TRI-PP-88-17
Mar 1988

Contributed paper to the "International Conference on Spin Observables of Nuclear Probes," Telluride, Colorado, March 14-17, 1988.

8 pages

Relativistic Treatment of Mesonic Contributions to Quasielastic (e, e')

P. G. Blunden and M. N. Butler

TRIUMF
4004 Wesbrook Mall
Vancouver, BC, Canada V6T 2A3

It is clear now that meson exchange currents play an important role in the description of observables in electron scattering. This has been shown most dramatically in the description of the high momentum components of the elastic form factors of A=3 nuclei.[1] For quasielastic scattering, two calculations exist, one which examines the effects at the quasielastic peak for 1p–1h exchange currents,[2] and one which examines 2p–2h contributions.[3] (It should be pointed out that the results of Ref. [2] are somewhat confusing, since they show a suppression of the quasielastic cross section when exchange currents are included, rather than the expected enhancement.) All of these calculations have been performed with the approximation that the system is non-relativistic. The validity of this may be called into question by the fact that the exchange-current effects are most significant at high momentum transfers and often are coupled to high momentum components of the nuclear wavefunction. It can also be unclear which exchange current processes should be included and which are already included in the bound state structure of the nucleus.

Another problem is that current conservation, evaluated using Siegert's theorem, is preserved only to $\mathcal{O}(q/M)$ (q is the momentum transfer and M is the nucleon mass). Relativistic models for the bound state structure of the nucleus contain the mesons as explicit degrees of freedom, providing a framework for calculating their effects, and virtually all other dynamical effects, in a completely self-consistent manner. Since the mesons and nucleons are coupled to the electromagnetic field in a gauge invariant way, it is possible to write down an electromagnetic current for the nucleons and mesons that is conserved both in theory and in practice. We use a relativistic model with pseudovector pion coupling to study the exchange current contributions, with emphasis on quasielastic kinematics.

We begin with the Lagrangian for nucleons interacting with a scalar and vector meson along with pseudovector coupling to pions,

$$\begin{aligned}\mathcal{L} &= \bar\psi(i\partial\!\!\!/ - M)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\mathcal{D}_i^\mu\pi^i)^2 \\ &-\frac{1}{2}m_S^2\phi^2 - \frac{1}{2}m_\pi^2\vec\pi^2 - \frac{1}{2}m_V^2V_\mu^2 \\ &-g_S\phi\bar\psi\psi + e\bar\psi\Gamma_\mu A^\mu\psi + g_V\gamma_\mu V^\mu\bar\psi\psi - i\frac{g_\pi}{2M}\gamma_5\bar\psi\gamma_\mu\mathcal{D}_i^\mu\pi^i\psi,\end{aligned} \tag{1}$$

where

$$\mathcal{D}_i^\mu = \tau_i\partial^\mu - i\frac{e}{2}A^\mu\epsilon_{ji3}\tau_j\,, \quad \Gamma_\mu = F_1(Q^2)\gamma_\mu + i\frac{F_2(Q^2)}{2M}\sigma_{\mu\nu}q^\nu$$

and

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,, \quad G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu\,.$$

Detailed calculations of the bound state problem using this model have been studied elsewhere. Here, we use this Lagrangian to derive the one and two-body electromagnetic currents. In doing so, we can use Ward–Takahashi Identities, relating the Hartree-Fock self-energy to the vertex function, to verify our results.[4]

Figure 1 shows the irreducible diagrams that could contribute to the one-body electromagnetic current up to first order in g_π^2. Diagram (a) represents the standard impulse approximation, and diagrams (b–d) represent the meson-exchange current corrections. Diagram (e) gives the Hartree-Fock self energy. In a Fermi gas, the expressions for these diagrams are given by

$$\begin{aligned}\Lambda_\mu^a &= \Big(F_1 + \frac{M^*}{M}F_2\Big)\gamma_\mu - \frac{F_2}{2M}(p+p')_\mu\,, \\ \Lambda_\mu^b &= i\Big(\frac{g_\pi}{2M}\Big)^2 2\tau_3F_\pi\int\frac{d^4k}{(2\pi)^4}\frac{\gamma_5(p\!\!\!/' - k\!\!\!/)G_D(k^*)\gamma_5(p\!\!\!/ - k\!\!\!/)}{((p-k)^2 - m_\pi^2)((p'-k)^2 - m_\pi^2)}(p'+p-2k)_\mu\,, \\ \Lambda_\mu^{c+d} &= -i\Big(\frac{g_\pi}{2M}\Big)^2 2\tau_3F_\pi\int\frac{d^4k}{(2\pi)^4}\Big(\frac{\gamma_5\gamma_\mu G_D(k^*)\gamma_5(p\!\!\!/ - k\!\!\!/)}{((p-k)^2 - m_\pi^2)} + \frac{\gamma_5(p\!\!\!/' - k\!\!\!/)G_D(k^*)\gamma_5\gamma_\mu}{((p'-k)^2 - m_\pi^2)}\Big)\,, \\ \Sigma^{HF} &= i\Big(\frac{g_\pi^2}{2M}\Big)^2\vec\tau\cdot\vec\tau\int\frac{d^4}{(2\pi)^4}\frac{\gamma_5(p\!\!\!/ - k\!\!\!/)G_D(k^*)\gamma_5(p\!\!\!/ - k\!\!\!/)}{((p-k)^2 - m_\pi^2)}\,.\end{aligned}$$

