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A GENERAL ANALYSIS ON A COMPLETE DETERMINATION OF THE 3-VECTOR-BOSON COUPLINGS *

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A complete determination of three-vector-boson couplings ($W^+W^-Z^0$ and $W^+W^-\gamma$) is very important not only for testing the $SU(2)*U(1)$ gauge structure of the standard model (SM), but also for finding novelty beyond SM. It will be performed at high energy colliders (LEP, LHC at CERN, Tevatron at Fermilab, HERA, SSC etc.) through the various processes which contain the vertex with different production of the bosons such as W^+W^-, WZ^0, and $W\gamma$. All these processes have been extensively studied by some authors[1-8]. In order to do a complete determination for the vertex, firstly, one needs a general effective Lagrangian L_{eff} corresponding to the couplings which are given completely in SM. Secondly, one needs to calculate the cross section for these processes with general L_{eff} so that the couplings could be determined by the experiments

In literatures[1,3], L_{eff} for $W^+W^-V(V = \gamma,\ Z^0)$ ignoring the bosons on shell terms was concerned. It is not the most general case because some terms in the most general L_{eff} will vanish due to on-shell condition. Futhermore, the various processes with different on-shell condition will have different L_{eff} .

In this paper, starting from Lorentz invariance and CPT invariance, we write down the most general L_{eff} . That is:

$$
\begin{aligned}
\Gamma^{\mu\alpha\beta} &= \sum_{i=1}^{20} g_i \Gamma_i^{\mu\alpha\beta} \\
&= g_1 P^\mu g^{\alpha\beta} + g_2 P^\alpha g^{\mu\beta} + g_3 P^\beta g^{\alpha\mu} + g_4 q^\mu g^{\alpha\beta} + g_5 q^\alpha g^{\mu\beta} \\
&+ g_6 q^\beta g^{\alpha\mu} + g_7 P^\mu P^\alpha P^\beta + g_8 q^\mu q^\alpha q^\beta + g_9 P^\mu P^\alpha q^\beta + g_{10} P^\mu q^\alpha P^\beta \\
&+ g_{11} q^\mu P^\alpha P^\beta + g_{12} q^\mu q^\alpha P^\beta + g_{13} q^\mu P^\alpha q^\beta + g_{14} P^\mu q^\alpha q^\beta \\
&+ g_{15} \epsilon^{\mu\alpha\beta\rho} P_\rho + g_{16} \epsilon^{\mu\alpha\beta\rho} q_\rho + g_{17} P^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho q_\sigma \\
&+ g_{18} q^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho q_\sigma + g_{19}[P^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_\sigma + P^\beta \epsilon^{\mu\alpha\rho\sigma} P_\rho q_\sigma] \\
&+ g_{20}[q^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_\sigma + q^\beta \epsilon^{\mu\alpha\rho\sigma} P_\rho q_\sigma] + h.c.
\end{aligned} \tag{1}
$$

where $\Gamma^{\mu\alpha\beta}$ is the whole vertex, g_i (i=1,2,...) are the couplings and the corresponding subvertices $\Gamma_i^{\mu\alpha\beta}$, P, q_1, q_2 are the momentum of Z^0, W^+, W^- and $q = q_1 - q_2$.

It can be divided into three parts. The first part from $\Gamma_1 - \Gamma_6$ contains the metric tensor $g^{\alpha\beta}$. The second part from $\Gamma_7 - \Gamma_{14}$ is the product of the 3 momentum. The third part from $\Gamma_{15} - \Gamma_{20}$ contains antisymmetric tensor. For the third part, only 6 of 8 terms are indepentent due to the antisymmetric tensor identity[1]:

$$
g_{\lambda\mu}\epsilon_{\alpha\beta\rho\sigma} - g_{\lambda\alpha}\epsilon_{\mu\beta\rho\sigma} + g_{\lambda\beta}\epsilon_{\mu\alpha\rho\sigma} - g_{\lambda\rho}\epsilon_{\mu\alpha\beta\sigma} + g_{\lambda\sigma}\epsilon_{\mu\alpha\beta\rho} = 0 \tag{2}
$$

