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Electroweak Symmetry Breaking: Top Quark Condensates

by

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Abstract. The fundamental mechanisms for the dynamical breaking of the electroweak gauge symmetries remain a mystery. This paper examines the possible role of heavy fermions, particularly the top quark, in generating the observed electroweak symmetry breaking, the masses of the W and Z bosons and the masses of all observed quarks and leptons.

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1. Introduction.

Identifying the physical mechanism for the spontaneous breaking of the electroweak gauge symmetries has been a central challenge of elementary particle physics. During the present decade we should begin to establish the experimental evidence for the specific realization chosen by nature. Theoretical speculations have covered a wide range. The Standard Model presumes the existence of elementary scalar fields which condense to produce the observed symmetry breaking and result in a single physical Higgs particle whose mass is not determined. Alternative proposals have involved more complex Higgs sectors with multiple physical Higgs scalars, supersymmetric models with many additional physical states, or dynamical symmetry breaking models, such as technicolor, which rely on condensates of new technifermions and an entirely new sector of strong dynamics. The specific models are motivated by a variety of physical issues including renormalization, strong CP, natural gauge hierarchies, supersymmetry and superstrings. These models generally require the introduction of many new fundamental particles and their interactions.

This paper will focus on a different alternative which relies only on the presently observed particles and a heavy top quark to generate the electroweak symmetry breaking. Our work is motivated by Y. Nambu [1] who suggested that short range, attractive interactions between the fermions could generate the symmetry breaking, in analogy to the BCS theory of superconductivity. We will explore the physical consequences of these ideas. Fermion condensates of a heavy top quark will play a central role, and the expected masses for the top quark and physical Higgs particle, a top quark - antitop quark bound state, are principal predictions of the model. We will emphasize the reliability of these predictions for the minimal model and examine possible extensions to more complex situations.

The presentation of this paper follows the results of Bardeen, Hill and Lindner (BHL) [2]. The role of top quark condensates in electroweak symmetry breaking was also the central theme of the top mode model of Miransky, Tanabashi and Yamawaki (MTY) [3]. The more general theme of composite gauge vector bosons and composite Higgs bosons has been a recurrent theme in the theoretical literature [4].

After a brief discussion of the elements of the original BCS mechanism and its relativistic NJL generalization, the dynamical basis of the top condensate model is analyzed by comparing three different approaches to understanding the essential elements of the minimal version of the theory. In the following section, various critical elements of the theory are discussed along with possible generalizations of the minimal model.

2. The BCS and NJL Models.

The electroweak interactions are generated by gauge interactions where the electroweak symmetries are spontaneously broken by the vacuum structure. All of the masses of the observed elementary particles, from gauge bosons to quarks and leptons, are generated by this symmetry breaking. The Standard Model relies on a fundamental Higgs field to provide the symmetry breaking. Dynamical symmetry breaking replaces the elementary Higgs field by condensates of the more fundamental degrees of freedom. Dynamical symmetry breaking forms the basis of the BCS theory of superconductivity [5].

2.1. The BCS Theory.

In the BCS theory of superconductivity, the complex interactions between the electrons result in a residual local, attractive interaction between the electrons. This attractive interaction can cause the electrons to bind into Cooper pairs. Dynamical symmetry breaking occurs when the energy of a Cooper pair becomes negative, and the normal vacuum becomes unstable to spontaneous creation of electron pairs. The vacuum structure is modified, and a new BCS ground state forms with a condensate of electron pairs, $\langle e_\uparrow e_\downarrow \rangle \neq 0$. A gap forms at the fermi surface, the electron becomes "massive", and the Meisner effect excluding the magnetic field from a superconductor reflects the dynamical mass generated for the magnetic part of the photon interactions. It is clear that the basic elements of the BCS theory could provide the framework for dynamical symmetry breaking in the electroweak interactions as suggested by Nambu [1].

2.2. The NJL Model.

The attractive, local interaction which induced the dynamical symmetry breaking was given a relativistic generalization through the Nambu - Jona-Lasinio (NJL) model [6]. This model considers the effects of a local, chiral invariant interaction between the fermions of the theory.

The model is described by the following Lagrangian,

$$L = \overline{\Psi} \{i\partial\} \Psi + G (\overline{\Psi}_L \Psi_R)\cdot(\overline{\Psi}_R \Psi_L) \tag{1}$$

where a cutoff, Λ, must be introduced to define the quantum theory. The interaction is attractive for $G > 0$.

If the interactions are not sufficiently attractive, $G < G_c$, the vacuum structure will choose the symmetric phase for the chiral symmetries. The fermions will remain massless, $m_f = 0$, and the fermions will not form chiral condensates, $\langle \overline{\Psi}_L \Psi_R \rangle = 0$.

For sufficiently attractive couplings, $G > G_c$, the four fermion interactions will induce a dynamical symmetry breaking. In the broken vacuum, the fermions will develop masses and chiral condensates which imply a gap equation the fermion masses, $m_f = - G \langle \overline{\Psi}_L \Psi_R \rangle \neq 0$. The symmetry breaking implies the existence of Nambu-Goldstone bosons. A scalar boundstate of the massive fermions is formed with the usual NJL relation, $m_s = 2\, m_f$.

The NJL model is usually "solved" by using a bubble approximation for the dynamics. Fermions can develop dynamical masses, but all vertex corrections are surpressed. This approximation corresponds to the use of the BCS wavefunction in superconductivity. The gap equation follows from the mass relation, $m_f = - G \langle \overline{\Psi}_L \Psi_R \rangle \neq 0$, where the condensate is computed using an internal free fermion loop with the dynamically generated mass, m_f, and the cutoff, Λ. In bubble approximation, the fermion - anti-fermion scattering amplitude is computed as a sum of diagrams where the fermion bubbles are iterated in the direct channel. Because of the sensitivity to the cutoff, the bubble contributions to the condensate must be computed consistently with those in the scattering amplitudes, otherwise the cutoff will introduce an explicit breaking of chiral symmetries. A direct calculation using the bubble approximation confirms the existence of the appropriate Goldstone bosons and the NJL prediction of the scalar meson boundstate mass. The NJL model has been previously invoked for generating composite structures [4] and provides the fundamental basis for models involving condensates of the top quark.

3. Top Quark Condensate Models.

In the Standard Model, the electroweak symmetries are spontaneously broken by condensates of an elementary scalar Higgs field. As the allowed mass for the top quark has systematically increased, it is natural to speculate on a possible connection between a large top quark mass and source of electroweak symmetry breaking. Top quark condensate models carry this idea to the extreme. The Higgs sector of the Standard Model is totally eliminated in favor of local, attractive interactions between the fermions of the theory which will induce the electroweak symmetry breaking as in the NJL model. Because of its large mass, the top quark plays the central dynamical role. In the minimal model, electroweak symmetry breaking follows from top quark condensate alone.

The analysis of this section follows that of Bardeen, Hill and Lindner (BHL) [2] which was motivated directly by the suggestions of Nambu [1]. The idea of top quark condensates as the mechansim of electroweak symmetry breaking was also advocated by Miransky, Tanabashi and Yamawaki (MTY) [3]. MTY use a somewhat different dynamical basis for their analysis of the four-fermion interactions than that presented in the BHL paper and reach somewhat different results.

As mentioned above, the Higgs sector of the Standard Model is replaced by local, attractive interactions of the fundamental fermion fields. In the minimal model, the top quark plays an essential role in generating the electroweak symmetry breaking and the masses for the physical W and Z gauge bosons. The minimal model is described by the Lagrangian,

$$L_{fermion} = L_{kinetic} + G\,(\overline{\Psi}_{La}{}^{A}\, t_R{}^{a})\cdot(\overline{t}_{Rb}\, \Psi_{LA}{}^{b}) \qquad (2)$$

where the composite operators are defined using a cutoff but preserving the electroweak gauge symmetries. $L_{kinetic}$ contains the kinetic terms for the fermions with the usual gauge couplings of the Standard Model. The four-fermion interactions in Eq.(2) represent the residual attractive interactions generated by a more fundamental dynamics existing above the cutoff scale. The four-fermion theory is not renormalizable, and physical quantities will be expected to depend on the cutoff scale even after renormalization of the independent coupling constants. Additional four-fermion interactions with weaker couplings could be added to generate masses for the lighter quarks and leptons but will have little effect on the dynmical symmetry breaking.

If these interactions are sufficiently attractive, $G > G_c$, then the electroweak symmetries will be spontaneously broken generating a mass for the top quark, $m_{top} > 0$, and a nontrivial top quark condensate, $\langle \bar{t} t \rangle \neq 0$. This symmetry breaking will also induce masses for the electroweak gauge bosons, $m_W \neq 0$ and $m_Z \neq 0$. We will also find that the physical Higgs particle of the Standard Model will be formed as a top quark - antitop quark bound state.

3.1. Bubble Approximation (NJL).

The standard method for analyzing the Nambu - Jona-Lasinio model (NJL) makes use of the bubble approximation. This method can be used as a first appoximation to the top quark condensate model as it contains the basic features of the composite structure produced by the dynamical structure of the theory. Phenomenological predictions will require the more complete analysis given in subsequent sections. The bubble approximation can be viewed as the large N_c limit of the theory where N_c is the number of colors, and G^*N_c is held fixed but all gauge couplings are neglected. The bubble theory has an exact solution in leading N_c approximation and $1/N_c$ corrections can be systematically computed.

