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A Criterion for Existence of Finite to All Orders N=1 SYM Theories*

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ABSTRACT

We investigate the general conditions for an all-orders finite theory by redefi-ning the coupling constants such that both the gauge coupling β-function and the anomalous dimensions of the gauge superfield and chiral superfields vanish. These explicit expressions for the conditions of all-orders finiteness involve solutions of an infinite number of equations. Both a solving process and a criterion for existence of the solutions of the equations are given.

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1. Introduction

Over the past few years, the search for perturbatively finite quantum field theories always retains a certain interest. A number of four-dimensional super-symmetric Yang-Mills (SYM) theories have been shown to be free of ultraviolet divergences. At first, a large class of N=4 (Sohnuis and West 1981, Mandelstam 1983, Brink et al 1983) and N=2 (Howe et al 1983) theories was found. Many attempts were then made to find finite theories for N=1 case. Through direct calculating (Parkes and West 1984, West 1984) and considering involving the chiral anomaly (Jones 1983, Jones and Mezincescu 1984, Breitenlohner et al 1984, Jones et al 1985, Grisaru and West 1985) based upon the Adler-Bardeen theorem (Adler and Bardeen 1969, Bardeen 1969, Zee 1972), the conditions which guarantee one-loop finiteness ensure two-loop finiteness as well were proved. Then later on all two-loop finite N=1 SYM theories of simple groups were found (Hamidi et al 1984, Jiang and Zhou 1986, Dong et al 1986). Naturally, people attempt to find the finite N=1 theories to all orders. Analyses of three-loop approximation have shown that two-loop finiteness automatically keeps the gauge superfield propagator finite at three-loop level, but the chiral superfield one is in general divergent (Parkes and West 1985, Parkes 1985, Jones and Parkes 1985, Lucha and Neufeld 1986, Lucha 1987, Bohm and Denner 1987). Fortunately, a new algorithm, redefining the Yukawa coupling constants in the theory as a Taylor series of the gauge coupling constant g, is proposed by Jones and Ermushev et al (Jones 1986, Ermushev et al 1987, Kazakov 1986), since then one can make the N=1 theory finite to all orders.

In this paper, using a method similar to above, we further investigate the general conditions for an all-orders finite theory. Section 2 deals with the analyses of the self-energy graphs of the chiral superfields and with a proof of a theorem which is important for later solving the equations of the conditions of the finiteness. The relationship between vanishing both the gauge coupling β-function and the anomalous dimensions of the gauge superfield and chiral superfields is given in section 3. These explicit expressions for the conditions of all-orders finiteness involve solutions of an infinite number of equations. Both a solving process and a criterion for existence of the solutions of the equations are concluded in section 4.

2. Analyses of the self-energy graphs

In a general N=1 SYM theory, the fundmental superfields are the N=1 vector multiplet A^r (in a adjoint representation, r is the component-index), and a set of N=1 scalar multiplets ϕ_α (in reducible representations R^r). Matrices R^r satisfy the following condition:

$$[R^r, R^s] = i f^{rst} R^t . \quad (1)$$

In general, R^r can be reduced to the sum of irreducible representations and also obey eq.(1). We denote these irreducible representations by i, j, k (i,j, k=1,2,...n), their components by a and b, i.e. $\alpha=(i,a)$, $\beta=(j,b)$, and the classes of inequivalent irreducible representations by x, y, z. The matrix elements of R^s can be written as

$$(R^s)^\alpha_\beta = (R^s)^{ia}_{jb} = (R^s(i))^a_b \delta^i_j = (R^s(x))^a_b \delta^i_j \qquad i \in x . \quad (2)$$

The superpotential is defined as

$$W = \frac{1}{3!} \sum_{\mu,i,j,k,a,b,c} d^{ia,jb,kc}(\mu)\, \phi_{ia} \phi_{jb} \phi_{kc}$$

$$= \frac{1}{3!} \sum_{\alpha\beta\gamma} d^{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma \qquad , \quad (3)$$

where W is invariant under the gauge group G. $d^{\alpha\beta\gamma}$ is a Yukawa coupling constant and totally symmetric in α, β and γ. We denote complex conjugation by raising and lowering indices, thus $d^*_{\alpha\beta\gamma} = d^{\alpha\beta\gamma}$. In some cases R_i, R_j and R_k can be coupled into a singlet of G in several different ways which the index μ refers to.