In fact, as pointed in literatures[1], multiplying it by $P^\lambda P^\rho q^\sigma$ and $q^\lambda P^\rho q^\sigma$, two constraints can be obtained:

$$
\begin{aligned}
P^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_\sigma - P^\beta \epsilon^{\mu\alpha\rho\sigma} P_\rho q_\sigma &= P^2 \epsilon^{\mu\alpha\beta\rho} q_\rho - P^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho q_\sigma \\
q^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_\sigma - q^\beta \epsilon^{\mu\alpha\rho\sigma} P_\rho q_\sigma &= q^2 \epsilon^{\mu\alpha\beta\rho} P_\rho - q^\mu \epsilon^{\alpha\beta\rho\sigma} P_\rho q_\sigma
\end{aligned} \tag{3}
$$

On the basis of the most general $\Gamma^{\mu\alpha\beta}$, we consider the processes with one boson on shell and two bosons on shell respectively. Here we denote the vertex L_{eff} being composed by $G_i\Gamma_i^{\mu\alpha\beta}$, $Fi\Gamma_i^{\mu\alpha\beta}$ and $Di\Gamma_i^{\mu\alpha\beta}$ corresponding to the W^+ on shell, W^- on shell and Z^0 on shell respectively. If W^+ is on shell, we have the on shell condition:

$$\epsilon^{\alpha}_{W^+}.q_{1\alpha} = 0 \tag{4}$$

here $q_{1\alpha} = \frac{1}{2}(P_\alpha + q_\alpha)$.

That means we can simplify the $\Gamma^{\mu\alpha\beta}$ through replacing P_α by $-q_\alpha$. However, in the cases of Γ_{19}, Γ_{20} we need to reconsider the antisymmtric condition (3) due to the additional condition:

$$P^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_\sigma = -q^\alpha \epsilon^{\mu\beta\rho\sigma} P_\rho q_\sigma \tag{5}$$

Thus, the number of independent effective couplings containing antisymmetric is $8 - 2 - 1 = 5$.

If we take $\Gamma_{15} - \Gamma_{19}$ as the independent subvertex, solving the 3 equations, we can express Γ_{20} by $\Gamma_{15} - \Gamma_{20}$:

$$\Gamma_{20} = -\Gamma_{19} + \frac{P^2}{M_W^2}\Gamma_{15} + \frac{q^2}{M_W^2}\Gamma_{16} - \Gamma_{17} - \Gamma_{18} \tag{6}$$

Thus we have the following relationship between G_i and g_i :

$$\begin{aligned}
&G_i = g_i \;\; for \;\; i = 1,3,4,6; \;\; G_2 = g_2 - g_5, \;\; G_7 = g_7 - g_{10} \\
&G_8 = -g_8 + g_{13}, \;\; G_9 = g_9 - g_{14}, \;\; G_{11} = g_{11} - g_{12} \\
&G_{15} = g_{15} + P^2 g_{20}, \;\; G_{16} = g_{16} + q^2 g_{20} \\
&G_{17} = g_{17} - g_{20}, \;\; G_{18} = g_{18} - g_{20}, \;\; G_{19} = g_{19} - g_{20}
\end{aligned} \tag{7}$$

Similarly, if W^- is on shell, we obtain the relationship between D_i and g_i as follows:

$$\begin{aligned} &F_i = g_i \ \ for \ \ i = 1,2,4,5; \ \ F3 = g_3 + g_6, \ \ F_7 = g_7 + g_9 \\ &F_8 = g_8 + g_{12}, \ \ F_{10} = g_{10} - g_{14}, \ \ F_{11} = g_{11} + g_{13} \\ &F_{15} = g_{15} + P^2 g_{20}, \ \ F_{16} = g_{16} - q^2 g_{20} \\ &F_{17} = g_{17} + g_{20}, \ \ F_{18} = g_{18} - g_{20}, \ \ F_{19} = g_{19} + g_{20} \end{aligned} \tag{8}$$

For the case of Z^0 on shell, the relationship between D_i and g_i is simply as:

$$D_i = g_i \ \ for \ \ i = 2,3,4,5,6,8,11,12,13,15,16,18,19,20 \tag{9}$$

In each case, we found only 14 of 20 terms are independent. Ideally all of them can be measured by the experiments, although some of them are difficult to be detected