Here, M^* is the effective mass and $G_D(k^*) = 2\pi i(k\!\!\!/^* + M^*)\delta(k^{*2} - M^{*2})\theta(k_F - k)\theta(k_0)$ is the propagator for an occupied single particle state. F_1, F_2 and F_π are parameterizations of the nucleon and pion form factors,[5] given by

$$F_1^n(Q^2) = \frac{5.6\tau^2\mu_n}{(1+\tau)(1+5.6\tau)}G_E^p(Q^2), \quad F_2^n(Q^2) = \mu_nG_E^p(Q^2) - F_1^n,$$

$$F_1^p(Q^2) = \frac{1+\tau\mu_p}{1+\tau}G_E^p(Q^2), \quad F_2^p(Q^2) = \mu_pG_E^p(Q^2) - F_1^p\,,$$

$$F_\pi(Q^2) = F_1^p - F_1^n, \quad G_E^p(Q^2) = \frac{1}{(1+Q^2/(0.71\,\mathrm{GeV}^2))^2}$$

Q^2 is the four momentum transfer, given as $Q^2 = q^2 - \omega^2$, $\tau = Q^2/4M^2$ and μ is the magnetic moment. In order to simplify the calculation, we neglect any momentum dependence in the self energies and approximate $k^*_\mu = (k_0 + \Sigma_0, \boldsymbol{k})$. This is an excellent approximation justified by relativistic Hartree-Fock calculations. One can now replace $p_\mu - k_\mu$ by $p^*_\mu - k^*_\mu$ in the above expressions. We can then evaluate the effective one-body current between on-shell spinors as

$$J_\mu = \bar{u}(p'^*)[\Lambda^{1a}_\mu + \Lambda^{1b}_\mu + \Lambda^{1c}_\mu + \Lambda^{1d}_\mu]u(p^*)\,.$$

Making the replacement $\not{p}^* u(p^*) = M^* u(p^*)$, one finds general forms for the time–like and space–like parts of the current, given by

$$\begin{aligned} J_0 &= A_1\gamma_0 + A_2 \\ J_i &= B_1\gamma_i + B_2 p_i + B_3\gamma_i\gamma_0 + B_4\gamma_0 p_i\,. \end{aligned}$$

The factors A_j and B_j can be loosely thought of as medium form factors and are functions of q, ω, k_F and p. The lack of manifest Lorentz invariance is due to the fact we have evaluated the expressions for the vertex functions in a specific frame. In general, the current is indeed Lorentz invariant.

We can now use these expressions for the current to calculate the longitudinal (R_L) and transverse (R_T) pieces of the quasielastic cross section for various nuclei and kinematics. These are given as

$$\begin{aligned} R_L &= \frac{1}{V}\sum_{\text{isospin}}\int\frac{d\boldsymbol{p}}{(2\pi)^3}\frac{\mathrm{Tr}[J_0^\dagger J_0]}{4E^*_p E^*_{p+q}}\theta(k_F - p)\theta(|\boldsymbol{p}+\boldsymbol{q}| - k_F)\delta(\omega + E_p - E_{p+q})\,, \\ R_T &= \frac{1}{V}\sum_{\text{isospin}}\int\frac{d\boldsymbol{p}}{(2\pi)^3}\left(\delta^{ij} - \frac{q^i q^j}{q^2}\right)\frac{\mathrm{Tr}[J_i^\dagger J_j]}{4E^*_p E^*_{p+q}}\theta(k_F - p)\theta(|\boldsymbol{p}+\boldsymbol{q}| - k_F)\delta(\omega + E_p - E_{p+q})\,. \end{aligned}$$

M^* and k_F are parameters which we use to fit the position of the quasielastic peak. In the future, we would like to input these from nuclear structure calculations of the initial and final states.