According to the Feynman rule in an N=1 SYM theory, the shrinkage of two vector multiplets A^r and A^s gives a factor of the propagator as

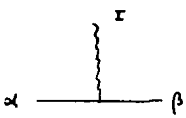
$$r \text{ - - - - - - - - - - } s \qquad \sim \delta^{rs} \quad . \tag{4a}$$

The shrinkage of scalar multiplets ϕ^α and ϕ_β gives

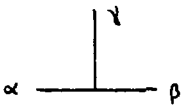
$$\alpha \text{ —+——— } \beta \qquad \sim \delta^\alpha_\beta = \delta^i_j \delta^a_b \quad , \tag{4b}$$

but no shrinkage exists between ϕ^α and ϕ^β or ϕ_α and ϕ_β .

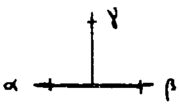
Some vertices are as follows:

(a) 

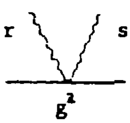
$$\sim g(R^r)^\alpha_\beta = g[R^r(x)]^a_b\, \delta^i_j \qquad i \in x \ , \tag{5a}$$

(b) 

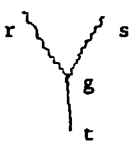
$$\sim d_{\alpha\beta\gamma} \quad , \tag{5b}$$

(c) 

$$\sim d^{\alpha\beta\gamma} \quad , \tag{5c}$$

(d) 

$$\sim g^2\,(R^r R^s)^\alpha_\beta \quad , \tag{5d}$$

(e) 

$$\sim g\, f^{rst} \quad . \tag{5e}$$

For exploring a finite N=1 SYM theory,we are interested in calculating the Faynman diagram as α —+—[ ]—— β . This self-energy diagram of chiral superfields concerns with γ^α_β ,the anomalous dimensions of the chiral superfields. The calcu -lated result of each self-energy diagram always can be disassembled to two factors. One factor involves the group quantities which including the coupling constant. Another one involves the momentum integral which is in general divergent. Using the calculated results of these diagrams we can get the expression of γ^α_β. In general, there are more than one diagrams which contribute to γ^α_β at same order. If we want to vanish γ^α_β at any orders we try to make the contributions of these diagrams can be canceled. For example, one-loop anomalous dimension of chiral superfields is given by (Jones and Mezincescu 1984, Parkes and West 1985, Jones 1986, Capper and Jones 1985, Lucchesi et al 1988):

$$\gamma^{(1)\alpha}_{\beta} = \frac{1}{32\pi^2}[d^{\alpha\delta\gamma} d_{\beta\delta\gamma} - 4g^2 (R^s R^s)^{\alpha}_{\beta}] \quad , \tag{6}$$

where two terms in the bracket are from follow two diagrams

Now let us give a analysis for the properties of group factors of a arbitrary diagram. It is easy to see that the group factors in general consist of the products of group quantities such as $d^{\alpha\beta\gamma}$ (or $d_{\alpha\beta\gamma}$), $(R^s)^{\alpha}_{\beta}$ and f^{rst}. The indices of group quantities are shrinked in pair. At last only up-index α and down-index β do not shrink. For the indices of adjoint representation, the r, s, t, two arbitrary indices can be shrinked. But for the indices of chiral fields, the α, β, γ, only a up-index and a down-index can be shrinked.

For later using, we give a few definitions:

$$\sum_s [R^s(x)R^s(y)]^a_b = C_2(x)\, \delta^a_b\, \delta^x_y \quad , \tag{7}$$

$$Tr[R^s(x)R^t(x)] = \delta^{st}\, T[R(x)] = \delta^{st}\, T(x) \quad , \quad \text{(not sum for x)} \tag{8}$$

where $C_2(x)$ and $T(x)$ are the value of the quadratic Casimir operator and the Dykin index for the irreducible representation x, respectively.