In the same way, let us consider the processes with two bosons on shell. For the cases of W^+W^- on shell, W^+Z^0 on shell and W^-Z^0 on shell , we denote the $\Gamma^{\mu\alpha\beta}$ being composed by $c_i\Gamma_i^{\mu\alpha\beta}$, $d_i\Gamma_i^{\mu\alpha\beta}$ and $e_i\Gamma_i^{\mu\alpha\beta}$ respectively. Firstly, let us consider the case of W^+W^- on shell. From the on shell conditions, we have the following relationship between c_i and g_i:

$$\begin{aligned} &c_1 = g_1, \ \ c_2 = -g_2 + g_5, \ \ c_3 = g_3 + g_6, \ \ c_4 = g_4, \\ &c_7 = g_7 + g_9 - g_{10} - g_{14}, \ \ c_{11} = -g_8 + g_{11} - g_{12} + g_{13} \\ &c_{15} = g_{15} + \frac{q^2}{M_W^2} g_{19}, \ \ c_{16} = g_{16} + \frac{P^2}{M_W^2} g_{19} \ \ c_{17} = g_{17} - g_{20}, \ \ c_{18} = g_{18} - g_{19} \end{aligned} \tag{10}$$

The equation $c_{15} - c_{18}$ comes from the reconsidering the antisymmetric tensor condition (3) and the on shell condition like (5) together and finally, we have:

$$\begin{aligned}\Gamma_{19} &= q^2\Gamma_{15} - \Gamma_{18} \\ \Gamma_{20} &= P^2\Gamma_{16} - \Gamma_{17}\end{aligned} \tag{11}$$

Therefore, we can take the 4 of 6 Γ_i's, $\Gamma_{15} - \Gamma_{18}$, as the independent terms.

For the case of Z^0W^+ on shell, we simply have:

$$d_i = G_i \;\; for \;\; i = 2,3,4,6,8,11,15,16,18,19 \tag{12}$$

Similiarly, for the case of Z^0W^- on shell, we have:

$$e_i = F_i \;\; for \;\; i = 2,3,4,5,8,11,15,16,18,19 \tag{13}$$

In each case, we have only 10 of 20 terms are independent. That means only 10 of the g_i or their combinations can be determined through the one experiment with two bosons on shell. In order to determine the most general couplings g_i, one has to combine various experimental results from the processes with different on-shell conditions. If we consider all of the processes with various two vector bosons on shell. We can determine g_i's by using the values of c_i's, d_i's and e_i's:

$$\begin{aligned}&g_1 = c_1, \;\; g_2 = e_2, \;\; g_3 = d_1, \;\; g_4 = e_4, \;\; g_5 = e_5, \;\; g_6 = e_6 \\ &g_8 + g_{11} = e_{11} - d_8, \;\; g_{12} + g_{13} = e_8 + d_8, \;\; g_8 + g_{12} = e_8 \\ &g_7 + g_9 - g_{10} - g_{14} = c_7, \;\; g_{20} = \tfrac{1}{2q^2}(d_{16} - e_{16}), \;\; g_{15} = e_{15} - P^2 g_{20} \\ &g_{16} = \tfrac{1}{2}(d_{16} + e_{16}), \;\; g_{17} = c_{17} - g_{20} \\ &g_{18} = e_{18} - g_{20}, \;\; g_{19} = \tfrac{1}{2}(d_{19} + e_{19})\end{aligned} \tag{14}$$

In addition, the measurements on c_i's, d_i's and e_i's from different processes should satisfy the consistent relationship as follows:

$$
\begin{aligned}
&c_2 = d_2 = e_2 - e_5, \quad c_3 = e_3 = d_3 + d_6, \quad d_4 = e_4 = c_4 \\
&c_{11} = e_{11} - e_8 = d_{11} + d_8, \quad d_{15} = e_{15}, \quad d_{18} = e_{18} \\
&d_{16} - e_{16} = -q^2(d_{19} - e_{19}), \quad c_{15} = -\frac{p^2}{2q^2}(d_{16} - e_{16}) + \frac{q^2}{2}(d_{19} + e_{19})
\end{aligned}
\tag{15}
$$

It shows that if combining the experimental results from all of the processes with two bosons on shell, one can determine only 14 g_i or their combination and then the experimental values from the different processes should be checked each other. From the other point of view, it means that one can enchance the precision of the measurements on the 3-vector-boson couplings by putting the experimental results from the different processes together. Moreover, to perform a complete determination on the 20 couplings g_i's (most general) of the 3-vector-boson couplings, the measurements of all those processes with two bosons on shell only are not sufficient. One has to do some measurements on the processes with one boson on shell, for complementarity.

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