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STRING MODEL IN DIFFERENTIAL FORMS *

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ABSTRACT

The string model in terms of the two-dimensional differential forms of arbitrary rank is formulated. The local sypersymmetric string action with local conformal and Lorentz symmetries is constructed. The connection with topological quantum field theory is discussed. Covariant quantization of the model is investigated. The critical space-time dimension is found to be $d = 4$.

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I. Introduction.

Recent hope about the unifying of the fundamental interactions is connected with the string theory. In the low energy limit this theory gives good description of the gravity and the gauge fields. The cancellation condition of gauge anomalies determines the gauge group to be either $SO(32)$ or $E_8 \otimes E_8$ [I]. On the other hand one determines the dimension of the space-time, in which the string is embedded, from a condition of cancellation of the conformal anomalies [2]. For known models this dimension is equal to d=IO or d=26. Hence compactification of the additional dimensions is necessary. But the mechanism of such a compactification is too arbitrary [I] for the theory to be fundamental. Thus the problem of construction of a string model dicectly in the four-dimensional space-time is actual. This approach attract the raising attention in literature [3].

The standard string models are formulated in terms of two-dimensional scalar and spinor fields [2]. We develop here the string model on the basis of our programme [4] to use only two-dimensional differential forms of arbitrary rank. It is known that antisymmetric tensor fields (or differential forms) play an important role in various aspects of string theory [5]. On the other hand connection of external forms with topology makes such consideration especially attractive. We widely exploit the possibility of description of fermions in terms of differential forms using the equation firstly suggested by D.Ivanenko and L.Landau [6] and intensively discussed in the literature [7].

2. Bosonic sector of the model.

In standard string models [I,2,IO] the string is described by the action

$$S = \frac{1}{2} \int d^2 z \sqrt{g}\, g^{\mu\nu} \partial_\mu X^i \partial_\nu X^i, \qquad (1)$$

where $g_{\mu\nu}$ is the metric on the world-sheet of a moving string and scalar functions $X^i(z)$, $i=1,\dots,d$ realise the embedding of this world-sheet into the d-dimensional space-time.

It is easy to see that the action (I) might be transformed into:

$$S = \frac{1}{2}\int * dX^i \wedge dX^i. \qquad (I')$$

Natural generalization of this expression is to substitute zero-forms X^i by the full inhomogenous exterior forms on the world-sheet:

$$\Phi^i = \varphi^i + \varphi^i_\mu dz^\mu + \frac{1}{2}\varphi^i_{\mu\nu} dz^\mu \wedge dz^\nu.$$

Thus one can suggest the following generalization of (I'):

$$S = \frac{1}{2}\int * (d-\delta)\Phi^i \wedge (d-\delta)\Phi^i, \qquad (2)$$

where d and δ are correspondingly external differential and co-differential operators [8].

In the string theory conformal symmetry plays special role [I]. It is easy to see that (2) is not conformal invariant in general. However it is invariant under global transformations:

$$\begin{aligned} g_{\mu\nu} &\to e^{2\sigma} g_{\mu\nu}, \quad & \varphi_\mu &\to e^{\sigma}\varphi_\mu \\ \varphi &\to \varphi, \quad & \varphi_{\mu\nu} &\to e^{2\sigma}\varphi_{\mu\nu}. \end{aligned} \qquad (3)$$

Localizing these we find that under conformal transformation

$$\begin{aligned}(d-\delta)\Phi \to [(d-\delta)\Phi]_{glob.} + e^{\sigma}\nabla^\mu\sigma\,\varphi_\mu + \\ + e^{\sigma}\partial_\mu\sigma\,\varphi_\nu\, dz^\mu\wedge dz^\nu,\end{aligned} \qquad (4)$$

where $[(d-\delta)\Phi]_{glob.}$ is expression, which remains in (4) if σ =const.

In order to compensate additional terms in (4), let us consider the extended operator $\hat{D}$, which acts on the space of the one-forms Φ_1 (hereafter Φ_1 means odd form and Φ_2 - even form):

$$\hat{D}\Phi_1 = (d-\delta)\Phi_1 + \tilde{\omega}\vee\Phi_1, \qquad (5)$$

where $\tilde{\omega} = *\frac{1}{2}\omega^{ab}\epsilon_{ab}$ is the dual Lorentz connection one-form. If

h^a_μ is a two-dimensional orthonormal basis then $\tilde{\omega}_\mu = h^\nu_a(\partial_\nu h^a_\mu - \partial_\mu h^a_\nu)$. Let us note that $\tilde{\omega}$ like the Lorentz connection form ω^{ab}, corresponding to the localizing of the two-dimensional Lorentz group, plays important role in the structure of the conformal and Lorentz anomalies in the two-dimensions [9].