The top quark mass is determined by the appropriate solution of the gap equation,

$$\begin{aligned} m_t &= -(1/2)\, G \langle \bar{t} t \rangle \\ &= G\, (N_c/8\pi^2)\, \{\, \Lambda^2 - m_t^2 \log\, (\Lambda^2/m_t^2)\, \}\, m_t \end{aligned} \tag{3}$$

with solutions,

$$m_t = 0$$

or

$$m_t^2 \log\, (\Lambda^2/m_t^2) = \Lambda^2 - 8\pi^2/(N_c{*}G) \tag{4}$$

with the massive solution being the preferred vacuum solution. Dynamical symmetry breaking can only occur for sufficiently attractive couplings, $G > G_c = (8\pi^2/N_c)\ (1/\Lambda^2)$. If the top quark mass is to be much below the cutoff, $m_{top} << \Lambda$, then a fine tuning of the four-fermion coupling, G, is required to cancel the quadratic cutoff dependence.

In the broken symmetry phase, the vector bosons become massive through the effects of the vacuum polarization diagrams including the contributions of the fermion bubbles which are needed to preserve the transversality consistent with the underlying electroweak gauge symmetry. The inverse W-boson propagator is given by

$$(1/g_2^2)D_W^{-1\mu\nu}(k) = (k^\mu k^\nu/k^2 - g^{\mu\nu}) \{ (1/g_2^2) k^2 - f_W^2(k) \} \qquad (5)$$

where the effective decay constant, f_W, is

$$f_W^2(k) = N_c (1/16\pi^2) \int_0^1 dx\, x\, m_t^2 \log(\Lambda^2/(x m_t^2 - x(1-x)k^2)) \qquad (6)$$

and $m_W^2 = g_2^2 f_W^2(m_W)$. The Z boson mass can also be computed in terms of its effective decay constant.

The computed values of the effective decay constants can be determined by the observed Fermi constant

$$\text{W:}\quad f_W^2 = (N_c/32\pi^2)\, m_t^2 \{ \log(\Lambda^2/m_t^2) + 1/2 \} = 1/4\sqrt{2}\, G_F$$
$$\text{Z:}\quad f_Z^2 = (N_c/32\pi^2)\, m_t^2 \{ \log(\Lambda^2/m_t^2) \} \approx f_W^2. \qquad (7)$$

For a large cutoff, $\Lambda \approx 10^{15}$ GeV, the bubble theory predicts a value for the top quark mass, m_{top} = 163 GeV, while m_{top} = 1 TeV for a smaller cutoff, $\Lambda \approx 10$ TeV. The bubble theory also predicts the mass of the physical Higgs scalar boson. It is given by $m_{higgs} = 2\, m_{top}$ which is the result usually quoted in the pure NJL theory.

We have seen that the elimination of the elementary Higgs sector has resulted in predictions for masses of both the top quark and the physical Higgs particle. Although qualitatively correct, the above predictions are strongly modified by the full electroweak dynamics.

3.2. Effective Field Theory.

From the bubble theory, we can infer that the effective low energy theory is the full Standard Model with composite Higgs fields. When viewed as the Standard Model, the coupling constants run with momentum scale at low energy, and the Higgs becomes static at high energy.

The effective field theory can be defined through the introduction of a static Higgs field, $H_A(x)$,

$$L_{fermion} = L_{kinetic} + G\,(\overline{\Psi}_{La}{}^{A}\,t_R{}^{a})\cdot(\overline{t}_{Rb}\,\Psi_{LA}{}^{b})$$

$$= L_{kinetic} - (\overline{\Psi}_{La}{}^{A}\,t_R{}^{a})H_A - H^{+A}(\overline{t}_{Rb}\,\Psi_{LA}{}^{b}) - (1/G)\,H^{+}H. \quad (8)$$

Instead of integrating out the static Higgs field to produce the four-fermion interaction, we can instead integrate out the short distance physics, replacing the cutoff scale Λ with a lower normalization scale μ. The short distance physics will generate contributions to the effective action defined at scale μ,

$$L_{fermion} = L_{kinetic} - (\overline{\Psi}_{La}{}^{A}\,t_R{}^{a})H_A - H^{+A}(\overline{t}_{Rb}\,\Psi_{LA}{}^{b})$$

$$+ Z_H\,(D_\mu H)^2 - (1/2)\,\lambda_0\,(H^{+}H)^2 - (1/G + \Delta M^2)\,(H^{+}H) + O(1/\Lambda^2). \quad (9)$$

In the bubble theory, the induced couplings are given by

$$Z_H = (N_c/16\pi^2)\,\log(\Lambda^2/\mu^2)$$

$$\lambda_0 = (2N_c/16\pi^2)\,\log(\Lambda^2/\mu^2) \quad (10)$$

and ΔM^2 has a quadratic dependence on the cutoff. From these results we can infer compositeness conditions on the running coupling constants,

$$\left.\begin{aligned} Z_H &= 1/g_t^2 \to 0 \\ \lambda_0 &= \lambda/g_t^4 \to 0 \end{aligned}\right\} \quad \mu^2 \to \Lambda^2 \quad (11)$$

These conditions are exact in the bubble theory and are abstracted to the full theory where they should refect the approximate behavior of the effective running couplings. If Z_H becomes sufficiently large, then the effective top quark Yukawa coupling, g_t, is small, and the effective field theory below scale μ is the weakly coupled Standard Model with a dynamical Higgs field.

3.3. Renormalization Group.

The long distance behavior of the four-fermion theory can be described by a weakly coupled gauge theory with a composite Higgs field. Renormalization group methods are an efficient way to sum infinite sets of diagrams. The leading terms are just the leading log contributions which are expected to dominate if there is a large hierarchy of scales, ie. $m_{top} << \Lambda$. The renormalization group can be used to evolve the running couplings to high scales where they must be matched to the appropriate boundary conditions of the composite theory.

We can compare various treatments of the coupling constant evolution:

1] Bubble (NJL) theory includes only the fermion loop contributions. This theory generates a composite Higgs but suppresses gauge and Higgs loop contributions.

2] The usual large N_c limit of QCD requires $N_c \to \infty$, with $G*N_c$ and α_3*N_c fixed, neglecting all other gauge couplings in loops. This theory includes all planar QCD corrections, and the low energy behavior is affected by infrared fixed points and is ultimately a theory of hadrons, not quarks and gluons.

3] The full Standard Model includes the effects of virtual Higgs contributions as well as the full gauge boson corrections. The theory is dominated by infrared fixed points of the renormalization group.

Figure 1 compares the three treatments for the running of $Z_H = 1/g_t^2$, from the Z boson mass scale to the cutoff scale, $\Lambda = 10^{15}$ GeV. This coupling directly determines the top quark mass as $m_{top} = g_t(m_{top})*v$. The turnover observed at low scales reflects the infrared fixed point behavior. The naive composite boundary condition was used at high scales although the perturbative renormalization group methods will break down as g_t becomes large. For a cutoff of 10^{15} GeV, the top quark mass predictons are

$$m_t(N_c \to \infty) \approx 270 \mathrm{GeV} > m_t(SM) \approx 230 \mathrm{GeV} > m_t(NJL) \approx 165 \mathrm{GeV}.$$

Figure 2 compares various renormalization group trajectories for the coupling constant of the Higgs self-interaction which determines the physical Higgs particle mass. The infrared fixed point structure sharply focusses the running at low energy and provides a precise prediction of the mass. The trajectories which flow to negative couplings at high energy are

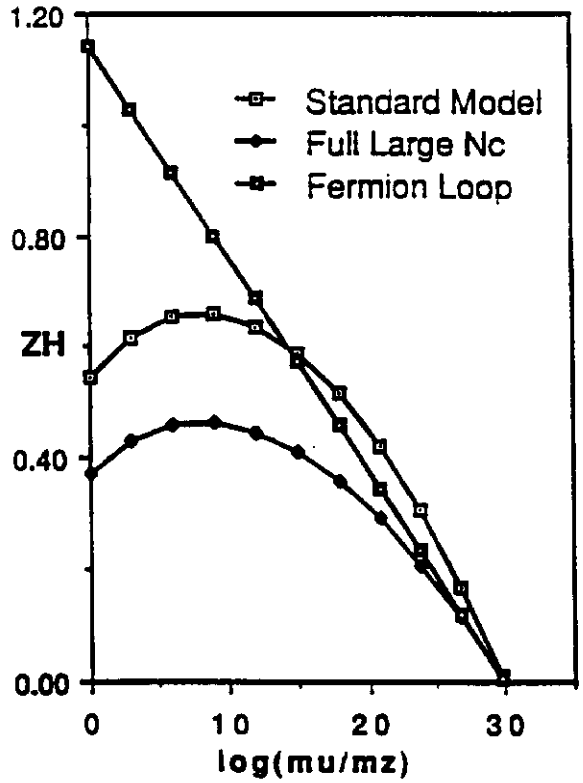

Figure 1. Higgs wavefunction/ top Yukawa coupling evolution.