As an example, let us consider the Saclay results for ^{40}Ca at q=410 and 550 MeV.[6] The results of our calculation on R_L for the relativistic impulse approximation, and for the inclusion of exchange currents can be seen in Figure 2 (the parameters used are $M^* = 0.8M$ and $k_F = 1.3\ \mathrm{fm}^{-1}$). It is clear that the exchange currents have a negligible effect in this channel, as would be expected. On the other hand, in Figure 3 we see the results for R_T and here the effects are quite dramatic. At both energies, at the quasielastic peak, the enhancement is on the order of 40%. This is much larger than the effect found non-relativistically, typically on the order of 25–30%. This effect comes in large part from a correct treatment of the relativistic kinematics, as the retardation in the pion propagators turns out to be a fairly small enhancement and contributes mainly to the low ω region of the structure function. Lastly, we show results for the transverse structure function of ^{12}C, at

q=300 and 550 MeV.[6] In Figure 4, we see the same sort of enhancement that we saw in the case of ^{40}Ca. The fit at q=550 MeV is surprisingly good, even though a Fermi gas is a weak approximation for ^{12}C.

We can also apply our model to the question of the ratio of R_T/R_L in quasielastic $(e, e'p)$ for parallel kinematics. Specifically, we study

$$R_G = \left(2M^2 \frac{q^2}{Q^4} \frac{R_T}{R_L}\right)^{1/2} .$$

Significant deviations from the expected value of $R_G = G_M^p/G_E^p$ were found at NIKHEF for ^{12}C$(e, e'p)^{11}$Be, and in Saclay data on the inclusive reaction ^{12}C(e, e').[9] Several attempts have been made to explain this, among them studies of the final-state interaction of the struck proton with the residual nucleus.[10] We have found that, in our model, the inclusion of exchange currents into R_T is sufficient to bring the theory into reasonable agreement with the experimental results (Figure 5).

In conclusion, we have found that the effects of meson-exchange currents are much more important in a relativistic model than in a non-relativistic one. However, there are a few points which need to be pursued further. It should be noted that the Lagrangian of Eqn. (1) does not contain explicit information about the Δ. While this has sometimes been found to be an important contribution to the exchange current in non-relativistic calculations, we do not include it in our calculation. In non-relativistic calculations, the delta contribution has been found to interfere destructively with the diagrams we have studied. Another point is that we do not include RPA corrections in either R_L or R_T. These corrections might be balanced, however, were we to include the delta. The contribution to the structure functions of two-nucleon knockout due to exchange currents is also being studied, expecially with regard to the so-called "dip" region, and we hope to have results on that soon.

This work has been supported by the Natural Sciences and Engineering Research Council, Canada.

REFERENCES

[1] See, for example, J.L. Friar, *New Vistas in Electro-Nuclear Physics*, Nato ASI Series (Plenum, New York NY, 1986) p. 213.

[2] M. Kohno and N. Ohtsuka, Phys. Lett. **98B**, 335 (1981).

[3] J.W. Van Orden and T.W. Donnelly, Ann. Phys. **131**, 451 (1981).

[4] W. Bentz, A. Arima, H. Hyuga, K. Shimizu and K. Yazaki, Nucl. Phys. **A436**, 593 (1985).

[5] M.A. Preston and R.K. Bhaduri, *The Structure of the Nucleus* (Addison-Wesley, Reading, Mass., 1975).

[6] Z.E. Meziani *et al.*, Phys. Rev. Lett. **52**, 2130 (1984).

[7] Z.E. Meziani *et al.*, Phys. Rev. Lett. **54**, 1233 (1985).

[8] P. Barreau *et al.*, Nucl. Phys. **A402**, 515 (1983).

[9] G. van der Steenhoven *et al.*, Phys. Rev. Lett. **57**, 182 (1986).

[10] T.D. Cohen, J.W. Van Orden and A. Picklesimer, Phys. Rev. Lett. **59**, 1267 (1987), and T. Suzuki, Phys. Rev. **C37**, 549 (1988).

FIGURE CAPTIONS

Fig. 1: Irreducible diagrams contributing to the one-body current (a–d) and the pionic contribution to the Hartree-Fock self energy (e).

Fig. 2: $R_L(q,\omega)$ for ^{40}Ca at q=410 and 550 MeV. The solid curve represents the relativistic impulse approximation and the dashed curve the result including the mesonic effects. Data from Ref. [6].

Fig. 3: Same as Fig. 2, but for R_T. Data from Ref. [7].

Fig. 4: The structure function R_T for ^{12}C at q=300 and 550 MeV. The solid curve represents the impulse approximation result, and the dashed line the result including exchange currents. Data from Ref. [8].

