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VACUUM ENERGY AND THE FIXED POINT OF QED

A. A. Natale

Instituto de Fisica Teorica - Universidade Estadual Paulista

Rua Pamplona, 145 - 01405 - Sao Paulo - SP - Brazil

ABSTRACT

The vacuum energy of QED, as a function of the coupling constant α, is shown to have an absolute minimum at the critical coupling $\alpha_c = \pi/3$. The effect of chiral symmetry breaking diminishes as the coupling is increased. We argue that these aspects of the vacuum energy shall remain unaltered beyond the ladder approximation.

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The vacuum energy of QED, as a function of the coupling constant α, is shown to have an absolute minimum at the critical coupling $\alpha_c = \pi/3$. The effect of chiral symmetry breaking diminishes as the coupling is increased. We argue that these aspects of the vacuum energy shall remain unaltered beyond the ladder approximation.

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Based on analysis of the Schwinger-Dyson equation for the fermion propagator, it has been conjectured that quantum electrodynamics (QED) has an ultraviolet-stable fixed point at strong coupling[1]. This hypothesis is also supported by numerical simulations[2]. These results are quite important, because they may change the old argument based on perturbation theory, that renormalized QED is a trivial theory[3], providing to us an entirely new view of this theory at short distances. In another hand the existence of such behavior, shall allow us to build models of dynamical mass generation with U(1) technicolor theories[4].

If the theory has one fixed point, or a phenomenon of collapse of the wave function[1,2], one should be able to verify that this point corresponds to an extremum of energy. To look for an absolute minimum of the vacuum energy as a function of the coupling constant is another way to find a fixed point. This idea goes back to the seminal work of Wilson on renormalization group[5], where the existence of a critical point is directly related to the extrema of a potential. One should note that the Miransky's hypothesis about the existence of a non-trivial fixed point in QED[1], is a necessary condition for the existence of a non-trivial solution of the Schwinger-Dyson equation for the fermionic propagator, and a verification that this point is also energetically prefered is complementary to his work. Actually, the theory makes sense only at the critical point $\alpha = \alpha_c \equiv \pi/3$, where the dynamical mass remains finite in the limit we set an ultraviolet cutoff Λ to infinity[1,2]. Furthermore, at this point the fermionic self-energy has not an oscilatory character, but behaves asymptotically as $(m^2/p)\ ln\ (p/m)$, which gives maximal suppression of flavor-changing neutral currents in U(1) technicolor theories. In this Letter, starting with the solution of the Schwinger-Dyson equation, we will compute the effective potential of Cornwall, Jackiw and Tomboulis[6] at stationary points as a function of the coupling constant, and show that the vacuum energy has an absolute minimum when the coupling is equal to the critical value α_c, what corroborates the hypothesis of existence of this fixed point.

It has been shown that the Schwinger-Dyson equation of the fermionic self-energy in massless QED, in the ladder approximation

$$\Sigma(p) = \frac{-3\ \alpha}{4\pi^3}\ i \int d^4k\ \frac{\Sigma(k)}{(p-k)^2\left[k^2-\Sigma^2(k)\right]} \tag{1}$$

has only the trivial solution when $\alpha < \alpha_c = \pi/3$[7,8]. For $\alpha \geq \pi/3$ the self-energy has also a non-trivial solution, given by a hipergeometric function whose asymptotic behavior at large momenta is[8,9]

$$\Sigma(p) \sim \frac{m^2}{p}\left[\frac{\coth\ \pi\gamma}{\pi\gamma(\gamma^2+1/4)}\right]^{1/2} \sin\left(2\gamma \ln\frac{p}{m} + \Xi(\gamma) - \text{arctg}\ 2\gamma\right) , \tag{2}$$

where m is the dynamical mass, and

$$\gamma = \frac{1}{2}\sqrt{\frac{\alpha}{\alpha_c} - 1} \tag{3a}$$

$$\Xi(\gamma) = \arg\left[\frac{\Gamma(1+2i\gamma)}{\Gamma^2(1/2+i\gamma)}\right] \tag{3b}$$

When $\gamma = 0$ (or $\alpha = \alpha_c$) instead of equation (2) we obtain

$$\Sigma(p) \sim \frac{m^2}{p} \ln(p/m) \qquad (4)$$

At low momenta $\Sigma(p)$ behaves as a constant, and does not depend on α at leading order[1,8].

To compute the vacuum energy we start with the effective action for composite operators as obtained by Cornwall, Jackiw and Tomboulis[9]

$$\Gamma(S) = - \mathrm{Tr}\,(\ln S^{-1}) + \mathrm{Tr}\,[(S^{-1} - \not{\partial})S] \; - (\text{2PI diagrams}) \qquad (5)$$

where $S(p)$ is the full fermion propagator

$$S^{-1}(p) = S_0^{-1}(p) - \Sigma(p) \qquad (6)$$

$S_0(p)$ the bare one, and the 2PI (in eq. (5)) means all the two-particle irreducible diagrams[9]. It is important to remember that the stationary condition for $\Gamma(S)$, i.e., $\delta\Gamma(S)/\delta S(p) = 0$, leads exactly to the Schwinger-Dyson equation (eq.(1)). Considering translation invariant propagator configurations, equation (5) is reduced to the effective potential $(V(S))$, from which we define the vacuum energy density as

$$\Omega = V(S) - V(S_n) \qquad (7)$$

where we are subtracting from the asymmetric potential the symmetric one, denoted by $V(S_n)$. With one given expression for $\Sigma(p)$ we are ready to compute Ω. However, as pointed out by Castorina and Pi[8], a much better approximation to study the vacuum energy results when we plug the stationary condition (given by equation (1)) into Ω, obtaining an expression for the values of Ω at its minimum

$$\langle \Omega \rangle = 2i \int \frac{d^4p}{(2\pi)^4} \left\{ \ln\left[1 - \Sigma^2(p)/p^2\right] + \Sigma^2(p)/[p^2 - \Sigma^2(p)] \right\} \tag{8}$$

Once we assume Ω satisfying the complete Schwinger-Dyson equation, and reduce it to the "one-loop" expression $\langle\Omega\rangle$, we expect it to be much less sensible to possible deviations of our approximations to $\Sigma(p)$ from the exact solution of equation (1).