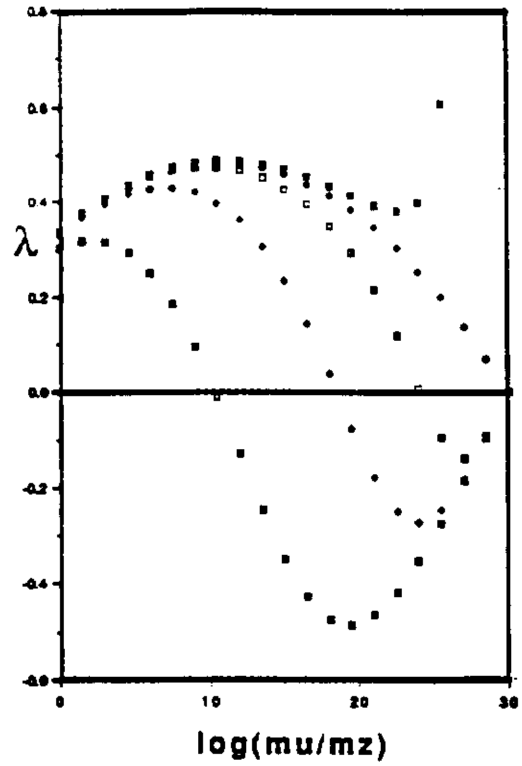

Figure 2. Higgs self-coupling evolution.

ruled out by the expected vacuum instability of these solutions at short distance.

Using the naive compositeness conditions but the full Standard Model evolution, BHL [2] obtained the following predictions for the top and physical Higgs particle masses for various composite scales, Λ. From a study of the compositeness conditions and the Standard Model evolution, it is expected that the theoretical ambiguities in the top quark mass predictions are only a few GeV [2].

Λ(GeV)	10^{19}	10^{15}	10^{13}	10^{9}	10^{4}
m_{top}	218	229	237	264	455
m_{higgs}	239	259	268	310	605

Table 1. Top quark and Higgs mass predictions in minimal model.

3.4. Conclusions for Minimal Model.

In the minimal model, the Higgs sector of the Standard Model is replaced by short range interactions of the top quark. If these interactions are sufficiently attractive, the electroweak symmetries are dynamically broken and the top quark becomes massive. The effective low energy field theory is the full Standard Model. The renormalization group methods give precise predictions for the top quark and the physical Higgs particle. The top quark must be quite heavy, $m_{top} > 220$ GeV. A high composite scale is favored, 10^{15} GeV $\rightarrow$ 10^{19} GeV, which might be identified with GUT or string model physics. The physical Higgs particle is expected to be only slightly heavier than the top quark, $m_{higgs} \approx 1.1\ m_{top}$, in contrast to the NJL (bubble) prediction of $m_{higgs} \approx 2\ m_{top}$. The infrared fixed point structure of the renormalization group equations stabilizes the predictions of the minimal model.

The minimal model predictions can be compared with constraints on the top quark mass coming from the various precision tests of the Standard Model. CDF provides a direct lower limit for the top quark mass, $m_{top} > 91$ GeV [7]. The strongest upper limits on the top quark mass come from deep inelastic neutrino scattering experiments and the collider measurements of the W boson mass. Langacker has reported the results of global fits to the present electroweak data [8]. He obtains the following upper limits,

$$m_{top} < 180 \text{ GeV (90\%CL)}, < 190 \text{ GeV (95\%CL)}, 210 \text{ GeV (99\%CL)}$$

where a Higgs particle mass of 250 GeV is assumed. This analysis depends on the careful understanding of the systematic errors for both theory and experiment for a wide range of processes. It is clear that the present analysis favors a lighter top quark than the expectations of the minimal top quark condensate model. However, we must await the discovery of the top quark as the determination of its mass will have crucial implications for the minimal top quark condensate model and perhaps the structure of Standard Model radiative corrections.

4. Comments and Extensions.

The minimal model of electroweak symmetry breaking discussed in the previous section can be related to a number of other approaches. It is important to examine the theoretical structure that makes it possible to have rather precise predictions of the top quark and Higgs particle masses. There are also many alternatives to the minimal model which still rely on the basic idea of short range, attractive interactions to generate the composite Higgs structure. In this section we will make a number of comments on the theoretical foundations of the model and discuss some of the most obvious extensions of the theory including a fourth generation and supersymmetry.

4.1. Infrared Fixed Points and Triviality.

The low energy behavior of the minimal model is governed by the full dynamics of the usual Standard Model. The low energy predictions are stabilized by the infrared fixed points (or more precisely pseudo-fixed points) of Standard Model renormalization group equations.

Fixed points were originally analyzed by Pendleton and Ross [9] and shown to provide a relation between the top quark Yukawa coupling constant and the running of the gauge coupling constants. If the evolution is to match the compositeness condition at high energy, then a different, but similar, relation between the couplings is achieved and the pseudo-fixed point discovered by Hill [10] dominates the low energy behavior. Figure 3 show the running top quark mass, or Yukawa coupling constant, as a function of normalization scale using a variety of couplings at a high energy cutoff scale, (A): $\Lambda = 10^{15}$ GeV or (B): $\Lambda = 10^{19}$ GeV. The renormalization flow of the top quark and Higgs coupling constants to the pseudo-fixed point was analyzed by Hill, Leung and Rao [11] and is shown in Figure 4 for a variety of intial conditions. The pseudo-fixed point structure makes the low energy predictions very insensitive to high energy boundary conditions and the precise value of the cutoff.

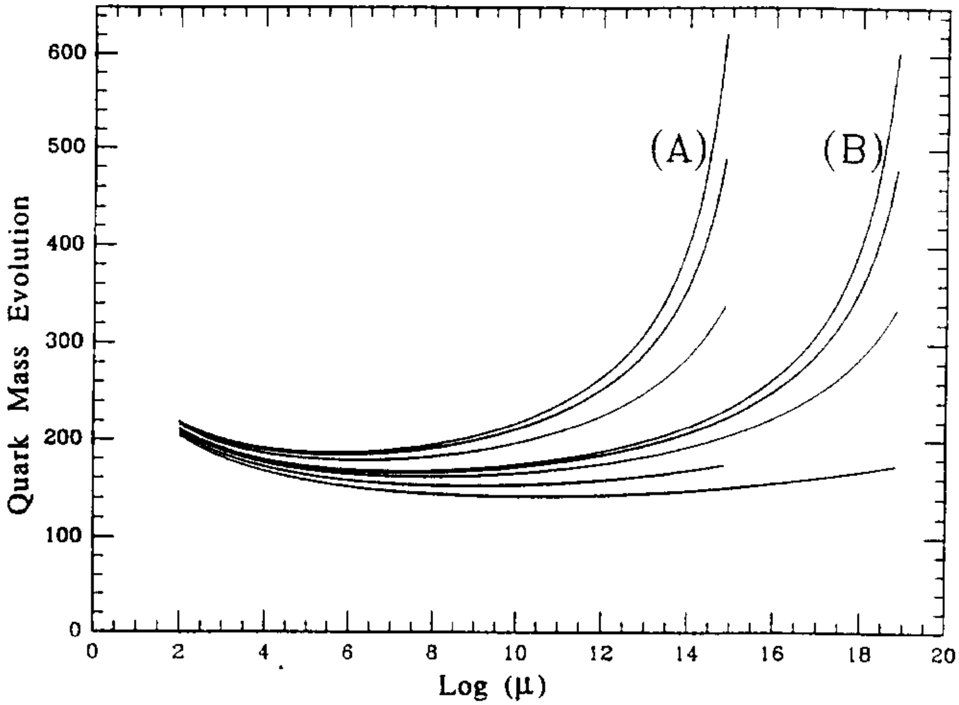

Figure 3. The quark mass evolution showing the pseudo-fixed point behavior for composite scales: 10^{15} (A) and 10^{19} (B) GeV.

