[V_2]. \tag{A.1}$$

It has to be stressed that, for *a given potential* in the time interval in which the reduction of the macro-apparatus takes place, the outcome of the measurement depends in a crucial way on the state of the system which triggers it. According to CSL, the probability of occurrence of a $V_1(\mathbf{x},t)$ leading, when M_1 is triggered by the eigenstate of σ_x corresponding to the value +1, to the outcome " M_1 registers that σ_z =+1", is 1/2. Similarly, the probability of occurrence of a $V_2(\mathbf{x},t)$ leading, when M_2 too is triggered by the eigenstate of σ_x corresponding to the value +1, to the outcome " M_2 registers that σ_z= -1", is 1/2. [Note that the same $V_2(\mathbf{x},t)$, when M_2 is triggered by the eigenstate of σ_z corresponding to the value +1 leads to the outcome " M_2 registers that σ_z =+1"]. It follows then from eq.(A.1) that the probability of occurrence of a potential leading when $g_1 \neq 0$ and $g_2 \neq 0$ to the result +1 and when $g_1 = 0$ and $g_2 \neq 0$ to the result -1 for M_2 is 1/4.

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