Fig. 5: The ratio R_G. The solid curve is the naive expectation from scattering off a free proton, and the dashed curve is our result including exchange currents in R_T. Data from Ref. [9]

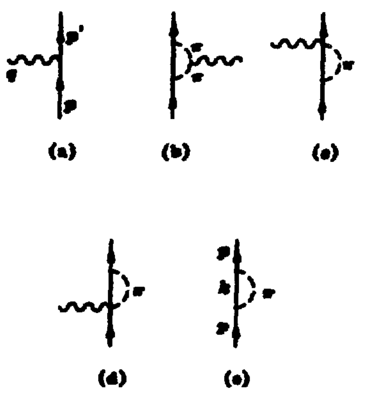

Fig. 1

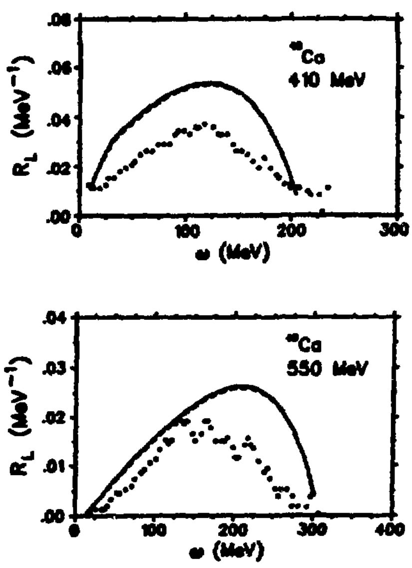

Fig. 2

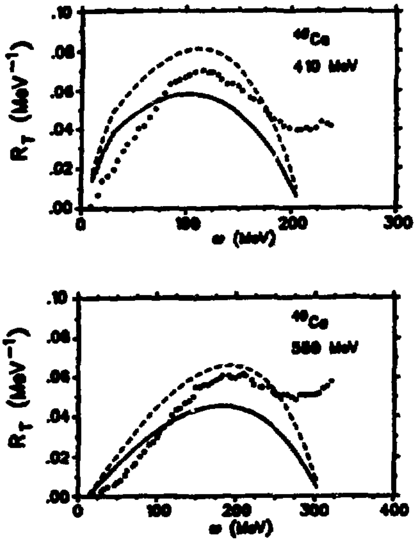

Fig. 3

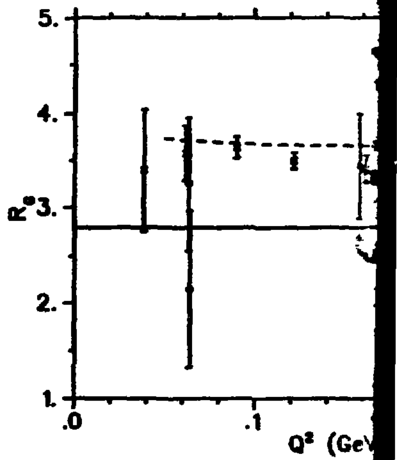

Fig.

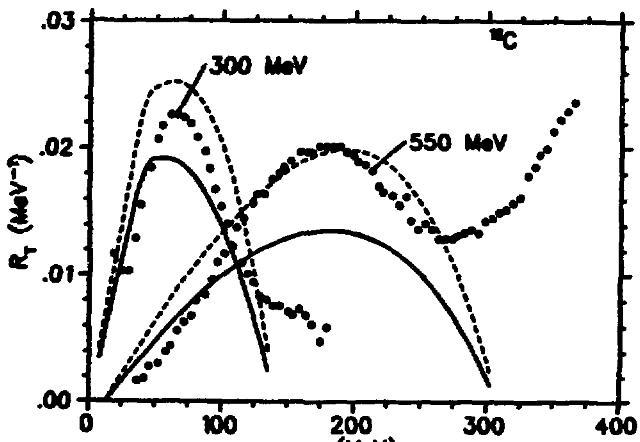

Fig. 4

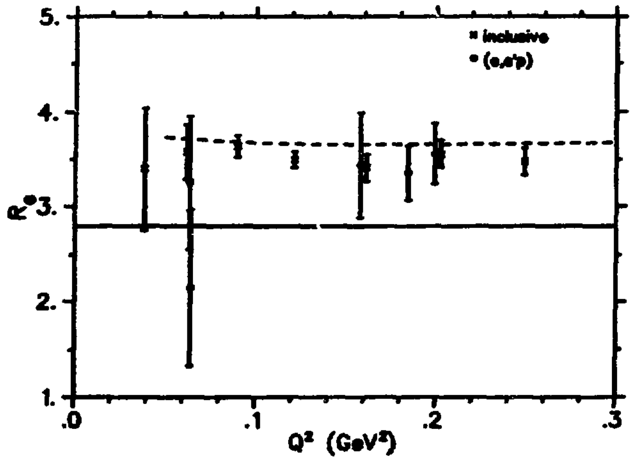

Fig. 5