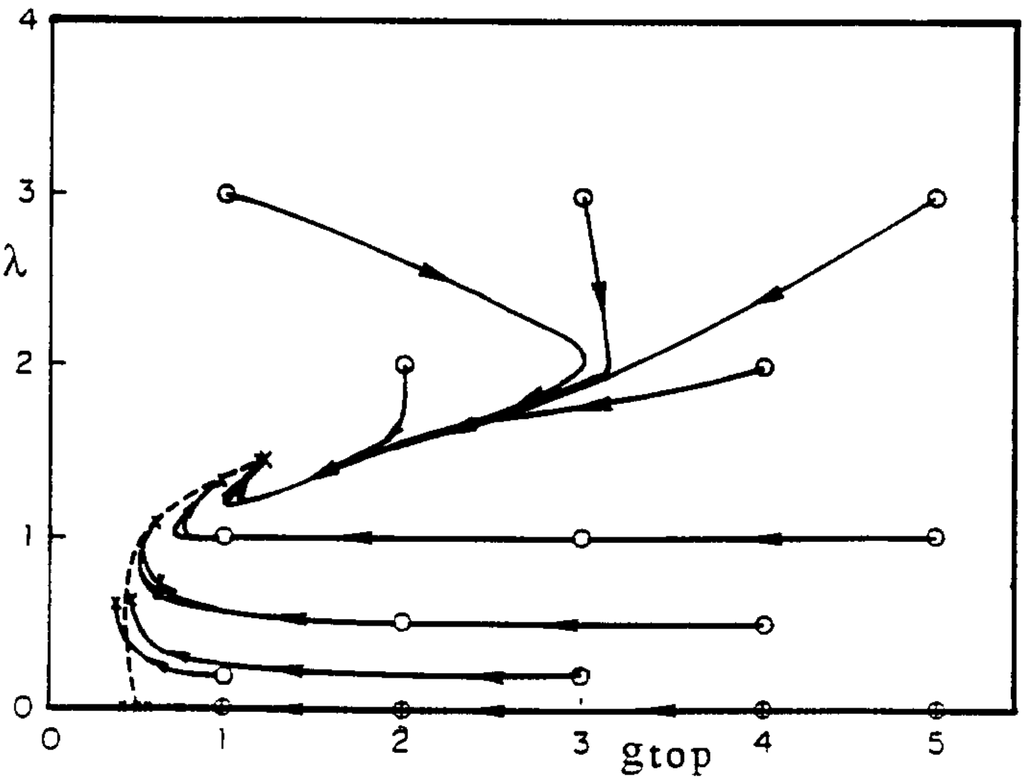

Figure 4. Renormalization flow to the pseudo-fixed point [11].

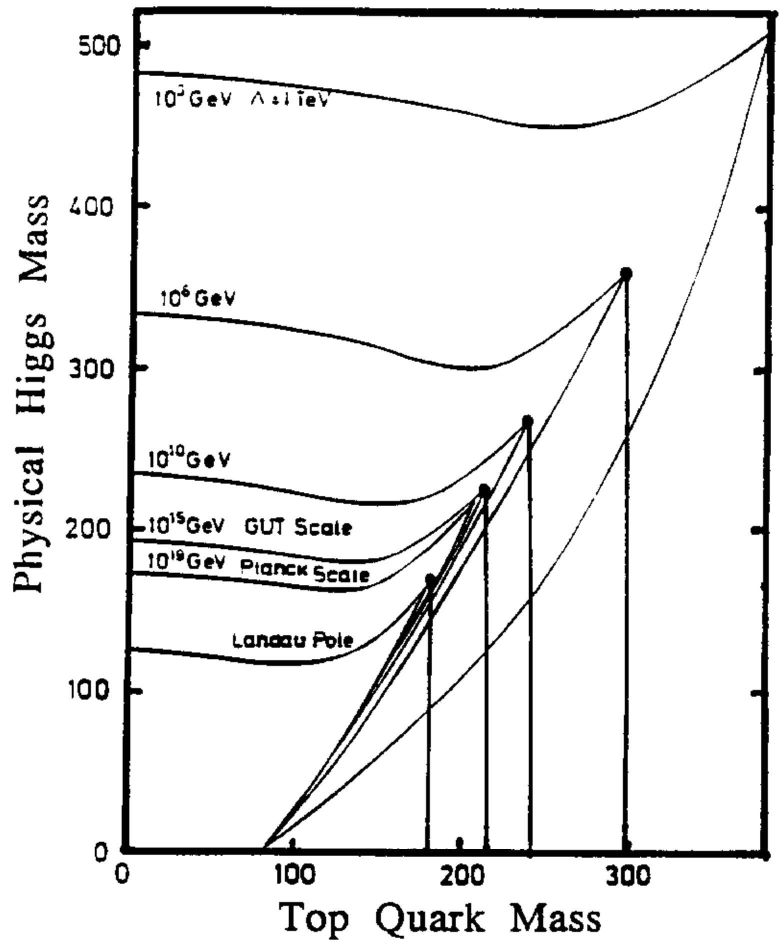

Figure 5. The triviality diagram for the Standard Model [12]. For each cutoff, the physical values of the top quark and Higgs particle masses must lie within the triviality domain. The compositeness condition is shown for each cutoff by the vertical line .

The Standard Model is said to be a trivial quantum field theory [12] as the running coupling constants grow at high energy and that the low energy couplings would vanish in a theory without cutoff. In this sense the Standard Model is not fully renomalizable as the cutoff can not be removed. Triviality diagrams, as in Figure 5, show the limitations on the low energy parameters, m_{top} and m_{higgs}, which follow from requiring that the effective theory remain perturbative up to the cutoff scale. Since the minimal model requires the naive compositeness condition, $Z_H \to 0$ or $g_t^2 \to \infty$ as $\mu \to \Lambda$, be satisfied, the top quark condensate model will lie on the boundary of the triviality diagram. In fact, it will only be consistent with the tip of the diagram with the largest allowed values of the top quark and

Higgs masses for a given cutoff scale because the vacuum instability noted in Figure 2 eliminates possible solutions for lighter Higgs particle masses. For a wider class of models, the boundaries of the triviality domains may be interesting to analyze for the possible interpretations of composite structure.

4.2. Compositeness Conditions.

In the bubble (NJL) approximation, an exact connection was made between the fundamental four-fermion dynamics at short distance and the effective Standard Model theory relevant to the long distance dynamics. We have abstracted an ultraviolet boundary condition on the running of the Standard Model coupling constants to reflect the composite structure. This connection is made in a domain where both the effective Standard Model couplings, $g_t \to \infty$, and the four-fermion couplings, $G \to G_c$, are becoming nonperturbative. Physics near the composite scale, Λ, is expected be very sensitive to renormalization effects, strong operator mixing, etc.

The basic physical structure of the theory will be preserved so long as the critical coupling, $G \to G_c$, remains a second order phase transition. The fine tuning required to produce an electroweak scale much below the composite scale can always be achieved. The second order transition implies the existence of a dynamical Higgs field and the effective field theory to describe the low energy physics. The precise bound state structure (one Higgs doublet, two Higgs doublets, etc) may depend on the nonperturbative aspects of the fundamental theory. However, the effective field theory which includes the bound states as independent degrees of freedom, should provide a good description of the dynamics at scales sufficiently below the composite scale, $\mu < \Lambda/10$, $\Lambda/100$. The physics near the composite scale, $\Lambda/10 < \mu < \Lambda$, is nonperturbative and must be properly integrated out. Corrections to the bubble theory can be expected to be large, O(1). However, these effects are expected to be small compared to the large logs generated by integrating out the physics below the the scale where the effective field theory becomes perturbative, eg. $\Lambda/10$. This expectation should be valid for the running couplings but not for the effective Higgs mass parameter which is subject to fine tuning and remains quite sensitive to even small modifications of the full theory. This sensitivity is irrelevant so long as fine tuning is possible and so long as a dynamical explanation of the fine tuning mechanism is not demanded.

To test the sensitivity to the specific choice of compositeness conditions, a model with higher derivative four-fermion interactions suggested by Suzuki [13] can be analyzed. The Lagrangian of Eq.(2) is replaced by

$$L = L_0 + G_0\cdot\{ (\overline{\Psi}_L t_R + (\chi/\Lambda^2)\cdot D\overline{\Psi}_L D t_R)(\overline{t}_R \Psi_L + (\chi/\Lambda^2)\cdot D\overline{t}_R D\Psi_L \}$$

$$= L_0 - (1/G_0)\cdot\{H^+H\} - (\overline{\Psi}_L t_R + (\chi/\Lambda^2)\cdot D\overline{\Psi}_L D t_R)H - H^+(\overline{t}_R \Psi_L + (\chi/\Lambda^2)\cdot D\overline{t}_R D\Psi_L) \quad (12)$$

By integrating out the high momentum components of the fermion loops, the effective action becomes

$$L \to L_0 - (\overline{\Psi}_L t_R + (\chi/\Lambda^2)\cdot D\overline{\Psi}_L D t_R)H - H^+(\overline{t}_R \Psi_L + (\chi/\Lambda^2)\cdot D\overline{t}_R D\Psi_L)$$

$$+ Z_H\cdot\{DH^+DH\} - m^2\{H^+H\} - (1/2)\cdot\lambda_0\cdot\{(H^+H)^2\} \quad (13)$$

where the running couplings are given by

$$Z_H = (N_c/(4\pi)^2)\cdot\{ \ln(\Lambda^2/\mu^2) - 2\cdot\chi + \chi^2/4 \}$$

$$\lambda_0 = 2\cdot(N_c/(4\pi)^2)\cdot\{ \ln(\Lambda^2/\mu^2) - 4\cdot\chi + 3\cdot\chi^2 - (4/3)\cdot\chi^3 + \chi^4/4 \} \quad (14)$$

$$m^2 = 1/G_0 + \cdots \quad \text{(fine tuned)}$$

using bubble approximation for the explicit calculations. For scales sufficiently below the composite scale, the higher derivative Yukawa couplings may be neglected, $D\mu/\Lambda \ll 1$, and the theory evolves as the normal Standard Model as in the case of minimal model. However, the presence of the higher derivative interactions has modified the compositeness boundary conditions.