Writting the group factor of one self-energy diagram of chiral superfields as P^{α}_{β}, we find a group factor theorem as follows:

$$P^{\alpha}_{\beta} = q^i_j\, \delta^a_b\, \delta^x_y \quad , \quad \alpha=(i,a),\ \beta=(j,b),\ i\in x,\ j\in y, \tag{9}$$

where the quantity q^i_j only depends on the indices of the irreducible representations. We give a brief proof of this theorem as following.

In general, the interaction vertex of N superfields can be denoted as

$$R_{\mu_1\mu_2\cdots\mu_N}\,\phi^1_{\mu_1}\,\phi^2_{\mu_2}\cdots\phi^N_{\mu_N} \quad . \tag{10}$$

where $R_{\mu_1\mu_2\cdots\mu_N}$ could be Yukawa coupling coefficient of chiral superfields, also could be the C-G coefficient of chiral superfields and vector superfields.

Because of the theory is invariant under the transformation (local, global) of gauge group G, so that the interaction vertex is a siglet of the group. We assume that each field above belongs to a irreducible representation of G and its corresponding generator matrices are:

$$(T^s_i)_{\mu_i \mu_i'} \qquad\qquad i=1,2,\cdots N ,$$

where i denotes irreducible representation. T^s_i can be selected as Hermitian matrix:

$$(T^{s*}_i)_{\mu_i \mu_i'} = (T^s_i)_{\mu_i' \mu_i} .$$

A self-energy diagram consists of several interaction vertices, like (10), contracting all pairs of superfields with the exception of one pair, correspond-ing to the external lines. Its group factor $P_{\mu_1\mu_2}$ is a product of several R coefficients in (10), contracting all pairs with the exception of one pair μ_1, μ_2. For a chiral superfield self-energy diagram, $P_{\mu_1\mu_2} = P^\alpha_\beta = P^{ia}_{jb}$. It is not difficult to see P^α_β is a C-G coefficient, which couples ϕ_{ia} and ϕ^{jb} to a singlet:

$$\sum_{a,b} P^{ia}_{jb} \phi_{ia} \phi^{jb} ,$$

where $\phi_{ia} = (\phi^{ia})^\dagger$, belonging to the contragredient representation of ϕ^{ia} . The generator matrices $\tilde{T}^s_i$ of this representation is equal to $-(T^s_i)^t$, where $(T^s_i)^t$ is the transposition of matrix T^s_i . According to the property of C-G coefficients, we have:

$$\sum_{a'} P^{ia'}_{jb} (\tilde{T}^s_i)^a{}_{a'} + \sum_{b'} P^{ia}_{jb'} (T^s_j)^{b'}{}_b = 0 , \qquad (11)$$

or in matrix form:

$$(T^s_i \cdot P^{ia}_{jb}) - (P^{ia}_{jb} T^s_j) = 0 \quad \text{(no sumation over i or j)} . \tag{12}$$

Now using Schur Lemma, if T^s_i and T^s_j belong to two inequivalent irreducible representations x and y, $P^{ia}_{jb} = 0$; if T^s_i and T^s_j belong to the same irreducible representation x, $P^{ia}_{jb} = q^i_j \delta^a_b$, where q^i_j are independent of a. So we have:

$$P^{ia}_{jb} = q^i_j \delta^a_b \delta^x_y \quad , \qquad i \in x,\ j \in y . \tag{13}$$

The group factor theorem shows: Two extenal lines belong to either inequivalent irreducible representations or to same class of irreducible representations but to different components of the representations, while the group factor P^α_β is equal to zero. If two extenal lines belong to same class of irreducible represen-tations and same components but to different equivalent irreducible representa-tions, i.e. i ≠ j, while P^α_β is not equal to zero in general. Thus $P = q^i_j$ depends on the indices i and j, but does not depend on the indices a and b.