For $\tilde{\omega}$ we have the following transformation law under the local transformations (3): $\tilde{\omega}_\mu \to \tilde{\omega}_\mu - \partial_\mu\sigma.$

It is interesting to note the following expression for scalar curvature [9]:

$$R = 2\nabla_\mu\tilde{\omega}^\mu. \qquad (6)$$

Thus we come to the action

$$S = \int *\frac{1}{2}(d-\delta)\Phi_2 \wedge (d-\delta)\Phi_2 + \frac{1}{2}*\hat{D}\Phi_1\wedge\hat{D}\Phi_1, \qquad (7)$$

which is conformal invariant generalization of the standard string action (I).

The co-differential operator δ is conjugated to the operator d under natural scalar product: $(\Phi,\Psi) = \int *\Phi\wedge\Psi.$

The operator $\hat{D}^+$, conjugated to $\hat{D}$ acts on the space of even forms, and reads

$$\hat{D}^+\Phi_2 = -(d-\delta)\Phi_2 + \tilde{\omega}\vee\Phi_2, \qquad (8)$$

where $\vee$ is the Clifford multiplication, defined for the one-forms basis as follows [7]: $dz^\mu \vee dz^\nu \equiv dz^\mu\wedge dz^\nu + g^{\mu\nu}.$

The action (7) is invariant also under the local lorentz rotations of the two-dimensional orthonormal basis:

$$\begin{aligned} h^a_\mu &\to h^a_\mu + \epsilon^a{}_b h^b_\mu\beta \\ \tilde{\omega}_\mu &\to \tilde{\omega}_\mu + \epsilon_\mu{}^\nu\partial_\nu\beta, \end{aligned} \qquad (9)$$

if one determines the action of the lorentz rotations on the form Φ_1 as dualization:

$$\psi_\mu \to \psi_\mu + \epsilon_\mu{}^\nu \psi_\nu \beta, \qquad (10)$$

where β is the parameter of the transformation.

3. Fermionic sector of the model.

In order to receive the fermionic excitations of the string one should include the fermionic sector. In the standard string model [1,2] fermions are described by the two-dimensional Weyl spinors. In our model, following to the programme to use only differential forms, we will describe fermion fields on the world-sheet by inhomogenous differential form:

$$\Psi = \psi + \psi_\mu dz^\mu + \frac{1}{2}\psi_{\mu\nu} dz^\mu \wedge dz^\nu.$$

The action is as follows [7]:

$$S = \frac{1}{2}\int * \Psi \wedge (d-\delta)\Psi, \qquad (11)$$

where Ψ components are real and anticommutating.

One can rewrite (11) in the form

$$S = \int d^2 z \sqrt{g}\,(\psi_\mu \partial^\mu \psi + \psi^{\mu\nu}\partial_\mu \psi_\nu). \qquad (12)$$

This action is invariant under the global scaling:

$$\begin{aligned} g_{\mu\nu} &\to e^{2\sigma} g_{\mu\nu}, \quad \psi_\mu \to e^{p\sigma}\psi_\mu \\ \psi &\to e^{k\sigma}\psi, \quad \psi_{\mu\nu} \to e^{n\sigma}\psi_{\mu\nu}, \end{aligned} \qquad (13)$$

where k,p,n are real numbers, such that

$$k+p=0, \quad n+p-2=0. \qquad (14)$$

Let us assume $\sigma = \sigma(z)$ and p=1,k=(-1),n=1. Then for compensation of $\partial_\mu \sigma$ depending terms under the variation of (12), one should again consider the operator $\hat{D}$: $\hat{D}\Psi_1 = (d-\delta)\Psi_1 + \tilde{\omega} \vee \Psi_1$.

In this case the action

$$S = \int * \Psi_2 \wedge \hat{D}\Psi_1 \qquad (15)$$

is invariant under the local conformal transformations (13) and also under the local Lorentz rotation (9) if we assume that

$$\begin{aligned} \psi &\to \psi + \frac{1}{2}\epsilon_{\mu\nu}\psi^{\mu\nu}\beta \\ \psi_\mu &\to \psi_\mu + \epsilon_\mu{}^\nu \psi_\nu \beta \\ \psi_{\mu\nu} &\to \psi_{\mu\nu} - \epsilon_{\mu\nu}\psi\beta. \end{aligned} \qquad (16)$$

On the other hand the action (11) is invariant under the local conformal transformations (13) without addition of $\tilde{\omega}$, if we assume k=0,p=0,n=2 in (13).