Using Eq.(14) for the evolution between scales Λ and $\Lambda/5$, the low energy effective theory can be computed using various approximations for the evolution (NJL(bubble), large Nc, Standard Model) below the scale, $\Lambda/5$. Figures 6 and 7 show the effects of the higher derivative interactions on the predictions of the top quark mass and the Higgs mass. For reasonable

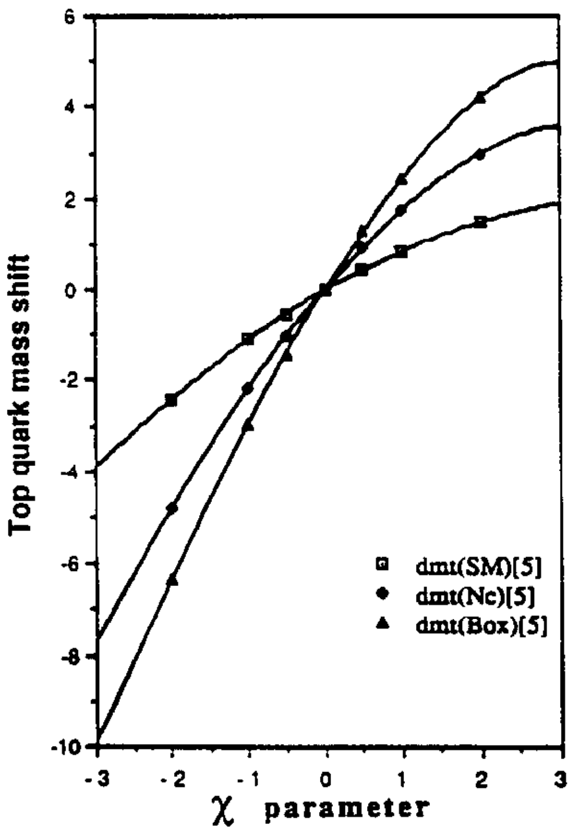

Figure 6. Top quark mass shift.

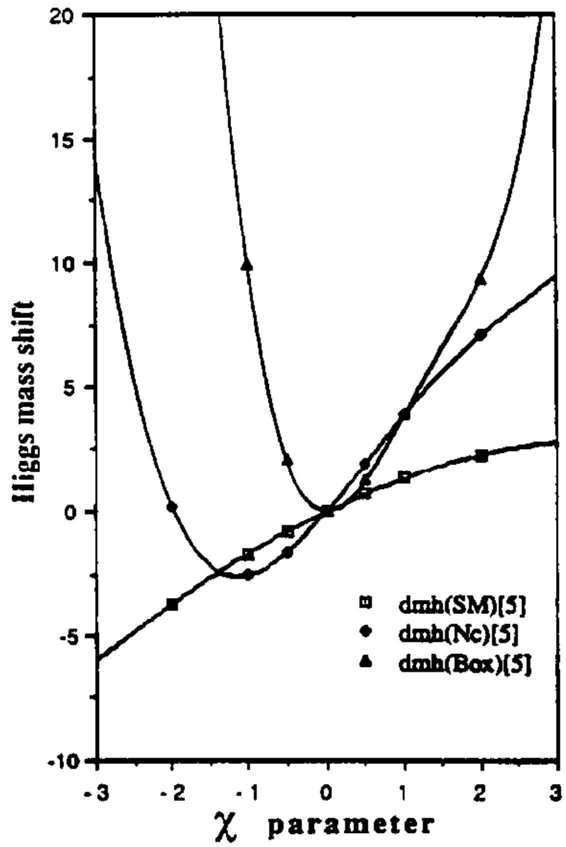

Figure 7. Higgs mass shift.

Shift due to modified UV boundary conditions for various treatments of the coupling constant evolution (SM, large N_c, box (NJL)) where χ is the coupling constant of the higher derivative interactions.

variations of the higher derivative coupling strength, $0 < \chi < O(1)$, the predictions of the Standard Model evolution are very stable with at most a few GeV shift in the masses. It is the fixed point structure of the full Standard Model evolution that provides this stability. This example is used only to indicate the possible effects of the physical evolution of the effective theory near the composite scale. As mentioned earlier, the initial evolution from the composite scale is likely to require nonperturbative analysis. This initial evolution modifies the boundary conditions for the subsequent Standard Model evoluton but has a limited effect on the ultimate predictions.

4.3. Fourth Generation Models.

If the top quark is found to be light, $m_{top} < 200$ GeV, then the top quark dynamics can not produce all the observed electroweak symmetry breaking, for $\Lambda < m_{planck}$. Additional symmetry breaking could come from a number of sources. An obvious extension would be to consider condensates involving a fourth generation of quarks and leptons assuming the masses satisfy the ρ parameter bounds. If the fourth generation is very heavy, then the composite scale could be much lower than the GUT scale considered for the minimal top quark condensate model. For low composite scales, the fine tuning problem is reduced, and the composite scale physics could be observable through the study of rare decays, FCNC, etc.

A degenerate fourth generation was considered by BHL [2]. The top contribution to electroweak symmetry breaking was neglected, and the degenerate mass for the fourth generation quark and the Higgs particle mass were computed for different composite scales, Λ, and are shown in Table 2.

Λ (GeV)	10^{19}	10^{15}	10^{13}	10^{6}	10^{4}
m_{quark}	199	206	212	277	388
m_{higgs}	235	248	258	365	553

Table 2. Mass predictions for the fourth generation model.

A fourth generation model with maximal mixing of the fourth generation quarks with the top quark was considered by Marciano [14]. He found that the top quark and the fourth generation up quark were nearly degenerate with a mass of 140 GeV. The fourth generation bottom quark was somewhat heavier at 160 GeV. Clearly the precise nature of the weak mixing will have an important impact on the predictions for a fourth generation model, and the compositeness conditions will only partly constrain these mixings.

The recent bounds from LEP on the number of light neutrinos implies that the fourth generation neutrino, if it exists, must be rather heavy, $m_{\nu 4} > 45$ GeV. Heavy neutrinos could result from mixing structure in the

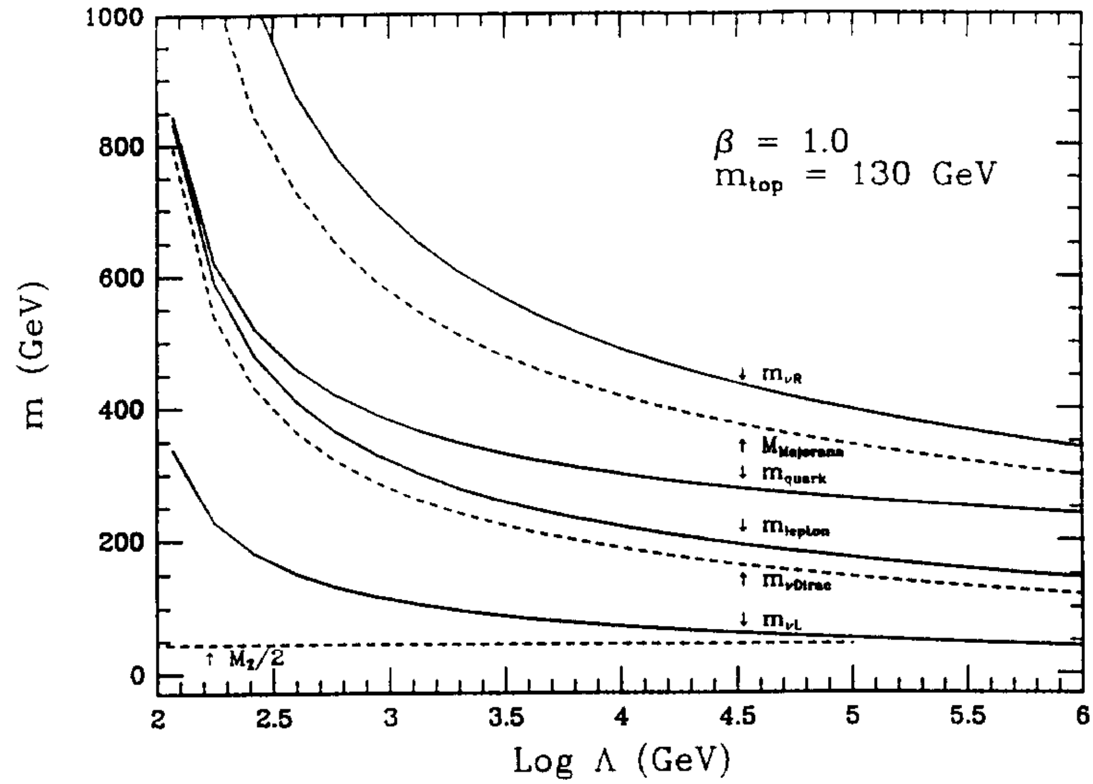

Figure 8. Quark and lepton masses of the four generation model with neutrino and quark condensates as functions of the cutoff.