As our main intention is to determine $\langle\Omega\rangle$ as a function of α, we see that only the ultraviolet part of equation (8) is important. The infrared part, at least in this approximation, gives the same contribution to $\langle\Omega\rangle$ for any value of α. If we also consider that $\Sigma(p)$ naturally damps the integrals in (8), we come to the conclusion that we may expand the ultraviolet part of $\langle\Omega\rangle$ in powers of $\Sigma(p)/p$. Introducing the variable $x = p^2/m^2$ and defining $\bar{\Sigma} = \Sigma/m$, we obtain the leading term of $\langle\Omega\rangle$

$$\frac{8\pi^2}{m^4} \langle \Omega \rangle \approx -\frac{1}{2} \int_1^\infty dx \; \frac{\bar{\Sigma}^4}{x} + O\left[\frac{\bar{\Sigma}^6}{x^2}\right] \tag{9}$$

Notice that we are taking into account only the contribution of $\Sigma(p)$ for $p \geq m$. The substitution of equation (2) into (9) yields

$$\frac{8\pi^2}{m^4} \langle \Omega \rangle \approx \frac{-1}{32} \left[\frac{\coth(\pi\gamma)}{\pi\gamma(\gamma^2+\frac{1}{4})} \right]^2 \times \left[3 - \frac{(\gamma^2+1)(2\gamma\sin 4\beta - \cos 4\beta) + 4(4\gamma^2+1)(\cos 2\beta - \gamma \sin 2\beta)}{(4\gamma^4+5\gamma^2+1)}\right] \tag{10}$$

where

$$\beta = \Xi(\gamma) - \mathrm{arctg}(2\gamma)$$

In table 1 we give the value of $\langle\Omega\rangle$ for a series of values of

α/α_c. Equation (10) is peaked at $\gamma = 0$, and the numbers of table 1 show that we approach the deeper minimum as $\alpha \to \alpha_c$. We should not expect any qualitative modification in the overall behavior of $\langle\Omega\rangle$, if we had used a more complete approximation to $\Sigma(p)$ as, for example, the hypergeometric function[4]. The vacuum energy, as a function of the coupling constant, is better seen in Fig. 1, where the minimum up to $\alpha = \alpha_c$ is zero and we do not have dynamical symmetry breaking. At $\alpha = \alpha_c$ the symmetry is broken and the deeper minimum of energy happens at that point. When α $(>\alpha_c)$ is increased the effect of condensation is diminished, therefore any change of the coupling constant towards the critical point will be energetically preferred. Our result is valid in the ladder approximation, however as it relies on the form of $\Sigma(p)$ (with the absolute minimum occurring when $\Sigma(p)$ is reduced to eq. (4)), and since this form has been shown to hold at higher orders[10], we also expect the result to remain unaltered beyond the ladder approximation.

In conclusion, we have shown that the vacuum energy of QED has an absolute minimum at the critical coupling $\alpha_c = \pi/3$, this is in agreement with Miransky's conjecture[1] that this is an ultraviolet stable fixed point. For $\alpha < \alpha_c$, since we do not have any non-trivial self-energy solution[7,8], the vacuum energy is zero and the chiral symmetry is unbroken. For $\alpha > \alpha_c$ the chiral symmetry is allways broken , but its effect is diminished as we increase the coupling constant.

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α/α_c	$-\frac{8\pi^2}{m^4}\langle\Omega\rangle$
1.001	0.282
1.1	0.245
1.5	0.148
2.0	0.089
5.0	0.011

Table 1. Values of the vacuum energy as a function of α/α_c

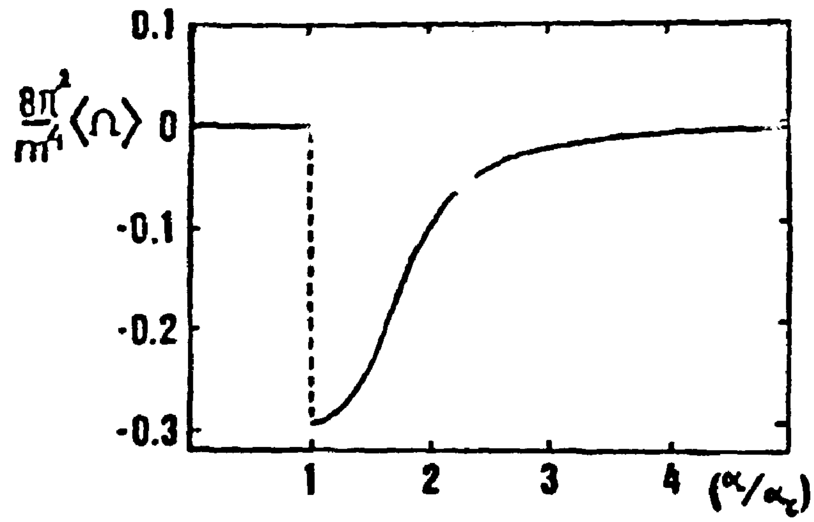

Fig. 1 Behavior of $\langle\Omega\rangle$ as a function of α/α_c