3. The conditions for all-orders finiteness

The conditions for one-loop finiteness are (West 1984, Parkes and West 1984, Jiang and Zhou 1986):

$$T(R) = \sum_{i=1}^{N} T(R_i) = 3C_2(G) \quad , \tag{14}$$

$$\sum_{b,c,j,k,\mu,\mu'} d^{abc}_{ijk}(\mu)\, d^{*a'bc}_{i'jk}(\mu') = 2g^2 \delta_{aa'} \delta_{ii'} C_2(R_i) \quad \text{(for all i,i')} , \tag{15}$$

where $T(R_i)$ and $C_2(R_i)$ are the Dykin index and the value of the quadratic Casimir oprator for the irreducible representation R_i, respectively. $C_2(G) = C_2(R_a)$, where R_a is the adjoint representation of G.

The ultraviolet divergences of a general renormalizable N=1 supersymmetric gauge theory are controlled by two functions: the gauge β-function β_g and the anomalous dimension matrix of the chiral superfields, γ^α_β. A theory is finite if β_g and γ^α_β vanish. Using a consequence of a general theorem: (if an N=1 super

-symmetric gauge theory is finite up to n-loop, then the gauge propagator is finite in (n+1)-loop)(Grisaru et al 1985), the condition for one-loop β-function $\beta_j^{(1)} = 0$ is known, and if $\gamma_\beta^{\alpha(l)} = 0$ $(l=1,2,\cdots,n)$ are satisfied, then $\beta_j^{(n+1)} = 0$. Therefore, in this way $\beta_j^{(1)} = 0$ and $\gamma_\beta^{\alpha(n)} = 0$ to all orders guarantee a theory finite to all orders. If one wants to construct a finite theory based on the one-loop finite theory, one only demands:

$$\sum_{n=1}^{\infty} \gamma_\beta^{\alpha(n)} = \gamma_\beta^{\alpha} = 0. \tag{16}$$

Writing the Yukawa coupling $d^{\alpha\beta\gamma}$ into two factors:

$$d^{\alpha\beta\gamma} = d^{ia,jb,kc}(\mu) = h^{abc}(x,y,z,\mu) g^{ijk}(\mu), \tag{17}$$

where the first factor h^{abc} depends on the class indices x,y,z of irreducible representations, the component indices a,b,c and the coupling ways μ. In fact, h^{abc} is the C-G coefficient of gauge group. The second factors g^{ijk} are independent of the property of the group, which are just the usual Yukawa coupling constants.

Using a method similar to that used by Jones et.al., we expand the Yukawa couplings g^{ijk} as a Taylor series of the gauge coupling constant g:

$$g^{ijk} = g\triangle_1^{ijk} + g^3\triangle_2^{ijk} + g^5\triangle_3^{ijk} + \cdots . \tag{18}$$

Because γ_β^α is a function of g and $g^{ijk}(g)$, so it can be written as follows:

$$\gamma_\beta^\alpha = g^2\Gamma_{1\beta}^\alpha + g^4\Gamma_{2\beta}^\alpha + g^6\Gamma_{3\beta}^\alpha + \cdots . \tag{19}$$

Then, the conditions for a finite to all-orders N=1 SYM theory are:

$$\Gamma_{n\beta}^\alpha = 0 \qquad (n = 1,2,3,\cdots) . \tag{20}$$

Substituting eqs.(17) and (18) into (6), and according to the property of the coefficient h^{abc}, the anomalous dimentions of one-loop, $\gamma_\beta^{\alpha(1)}$, can be written as