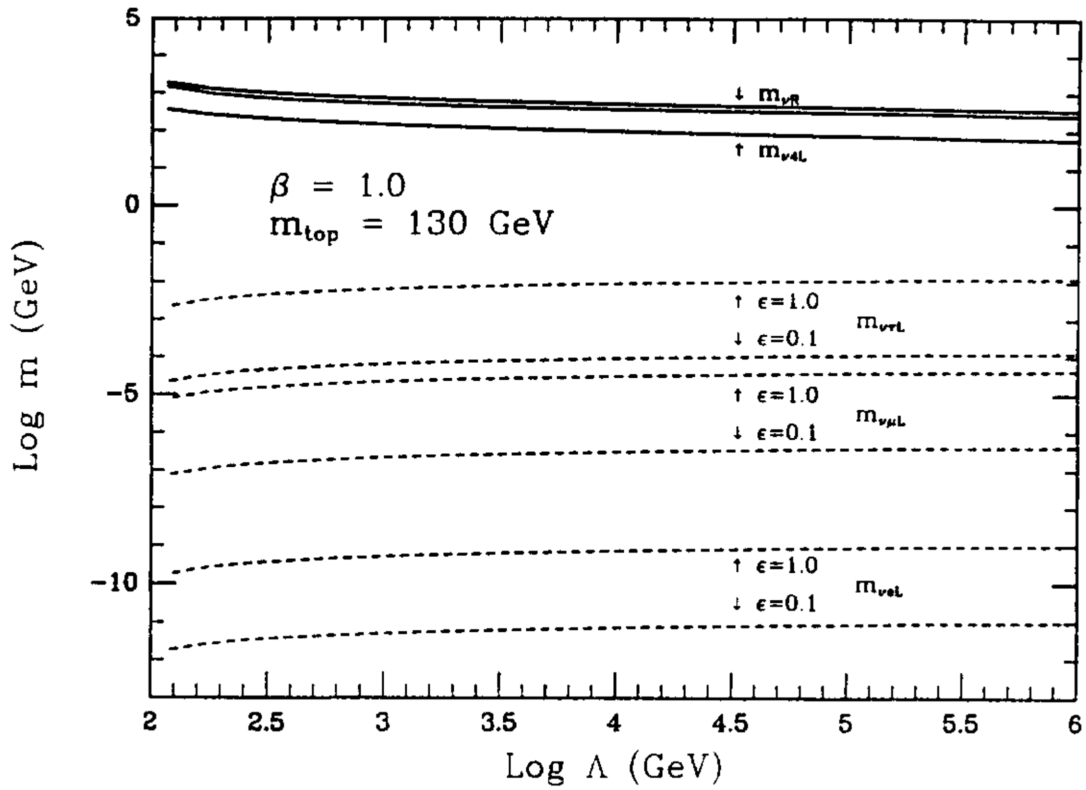

Figure 9. Neutrino mass spectrum of the four generation model assuming comparable Dirac masses for charged and neutral leptons.

neutrino mass matrix [15]. With the addition of right-handed neutrinos, it might be natural to expect that the dirac masses of the neutrinos are comparable to the charged lepton masses. A large Majorana mass for the neutrinos which does not break the usual electroweak symmetries could then be invoked which would suppress the masses of the observed neutrinos of the first three generations leaving the fourth generation neutrino heavy. Hill et al [16] have suggested a class of models where the Majorana mass results from neutrino condensates produced by attractive interactions of the right-handed neutrinos. Their predictions for the fourth generation masses are given as a function of the composite scale, Λ, in Figure 8 while the spectrum of neutrino masses are given in Figure 9. These results are for the case where the Majorana and normal Higgs VEV's are taken equal, $\beta = V_m/V_h = 1$; note that a low composite scale, $\Lambda < 10^6$ GeV, is required in this scheme. Right-handed neutrino condensates were also considered by Achiman and Davidson [17] as a mechanism for neutrino mixing with implications for composite DFS axions.

4.4. SUSY Extensions.

It is natural to consider the possible extension of the fermion condensate model to theories with supersymmetry. At the fundamental level, a supersymmetric extension of the local four-fermion interaction replaces the fundamental Higgs fields. This minimal supersymmetric extension of the BHL model would generate two composite Higgs supermultiplets. The naive compositeness boundary conditions require that only one of the the wavefunction normalization factors need vanish, $Z_H = 0$ and $Z_{H'} \neq 0$, to have both Higgs supermultiplets, H and H', be composite.

The results of even the minimal model are sensitive to the nature of supersymmetry breaking mechanisms. The renormalization group methods can be applied in two stages. From the composite scale, Λ, to the SUSY breaking scale, Δ, the couplings evolve supersymmetrically. At low energies the theory evolves as a normal gauge theory with additional Higgs representations.

A minimal version of a supersymmetric top quark condensate model was considered by Clark, Love and Bardeen (CLB) [18]. Only the top quark supermultiplet was involved in the dynamics with minimal SUSY breaking. The top quark becomes massive with the development of the corresponding condensate. CLB found that the top quark remained rather

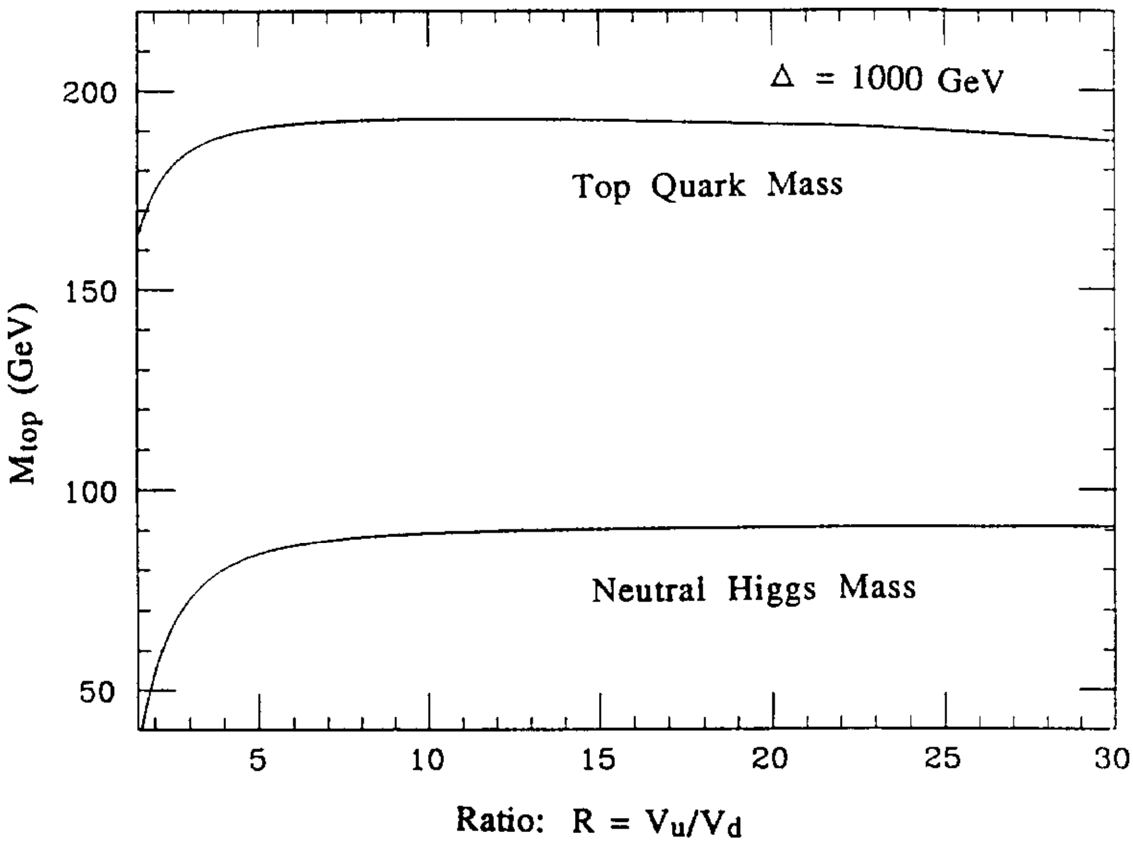

Figure 10. Top quark and physical Higgs particle masses for the minimal SUSY condensate model as a function of the ratio of the vacuum expectation values of the effective Higgs fields.

heavy with $m_{top} \approx$ 200 GeV. The fine tuning problem is reduced as the SUSY breaking scale, Δ, determines the fine tuning rather than the composite scale, Λ, which could be much larger.

The minimal model of CLB was somewhat too minimal as only the effective Higgs coupled to the top quark received a vacuum expectation value. The bottom quark would remain massless even if the effective Yukawa coupling were nonzero. Additional SUSY breaking must be added, as in the usual SUSY models, to the original interactions to produce vacuum expectation values for both effective Higgs multiplets.

The prediction of the top quark mass is reduced by the ratio of the VEV seen by the top quark, V_u, to the VEV seen by the gauge bosons, $V = \sqrt{V_u^2+V_d^2}$. The top quark mass depends on both the composite scale and SUSY breaking scale. The predicted values are given in Table 3 where the mass value must still be multiplied by the VEV ratio, m_{top} = [Table 3] * (V_u/V).

With this standard SUSY breaking scheme, the top quark mass predictions [19] are shown in Figure 10 as a function of the Higgs VEV ratio, V_u/V_d. Also plotted are the mass predictions for the lightest neutral Higgs particle. These results can be related to the boundaries of standard renormalization group studies of supersymmetric models [20]. In the minimal models considered above, the Higgs particle mass must be less than the Z boson mass, 91 GeV, and more than the recent LEP lower bounds, 40 GeV [21]. These results imply that the top quark should be heavier than about 170 GeV in the minimal composite SUSY model with the standard SUSY breaking scheme.