4. Local supersymmetry.

Thus let us consider the set of the boson forms

$$\Phi_2^i, \; i=1,\dots,d$$
$$\Phi_1^A, \; A=1,\dots,N$$

and also the set of the fermion forms

$$\Psi^i, \; i=1,\dots,d$$
$$\tilde{\Psi}^A, \; A=1,\dots,N$$

For these fields action

$$\begin{aligned} S_0 = \int \frac{1}{2} * (d-\delta)\Phi_2^i \wedge (d-\delta)\Phi_2^i + \frac{1}{2} * \hat{D}\Phi_1^A \wedge \hat{D}\Phi_1^A + \\ + * \Psi_1^i \wedge (d-\delta)\Psi_2^i + * \tilde{\Psi}_2^A \wedge \hat{D}\tilde{\Psi}_1^A \end{aligned} \qquad (17)$$

is invariant under the global supersymmetric transformations:

$$\begin{aligned} &\Delta_\alpha \Phi_2^i = \alpha^{ij}\Psi_2^j \; ; \; \Delta_\alpha \tilde{\Psi}_2^A = -\alpha^{BA}\hat{D}\Phi_1^B \; ; \; \Delta_\alpha \tilde{\Psi}_1^A = 0 \\ &\Delta_\alpha \Phi_1^A = \alpha^{AB}\tilde{\Psi}_1^B \; ; \; \Delta_\alpha \Psi_1^i = -\alpha^{ji}(d-\delta)\Phi_2^j \; ; \; \Delta_\alpha \Psi_2^i = 0, \end{aligned} \qquad (18)$$

where (α^{ij}, α^{AB}) is the set of anticommutating variables.

Localizing these transformations we find that

$$\Delta_\alpha S_0 = \int * d\alpha^{ij} \vee \Psi_2^j \wedge (d-\delta)\Phi_2^i + * d\alpha^{AB} \vee \tilde{\Psi}_1^B \wedge \hat{D}\Phi_1^A.$$

In order to compensate new terms it is necessary to introduce the set of anticommutating one-forms (the analogue of the Rarita-Schwinger field) with the transformation law:

$$\Delta_\alpha \chi^{ij} = -d\alpha^{ij}, \quad \Delta_\alpha \chi^{AB} = -d\alpha^{AB} \tag{19}$$

and add to the action the following term:

$$S_1 = \int * \chi^{ij} \vee \psi_2^j \wedge (d-\delta)\phi_2^i + * \chi^{AB} \vee \tilde{\psi}_1^B \wedge \hat{\mathcal{D}} \phi_1^A .$$

Then we have

$$\Delta_\alpha (S_0 + S_1) = \int * \chi^{ij} \vee \psi_2^j \wedge \alpha^{ik} (d-\delta) \psi_2^k + \\ + * \chi^{AB} \vee \tilde{\psi}_1^B \wedge \alpha^{AC} \hat{\mathcal{D}} \tilde{\psi}_1^C + * \chi^{ij} \vee \psi_2^j \wedge d\alpha^{ik} \vee \psi_2^k + \\ + * \chi^{AB} \vee \tilde{\psi}_1^B \wedge d\alpha^{AC} \vee \tilde{\psi}_1^C . \tag{20}$$

For compensation of two last terms one should add to the action:

$$S_2 = \int \frac{1}{2} * \chi^{ij} \vee \psi_2^j \wedge \chi^{ik} \vee \psi_2^k + \frac{1}{2} * \chi^{AB} \vee \tilde{\psi}_1^B \wedge \chi^{AC} \vee \tilde{\psi}_1^C$$

and for compensation of two first terms in (20) one should assume:

$$\Delta_\alpha \psi_1^i = -\alpha^{ji} (d-\delta) \phi_2^j - \alpha^{ki} \chi^{kj} \vee \psi_2^j$$

$$\Delta_\alpha \tilde{\psi}_2^A = -\alpha^{BA} \hat{\mathcal{D}} \phi_1^B - \alpha^{CA} \chi^{CB} \vee \tilde{\psi}_1^B .$$

Thus we come to the complete local supersymmetric action:

$$S_{tot} = S_0 + S_1 + S_2 , \tag{21}$$

which is the analogue of the complete standard string action [10]. However contrary to the standard model the supersymmetry (18),(19) does not touch the gravitational variables (metric).