$$\begin{aligned}\gamma_\beta^{\alpha(1)} &= \left[g^{ilk}g_{jlk}\delta_y^x - g^2\frac{1}{8\pi^2}C_2(x)\delta_j^i\right]\delta_b^a \\ &= g^2\left[\Delta_1^{ilk}\Delta_{1jlk}\delta_y^x - \frac{C_2(x)}{8\pi^2}\delta_j^i\right]\delta_b^a + \left[g^4(\Delta_1^{ilk}\Delta_{2jlk} + \Delta_2^{ilk}\Delta_{1jlk})\right. \\ &\quad \left. + g^6(\Delta_1^{ilk}\Delta_{3jlk} + \Delta_3^{ilk}\Delta_{1jlk} + \Delta_2^{ilk}\Delta_{2jlk}) + \ldots\right]\delta_y^x\delta_b^a ,\end{aligned} \tag{21}$$

where g^{ijk} may have a constant factor different from that in eq.(17). For the expressions $\gamma_\beta^{\alpha(2)}, \gamma_\beta^{\alpha(3)}, \cdots$, all can be inferred by analogy. In general, $\gamma_\beta^{\alpha(n)}$ contains such terms: a product of 2n Yukawa coupling coefficient factors (i.e. g^{ijk}), a product of (2n-1) Yukawa coupling coefficient factors and g^2; the rest of the products, in turn decrease two Yukawa coupling coefficients but increase g^2, until the last term only has g^{2n} but without the Yukawa coupling coefficient factor.

Therefore, in $\gamma_\beta^{\alpha(n)}$, the term of lowest degree of g is the g^{2n} term in which only appear the expanding coefficient Δ_1^{ijk}, but no Δ_2^{ijk}, Δ_3^{ijk} and so on; in the $g^{2(n+1)}$ term, there are Δ_1^{ijk} and Δ_2^{ijk}, but no Δ_3^{ijk}, Δ_4^{ijk}, $\cdots$; the $g^{2(n+2)}$ term just contains Δ_1^{ijk}, Δ_2^{ijk} and Δ_3^{ijk}; the rest can be inferred by analogy.

Now, the conditions for finiteness, eq.(20), can be written as follows (Jiang and Zhou 1988):

1-loop:
$$\Gamma_{1\beta}^{a} = \left(\Delta_1^{ilk}\Delta_{1jlk}\delta_y^x - \frac{C_2(x)}{8\pi^2}\delta_j^i\right)\delta_b^a = 0 \quad , \tag{22}$$

n-loop:
$$\Gamma_{n\beta}^{a} = (\Delta_1^{ilk}\Delta_{njlk} + \Delta_n^{ilk}\Delta_{1jlk})\delta_y^x\delta_b^a + \tilde{\Gamma}_{n\beta}^{a}(\Delta_1, \Delta_2, \ldots, \Delta_{n-1}) = 0 , \qquad (n = 2, 3, 4, \ldots) , \tag{23}$$

where $\tilde{\Gamma}_{n\beta}^{\alpha}$ denotes the remaining terms of $\Gamma_{n\beta}^{\alpha}$ that do not contain Δ_n's; their particular expressions depend on the values of the γ_β^α to n orders.

According the group factor theorem expressed by eq.(9), the second term in eq.(23) can be written as:

$$\tilde{\Gamma}_{n\beta}^{a}(\Delta_1, \Delta_2, \ldots, \Delta_{n-1}) = \tilde{\Gamma}_{nj}^{i}(\Delta_1, \Delta_2, \ldots, \Delta_{n-1})\delta_y^x\delta_b^a . \tag{24}$$

Thus, eqs.(22) and (23) become:

$$\Delta_1^{ijk}\Delta_{1jlk} = \frac{C_2(x)}{8\pi^2}\delta_j^i , \tag{25}$$

$$(\Delta_1^{ijk}\Delta_{njlk} + \Delta_n^{ilk}\Delta_{1jlk}) + \tilde{\Gamma}^{i}_{nj}(\Delta_1, \Delta_2, \ldots, \Delta_{n-1}) = 0$$

$$(i, j \text{ belong to same irreducible representation, } n = 2, 3, \ldots) . \tag{26}$$

It is easy to see that the number of eqs.(25) and (26) is less than the one of eqs.(22) and (23), because of the equations in (22) and (23) with indices $x \neq y$, $a \neq b$ now automatically satisfied.