$\Lambda \backslash \Delta$	10^2	10^4	10^6	10^8	10^{10}
10^6	259	290			
10^9	222	241	254	263	
10^{13}	203	215	224	231	236
10^{15}	198	208	216	222	227
10^{19}	191	200	206	211	214

Table 3. SUSY model predictions of the top quark mass (GeV) as functions of the composite scale Λ and SUSY breaking scale, Δ.

Supersymmetry is usually invoked to help explain the gauge hierarchy problem. The composite Higgs models considered in this section do not directly address this problem although the fine tuning scale is set by the SUSY breaking scale instead of the higher compositeness scale. However this scale dependence also implies that the effective couplings of the fundamental, higher dimension operators may have greatly enhanced couplings if they are to generate the composite Higgs structure; this may cause problems with the perturbative unitarity of the theory. Although these models focus on composite Higgs structure, other supermultiplets could be considered for compositeness on the same basis.

4.5. Multiple Composite Higgs.

Another natural extension of the minimal top quark condensate model is the possible formation of additional composite Higgs bound states. This extension can be achieved by considering additional local, four-fermion interactions between the fundamental fermion fields. These additional interactions can produce attractive interactions in more than one channel and result in new bound states. Normally one would expect that additional fine tuning would be required for the new states to generate physics at a sufficiently low scale.

The simplest model involves interactions of both the top and bottom quarks as given in the following Lagrangian,

$$L_{fermion} = G_u\,(\overline{\Psi}_{La}{}^{A}\, t_R{}^{a})\cdot(\overline{t}_{Rb}\, \Psi_{LA}{}^{b}) + G_d\,(\overline{\Psi}_{La}{}^{A}\, b_R{}^{a})\cdot(\overline{b}_{Rb}\, \Psi_{LA}{}^{b})$$

$$+ G_{ud}\,(\overline{\Psi}_{La}{}^{A}\, t_R{}^{a})\cdot(\overline{\Psi}_{Lb}{}^{B}\, b_R{}^{b})\varepsilon_{AB} + \text{h.c.} \qquad (14)$$

where the last term is needed to provide mixing and break the chiral symmetries which would result in electroweak axions that are presently ruled out by experiment. With sufficient fine tuning, this theory generates two Higgs doublets, $H_{uA} = (\overline{t}_{Rb}\, \Psi_{LA}{}^{b})$ and $H_{bA} = (\overline{\Psi}_{Lb}{}^{B}\, b_R{}^{b})\varepsilon_{AB}$. Whether the degree of fine tuning required to keep both Higgs multiplets light is possible to achieve may require a nonperturbative study of the phase transition structure of the fundamental four-fermion theories. Studies using bubble approximation and modified renormalization group methods seem to indicate a consistent picture with additional light composite Higgs states. Two doublet models were considered by a number of authors [22] with consequences for the mass predictions of the top and bottom quarks as well as the spectrum of observable Higgs particles. The compositeness conditions place interesting constraints on the effective potential of the composite Higgs fields with implications for their low energy dynamics.

As mentioned before, four generation models require a mechanism for the producing a massive fourth neutrino. This may be accomplished [15,16] by introducing right-handed neutrinos. Possible condensates of these right handed neutrinos would produce electroweak singlet composite Higgs fields [16,17]. Hill, Luty and Paschos [16] have studied a unified picture of singlet and nonsinglet condensates which could play a role in realistic four generation models. Achiman and Davidson [17] have emphasized the possible role of right-handed neutrino condensates in producing composite DFS axions and their role in model building.

4.6. UV Fixed Points and Reduction.

We have emphasized the role of infrared (pseudo-) fixed points in generating stable predictions for the top quark condensate models. The pseudo-fixed points [10] control the renormalization group evolution at low energy where the gauge coupling constants control the running of the top quark Yukawa coupling constant. If the composite scale is large, then the top quark mass predictions are insensitive to the composite boundary conditions and are dictated by the infrared pseudo-fixed points.

A more ambitious analysis of the relation between the various electroweak coupling constants is made in the reduction approach advocated by Zimmerman, et al. [23] and Marciano[24]. In this approach, it is assumed that the Standard Model couplings are not all independent but have a functional interdependence. For example, the top quark Yukawa coupling constant is determined as a function of the gauge coupling constants. The renormalization group is used to determine the possible relationships between the various Standard Model couplings. For the top quark, the results are similar to those obtained in the orginial fixed point analysis of Pendleton and Ross [9].

Focussing on the top quark mass, the renormalization group provides a relation between the running top quark Yukawa coupling, K_t, and the color coupling constant, α_3. The renormalization group equations can be integrated to give

$$K_t = 2\,\alpha_3^{8/7} \,/\, (C + 9\,\alpha_3^{1/7}) + \text{electroweak corrections} \qquad (15)$$

where C is an integration constant with special values, 0 and ∞. $C = \infty$ might correspond to the situation of the light quarks. For $C = 0$, the coupling relation becomes analytic, and $C > 0$ is required if the Eq.(15) is to be nonsingular. If we identify $C = 0$ with the top quark situation, then the top quark mass is predicted to be 90-95 GeV [23, 24] which is just at the present lower limit of the direct search by CDF [7]. The same analysis would predict the Higgs particle mass of about 64 GeV which is still somewhat above the recent LEP lower bounds [21]. Refinements are not expected to change these predictions by large amounts.

The reduction approach chooses the special value, $C = 0$, to preserve the analytic structure of Eq.(15). From the renormalization group point of view, this constraint comes from requiring a nonsingular behavior up to infinite energies where α_3 vanishes. This contrasts with the top quark condensate models where the coupling constant becomes large at the composite scale which is taken to be less than the Planck scale. Hence, the solutions with $C < 0$ are required for top condensate models. The reduction method requires knowledge of the gauge coupling constant, α_3, for physical values corresponding to energies far beyond the Planck scale. Hence we can only view the constraints imposed by the reduction method as mathematical conditions imposed on the theory (perhaps as a result of hidden symmetries) and not as conditions on the physical running couplings. We will soon be able to determine the validity of the top quark mass predicition as the limits from CDF are improved or the top quark is discovered.

4.7. Schwinger-Dyson Equation Approach.

We have emphasized the use of renormalization group methods to give reliable predictions for the low energy parameters of the top condensate theory. The renormalization group is used to sum the leading contributions from an infinite set of Feynman diagrams describing the short distance physics. An alternate approach involves the direct solution of the Schwinger-Dyson equations for the behavior of the top quark self-energy function and use it to predict the top quark mass and the related electroweak symmetry breaking.

The Schwinger-Dyson equations have been studied by Barrios and Mahanta [25] and by King and Mannan [26]. Both groups study the effects of local four-fermion interactions on the solutions to the Schwinger-Dyson equations with the gauge interactions included in ladder approximation. The four-fermion coupling must be fine-tuned to generate a light top quark, $m_{top} \ll \Lambda$. The color gauge interactions increase the predicted value of the top quark mass from that obtained from the pure four-fermion theory in bubble approximation. The results are given in Table 4 for various values of the cutoff scale, Λ. Barrios and Mahanta have compared their Schwinger-Dyson results with the standard renormalization group procedure, as used by BHL [2], and achieve good agreement between the two methods so long as the same physics is considered.

Λ (GeV)	$m_t(\alpha_3=0)$	$m_t(\alpha_3)$	$m_t(RG)$
10^5	379.5	442.9	438.2
10^9	229.2	306.2	310.6
10^{15}	165.2	257.3	261.6
10^{19}	143.9	242.0	246.1

Table 4. Top mass predictions from the Schwinger-Dyson approach compared with the renormalization group results.

The Schwinger-Dyson equation method as implemented by both groups [25,26] includes only the effects of the local four-fermion interactions and the color gauge interactions, both in ladder approximation. These results must be compared to the equivalent renormalization group calculations which are similar to the "large Nc" results of BHL. As emphasized by BHL, the full low energy dynamics must be included to make meaningful physical predictions. The virtual Higgs contributions and the full electroweak gauge interactions were essential to obtain reliable predictions of the top quark and Higgs particle masses. These physical contributions must be included in the Schwinger-Dyson approach before its results can be compared directly to data. The Schwinger-Dyson approach must also confront possible nonperturbative aspects of the short distance physics associated with the four-fermion interactions which may also affect the intial evolution of the renormalization group method. The low energy predictions are expected to be somewhat insensitive to the short distance structure because of the infrared (pseudo-) fixed point behavior at long distance, but it may be difficult to separate these effects in the more direct analysis of Schwinger-Dyson equations.