5. Quantization and connection with topological quantum field theory.

The quantization of our model is defined by the functional integral:

$$Z = \int [\mathcal{D} h^q_\mu][\mathcal{D}\chi][\mathcal{D}\psi][\mathcal{D}\tilde{\psi}][\mathcal{D}\phi_1][\mathcal{D}\phi_2] \exp(-S_{tot}).$$

Let us start with calculation of the integral:

$$Z' = \int [\mathcal{D}\psi][\mathcal{D}\tilde{\psi}][\mathcal{D}\phi_1][\mathcal{D}\phi_2] \exp(-S_{tot}),$$

which is the partition function for the action (21) if we consider the gravitational (h^q_μ) and supergauge (χ) fields as external.

Since S_{tot} is quadratic over fields the calculation of the functional integral gives the expression of Z' in terms of superdeterminants [11]:

$$Z' = \left(\mathrm{sdet}\, R \cdot \mathrm{sdet}\, \tilde{R} \right)^{-1/2}, \tag{22}$$

where the operator R ($\tilde{R}$) has the following structure:

$$R = \begin{pmatrix} R_1 & R_2 \\ R_3 & R_4 \end{pmatrix}, \quad \tilde{R} = \begin{pmatrix} \tilde{R}_1 & \tilde{R}_2 \\ \tilde{R}_3 & \tilde{R}_4 \end{pmatrix}$$

with

$$R_1 = \left(-(d-\delta)^2_{(2)} \delta^{ij} \right); \quad R_2 = \begin{pmatrix} 0 & -(d-\delta)_{(1)} \chi^{ij} \end{pmatrix}$$

$$R_3 = \begin{pmatrix} 0 \\ -\chi^{ji} \vee (d-\delta)_{(2)} \end{pmatrix}; \quad R_4 = \begin{pmatrix} 0 & (d-\delta)_{(1)} \delta^{ij} \\ (d-\delta)_{(1)} \delta^{ij} & -\chi^{ki} \vee \chi^{kj} \end{pmatrix}$$

$$\tilde{R}_1 = \left(\hat{\mathcal{D}}^+ \hat{\mathcal{D}} \delta^{AB} \right); \quad \tilde{R}_2 = \begin{pmatrix} 0 & \hat{\mathcal{D}}^+ \chi^{AB} \end{pmatrix}$$

$$\tilde{R}_3 = \begin{pmatrix} 0 \\ -\chi^{BA} \vee \hat{\mathcal{D}} \end{pmatrix}; \quad \tilde{R}_4 = \begin{pmatrix} 0 & \hat{\mathcal{D}} \delta^{AB} \\ -\hat{\mathcal{D}}^+ \delta^{AB} & -\chi^{CA} \vee \chi^{CB} \end{pmatrix}.$$

The superdeterminant is expressed as [11]:

$$\mathrm{sdet}\, R = \det R_1 \det{}^{-1} (R_4 - R_3 R_1^{-1} R_2).$$

Hence

$$\mathrm{sdet}\, R = \left[\det \Delta_0 \det \Delta_2 \det{}^{-1} \Delta_1 \right]^{d/2} \tag{23}$$

$$\mathrm{sdet}\, \tilde{R} = \left[\det \hat{\mathcal{D}}^+ \hat{\mathcal{D}} \cdot \det{}^{-1} \hat{\mathcal{D}} \hat{\mathcal{D}}^+ \right]^{N/2}, \tag{24}$$

where $\Delta_k = -(d-\delta)^2_{(k)}$ is the Beltrami-Laplace operator on the k-forms.

Thus we have for Z':

$$Z' = [T(M)]^{d/2}\,[\tilde{T}(M)]^{-N/2}, \tag{25}$$

where

$$T(M) = [\det{}^{-1}\Delta_1 \det\Delta_0 \det\Delta_2]^{-1/2} \tag{26}$$

$$\tilde{T}(M) = [\det{}^{-1}\hat{D}^{+}\hat{D}\ \det\hat{D}\hat{D}^{+}]^{-1/2} \tag{27}$$

It is interesting to note that Z' is independent on the supergauge field $\mathcal{X}$ and is expressed in terms of T(M), the topological invariant of the manifold M (the string world-sheet) - the Ray-Zinger torsion [I2]. For the two-dimensional manifold T(M)=I, because in this case $\det\Delta_1 = \det\Delta_0 \det\Delta_2$. Similarly we have $\det\hat{D}^{+}\hat{D} = \det\hat{D}\hat{D}^{+}$ and consequently T(M)=I. It should be noted that the mutual cancellation of the boson and fermion determinants as typical property of the so-called topological quantum field theory [I3].