We disassemble Δ_1^{ijk}, Δ_n^{ijk} and $\tilde{\Gamma}^{i}_{nj}$ to real and imaginary parts, respectively:

$$\Delta_1^{ijk} = a^{ijk} + i\, b^{ijk} , \tag{27}$$

$$\Delta_n^{ijk} = x_n^{ijk} + i\, y_n^{ijk} , \tag{28}$$

$$\tilde{\Gamma}^{i}_{nj} = s^{i}_{nj} + i\, t^{i}_{nj} \equiv s^{ij}_{n} + i\, t^{ij}_{n} . \tag{29}$$

Now eq.(26) becomes:

$$a^{ilk}x_n^{jlk} + a^{jlk}x_n^{ilk} + b^{ilk}y_n^{jlk} + b^{jlk}y_n^{ilk} - s_n^{ij} = 0$$

$$-a^{ilk}y_n^{jlk} + b^{ilk}x_n^{jlk} + a^{jlk}y_n^{ilk} - b^{jlk}x_n^{ilk} + t_n^{ij} = 0$$

$$(n = 2, 3, \ldots ; \quad i, j \in x) . \tag{30}$$

In eqs.(30), if $i \neq j$, it does not give any new equations once i and j are exchanged. If eq.(25) gives a solution of Δ_1^{ijk}, a^{ijk} and b^{ijk} also become known quantities, then eqs.(30) just are linear equations with unknowns x_n^{ijk} and y_n^{ijk}. The coefficient matrix of the equations depends on a^{ijk} and b^{ijk}. Furthermore, there are 2M independent unknowns in eqs.(30), where M denotes the number of the independent possible non-zero Yukawa coupling coefficient factors d^{ijk}. The number of the independent equations is:

$$L = \sum_{i=1}^{s} n_i^2 , \tag{31}$$

where $n_1, n_2, \ldots, n_s$ denote the number of the chiral superfields which belong to irreducible representations x, y, $\cdots$, z, respectively. Anyway, the coefficient matrix A is a $L \times 2M$ matrix; L is the number of the rows and 2M is the number of the columns.

The details of a explanation of the general solutions of eqs.(30) was given elsewhere (Jiang and Zhou 1988). Here, we just summarize the conditions for existence of the solutions as follows:

(a) $L \leq 2M$;

(b) There exist solutions of eq.(25) which make the rank of the coefficient matrix A in eqs.(30) is equal to the number of the equations, i.e. $r(A) = L$.

4. A criterion for existence of finite N=1 SYM theory

As above section shown, the group factor theorem reduces the eqs.(22) and (23) to more simple eqs.(25) and (26) or (30). In principle, we can solve these equa -tions order by order once the anomalous dimensions $\gamma_\beta^{\alpha(n)}$ are calculated. The interesting question is whether there is a set of values of the unknown Δ_n^{ijk} which satisfy these equations of finiteness. From now on, we will show both of a solving process and a criterion for existence of the solutions for these equa -tions in particular.

Now, we introduce two diagonal conditions which make the non-diagonal equations (i.e. $i \neq j$) of eqs.(25) and (26) are automatically valid. The diagonal condition means choosing some Yukawa coupling coefficients g^{ijk} in the theory are equal to zero. By this way, it is easier than before to solve the equations, but in gene- ral will lose some of the solutions.

At first, imposing a one-loop diagonal condition (Jiang and Zhou 1986, Dong et al 1986), i.e. choosing a specific set of nonvanishing Yukawa coupling coeffi- cients g^{ijk} such that there are no two g^{ijk} in the set which have two equal indices, then eqs.(25) with $i \neq j$ are automatically valid.