4.8. Long Distance Contributions.

The renormalization group method can be used to systematically study the contributions of physics below the composite scale. The BHL analysis uses the one loop anomalous dimensions to compute the effective action to use at low energies, $\mu \approx m_Z$. The low energy radiative corrections must be used for accurate comparison of the theory with experiment. For example, top quark self-energy will receive a low energy contribution from its QCD interactions which result in a mass shift,

$$\delta m_{top} = m_0 (\alpha_3/3\pi) (4 + 3 \ln(\mu^2/m_0^2)) \qquad (16)$$

where μ is the low energy normalization scale, α_3 is the QCD coupling constant and m_0 is the lowest order top quark mass. In the BHL calculation, the ln contributions were absorbed by evolving the top quark Yukawa coupling constant to the top quark mass scale instead of the m_Z normalization scale. A remaining $O(\alpha_3)$ contribution could affect the top quark mass predictions as emphasized by Kugo [27]. However, these QCD corrections may be largely cancelled by similar electroweak corrections as they are for the log corrections which are largely cancelled because of the infrared fixed point structure. A consistent calculation must incorporate the two loop contributions to the renormalization group evolution as well as the finite part of the one loop effects at the low energy scale. Since the two loop anomalous dimensions are known, the top quark mass predictions can be systematically improved.

5. Conclusions.

We have shown that the BCS mechanism can be used to connect the electroweak scale (m_W, m_Z) with the masses of the top quark and the physical Higgs particle. Electroweak symmetry breaking is produced by condensates of the top quark which are triggered by attractive, local interactions of the top quark. Stable predictions for the top quark and Higgs masses are achieved using the full Standard Model evolution which reflects the presence of infrared (pseudo-) fixed points in the renormalization group equations.

The minimal model predicts a heavy top quark, $m_{top} > 200$ GeV. For large composite scales, the physical Higgs particle is predicted to be only slightly heavier than the top quark, $m_{higgs} \approx 1.1 * m_{top}$ which contrasts with the NJL prediction of $m_{higgs}/m_{top} = 2$. The renormalization group analysis results in a rather precise prediction of the top quark mass, m_{top} = 229 ± 5 GeV ($\Lambda \approx 10^{15}$ GeV) where the error reflects an estimate of the theoretical error coming from the composite boundary conditions. To achieve a relatively light top quark in the minimal model, the composite scale must be taken to be quite large and could be associated with the GUT scale, 10^{15} GeV, or the Planck scale, 10^{19} GeV. Even with this choice of composite scale, the minimal top quark model predicts masses which are somewhat larger than the range estimated by recent fits to the Standard Model radiative corrections, $m_{top} = 137 \pm 40$ GeV [8]. While the minimal top condensate model seems to be somewhat disfavored by the present data, we should wait until the top quark is discovered before reaching final conclusions about the validity of the minimal model or the normal radiative corrections.

There are many extensions to the minimal model which preserve the basic idea of electroweak symmetry breaking being generated by new short distance dynamics rather than additional fundamental degrees of freedom. The four generation version of the theory relaxes the constraint on a heavy top quark but requires a mechanism for producing a heavy neutrino for the fourth generation. Right-handed neutrinos could have an important dynamical role in generating mixing and producing a spectrum of neutrino masses. Condensates of right-handed neutrinos could have important phenomenological consequences and could be responsible for a new mechanism for axions.

Fermion condensates may also generate a more complex Higgs structure. Models with two Higgs doublets have been studied in some detail. The compositeness conditions place interesting constraints on the effective potentials of the dynamical Higgs fields. As stated above, neutrino condensates may also play a role in the low energy dynamics.

We have briefly discussed the relation of the condensate models with alternative approaches including coupling constant reduction which has much different predictions for the top quark and Higgs masses and the direct Schwinger-Dyson equation method which yields the same results as the renormalization group methods so long as the same physical input is achieved. We have not discussed early work [4] which focussed on the possible role of four-fermion interactions in generating composite vector mesons using methods associated with the NJL models. The role of electroweak gauge symmetry plays a crucial role in these models, and the dynamical structure of these models remains to be understood.

There are many remaining questions for the implementation of condensate models. The models require fine-tuning to produce an electroweak scale that is much below the composite scale as was the case in the normal Standard Model. It is interesting to speculate about possible mechanisms which could generate this fine-tuning dynamically. The physics at the composite scale is not renormalizable and is presumably generated by physics beyond the composite scale such as the fragments of a GUT model or superstring theory. These theories should generate many higher dimension interactions which are mostly irrelevant to the low energy physics. If the coupling constants of these interactions are dynamically determined, it is possible to imagine feed-back mechanisms could produce the apparent fine-tuning needed to produce the desired infrared structure of the full theory. The top quark condensate model makes definite predictions for the top quark and Higgs particle masses the flavor mixing structure observed in the quark sector can only be accommodated. The understanding of flavor mixing whether it is in the explored domain of the quarks or the unexplored domain of the neutrinos remains an outstanding problem for any fundamental theory.

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7. References.

1. Y. Nambu, Proceedings of the 1988 Kazimierz Workshop, Z. Ajduk et al., eds. (World Scientific, 1989); Proceedings of the 1988 Nagoya Workshop, M. Bando et al., eds. (World Scientific, 1989); EFI Preprint 89-08 (1989) (unpublished).

2. W. Bardeen, C. Hill and M. Lindner, Phys. Rev. D41, 1647(1990).

3. V. Miransky, M. Tanabashi and K. Yamawaki, Mod. Phys. Lett. A4,1043(1989); Phys. Lett. B221, 177(1989).

4. H. Terazawa, Phys. Rev. D22,2921(1980); H. Terazawa, Y. Chikashige and K. Akama, Phys. Rev. D15, 480(1977), F. Cooper, G. Guralnik, and N. Snyderman, Phys. Rev. Lett. 40, 1620(1978); T. Eguchi, Phys. Rev. D14, 2755(1976); K. Kikkawa, Prog. Theor. Phys. 56, 947(1976); T. Kugo, Prog. Theor. Phys. 55, 2032(1976); J.D. Bjorken, Ann. Phys. 24, 174(1963).

5. J. Bardeen, L. Cooper and J. Schrieffer, Phys. Rev. 108, 1175(1957).

6. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345(1961); W. Bardeen, C. Leung, and S. Love, Phys. Rev. Lett. 56, 1230 (1986).

7. CDF Collaboration (F. Abe, et al.), Phys. Rev. Lett. 64, 142(1990); FERMILAB-PUB-90-137-E (1990) (to be published in Phys. Rev. D).

8. D. Kennedy and P. Langacker, "Precision Electroweak Experiments and Heavy Physics: A Global Analysis", U. Pennsylvania Preprint UPR-0436T (1990); P. Langacker, "Precision Tests of the Standard Model", U. Pennsylvania Preprint UPR-0435T (1990).

9. B. Pendleton and G. Ross, Phys. Lett. 98B, 291(1981).

10. C. Hill, Phys. Rev. D24, 691(1981).

11. C. Hill, C. Leung and S. Rao, Nucl. Phys. B262, 517(1985).

12. L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B136, 115(1978); N. Cabbibo et al, Nucl. Phys. B158,295(1979); M. Lindner, Z. Phys. C31, 295(1986).

13. M. Suzuki, Mod. Phys. Lett. A5,1205(1990); 1990 Nagoya Workshop (1990).

14. W. Marciano, "Dynamical Symmetry Breaking and the Top Quark Mass", Brookhaven Preprint, September, 1990.

15. C. Hill and E. Paschos, Phys. Lett. B241, 96(1990).

16. C. Hill, M. Luty and E. Paschos, "Electroweak Symmetry Breaking by Fourth Generation Condensates and the Neutrino Spectrum", FERMILAB-PUB-90-212-T (1990).

17. Y. Achiman and A. Davidson, "Dynamical Axion of Dynamical Electro-Weak Symmetry Breaking", Wuppertal Preprint WU B90-14 (1990).

18. T. Clark, S. Love and W. Bardeen, Phys. Lett. B237, 235(1990).

19. K. Sasaki, M. Carena, C. Wagner, T. Clark and W. Bardeen, (in prep.)

20. H.P. Nilles, Phys. Rep. 110, 1(1984); J. Bagger and S. Dimopoulos, Phys. Rev. Lett. 55 920(1985); H. Okada and K. Sasaki, Phys. Rev. D40, 3743(1989).

21. OPAL Collaboration (M. Akrawy, et al.), "Search for the Minimal Standard Model Higgs Boson in e+e- Collisions at Lep", CERN-PPE-90-150 (1990).

22. M. Luty, Phys. Rev. D41, 2893(1990); M. Suzuki, Phys. Rev. D41, 3457(1990); C. Froggatt, I. Knowles and R. Moorhouse, Phys.Lett. B249, 273(1990) ; J. Fröhlich and L. Lavoura, "Bootstrap Dynamical Symmetry Breaking with Top and Bottom Quarks", Dortmund Preprint DO-TH-90/11 (1990); C. Hill, M. Luty and E. Paschos, "Electroweak Symmetry Breaking by Fourth Generation Condensates and the Neutrino Spectrum", FERMILAB-PUB-90-212-T (1990).

23. J. Kubo, K. Sibold and W. Zimmerman, Phys. Lett. B220, 85(1989).

24. W. Marciano, Phys. Rev. Lett. 62 (1989).

25. F. Barrios and U. Mahanta, ITP Santa Barbara Preprint, UCTP-104/90 (1990).

26. S. King and S. Mannan, Phys.Lett. B241, 249(1990).

27. T. Kugo, contribution to Nishinomiya/Kyoto Symposium (1990).