Thus we find that Z', a functional of $\mathcal{X}$ and h^a_μ, turns out to be the topological invariant, i.e. it does not change under the local variations of these fields. Consequently the suggested model is example of the topological quantum field theory of Witten [I3]. (It is interesting to note that in the work [I3] the action (II) is also used for the description of the fermionic sector).

6. Cancellation of conformal anomalies.

Let us return to the calculation of the functional integral Z, and choose the conformal-Lorentz gauge, in which

$$h^a_\mu = e^{\sigma}(\delta^a_\mu \cos\alpha + \epsilon^a{}_\mu \sin\alpha)$$

$$d * \mathcal{X} = 0,$$

where $\mathcal{X} \equiv (\mathcal{X}^{ij}, \mathcal{X}^{AB})$. The latter equation means that $\mathcal{X} = * d\beta$, where $\beta \equiv (\beta^{ij}, \beta^{AB})$.

For the functional measure over h^a_μ we have the standard expression [2,I4]:

$$[Dh^a_\mu] = [D\alpha][D\sigma][D\xi^\mu]\det{}^{1/2}\hat{L}, \tag{28}$$

where ξ^μ is vector field, the generator of the diffeomorphism group

$$L^\mu{}_\nu \xi^\nu = (-\nabla_\alpha \nabla^\alpha \delta^\mu_\nu + [\nabla^\mu, \nabla_\nu])\xi^\nu.$$

Let us note that $\int[D\xi^\mu]$ gives the volume of the diffeomorphism group.

For the integration measure over $\mathcal{X}$ we obtain:

$$[D\mathcal{X}] = [D\beta](\det\Delta_0)^{-(P_1+P_2)},$$

where P_I is number of $\{\alpha^{ij}\}$ and P_2 is number of $\{\alpha^{AB}\}$

Thus we have:

$$Z = \int [D\alpha][D\beta][D\sigma]\det{}^{1/2}\hat{L}\,(\det\Delta_0)^{-(P_1+P_2)}\,Z', \tag{29}$$

where Z' is independent on σ, β and α.

The determinants are well kown,

$$\ln\det\Delta_0 = \frac{1}{6\pi} I_0[\sigma]$$

$$\ln\det\hat{L} = \frac{13}{3\pi} I_0[\sigma],$$

where $I_0[\sigma] = -\frac{1}{2}\int d^2z\, \partial_\mu\sigma\, \partial^\mu\sigma$.

The Lorentz and supergauge anomalies, as it is seen from (29), are absent, so the integral over $[D\alpha]$ and $[D\beta]$ gives volume of the corresponding groups.

Hence we get

$$Z = \int [D\sigma] \exp\frac{C}{6\pi} I_0[\sigma],$$

where $C = I3 - (P_I + P_2)$.

For the conformal anomalies cancellation it is necessary to have C=0 or $P_I + P_2 = I3$. Up to this point $\{\alpha^{ij}\}$ and $\{\alpha^{AB}\}$ were arbitrary matrices. Let us assume now that

$$\alpha^{ij} = \begin{cases} \alpha, & i=j \\ \alpha^{ij} = -\alpha^{ji}, & i \neq j;\ i=1,\dots,4. \end{cases}$$

$$\alpha^{AB} = \begin{cases} \alpha, & A=B \\ \alpha^{AB} = -\alpha^{BA}, & A \neq B \; ; \; A=1,\dots,4. \end{cases}$$

In this case the infinitesimal supersymmetry parameters take values in the algebra of the group $O(4) \otimes O(4) \otimes O(I)$ and consequently $P_I + P_2 = I3$. Thus we finaly have that $N=d=4$.

In our model the zero-forms φ^i realise the embedding of the string world-sheet into the d-dimensional space-time, so the suggested here model is consistent (the anomalies are absent) directly in the four-dimensional space-time.

7. Conclusion.

We have considered the string model using differential forms on the string world-sheet. This model allows for the local supersymmetry, which like the supersymmetry in [I3], leads to the 'topological quantum field theory'.

Scalar components of the boson form Φ^i realise the embedding of string into d-dimensional space-time, and have the direct geometric sense. It was shown that the conformal anomalies are absent if $d=4$.

We will not discuss the geometric interpretation of the other components of the boson form Φ^i. Evidently one can consider them simply as the terms of the supermultiplet.

The canonical quantization and the no-ghost theorem will be considered elsewhere.

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