We introduce the all-orders diagonal condition as follows, which guarantees the nondiagonal equations ($i \neq j$) of eqs.(26) automatically to be valid. Suppose we choose P nonvanishing Yukawa coupling coefficients d^{ijk} ,denoted as d^μ ($\mu = 1,2,\cdots,P$), and m_i^μ is the number of the irreducible matter superfield ϕ_i ($i=1,2,\cdots,N$) appearing in d^μ. Obviously, $\sum_{i=1}^{n} m_i^\mu = 3$ for any μ. Suppose in a matter superfields self-energy diagram i —[ ]— j , b^μ is the number of d^μ appearing in the diagram, while b_μ is the number of $d_\mu = (d^\mu)^*$ appearing in the diagram, and $a_\mu = b^\mu - b_\mu$. Let $i \neq j$, but i,j belong to the same irreducible representation. If this diagram exists, the following equations must have a solution:

$$\sum_{\mu=1}^{P} a_\mu m_k^\mu = 0 \quad (k \neq i,j)\,, \qquad \sum_{\mu=1}^{P} a_\mu m_i^\mu = 1, \qquad \sum_{\mu=1}^{P} a_\mu m_j^\mu = -1 \quad (i \neq j)\,, \qquad (32)$$

where m_k^μ are known. Now let a_μ's be unknowns; if eqs.(32) have no integer solu-tion of a_μ, we call this condition an all-orders diagonal condition. If this condition is satisfied, the nondiagonal eqs.($i \neq j$) in (25) and (26) automatically hold. An all-orders diagonal condition must be a one-loop diagonal one, but the inverse is in general invalid. Whether an all-orders diagonal condition holds, it depends on which set of nonvanishing d^μ's we choose. Now suppose an all-orders diagonal condition holds for a particular choice of nonvanishing d^μ's we only need to solve the diagonal equations of (25) and (26) which are as follows:

$$\sum_{jk} H^{ijk} = T(x) \qquad (i \in x)\,, \qquad (33)$$

$$\sum_{jk} H_n^{ijk} = \tilde{\Gamma}_n^i(\Delta_1, \Delta_2, \cdots, \Delta_{n-1}), \quad (i=1,2,\cdots,N;\ n=2,3,\cdots)\,, \qquad (34)$$

where

$$H^{ijk} = a\,\Delta_1^{ijk}\,\Delta_{1ijk}\,, \quad a = 8\pi^2\frac{\dim x}{\dim G}\,, \text{ (no sum over } i,j \text{ and } k), \qquad (35)$$

$$H_n^{ijk} = \Delta_1^{ijk}\,\Delta_{nijk} + \Delta_n^{ijk}\,\Delta_{1ijk}\,, \qquad \text{(no sum over } i,j \text{ and } k). \qquad (36)$$

It is interesting to see that eqs.(33) and (34) have the same coefficient matrix A'(N×P-matrix), only have a difference from thire augmented matrices. The sufficient (and almost necessary) condition for these linear equations having a solution is: $\mathrm{rank}(A') = N \leq P$, which means $\gamma_\beta^\alpha = 0$ to all orders.

Based on our previous work of two-loop finite SYM theories and according to the above criterion, the procedure for finding the finite in all orders $N = 1$

SYM theory is as follows:

1. Using eqs.(33) check the solutions of two-loop finite theories. For a set of representations in two-loop finite theory,if the Yukawa coupling coefficients d^{ijk} were selected to be making eqs.(33) have a non-negative solution and in the solution without $H^{ijk} = 0$, then go to next step. In general there may be some $H^{ijk} = 0$ for this solution. In such case we will delete the corresponding vanish -ing d^{ijk} from the set of nonvanishing d^{μ}'s.

2. Using all-orders diagonal condition examine the set of d^{μ}'s. If this set of d^{μ}'s make eqs.(32) having a integer solution of a_{μ}, we have to renew a new set and to check it again from first step,until the all-orders diagonal condition holds for the new set.

3. If $N > P$, this means the solution although is two-loop finite but in general can not continue be finite to all orders.

4. If $N \leq P$, then using above criterion check the rank of the coefficient matrix A'. Those solutions of two-loop finite theories with rank(A')=N, just are candi -dates of finite to all orders theory.

According to above procedure,we obtain a large class finite in all orders N=1 SYM theories of representations of all classical groups (Jiang and Zhou 1987, 1988).

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